# Results of research

Takeshi Oota

In the generalized Kerr-NUT-de Sitter spacetime, it is shown that certain type of tensor perturbation admits separation of variables and the linearized Einstein equation (the Lichnerowicz equation) turns into ordinary differential equations ([26] of the Publication List). These ordinary differential equations contain the information of eigenvalues of the gauge-covariant Lichnerowicz operator on the base complex projective spaces. The form of eigenvalues has been guessed for rank-2 tensors. But the range of them is not determined. We have determined the eigenvalues of the gauge-covariant Lichnerowicz operator on $\mathbb{C}P^n$ and their multiplicities not only for rank-2 tensors, but for general rank-$r$ tensors.

We consider $\beta$-deformation of the quiver matrix models in the light of the recent progress on 2d-4d connection of conformal field theories, in particular, the Alday-Gaiotto-Tachikawa (AGT) conjecture. A quantum spectral curve is introduced in the case of $SU(n)$ quiver matrix model. Residue analysis is provided both for the curve of the $SU(n)$ matrix model with the "multi-log" potential and for the Seiberg-Witten curve in the case of $SU(n)$ with $N_f = 2n$ flavors, leading to the matching of the mass parameters. The isomorphism of two curves is made manifest ([27]).

We observe that the Dotsenko-Fateev integral representation of the conformal block of 2d conformal field theory can be interpreted as a $\beta$-deformed matrix model of Selberg type. Using the formula associated with the Jack polynomials, we established the method of generating $q$-expansion coefficients for conformal block and Nekrasov function for $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with four flavors ([28]).

We then consider a series of massive scaling limits of the $\beta$-deformed matrix model of Selberg type ($SU(2)$ with $N_f = 4$) which reduce the number of flavors to $N_f = 3$ and subsequently to $N_f = 2$ ([29]).

We consider $\beta$-deformed quiver matrix model based on the affine Lie algebra $A_n^{(1)}$. The Virasoro constraint of this model is determined. For $n = 1, 2$ cases, the explicit forms of the loop equations are obtained ([30]).

The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The "$q$-deformed" version of (W)AGT conjecture states that the $q$-lifted version of the partition function of five-dimensional gauge theories and the "conformal blocks" of the two-dimensional theories with the $q$-deformed Virasoro/W symmetries. Starting from this $q$-version of (W)AGT conjecture, we demonstrate by taking a certain $r$-th root of unity limit in $q$, the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the $r$-th root of limit of the $q$-deformed Virasoro and the $q$-deformed W algebra ([34]).