TOPOLOGICAL IMITATION OF A COLORED LINK WITH THE SAME DEHN SURGERY MANIFOLD

Akio KAWAUCHI
Department of Mathematics, Osaka City University
Sumiyoshi-ku, Osaka 558-8585, Japan
kawauchi@sci.osaka-cu.ac.jp

Abstract

By the topological imitation theory, we construct, from a given colored link, a new colored link with the same Dehn surgery manifold. In particular, we construct a link with a distinguished coloring whose Dehn surgery manifold is a given closed connected oriented 3-manifold except the 3-sphere. As a result, we can naturally generalize the difference between the Gordon-Luecke theorem and the property P conjecture to a difference between a link version of the Gordon-Luecke theorem and the Poincaré conjecture. Similarly, we construct a link with a π_1-distinguished coloring whose Dehn surgery manifold is a given non-simply-connected closed connected oriented 3-manifold. We also construct a link with just two colorings whose Dehn surgery manifolds are the 3-sphere.

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0. Introduction

The main purpose of our argument is to change a disconnected link L in the 3-sphere S^3 with a coloring f_L into an imitation link L^* in S^3 with the induced coloring f_L^* without changing the Dehn surgery manifold by the topological imitation theory developed in $[4-15]$. An early imitation technique in [6] enabled us to make the Dehn surgery manifolds of S^3 along (L,f) and (L^*,f_L^*) the same manifold, and since then we have looked for additional conditions on (L^*,f^*) in answers to our needs (see [5,10, 13]). We say that a coloring f_L of a link L in S^3 is *distinguished* (or π_1-*distinguished*, respectively) if $\chi(L,f)\neq\chi(L,f_L)$ (or $\pi_1(\chi(L,f))\neq\pi_1(\chi(L,f_L))$, respectively) for every coloring f of L with $f\neq f_L$ (see Definition 2.3 for the accurate definition). A typical result of this paper is that *for every disconnected colored link (L,f_L) such that $f_L(L)\subset\mathbb{Q}$ and $\chi(L,f_L)\neq S^3$, there is a normal imitation $q:(S^3,L^*)\to(S^3,L)$ such that $\chi(L^*,f_Lq)=\chi(L,f_L)$, every sublink of L^* is a hyperbolic link and the colorings f_Lq and ∞ of L^* are distinguished* (see Corollaries 3.3 and 3.4). By this result, we can generalize the difference between the Gordon-Luecke theorem [1] (saying that the coloring ∞ of every non-trivial knot in S^3 is distinguished) and the property P conjecture to a difference between a link version of the Gordon-Luecke theorem and the Poincaré conjecture. In other words, the coloring ∞ of every non-trivial knot (with ∞ distinguished by the Gordon-Luecke theorem) is π_1-distinguished if and only if the property P conjecture is true, and our result shows that the coloring ∞ of every link in S^3 with ∞ distinguished is π_1-distinguished if and only if the Poincaré conjecture is true (see Remark 3.6 later). On the other hand, *for every disconnected colored link* (L, f_L) *such that* $f_L(L) \subset \mathbb{Q}$ *and* $\chi(L, f_L) = S^3$, *we have a normal imitation* $q : (S^3, L^*) \to (S^3, L)$ *such that* $\chi(L^*, f_L q) = \chi(L, f_L) = S^3$ *and every sublink of* L^* *is a hyperbolic link and distinct from any sublink of* L (see Corollary 3.5 for the detail). The existence of a colored link imitation (L^*, f^*) of a colored link (L, f) with the same Dehn surgery manifold but with L^* distinct componentwise from L has been promised in [7, p.151].

Throughout this paper, we extensively consider a link as a link in an *ambient manifold* M that is a compact connected oriented 3-manifold such that the boundary ∂M is empty or consists of tori. To save words, we regard two ambient manifolds M_i $(i = 1, 2)$ with an orientation-preserving homeomorphism $M_1 \cong M_2$ as the same manifold $M_1 = M_2$, and two links L_i in ambient manifolds M_i $(i = 1, 2)$ with an orientation-preserving homeomorphism $h : (M_1, L_1) \cong (M_2, L_2)$ as the same link $L_1 = L_2$ in the same ambient manifold $M_1 = M_2$, unless confusion might occur.

In §1, we review some basic concepts and results in the topological imitation theory which are used in this paper. In §2, we explain links with a distinguished coloring and with a π_1-distinguished coloring. In §3, the statement of the main theorem (Theorem 3.1) and the corollaries cited above are proved here by assuming Theorem 3.1. §4 is devoted to the proof of Theorem 3.1.

The content of this paper is a revised version of a part of the research announcement "*Link corresponding to closed 3-manifold*" (see http://www.sci.osaka-cu.ac.jp/~kawauchi/index.htm) a growing up version of whose remaining part will appear in [17].

1. Reviews on topological imitations of links

We grant the link L to be empty only in this section. We briefly explain some concepts and results of topological imitations of the pair (M, L) for a link L in an ambient manifold M and refer more detailed accounts of topological imitations to [4 – 15].

Let $I = [-1, 1]$. The concept of a topological imitation arose from an interpretation of *reflection*. Namely, for a link L in M, an involution α on $(M, L) \times I = (M \times I, L \times I)$ is called a *reflection* in $(M, L) \times I$ if

(1) $\alpha((M, L) \times 1) = (M, L) \times (-1)$, and
(2) the fixed point set $\mathrm{Fix}(\alpha, (M, L) \times I)$ of α in $(M, L) \times I$ is the pair for a link in an ambient manifold.

The reflection α is *standard* if $\alpha(x, t) = (x, -t)$ for all $(x, t) \in M \times I$, and *normal* if $\alpha(x, t) = (x, -t)$ for all $(x, t) \in \partial(M \times I) \cup N(L) \times I$ for a tubular neighborhood $N(L)$ of L in M. The reflection α is *isotopically standard* if $h^{-1}\alpha h$ is standard for an auto-homeomorphism h of $M \times I$ which is isotopic to the identity by an isotopy keeping $\partial(M \times I) \cup N(L) \times I$ fixed for a tubular neighborhood $N(L)$ of L in M. Further, the reflection α is *isotopically almost standard* if $L \neq \emptyset$ and α defines an isotopically standard reflection in $(M, L-K) \times I$ for every component K of L. A *reflector* of a reflection α in $(M, L) \times I$ is an embedding

$$\phi_\alpha : (M^*, L^*) \longrightarrow (M, L) \times I$$

with $\phi_\alpha(M^*, L^*) = \mathrm{Fix}(\alpha, (M, L) \times I)$. We note that $L*$ is a link in the ambient manifold M^*.

Definition 1.1. An *imitation* of (M, L) is the composite

$$q : (M^*, L^*) \xrightarrow{\phi_\alpha} (M, L) \times I \xrightarrow{\mathrm{proj}} (M, L)$$

where $\phi_\alpha : (M^*, L^*) \to (M, L) \times I$ is reflector of a reflection α in $(M, L) \times I$.

We also call (M^*, L^*) an *imitation* of (M, L) (with *imitation map* q). The imitation map q induces epimorphisms

$$\pi_1(M^*) \longrightarrow \pi_1(M) \quad \text{and}$$
$$\pi_1(M^* - L^*) \longrightarrow \pi_1(M - L)$$

whose kernels are perfect groups, and isomorphisms

$$H_*(M^*; Z) \cong \pi_1(M; Z) \quad \text{and}$$
$$H_*(M^* - L^*; Z) \cong H_*(M - L; Z)$$

. Further, the restriction

$$q|_{\partial M^*} : \partial M^* \to \partial M$$

is a homotopy equivalence. These properties pass to any lift

$$\tilde{q} : (\tilde{M}^*, \tilde{L}^*) \longrightarrow (\tilde{M}, \tilde{L})$$

of q associated with every covering $\tilde{M}$ over M. The orientation of (M, L) induces an orientation of the pair (M^*, L^*) by q. If the reflection α is normal, then we say that the imitation

$$q : (M^*, L^*) \longrightarrow (M, L)$$

is a *normal imitation.* If α is isotopically almost standard, then we say that the imitation

$$q : (M^*, L^*) \longrightarrow (M, L)$$

is an *AID (=almost identical) imitation.* A normal imitation

$$q' : (M^{*\prime}, L^{*\prime}) \longrightarrow (M, L)$$

is *imitation-homotopic* to a normal imitation

$$q : (M^*, L^*) \longrightarrow (M, L)$$

if for a reflector $\phi : (M^*, L^*) \to (M, L) \times I$ of a normal reflection α in $(M, L) \times I$ with $q = \text{proj}\phi$ there is an auto-homeomorphism h of $M \times I$ isotopic to the identity by an ambient isotopy keeping $\partial(M \times I) \cup N(L) \times I$ fixed such that $q' = \text{proj}\phi'$ for a reflector $\phi' : (M^{*\prime}, L^{*\prime}) \to (M, L) \times I$ of the normal reflection $h^{-1}\alpha h$ in $(M, L) \times I$.

If $q : (M^*, L^*) \to (M, L)$ is an AID imitation, then the restricted normal imitation

$$q|_{(M^*, L^*-K^*)} : (M^*, L^* - K^*) \longrightarrow (M, L - K)$$

for every component K of L and $K^* = q^{-1}(K)$, is imitation-homotopic to the identical imitation $1_{(M,L-K)} : (M, L-K) \to (M, L-K)$. In particular, in the case of AID imitation, we can identify M^* with M and L^* with $(L-K) \cup K^*$ for every component K of L. From construction, we see that if $q^* : (M^{**}, L^{**}) \to (M^*, L^*)$ and $q : (M^*, L^*) \to (M, L)$ are normal (or AID, respectively) imitations, then there is a normal (or AID, respectively) imitation $q^{**} : (M^{**}, L^{**}) \to (M, L)$ with $q^{**} = qq^*$ on a tubular neighborhood $N(L^{**})$ of L^{**} in M^{**}. The *exterior* of a link (M, L) is the compact manifold $E(L) = \text{cl}(M - N(L))$. An ambient manifold M is called a *hyperbolic* 3-manifold if $M - \partial M$ is a complete hyperbolic 3-manifold. Except for the hyperbolic 3-manifolds $S^1 \times D^2$ and $S^1 \times S^1 \times [0, 1]$, the hyperbolic 3-manifold M has a finite volume (see [16, C.7.2] for an explanation). Unless otherwise stated, hyperbolic 3-manifolds are assumed to have finite volomes. The volume and the isometry group of a hyperbolic 3-manifold M are denoted by $\text{Vol}(M)$ and $\text{Isom}(M)$ respectively, which are topological invariants of M by the Mostow rigidity theorem (see G. D. Mostow [19], W. P. Thurston [22, 23]). A hyperbolic 3-manifold M is said to be *asymmetric* if the isometry group $\text{Isom}(M)$ is trivial. An imitation $q : (M^*, L^*) \to (M, L)$ is called a *hyperbolic asymmetric imitation* if the exterior $E(L^*)$ is hyperbolic and asymmetric. The following lemma is proved in [6] except the asymmetry condition which is proven in [7].

Lemma 1.2. Let L be a disconnected oriented link in an ambient manifold M. Then for any positive number C, there is a hyperbolic asymmetric AID imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

with $\text{Vol}(E(L^*)) > C$.

From a technical reason, we need the following lemma:

Lemma 1.3. Let $q : M^* \to M$ be an imitation such that M is a hyperbolic 3-manifold. Then there is a connected sum decomposition $M^* = M' \# S$ such that

(1) the connected summand M' is an irreducible 3-manifold, and if ∂M^* is not empty, then it is a Haken manifold with incompressible boundary (consisting of torus components),

(2) the restriction $q|_{M'_o} : M'_o \to M$ of q to the compact punctured manifold M'_o of M' used for the connected sum extends to a map $q' : (M', \partial M') \to (M, \partial M)$ whose lift

$$\tilde{q}' : (\tilde{M}', \partial\tilde{M}') \longrightarrow (\tilde{M}, \partial\tilde{M})$$

associated with every covering $\tilde{M}$ of M is a homology equivelence,

(3) the connected summand S is a homology 3-sphere and $q_{\#}(\pi_1(S_o)) = \{1\}$ for the punctured manifold S_o of S used for the connected sum.

Proof. We use the homology equivalence property of imitation in [4], saying that the lift $\tilde{q} : \tilde{M}^* \to \tilde{M}$ of the imitation map q associated with every covering $\tilde{M}$ of M induces isomorphisms

$$\tilde{q}_* : H_*(\tilde{M}^*; Z) \cong H_*(\tilde{M}; Z),$$
$$\tilde{q}_* : H_*(\tilde{M}^*, \partial\tilde{M}^*; Z) \cong H_*(\tilde{M}, \partial\tilde{M}; Z).$$

First, we show the following assertion:

(1.3.1) If there is a connected sum decomposition $M^* = M_1^* \# M_2^*$, then we have $q_{\#}(\pi_1((M_i^*)_o)) = \{1\}$ for some i where $(M_i^*)_o$ denotes the punctured manifold of M_i^* used for the connected sum.

If there is a connected sum decomposition $M^* = M_1^* \# M_2^*$ with $q_{\#}(\pi_1((M_i^*)_o)) \neq \{1\}$ for $i = 1$ and 2, then we consider the universal covering $\tilde{M}$ of M whose interior is homeomorphic to the 3-space $\mathbb{R}^3$. Let $\tilde{q} : \tilde{M}^* \to \tilde{M}$ be the associated lifting of q. Let S^2 be a 2-sphere in $\tilde{M}^*$ lifting the 2-sphere defining the connected sum $M^* = M_1^* \# M_2^*$. By the homology equivalence property, we see that $\tilde{M}^*$ is connected and $H_1(\tilde{M}^*; Z) = 0$, so that S^2 splits $\tilde{M}^*$ into two connected submanifolds X_i $(i = 1, 2)$. Using that $\pi_1(M)$ is a torsion-free group and hence $q_{\#}(\pi_1((M_i^*)_o))$ is an infinite group for $i = 1, 2$, we see that X_i is not compact for $i = 1$ and 2. This implies that S^2 represents a non-zero element of $H_2(\tilde{M}^*; Z)$, contradicting that $H_2(\tilde{M}^*; Z) = H_2(\tilde{M}; Z) = 0$. This proves (1.3.1).

By applying (1.3.1) and the homology equivalence property to a prime decomposition of M^* (cf. J. Hempel [2]), we can conclude that there is a connected sum decomposition $M^* = M' \# S$ such that M' is a prime 3-manifold and S is a closed 3-manifold with $q_{\#}(\pi_1(S_o)) = \{1\}$. Since $q_{\#}(\pi_1(M'_o)) = \pi_1(M)$ is a non-abelian hyperbolic group, we see that M' is an irreducible 3-manifold. If $\partial M'$ is not empty, then M' is a Haken manifold with incompressible boundary, because the restriction

$$q|_{\partial M^*} : \partial M^* = \partial M' \longrightarrow \partial M$$

of the imitation map q is a homotopy equivalence (see [4]) and ∂M is incompressible in M, showing (1) (cf. W. Jaco [3] for an account on Haken manifold and incompressibility). Let

$$W = M^* \times [0, 1] \cup_{j=1}^{s} h_j^2$$

be a cobordism from $M^* = M^* \times 0$ to M' such that h_j^2 $(j = 1, 2, \dots, s)$ are mutually disjoint 2-handles on the connected summand $S \times 1$ of $M^* \times 1$ whose surgery produces S^3 which is a connected summand of M'. Because $q_{\#}(\pi_1(S_o)) = \{1\}$, the map $q : M^* \to M$ extends to a map $F : W \to M$. Let $q' = F|_{M'} : M' \to M$. For every covering $\tilde{M} \to M$, we have a map

$$\tilde{F} : \tilde{W} \longrightarrow \tilde{M}$$

lifting F and extending the liftings $\tilde{q} : \tilde{M}^* \to \tilde{M}$ and $\tilde{q}' : \tilde{M}' \to \tilde{M}$ of q and q', respectively. By excision, we see that $H_d(\tilde{W}, \tilde{M}^*; Z) = H_d(\tilde{W}, \tilde{M}'; Z) = 0$ for $d \neq 2$. Assuming that S is a homology 3-sphere, shown later, we have natural isomorphisms

$$H_d(\tilde{M}^*; Z) \xrightarrow{\cong} H_d(\tilde{W}; Z) \xleftarrow{\cong} H_d(\tilde{M}'; Z)$$

for $d \neq 2$ and natural monomorphisms

$$H_2(\tilde{M}^*; Z) \longrightarrow H_2(\tilde{W}; Z) \quad \text{and} \quad H_2(\tilde{M}'; Z) \longrightarrow H_2(\tilde{W}; Z)$$

with the same image. Then the isomorphism $\tilde{q}_* : H_*(\tilde{M}^*; Z) \cong H_*(\tilde{M}; Z)$ induces an isomorphism $(\tilde{q}')_* : H_*(\tilde{M}'; Z) \cong H_*(\tilde{M}; Z)$. When $\partial M'$ is not empty, the restriction $\tilde{q}'|_{\partial M'} : \partial M' \to \partial M$ is a homotopy equivalence, and hence by the five lemma we have an isomorphism

$$(\tilde{q}')_* : H_*(\tilde{M}', \partial\tilde{M}'; Z) \longrightarrow H_*(\tilde{M}, \partial\tilde{M}),$$

showing (2) by assuming (3). For the universal covering space $\tilde{M}$ of M, we have that $H_1(\tilde{M}^*; Z) = 0$ and $\tilde{M}^*$ contains an infinitely many copies of S as connected summands. Thus, we have $H_1(S; Z) = 0$ and hence S is a homology 3-sphere, showing (3). □

2. Links with a distinguished coloring and with a π_1-distinguished coloring

Let $K_i (i = 1, 2, \dots, r)$ be the components of a link L in M. A meridian system $m(L)$ on a tubular neighborhood $N(L) = \cup_{i=1}^r N(K_i)$ of L in M is always defined as a system consisting of a meridian $m(K_i)$ of $N(K_i)$ for every $i = 1, 2, \dots, r$. On the other hand, a longitude system $\ell(L)$ on $N(L)$ is not uniquely specified in general. A *framed* link is a link L in an ambient manifold M such that a longitude system $\ell(L)$ of L in M is specified on a tubular neighborhood $N(L)$ as a system consisting of a longitude $\ell(K_i)$ of $N(K_i)$ for every $i = 1, 2, \dots, r$. By a *meridian-longitude system* of a framed link L, we mean a pair of a meridian system $m(L)$ and a longitude system $\ell(L)$ on $N(L)$ such that $m(K_i)$ meets $\ell(K_i)$ transversely in a single point for every i. We can specify the orientations of $m(L)$ and $\ell(L)$ from those of L and M uniquely. When $M = S^3$, we have a *canonical* meridian-longitude system $(m(L), \ell(L))$ of L by taking a canonical longitude $\ell(K_i)$ on $N(K_i)$ characterized by that $\ell(K_i)$ is null-homologous in the exterior $E(K_i) = \mathrm{cl}(S^3 - N(K_i)$. Unless otherwise stated, we will consider a link L in S^3 as a framed link by taking a canonical meridian-longitude system of L. We note that if (M, L) is a framed link and $q : (M, L^*) \to (M, L)$ is a normal imitation, then L^* is a framed link by a unique meridian-longitude system induced from that of L by q, so that a colored link (L, f) induces a unique colored link (L^*, fq).

Definition 2.1. A *coloring* f of a framed link L is a map

$$f : \{K_i | \, i = 1, 2, \dots, r\} \longrightarrow \mathbb{Q}^+,$$

where $\mathbb{Q}^+ = \mathbb{Q} \cup \{\infty, \emptyset\}$ for the set $\mathbb{Q}$ of rational numbers and the symbols ∞, $\emptyset$ with the identities $-\infty = \infty$, $-\emptyset = \emptyset$, $\infty + c = c + \infty = \infty$ and $\emptyset + c = c + \emptyset = \emptyset$ for all $c \in \mathbb{Q}$.

Let $f(L)$ be the subset of $\mathbb{Q}^+$ consisting of the elements $f(K_i) \in \mathbb{Q}^+$ for all i. A coloring f of L is the *constant coloring* c for an element $c \in \mathbb{Q}^+$ if f is the constant map to c, i.e., $f(K_i) = c$ for all i. A colored link (L', f') is *equivalent* to a colored link (L, f) if there is an orientation-preserving homeomorphism $h : M \to M$ such that $f(K_i) = f'h(K_i)$ for all i. A coloring f of L is *finite* if $f(L) \subset \mathbb{Q} \cup \{\emptyset\}$, and *regular* if $f(L) \subset \mathbb{Q} \cup \{\infty\}$.

The *size* of a rational number $c = \frac{a}{b}$ with a, b coprime integers is the integer $\rho(c) = |a| + |b|$. The sizes of the symbols $\emptyset$ and ∞ are defined as $\rho(\emptyset) = 0$ and $\rho(\infty) = 1$ by convention. The *size* $\rho(f)$ of a coloring f of L is the set of the sizes $\rho(f(K_i))$ for all i. For an integer J, we denote by $\rho(f) \geqq J$ or $\rho(f) > J$, respectively, the inequality $\rho(f(K_i)) \geqq J$ for all i or $\rho(f(K_i)) > J$ for all i, respectively. Similarly, we denote by $\rho(f) \leqq J$ or $\rho(f) < J$, respectively, the inequality $\rho(f(K_i)) \leqq J$ for all i or $\rho(f(K_i)) < J$ for all i, respectively. By re-indexing the components K_i $(i = 1, 2, \dots, r)$, let K_i $(i = 1, 2, \dots, u)$ be the components of L with $f(K_i) \neq \emptyset$. Let $f(K_i) = \frac{a_i}{b_i}$ for coprime integers a_i, b_i for $i \leqq u$ where we take $a_i = \pm 1$ and $b_i = 0$ when $f(K_i) = \infty$. Then we have a simple loop s_i on $\partial N(K_i)$ (unique up to isotopies) such that we have $[s_i] = a_i[m_i] + b_i[\ell_i]$ in $H_1(\partial N(K_i); Z)$ for the meridian-longitude pair (m_i, ℓ_i) of K_i on $N(K_i)$. We note that if the other choice $\frac{-a_i}{-b_i}$ of $f(K_i)$ is made, then only the orientation of s_i is changed.

Definition 2.2. The *Dehn surgery manifold* of a colored link (L, f) is the oriented 3-manifold

$$\chi(L, f) = E(L) \cup_{s_1 = 1 \times \partial D_1^2} S^1 \times D_1^2 \cdots \cup_{s_u = 1 \times \partial D_u^2} S^1 \times D_u^2$$

with the orientation induced from $E(L) \subset M$, where $\cup_{s_i = 1 \times \partial D_i^2}$ denotes a pasting of $S^1 \times \partial D_i^2$ to $\partial N(K_i)$ so that s_i is identified with $1 \times \partial D_i^2$.

By definition, we have $\chi(L, f) = E(L)$ if $f = \emptyset$ and $\chi(L, f) = M$ if $f = \infty$. In this construction, the oriented 3-manifold $\chi(L, f)$ up to orientation-preserving homeomorphisms is independent of choices of orientations of the simple loops s_i and hence determined uniquely from the colored link (L, f) up to equivalences. We see by W. B. R. Lickorish [18] and A. H. Wallace [24] that for every ambient manifold M, we have a link L in S^3 and a finite coloring f of L such that $\chi(L, f) = M$ where the number of the components K_i of L with $f(K_i) = \emptyset$ is equal to the number of components of ∂M.

Definition 2.3. A coloring f_L of a framed link L in an ambient manifold M is *distinguished* if we have $\chi(L, f_L) \neq \chi(L, f)$ for every coloring f' of L with $f \neq f_L$, and π_1-*distinguished* if the fundamental groups $\pi_1(\chi(L, f_L))$ and $\pi_1(\chi(L, f))$ are not isomorphic to each other for every coloring f of L with $f \neq f_L$.

For example, the coloring ∞ of any framed link L in M with a trivial component is not distinguished. A characterization of the distinguished coloring ∞ is given as follows:

Lemma 2.4. The constant coloring ∞ of a framed link L in an ambient manifold M is distinguished if and only if every homeomorphism $h : E(L') \to E(L)$ from the exterior $E(L')$ of any link L' in M to $E(L)$ sends every meridian system $m(L')$ to a meridian system $m(L)$ setwise, so that h extends to a homeomorphism $h^+ : (M, L') \to (M, L)$.

Proof. To prove the "only if"part, we consider a pair (M, L') with a homeomorphism $h : E(L') \to E(L)$. Since the constant coloring ∞ of (M, L) is distinguished, the image $h(m(L'))$ must be equal to a meridian-system $m(L)$ in $E(L)$ up to orientations of $m(L)$. Hence we can extend h to a homeomorphism $h^+ : (M, L') \to (M, L)$. To prove the "if"part, suppose that the constant coloring ∞ of a framed link L is not distinguished. Then there is a coloring $f \neq \infty$ of (M, L) such that $\chi(L, f) = M$, and the dual link of L in $\chi(L, f)$ is a link L' in M with $E(L') = E(L)$ such that the meridian system $m(L')$ of (M, L') up to orientations is not homologous to $m(L)$ in $\partial E(L)$. $\square$

A link L in M is *determined by the exterior* $E(L)$ if there is a homeomorphism $(M, L') \cong (M, L)$ for every link L' in M with a homeomorphism $E(L') \cong E(L)$. By Lemma 2.4, every link with the constant coloring ∞ distinguished is determined by the exterior. For example, a trivial knot and a Hopf link are examples of links determined by the exteriors but having the constant coloring ∞ not distinguished. A link (M, L) is *totally hyperbolic* if every non-empty sublink L_s is a hyperbolic link in M, that is, if the exterior $E(L_s)$ is a hyperbolic 3-manifold. If $q : (M, L^*) \to (M, L)$ is a normal imitation of a framed link L, then we can consider L^* as a framed link so that the imitation map q preserves meridian-longitude systems of L^* and L. Further, if $M = S^3$, then q preserves canonical meridian-longitude systems of L^* and L by the homology equivalence property in [4]. If f is a coloring of a framed link L in M, then fq is a coloring of the framed link L^* in M.

3. Statement of the main theorem (Theorem 3.1) and its consequences

In this section, we explain the basic result (Theorem 3.1) on the Dehn surgery description of a disconnected colored link. Its consequences are shown by assuming Theorem 3.1 here. The basic result is stated as follows:

Theorem 3.1. Let L be a disconnected framed link in an ambient manifold M. For every finite regular coloring f_L of L, any positive integer J with $\rho(f_L) \leqq J$ and any positive number C, we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) $\chi(L^*, f_L q) = \chi(L, f_L)$,
(2) the Dehn surgery manifolds $\chi(L^*, fq)$ for all distinct colorings f of L with $\rho(f) \leqq J$ and $f \neq f_L, \infty$ are mutually distinct hyperbolic asymmetric 3-manifold with volumes greater than C,
(3) the Dehn surgery manifold $\chi(L^*, fq)$ for every coloring f of L with $\rho(f) \not\leqq J$ is a normal imitation of a hyperbolic asymmetric 3-manifold with volume greater than C, and
(4) the fundamental group $\pi_1(\chi(L^*, fq))$ for every coloring f of L with $f \neq f_L, \infty$ is not isomorphic to the fundamental group $\pi_1(\chi(L, f'))$ for every coloring f' of L.
(5) the fundamental group $\pi_1(\chi(L^*, fq))$ for every coloring f of L with $f \neq f_L, \infty$ is not isomorphic to the fundamental group $\pi_1(\chi(L, f'))$ for every coloring f' of L.

It is convenient to add the following properties to Theorem 3.1:

Corollary 3.2. In Theorem 3.1, we have the following additional properties:

(1) L^* is a totally hyperbolic link in M,
(2) the fundamental group $\pi_1(\chi(L^*, fq))$ for every coloring f of L with $f \neq f_L, \infty$ admits an epimorphism onto a non-abelian hyperbolic group,
(3) L^* is distinct from L componentwise.

Proof. (1) follows from (2) of Theorem 3.1 by considering all colorings f of L such that $f(L) \subset \{\infty, \emptyset\}$ but $f \neq \infty$, for $\rho(f) \leqq 1 \leqq J$. (2) follows from the properties (2), (3) of Theorem 3.1 combined with a property of imitation map. To see (3), we consider any two components K, K' of L (possibly $K = K'$) and the colorings f and f' of L such that

$$f(L - K) = f'(L - K') = \{\infty\} \quad \text{and} \quad f(K) = f'(K') = \emptyset.$$

Then by (4), $\chi(L^*, fq) = E(K^*)$ is not homeomorphic to $\chi(L, f') = E(K')$. $\square$

Counting this corollary, we obtain the following three corollaries from Theorem 3.1:

Corollary 3.3. Assume that $\chi(L, f_L) \neq M$ for a finite regular coloring f_L of a disconnected framed link L in an ambient manifold M. Then we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) L^* is totally hyperbolic and distinct from L componentwise,
(2) $\chi(L^*, f_L q) = \chi(L, f_L)$, and
(3) the colorings $f_L q$ and ∞ of L^* are distinguished.

Corollary 3.4. Let L be a disconnected framed link in an ambient manifold M. Assume that the fundamental group $\pi_1(\chi(L, f_L))$ is not isomorphic to the fundamental group $\pi_1(M)$ for a finite regular coloring f_L of L. Then we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) L^* is totally hyperbolic and distinct from L componentwise,
(2) $\chi(L^*, f_L q) = \chi(L, f_L)$, and
(3) the colorings f_L and ∞ of L^* are π_1-distinguished.

Corollary 3.5. Let L be a disconnected framed link in an ambient manifold M, and f_L a finite regular coloring of L such that $\chi(L, f_L) = M$. Then we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) L^* is totally hyperbolic and distinct from L componentwise,
(2) $\chi(L^*, f_L q) = \chi(L, f_L) = M$, and
(3) the fundamental group $\pi_1(\chi(L^*, fq))$ for every coloring f of L with $f \neq \infty, f_L$ is not isomorphic to the fundamental group $\pi_1(\chi(L, f'))$ for every coloring f' of L.

Further, here are three remarks on Corollaries 3.3 and 3.5.

Remark 3.6. For every closed ambient manifold M with $M \neq S^3$, we have lots of disconnected links L in S^3 with finite regular colorings f_L such that $\chi(L, f_L) = M$ and the colorings f_L and ∞ of L are distinguished by combining a well-known fact of W. B. R. Lickorish [18] and A. H. Wallace [24] with Corollary 3.3. If the constant coloring ∞ is π_1-distinguished for every link in S^3 with the constant coloring ∞ distinguished, then the fundamental group $\pi_1(M)$ must be non-trivial, implying that the Poincaré conjecture is affirmative. Counting Corollary 3.2, we see that one can impose on the coloring f_L above that $\pi_1(\chi(L, f'))$ admits an epimorphism onto a non-abelian hyperbolic group for every coloring f' of L with $f' \neq f_L, \infty$.

Remark 3.7. In Corollary 3.5, we take $M = S^3$ and $f_L(L) \subset \mathbb{Q}$. Then by Corollary 3.5, we have that $\chi(L^*, f_L q) = S^3$ and L^* is a totally hyperbolic link. A similar example is recently given by M. Teragaito [21]. In our case, let (L', f'_L) be the dual colored link of $I, f_L)$ in S^3 (obtained by the Dehn surgery along (L, f_L)), so that we have also $\chi(L', f'_L) = S^3$. From the construction of normal imitation, we see that the normal imitation $q : (S^3, L^*) \to (S^3, L)$ induces a normal imitation

$$q' : (S^3, \bar{L}'^*) \to (S^3, L').$$

Further, we have $E(L'^*) = E(L^*)$, and by taking a large integer J in Theorem 3.1, we can see from Corollary 3.2 that L'^* is totally hyperbolic and componentwise distinct from L^*. We see also from (3) of Corollary 3.5 combined with the argument of Lemma 2.4 that every homeomorphism $h : E(L'') \to E(L^*)$ for any link L'' in S^3 sends the meridian system $m(L'')$ to the meridian system $m(L^*)$ or $m(L'^*)$ setwise, so that h extends to a homeomorphism $h^+ : (S^3, L'') \to (S^3, L^*)$ or $h^{+\prime} : (S^3, L'') \to (S^3, L'^*)$. Thus, *we have constructed infinitely many pairs of exactly two componentwise distinct totally hyperbolic links with the same exterior.*

4. Proof of Theorem 3.1

The following lemma is obtained by combining Lemma 1.2 with the idea of [13, Lemma 2.1]:

Lemma 4.1. For any disconnected framed link (M, L), any positive number C and any positive integer J, there is an AID imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) $\chi(L^*, fq)$ is a hyperbolic asymmetric 3-manifold with volume greater than C for every finite coloring f of L with $\rho(f) \leqq J$,
(2) $\chi(L^*, fq)$ and $\chi(L^*, f'q)$ are distinct, i.e., $\chi(L^*, fq) \neq \pm\chi(L^*, f'q)$, for every pair of distinct finite colorings f, f' of L with $\rho(f), \rho(f') \leqq J$.

Proof. When $M = S^3$, the proof is proved in [13, Lemma 2.1] except the volume condition which can be easily added in the topological imitation theory. Since the present proof is parallel to the argument of [13, Lemma 2.1], we give here only the outline of the proof. Let L^+ be a meridian addition link of L, that is a link obtained from L by adding a meridian loop to every component of L. We note that the sublink $L^+ - L$ is canonically framed by which we consider L^+ a framed link extending the framed link L. By Lemma 6.1 we have a hyperbolic asymmetric AID imitation

$$q^+ : (M, (L^+)^*) \longrightarrow (M, L^+)$$

with $\mathrm{Vol}(E(L^+)^*) > C$ for every given positive number C. For any finite coloring f of L and a positive integer n, let f^n be the finite coloring of L^+ such that

$$f^n(K) = \begin{cases} f(K) + n & (\text{if } K \subset L) \\ \frac{1}{n} & (\text{if } K \subset L^+ - L). \end{cases}$$

We note that $\chi(L^+, f^n) = \chi(L, f)$ and $\chi(L^+ - L, \frac{1}{n}) = M$. Since

$$\lim_{n \to +\infty} \rho(f^n(K)) = +\infty$$

for every component K of L^+ and there are only finitely many colorings f of L with $\rho(f) \leqq J$, we see from Thurston's hyperbolic Dehn surgery argument ([22, 23]) that if we take n sufficiently large, then the AID imitation

$$q = \chi(q^+; (L^+ - L, \frac{1}{n})) : (M, L^*) \longrightarrow (M, L)$$

obtained by taking the Dehn surgery manifold $\chi(L^+ - L, \frac{1}{n}) = M$ has the property that for every finite coloring f of L with $\rho(f) \leqq J$ the Dehn surgery manifold $\chi(L^*, fq)$ is a hyperbolic asymmetric 3-manifold with volume greater than C and the dual link in $\chi(L^*, fq)$ of the sublink obtained from L^* by removing the sublink $L^*_\emptyset \subset L^*$ consisting of a component K^* with $fq(K^*) = \emptyset$ consists of short geodesics. This last condition together with the Mostow rigidity theorem ([19,22,23]) implies that $\chi(L^*, fq) \neq \pm\chi(L^*, f'q)$ for every pair of distinct colorings f, f' of L with $\rho(f), \rho(f') \leqq J$ (see [13, Lemma 2.1]). $\square$

In Lemma 4.1, if f is an infinite coloring of L, then we have $\chi(L^*, fq) = \chi(L, f)$ by a property of the AID imitation q. We used first this property in an argument of Dehn surgery of [6, Corollary 4.1], which is also developed in the following lemma:

Lemma 4.2. Let L_s and $L_s^c = L - L_s$ be non-empty sublinks of a framed link (M, L). For any positive number C, any positive integer J and any finite regular coloring f_L of L, we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

where L^* is written as $L_s^* \cup L_s^c$ such that

(1) the restriction

$$q|_{(M, L^* - K^*)} : (M, L^* - K^*) \longrightarrow (M, L - K)$$

for every knot $K^* \subset L_s^*$ and the knot $K = q(K^*) \subset L_s$ is imitation-homotopic to the identical imitation,

(2) $\chi(L^*, fq) = \chi(L, f)$ for every coloring f of L such that $f(K) = f_L(K)$ for a knot $K \subset L_s^c$,

(3) the Dehn surgery manifolds $\chi(L^*, fq)$ for all distinct colorings f of L such that $\rho(f) \leqq J$, $f|_{L_s}$ is a finite coloring of L_s, and $f(K) \neq f_L(K)$ for any knot $K \subset L_s^c$ are mutually distinct hyperbolic asymmetric 3-manifolds with volumes greater than C.

Proof. We consider the Dehn surgery manifold $M' = \chi(L_s^c, f_L|_{L_s^c})$ and the framed link $L' = L_s \cup L_s'^c$ in M' where $L_s^{c\prime}$ denotes the dual framed link obtained from the link L_s^c by the Dehn surgery operation $M \to M'$. We apply Lemma 6.1 to (M', L') to obtain an AID imitation

$$q' : (M', (L')^*) \longrightarrow (M', L')$$

where $(L')^*$ can be written as $L_s^* \cup L_s^{c\prime}$. By the dual Dehn surgery operation $M' \to M$, the AID imitation q' induces a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

where L^* can be written as $L_s^* \cup L_s^c$. (1) follows directly, since the normal imitation

$$q'|_{(M', (L')^* - K^*)} : (M', (L')^* - K^*) \longrightarrow (M', L' - K)$$

is imitation-homotopic to the identical imitation. Since the coloring f of L in (2) changes into an infinite coloring f' of the framed link (M', L'), we obtain (2) from the remark preceding to this lemma. Since the coloring f of L in (3) changes into a finite coloring f' of the framed link (M', L'), (3) follows from the properties of Lemma 6.1 with a large positive integer J. $\square$

An important observation on Lemma 4.2 is that the coloring f of (3) may be ∞ on L_s^c.

Lemma 4.3. Let (M, L) be a disconnected framed link. For any positive number C, any positive integer J and any finite regular coloring f_L of L, we have a normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that

(1) $\chi(L^*, f_L q) = \chi(L, f_L)$,

(2) the Dehn surgery manifolds $\chi(L^*, fq)$ for all distinct colorings f of L such that $\rho(f) \leqq J$ and $f \neq f_L, \infty$ are mutually distinct hyperbolic asymmetric 3-manifolds with volumes greater than C.

Proof. Let L_i $(i = 1, 2, \ldots, m)$ be all the non-empty sublinks of L such that $L_i^c = L - L_i$ is not empty for all i. Inductively, we take positive numbers C_i $(i = 1, 2, \ldots, m)$ and m normal imitations

$$q_i : (M, L_i^* \cup L_i^{c*}) \longrightarrow (M, L_{i-1}^* \cup L_{i-1}^{c*}) \quad (i = 1, 2, \ldots, m)$$

which satisfy the following conditions:

(i) $L_0^* = L_1$, $L_0^{c*} = L_1^c$, $C_1 = C$.

(ii) When we regard L_{i-1}^*, L_{i-1}^{c*} and C_i as L_s, L_s^c and C in Lemma 6.2 respectively, we take q and L^* in Lemma 6.2 as q_i and $L_i^* \cup L_i^{c*}$ where we take L_i^* and L_i^{c*} so that

$$q_i q_{i-1} \ldots q_1(L_i^*) = L_i, \quad q_i q_{i-1} \ldots q_1(L_i^{c*}) = L_i^c.$$

(iii) $\|E(L_i^* \cup L_i^{c*})\| > C_i \geqq \|E(L_{i-1}^* \cup L_{i-1}^{c*})\| \quad (i = 1, 2, \ldots, m)$.

Taking $L^* = L_m^* \cup L_m^{c*}$, we have a composite normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that $q = q_m q_{m-1} \ldots q_1$ on a tubular neighborhood $N(L^*)$ of L^* in M. We show that this normal imitation q has the properties (1) and (2). (1) follows directly from (2) of Lemma 6.2. To see (2), let f be a coloring of L such that $\rho(f) \leqq J$ and $f \neq f_L, \infty$. Let $L_{f=f_L}$ be the sublink of L consisting of a component K of L with $f(K) = f_L(K)$, and $L_{f=\infty}$ the sublink of L consisting of a component K of L with $f(K) = \infty$. By the assumption that $f \neq f_L, \infty$, the sublinks $L_{f=f_L}$ and $L_{f=\infty}$ are disjoint proper sublinks of L (which may be empty). We take the largest index i such that $L_{f=f_L} \subset L_i$ and $L_{f=\infty} \subset L_i^c$. By (1) and (2) of Lemma 6.2, we have

$$\chi(L^*, fq) = \chi(L_i^* \cup L_i^{c*}, f q_i q_{i-1} \ldots q_1).$$

By (3) of Lemma 6.2, the Dehn surgery manifolds $\chi(L_i^* \cup L_i^{c*}, f q_i q_{i-1} \ldots q_1)$ for all distinct colorings f with $\rho(f) \leqq J$ and $f \neq f_L, \infty$ such that $L_{f=f_L} \subset L_i$ and $L_{f=\infty} \subset L_i^c$ are mutually distinct hyperbolic asymmetric 3-manifolds with volumes greater than C_i. Since the volumes of these hyperbolic 3-manifolds is smaller than or equal to C_{i+1}, we see (2). $\square$

Proof of Theorem 3.1 except (4). For $J_1 = J$, by Lemma 4.3 we have a normal imitation

$$q^1 : (M, L^{*1}) \longrightarrow (M, L)$$

such that $[\chi(L^{*1}, f_L q^1)] = [\chi(L, f_L)]$ and the Dehn surgery manifolds $\chi(L^{*1}, f q^1)$ for all distinct colorings f of L such that $\rho(f) \leqq J_1$ and $f \neq f_L, \infty$ are mutually distinct hyperbolic asymmetric 3-manifolds with volumes greater than C. Then by Thurston's argument on hyperbolic Dehn surgery, there exists an integer $J_1^+ > J_1$ such that

(*) the Dehn surgery manifolds $\chi(L^{*1}, fq^1)$ are mutually distinct hyperbolic 3-manifolds with volumes greater than C for all distinct colorings f of L such that $f \neq f_L, \infty$, $\rho(f|_{L_s}) \leqq J_1$ and $\rho(f|_{L-L_s}) > J_1^+$ for a (possibly empty) sublink $L_s \subset L$.

Let $J_2 = J_1^+$. Let L have the r components K_i $(i = 1, 2, \dots, r)$. Then by continuing this process, there are integers $J_j (j = 1, 2, \dots, r + 2)$ with $J_{r+2} > J_{r+1} > \cdots > J_1 = J$ and normal imitations

$$q^j : (S^3, L^{*j}) \longrightarrow (S^3, L^{*(j-1)}) \quad (j = 1, 2, \dots, r + 1)$$

where $L^{*0} = L$ such that

$$\chi(L^*, f_L q^{r+1} q^r \dots q^1) = \chi(L, f_L)$$

and we have the following condition for every $j = 1, 2, \dots, r + 1$:

(**) The Dehn surgery manifolds $\chi(L^{*j}, fq^j q^{j-1} \dots q^1)$ are mutually distinct hyperbolic 3-manifolds with volumes greater than C for all distinct colorings f of L such that $f \neq f_L, \infty$, $\rho(f|_{L_s}) \leqq J_j$ and $\rho(f|_{L-L_s}) > J_{j+1}$ for a (possibly empty) sublink $L_s \subset L$.

Since the component number of L is r, for every coloring f of L we can find an index j such that none of the sizes $\rho(f(K_i))$ for all i are in the half open interval $(J_j, J_{j+1}]$, so that every coloring f of L with $f \neq f_L, \infty$ satisfies the condition in (**) for some j and hence the Dehn surgery manifold $\chi(L^{*j}, fq^j q^{j-1} \dots q^1)$ is a hyperbolic 3-manifold with volume greater than C. Taking $L^* = L^{*(r+1)}$, we have a composite normal imitation

$$q : (M, L^*) \longrightarrow (M, L)$$

such that $q = q^{r+1} q^r \dots q^1$ on a tubular neighborhood $N(L^*)$ of L^* in M. Since the Dehn surgery manifold $\chi(L^*, fq)$ is a normal imitation of the Dehn surgery manifold $\chi(L^{*j}, fq^j q^{j-1} \dots q^1)$ for every coloring f and every j, the normal imitation q is a desired imitation with J_r as J. This completes the proof of Theorem 3.1 except (4). □

Proof of Theorem 3.1(4). For an ambient manifold M, the Gromov norm $\|M\|$ is defined and is a constant multiple of the hyperbolic volume $\mathrm{Vol}(M)$ when M is a hyperbolic 3-manifold (see W. P. Thurston [22, 23]). In Theorem 3.1, we consider the Dehn surgery manifold $M^* = \chi(L^*, fq)$ for every $f \neq \infty, f_L$ as a normal imitation of a hyperbolic 3-manifold H with the imitation map $q_H : M^* \to H$ such that the Gromov norm $\|H\| > C$, where we take $C \geqq \|E(L)\|$. Then we have $C \geqq \|\chi(L, f')\|$ for all colorings f' of L by a property of the Gromov norm (see W. P. Thurston [22, 23]). Suppose that $\pi_1(M^*)$ is isomorphic to the fundamental group $\pi_1(N)$ of the Dehn surgery manifold $N = \chi(L, f')$ for some coloring f' of L. By Lemma 4.3, there is a connected sum $M^* = M' \# S$ such that M' is an irreducible manifold with a degree one map $q'_H : (M', \partial M') \to (H, \partial H)$ and S is a homology 3-sphere. By a property of the Gromov norm (see W. P. Thurston [22, 23]), we have $\|M'\| \geqq \|H\| > C$. Since ∂N has only torus components and every compact oriented 3-manifold with a positive genus boundary component has a non-trivial first homology, we see from Kneser's conjecture (see J. Hempel [2]) that there is a connected sum $N = S' \# N'$ such that S' is a homology 3-sphere and N' is an irreducible 3-manifold homotopy equivalent to M'. Then we show that

$$\|N'\| = \|M'\|.$$

To see this, first, assume that ∂N is empty. Then $\partial M' = \partial M$ is empty and we have degree one maps $N' \to M'$ and $M' \to N'$, so that $\|N'\| = \|M'\|$ by a property of the Gromov norm. Next, assume that ∂N is not empty. Then M' and N' are Haken manifolds with incompressible boundary consisting of torus components. By the Johannson theorem (see W. Jaco [3, p.212]), the hyperbolic pieces of the torus decompositions of N' and M' are mutually homeomorphic. By T. Soma [20], $\|N'\|$ and $\|M'\|$ are equal to the sums of the Gromov norms of the hyperbolic pieces of the torus decompositions of N' and M', respectively. Hence we have $\|N'\| = \|M'\|$ as desired.

Since there is a degree one map $(N, \partial N) \to (N', \partial N')$, we have

$$\|N\| \geqq \|N'\| = \|M'\| > C$$

by a property of the Gromov norm, which contradicts $C \geqq \|N\|$. Thus, we see that $\pi_1(\chi(L^*, fq)) = \pi_1(M^*)$ is not isomorphic to $\pi_1(N) = \pi_1(\chi(L, f'))$. □

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