

Results of research

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We consider β -deformation of the quiver matrix models in the light of the recent progress on 2d-4d connection of conformal field theories, in particular, the Alday-Gaiotto-Tachikawa (AGT) conjecture. A quantum spectral curve is introduced in the case of $SU(n)$ quiver matrix model. Residue analysis is provided both for the curve of the $SU(n)$ matrix model with the “multi-log” potential and for the Seiberg-Witten curve in the case of $SU(n)$ with $N_f = 2n$ flavors, leading to the matching of the mass parameters. The isomorphism of two curves is made manifest (Ref.[27] of the Publication List).

We observe that the Dotsenko-Fateev integral representation of the conformal block of 2d conformal field theory can be interpreted as a β -deformed matrix model of Selberg type. Using the formula associated with the Jack polynomials, we established the method of generating q -expansion coefficients for conformal block and Nekrasov function for $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with four flavors ([28]).

We then consider a series of massive scaling limits of the β -deformed matrix model of Selberg type ($SU(2)$ with $N_f = 4$) which reduce the number of flavors to $N_f = 3$ and subsequently to $N_f = 2$ ([29]).

We consider β -deformed quiver matrix model based on the affine Lie algebra $A_n^{(1)}$. The Virasoro constraint of this model is determined. For $n = 1, 2$ cases, the explicit forms of the loop equations are obtained ([30]).

The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The “ q -deformed” version of (W)AGT conjecture states that the q -lifted version of the partition function of five-dimensional gauge theories and the “conformal blocks” of the two-dimensional theories with the q -deformed Virasoro/W symmetries. Starting from this q -version of (W)AGT conjecture, we demonstrate by taking a certain r -th root of unity limit in q , the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the r -th root of limit of the q -deformed Virasoro and the q -deformed W algebra ([34]).

A q -deformed vertex operator is determined from the five-dimensional $SU(2)$ Nekrasov partition function based on the q -AGT conjecture. We obtained a q -deformed version of Coulomb gas representation of the conformal block by using the vertex operators and q -screening charges. After slightly changing the position of one of the vertex operators, we have checked that the q -block coincides with 5D Nekrasov function in low degrees of instanton expansion ([35]).

A set of Schwinger-Dyson equations for the resolvents are considered in a class of supersymmetric Chern-Simons-matter matrix models. In the planar limit, these loop equations reduce to two independent algebraic cubic equations for the two planar resolvents ([36]).