Research program Kiyoki Tanaka

I continue to study the Bergman space theory and its related topics. The following two topics are short term target of my study.

1. Toeplitz operator with radial symbol on the radial measure weighted harmonic Bergman space on the ball

I consider the harmonic Bergman space with radial measure weight on the ball. The eigenvalue of the Toeplitz operator with radial measure symbol on the weighted harmonic Bergman space has the specific form. Moreover, its eigenvalue has the upper estimate for the averaging function and the lower estimate for the Berezin transform[2]. For analytic Bergman space, B. Korenblum and K. Zhu[1] gave that the three value are comparable. I would like to solve this problem.

2. Biharmonic Bergman space

I consider the space of all biharmonic functions $(\Delta^2 u = 0)$ satisfying that the integral $\int_{\mathbb{B}} |u|^2 dx$ are finite. This space is the reproducing kernel Hilbert space, because the biharmonic function has the mean-value property. The aims of study is to compare the harmonic Bergman space and the biharmonic Bergman space. The main targets are the estimate and the explicit form of the reproducing kernel and operator theory on the biharmonic Bergman space etc. (1) Alimansi decomposition. (2) The harmonic Bergman space is decomposed by the spaces of all harmonic homogeneous polynomials. By (1) and (2), I would like to calculate the reproducing kernel of the biharmonic Bergman space.

The long term target of my study is the extension of **2**. **Biharmonic Bergman space**. In the Bergman theory, the main function spaces are analytic, harmonic or caloric function spaces. I would like to consider the space consists of the solutions of certain linear partial differential equation.

References

- B. Korenblum and K. Zhu, An application of Tauberian theorems, J. Operator theory, Vol 33(1995), 353–361.
- [2] M. Nishio and K. Tanaka, *Harmonic Bergman spaces with radial measure weight on the ball*, submitted.