

# Plan

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## Determine a (minimal) generating set of Roseman moves

Seven types of Roseman moves are crucial to study surface-links in terms of diagrams. So, it is important to understand their properties. We have given the answer for the independence problem of seven types of Roseman moves. Therefore, I am planning to determine a (minimal) generating set of Roseman moves. Seven types of Roseman moves are often depicted by omitting over/under information. We first enumerate all possible Roseman moves whose over/under information is not omitted. Let  $\mathcal{R}$  denote the set of such all Roseman moves. The subset  $S$  of  $\mathcal{R}$  is called a *generating set* if all Roseman moves in  $\mathcal{R}$  are realized by a finite sequence of Roseman moves in  $S$ . A generating set  $S \subset \mathcal{R}$  is said to be *minimal* if  $S$  does not properly contain any generating set. To investigate a generating set, we need to transform a diagram into another one combinatorially, and then it is not easy to transform diagrams equipped with over/under information. Hence, by using motion picture method we investigate transformation of diagrams.

## Development of Roseman moves and quandle cocycle invariants for singular surface-knots

A quandle is a non-empty set with a binary operation whose conditions derive from Reidemeister moves, and it is compatible with classical knots and surface-knots. In 1990's Carter, Jelsovsky, Kamada, Langford and Saito introduced quandle (co)homology groups and defined the quandle cocycle invariant of a surface-knot as a generalization of the quandle coloring number of a surface-knot. It is known that the quandle cocycle invariant is effective to estimate the minimal triple point number of a surface-knot and to evaluate an invertibility of a surface-knot. Therefore, I am planning to develop the quandle cocycle invariant of a singular surface-knot. A *singular surface-knot* is a surface-knot admitting a finite number of transverse double points. The quandle coloring number and the quandle cocycle invariant are invariants of a surface-knot because they are defined for surface-knot diagrams and their values do not change under Roseman moves. However, it is not known local transformation of diagrams of singular surface-knots corresponding to Roseman moves. Hence, I try to construct of Roseman moves for singular surface-knots together with developing the quandle cocycle invariant. If a singular surface-knot is quandle colorable, quandle elements assigned to a neighborhood of a transverse double point are imposed on a certain condition. We modify a quandle chain complex by using the condition and construct of a new quandle homology group. Moreover, we investigate geometric and algebraic properties of its quandle 3-cocycles, and introduce local transformation on diagrams of singular surface-knots.