Friedmann宇宙に於ける
ブラックホールの時空構造

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Based on

arXiv:0912.281, 1003.2849
cowork with *Kei-ichi Maeda*

Talk @ Osaka City Univ. 4th June, 2010

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- 天体物理学におけるブラックホール

  恒星の進化の最終状態

  支えるエネルギーを失い重力崩壊

$M \gtrsim 20M_\odot$

超新星爆発

$M \gtrsim 8M_\odot$

ブラックホール

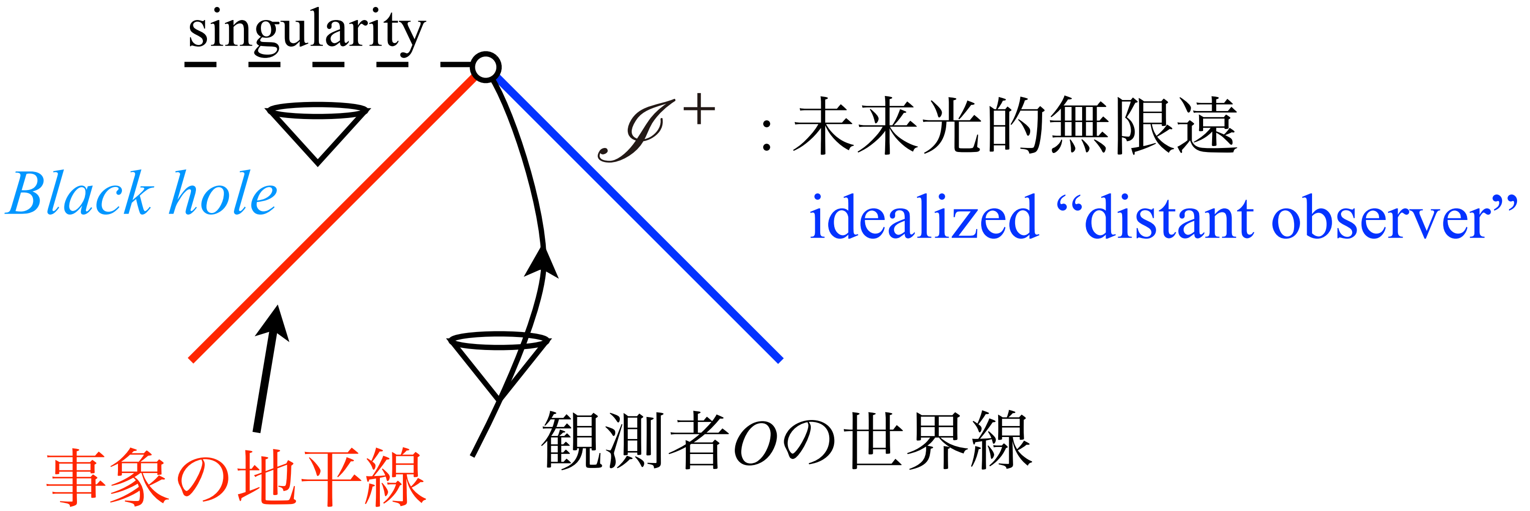
光さえも抜け出せない領域

Black holes: definition

ブラックホール = “no region of escape” c.f. *Hawking & Ellis 1973*

= 十分遠方(漸近平坦)の観測者(光的無限遠)と

因果的に繋がる曲線なし



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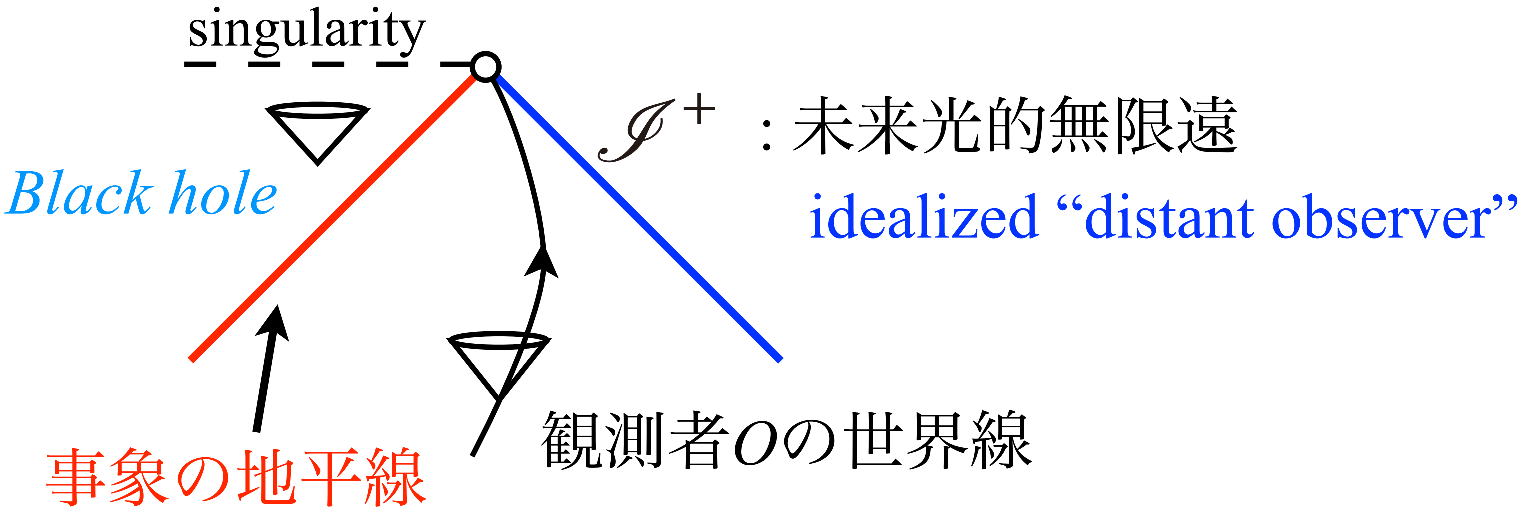

Black hole = $M/J^-(\mathscr{I}^+)$: causally disconnected region from $\mathscr{I}^+$

Event horizon = $\dot{J}^-(\mathscr{I}^+)$: boundary of black hole

Black holes: definition

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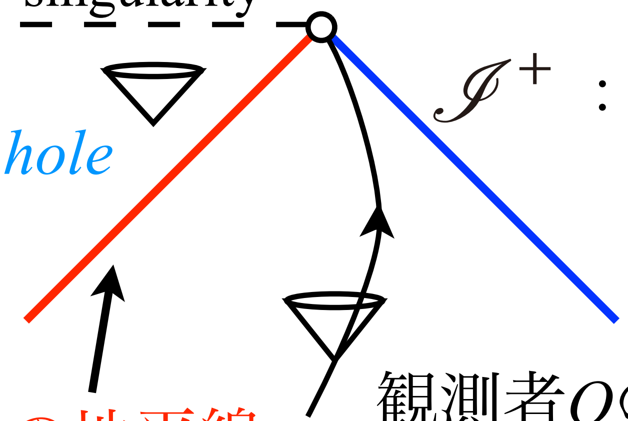
singularity



$\mathscr{I}^+$: 未来光的無限遠

idealized “distant observer”

Black hole

事象の地平線

観測者Oの世界線

Black hole $= M/J^-(\mathscr{I}^+)$: causally disconnected region from $\mathscr{I}^+$

Event horizon $= \dot{J}^-(\mathscr{I}^+)$: boundary of black hole

NB. Event horizon is a *null* surface & a *global* concept

Black holes in general relativity

一般相対論に於けるブラックホール

- “stellar sized” ブラックホール

孤立系 → •Einstein方程式の真空解(R_{ab}=0)で近似できる

•遠方で時空は平坦 (漸近的平坦性)

- 重力波放出等でダイナミカルな変化は減衰

重力崩壊から十分経過 → システムは平衡状態へ

1st step : 定常時空中の漸近平坦な真空ブラックホール

Stationary: there exists a Killing field t^a which is timelike at infinity

$$\mathcal{L}_t g_{ab} = \nabla_a t_b + \nabla_b t_a = 0\,, \qquad t^a t_a < 0 \text{ at infinity}$$

Stationary black holes in general relativity

- 厳密解の発見　　*Schwarzschild 1915, Kerr 1963*
  - Schwarzschild解: 静的球対称 (invariant under $t \to -t$, hole is round)
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We can focus on Kerr-family for analyzing equilibrium BHs

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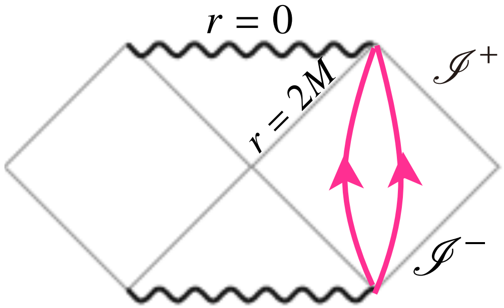
We can focus on Kerr-family for analyzing equilibrium BHs

Symmetry properties

✦ Schwarzschild解の地平線 (r=2M) はKillingベクトルで生成

ex. $$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2 \,.$$



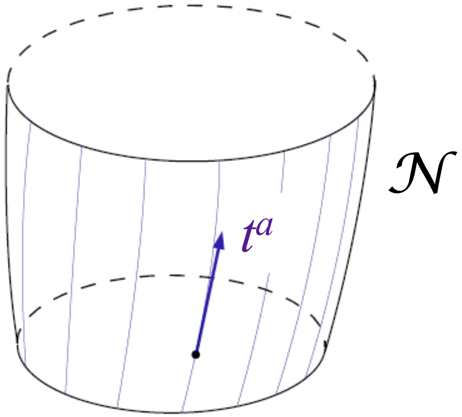
○ 静的観測者(t^a=($\partial/\partial t)^a$)は時空を加速運動

$$t^b \nabla_b t^a = \kappa(r)(\partial/\partial r)^a \,, \qquad \kappa\text{: 加速度}$$

○ 事象の地平線は特別な光的超曲面$\mathcal{N}$

$$t^b \nabla_b t^a = \kappa(r = 2M) t^a \,, \quad \text{on } \mathcal{N}$$

- Killingベクトルt^aは$\mathcal{N}$にnormal ($t^a t_a$=0) & tangent
- $\kappa|_{r=2M}$ =(4M)$^{-1}$: 地平線の表面重力



✦ Kerr解では事象の地平線上でξ^a=t^a+$\Omega_H \phi^a$ が光的 (Ω_H:地平線の角速度)

Killing horizon

Killing地平線 *Boyer 1969*

- Killingベクトル ξ^a を法線にもつ光的超曲面$\mathcal{N}$

$$\xi^a\xi_a=0 \text{ on } \mathcal{N}, \qquad \nabla_a\xi_b + \nabla_b\xi_a = 0\,,$$

$$\nabla_b(\xi^a\xi_a) = -2\kappa\xi_b\,, \quad \Leftrightarrow \quad \xi^b\nabla_b\xi^a = \kappa\xi^a\,,$$

Killing horizon

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- Killing地平線に流入するエネルギーなし　$0 \overset{H}{=} R_{ab}\xi^a\xi^b = 8\pi T_{ab}\xi^a\xi^b\,.$

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- 事象の地平線とは独立な概念

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▸対称性 *Hawking 1971, Moncrief-Isenberg 1973, Sudarsky-Wald 1993*

定常時空 (定常のKilling t^a=$(\partial/\partial t)^a$が存在) におけるBHの地平線は

(i) Killing 地平線と一致

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(ii-a) 回転していなければ($\xi^a=t^a$), 時空は静的 $t_{[a}\nabla_b t_{c]} = 0$

(ii- b) 回転していれば($\xi^a\neq t^a$), 時空は軸対称 $\xi^a = t^a + \Omega_{\mathrm{H}}\phi^a$

Black hole thermodynamics

ブラックホール熱力学 *Bekenstein 1971, Bardeen-Carter-Hawking 1973*

- 0th law: equilibrium *c.f. Racz-Wald 1992*

κ is constant on Killing horizon $\Longleftrightarrow$ $T = \text{constant}$

$\xi^b \nabla_b \xi^a = \kappa \xi^a$ κ: 表面重力 $\longrightarrow$ 定常BHは熱力学的にも平衡状態

- 1st law: energy conservation *c.f. Gao-Wald 2001*

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega \delta J \quad \Longleftrightarrow \quad \delta E = T\delta S + \text{work term}$$

A:地平線面積

- 2nd law: entropy increasing law *c.f. Flanagan et al 1999, Gao-Wald 2001*

$$\delta A \geq 0 \quad \Longleftrightarrow \quad \delta S \geq 0$$

$\longrightarrow$ 定常BHは面積不変 δA=0

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-Taking quantum effect into account, it turns out that BH emits *thermal radiation*

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→ 定常BHは面積不変 $\delta A=0$

-Taking quantum effect into account, it turns out that BH emits *thermal radiation*

$$T=\frac{\hbar\kappa}{2\pi ck_B},\quad S_{BH}=\frac{k_Bc^3A}{4G\hbar},$$

Hawking 1973

Stationary black holes

定常ブラックホール

漸近平坦，真空というセットアップのもとでは，

- Schwarzschild解, Kerr解などの重力的に安定な厳密解が存在
- Killing地平線で表されるような熱力学平衡状態に対応
- 本質的に１種類しか存在しない(Kerr族)

N.B Einstein-Maxwell系でも同様の性質

- Kerr-Newman族: (M,J,Q)の3パラメータファミリー　*Mazur 1982*

$$ds^2 = -dt^2 + \frac{2Mr - Q^2}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + (r^2 + a^2)\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \, .$$

$$\Sigma = r^2 + a^2\cos^2\theta \, , \quad \Delta = r^2 - 2Mr + a^2 + Q^2 \, , \qquad A = \frac{Qr}{\Sigma}(dt - a\sin^2\theta d\phi) \, ,$$

Contents

Black holes in the universe

- ダイナミカルブラックホール

定常性をはずす → *time-dependent*

応用: 原始ブラックホール　　　*Carr-Hawking 1974*

宇宙の初期に密度揺らぎでブラックホール形成

→ Hubble質量のブラックホールが生成

$$T_{\rm B} = \frac{\hbar c^3}{8\pi G k_{\rm B} M} \sim 10^{-7}(M/M_\odot)^{-1}\ {\rm K}, \quad \sim (M/10^{10}{\rm g})^{-1}\ {\rm TeV},$$

Hawking輻射が観測される可能性

✦ 宇宙論的背景の中でブラックホールを考える必要

漸近的平坦性や真空条件もはずすべき

Black holes in the universe

✦ 我々の宇宙は大きなスケールで一様等方

Robertson-Walker 計量: $$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)\,,$$

Friedmann方程式: $$H^2 := \left(\frac{1}{a}\frac{d}{dt}a\right)^2 = \frac{8\pi G}{3}\rho\,. \qquad \frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}(a^3)$$

✦ 1st step: exact black-hole solutions in *FRW universe*

唯一性は成り立たない → we expect much richer families of solutions

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- 我々の宇宙は大きなスケールで一様等方

Robertson-Walker 計量: $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)\,,$

Friedmann方程式: $H^2 := \left(\frac{1}{a}\frac{d}{dt}a\right)^2 = \frac{8\pi G}{3}\rho\,. \qquad \frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}(a^3)$

- 1st step: exact black-hole solutions in *FRW universe*

唯一性は成り立たない → we expect much richer families of solutions

Difficulties

Black holes in the universe

✦ 我々の宇宙は大きなスケールで一様等方

Robertson-Walker 計量: $$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,,$$

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○ Putting a BH in FRW universe ⇒ Universe becomes *inhomogeneous*

Black holes in the universe

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✦ 1st step: exact black-hole solutions in *FRW universe*

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Difficulties

- ○ Putting a BH in FRW universe ⇒ Universe becomes *inhomogeneous*
- ○ Matter accretion ⇒ BH will *grow & deform*

Black holes in the universe

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Robertson-Walker 計量: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,,$

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唯一性は成り立たない → we expect much richer families of solutions

Difficulties

○ Putting a BH in FRW universe ⇒ Universe becomes *inhomogeneous*

○ Matter accretion ⇒ BH will *grow & deform*

We must solve nonlinear PDE w/ space & time simultaneously.

FRW black holes with symmetry

Schwarzschild-de Sitter *Kottler 1918*

$$ds^2 = -\left(\frac{1-M/2ar}{1+M/2ar}\right)^2 dt^2 + a^2\left(1+\frac{M}{2ar}\right)^4 (dr^2 + r^2 d\Omega_2^2)\ ,\quad a(t) = e^{Ht}$$

$$R_{ab} = 3H^2 g_{ab}\ ,$$

✦ locally static (Birkhoff's theorem)

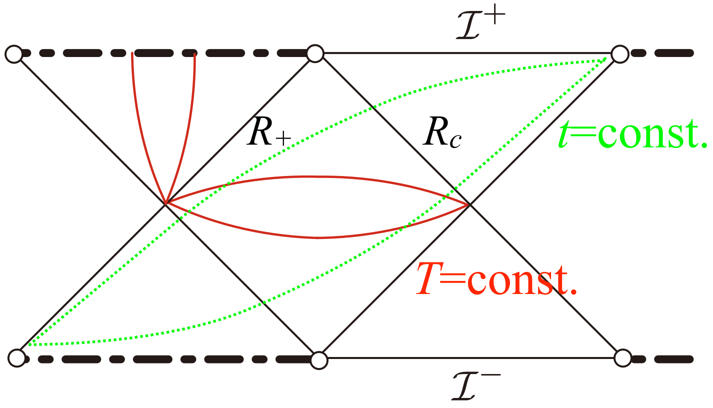
$$T = t + \int^R \frac{HR}{\sqrt{1-2M/R}(1-2M/R+H^2R^2)}\ ,\quad R = ar\left(1+\frac{M}{2ar}\right)^2$$

$$\rightarrow \quad \boxed{\begin{aligned} &ds^2 = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2 d\Omega_2^2\ , \\ &f(R) = 1 - \frac{2M}{R} + H^2R^2 \end{aligned}}$$

本質的にnon-dynamical

(R_+はKilling地平線)

$$f(R_+) = f(R_c) = 0,\quad R_+ < R_c$$

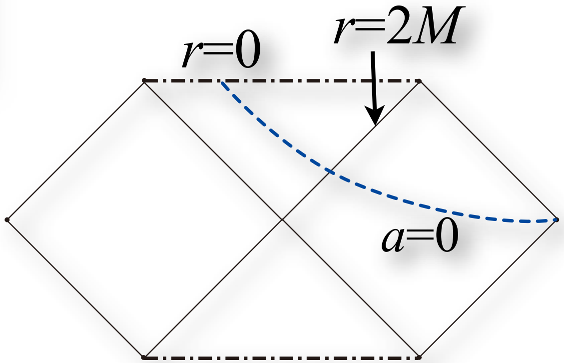


FRW black holes with symmetry

Sultana-Dyer solution

Sultana & Dyer 2005

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{2M}{r}(d\eta + dr)^2 + dr^2 + r^2 d\Omega_2^2\right] \qquad a(\eta) = \eta^2$$



c.f. Saida-Harada-Maeda 2007

- sourced by dust and null dust

$$T_{ab} = \rho_{\rm mat} u_a u_b + \rho_{\rm rad} l_a l_b \,,$$

- Schwarzschild 計量と共形

地平線はr=2M ⇒ $R_{\rm H}$=2Ma

- generated by a *conformal Killing vector* ξ^a=$(\partial/\partial\eta)^a$

$$\mathscr{L}_\xi g_{ab} = 2(a'/a) g_{ab}\,, \longrightarrow$$ 宇宙膨張と“同じ割合”でBHも進化

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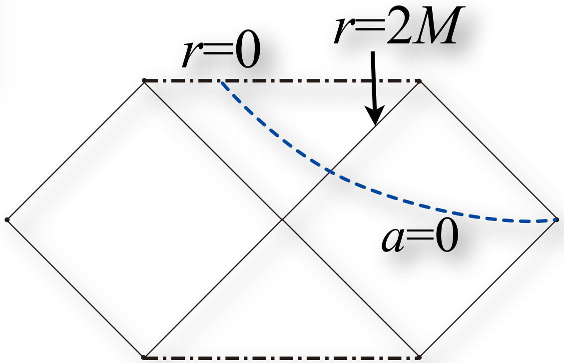

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- エネルギー条件の破れ

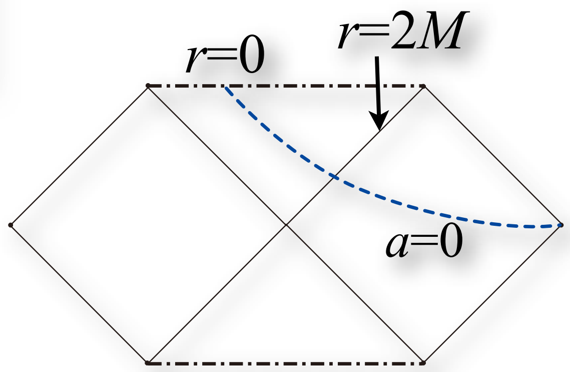
$\rho_{\rm mat} < 0,\ \rho_{\rm rad} < 0$ for $\eta > r(r+2M)/2M$

FRW black holes with symmetry

Sultana-Dyer solution

Sultana & Dyer 2005

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{2M}{r}(d\eta + dr)^2 + dr^2 + r^2 d\Omega_2^2\right] \qquad a(\eta) = \eta^2$$



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$$T_{ab} = \rho_{\rm mat} u_a u_b + \rho_{\rm rad} l_a l_b \, ,$$

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- generated by a *conformal Killing vector* $\xi^a=(\partial/\partial\eta)^a$

$\mathscr{L}_\xi g_{ab} = 2(a'/a) g_{ab}$, → 宇宙膨張と“同じ割合”でBHも進化

- エネルギー条件の破れ

$\rho_{\rm mat} < 0, \ \rho_{\rm rad} < 0$ for $\eta > r(r+2M)/2M$ → *physically unacceptable*

FRW black holes with symmetry

Self-similar black holes

自己相似性　$\mathscr{L}_\xi g_{ab} = 2g_{ab}\,,$

- 減速膨張のとき，BHは存在しない　*Harada-Maeda-Carr 2006*

McVittie's solution

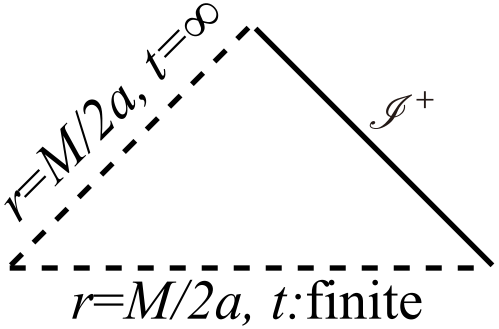
Nolan 2002, Kaloper et al 2010

$$\mathrm{d}s^2 = -\left(\frac{1-M/2ar}{1+M/2ar}\right)^2 \mathrm{d}t^2 + a^2\left(1+\frac{M}{2ar}\right)^4 (\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2)\ , \qquad a(t) = t^p\,,$$

→　r=$M/2a$は曲率特異点　$R = 12H^2 + 6\dot{H}\left(\frac{1+M/(2ar)}{1-M/(2ar)}\right),$

a=t^pのとき

(p<1)



FRW black holes

Black holes in FRW universe

時間依存性あり　⇒　ブラックホールは時間変化

厳密解に限ってもエネルギー条件をみたすような
ブラックホールを構築するのは(数学的にも)難しい

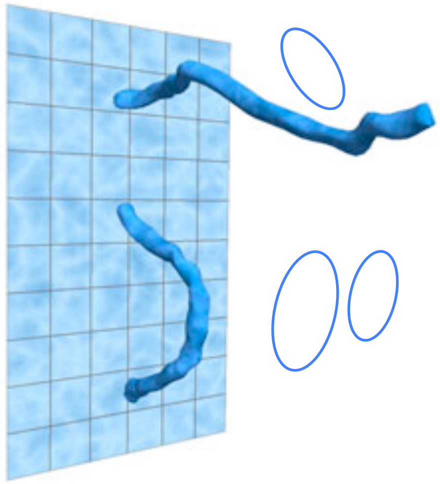
What we have done

高次元のダイナミカルな交差ブレーン解のコンパクト化により,
４次元のダイナミカルな“ブラックホール”解を得る.

Contents

Branes in string theory

String/M-theory



- Promising unified theory of all interactions
- 10/11 次元で定式化
- 基本的構成要素:

  string (open & closed), D-brane

超重力理論のブラックpブレーン解

Horowitz-Strominger 1991

- black “holes” w/ extended into spatial p-directions
- preserves a part of supersymmetries (BPS state)
- low energy description of D-branes (and M-branes)

M-Branes in 11D supergravity

- 11次元超重力

$$S = \frac{1}{2\kappa_{11}^2}\int \left(*R - \tfrac{1}{2}F\wedge *F - \tfrac{1}{6}F\wedge F\wedge A\right)$$

F=dA: 4-form

Fが電気的(4-form)に結合 ⇒ *M2-brane*

Fが磁気的(7-form)に結合 ⇒ *M5-brane*

c.f. 4次元点粒子(0-dim.)

electric $F_{\mu\nu}$ (0+2-dim.),

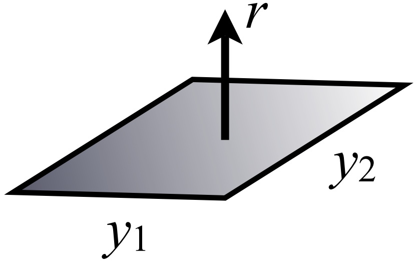
magnetic $*F_{\mu\nu}$ (0+(4-2)-dim.),

extremal M2-brane

$$\mathrm{d}s_{\mathrm{M2}}^2 = H_2^{1/3}\left[H_2^{-1}(-\mathrm{d}t^2 + \underbrace{\mathrm{d}y_1^2 + \mathrm{d}y_2^2}_{\text{extended directions}}) + \mathrm{d}r^2 + r^2\mathrm{d}\Omega_7^2\right]$$

$$F = \mathrm{d}(H_2^{-1})\wedge \mathrm{d}t\wedge \mathrm{d}y_1\wedge \mathrm{d}y_2$$

H_2: *harmonic* fun. on $\mathrm{d}r^2 + r^2\mathrm{d}\Omega_7^2$



- r=0 に点状源 $H_2 = 1 + \frac{Q}{r^6}$ → r=0 は正則地平線
- preserves 1/2-SUSY

Intersecting branes in supergravity

交差ブレーン

Tseytlin 1996, Ohta 1997

e.g., M2/M2/M5/M5 branes

$$\mathrm{d}s^2 = H_2^{1/3}H_{2'}^{1/3}H_5^{2/3}H_{5'}^{2/3}\left[-H_2^{-1}H_{2'}^{-1}H_5^{-1}H_{5'}^{-1}\mathrm{d}t^2 + H_5^{-1}H_{5'}^{-1}\left(\mathrm{d}y_1^2+\mathrm{d}y_2^2+\mathrm{d}y_3^2\right)\right.$$
$$\left. + H_5^{-1}H_2^{-1}\mathrm{d}y_4^2 + H_5^{-1}H_{2'}^{-1}\mathrm{d}y_5^2 + H_2^{-1}H_{5'}^{-1}\mathrm{d}y_6^2 + H_{2'}^{-1}H_{5'}^{-1}\mathrm{d}y_7^2 + (\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2)\right]$$

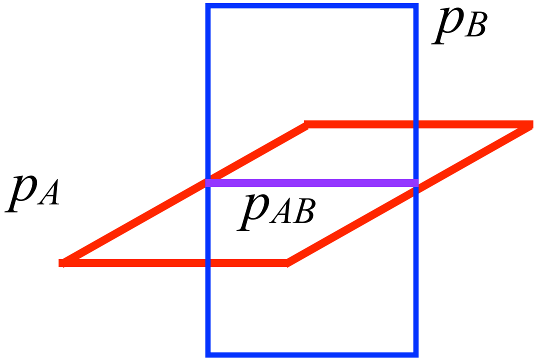
$$H_2 = 1 + \frac{Q_2}{r},\quad H_{2'} = 1 + \frac{Q_{2'}}{r}\quad H_5 = 1 + \frac{Q_5}{r},\quad H_{5'} = 1 + \frac{Q_{5'}}{r}$$

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			
M2	○				○		○				
M2	○					○		○			

M2∩M2=0, M2∩M5=1, M5∩M5=3

- [**] 内の計量でharmonics $H_n{}^{-1}$がかかっているところにM*n*-braneが存在 (intersection rule)

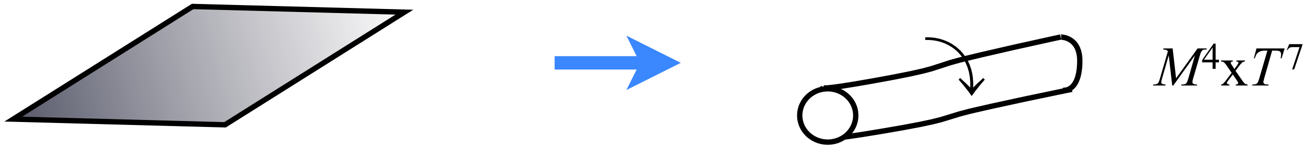
- 1/8-BPS状態



4D black hole from intersecting branes

ブレーン方向を丸めて，4Dへコンパクト化

$$S=\frac{1}{2\kappa_{11}^2}\int \mathrm{d}^{11}x\sqrt{-g_{11}}(R_{11}+\cdots) \longrightarrow S=\frac{1}{2\kappa_4^2}\int \mathrm{d}^4x\sqrt{-g_4}(R_4+\cdots)$$



4D Einstein frame metric from M2/M2/M5/M5

$$\mathrm{d}s_4^2=-\Xi\mathrm{d}t^2+\Xi^{-1}\left(\mathrm{d}r^2+r^2\mathrm{d}\Omega_2^2\right), \quad \Xi=(H_2H_{2'}H_5H_{5'})^{-1/2}$$

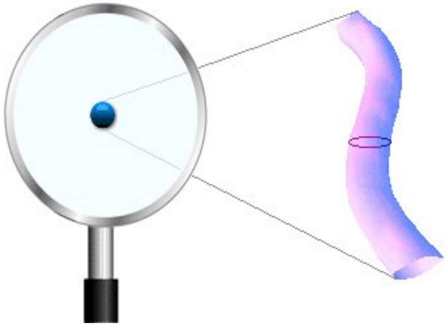
$$H_2=1+\frac{Q_2}{r},\quad H_5=1+\frac{Q_5}{r},\quad H_{2'}=1+\frac{Q_{2'}}{r},\quad H_{5''}=1+\frac{Q_{5''}}{r}.$$

- $Q_i \equiv Q$とすれば 極限 Reissner-Nordströmブラックホール

  otherwise: Einstein-Maxwell (x4)-dilaton (x3)

Advantages of intersecting brane picture

ブラックホールエントロピーのミクロな導出が可能



Dブレーンは開弦のendpoint

⇒ 弦の配位を数え上げ可能

$$S=\log W=2\pi(N_1N_5N_w)^{1/2}=A/4G_{10}=S_{BH}$$

Strominger-Vafa 1996, Callan-Maldacena 1996

- 4次元ブラックホールを得るには4電荷必要

$$R_H = r\left[\prod_i\left(1+\frac{Q_i}{r}\right)\right]^{1/4}_{r\to0},$$

M2/M2/M5/M5, M5/M5/M5/W, etc.

- 5次元ブラックホールを得るには3電荷必要

$$R_H = r\left[\prod_i\left(1+\frac{Q_i}{r}\right)\right]^{1/3}_{r\to0},$$

M2/M2/M2, M2/M5/W, D1/D5/W, etc.

Black hole from dynamically intersecting branes

Time-dependent branes in 11D SUGRA

Maeda-Ohta-Uzawa 2009

静的なM-ブレーンと同様な計量ansatzのもと,

時間依存性をもった交差ブレーン解を分類

ex. M2/M2/M5/M5 (4-charges) with evolving M2

$$\begin{aligned} \mathrm{d}s^2 \;=\; & H_2^{1/3} H_{2'}^{1/3} H_5^{2/3} H_{5'}^{2/3} \left[-H_2^{-1} H_{2'}^{-1} H_5^{-1} H_{5'}^{-1} \mathrm{d}t^2 + H_5^{-1} H_{5'}^{-1} \left(\mathrm{d}y_1^2 + \mathrm{d}y_2^2 + \mathrm{d}y_3^2 \right) \right. \\ & \left. + H_5^{-1} H_2^{-1} \mathrm{d}y_4^2 + H_5^{-1} H_{2'}^{-1} \mathrm{d}y_5^2 + H_2^{-1} H_{5'}^{-1} \mathrm{d}y_6^2 + H_{2'}^{-1} H_{5'}^{-1} \mathrm{d}y_7^2 + (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2) \right] \end{aligned}$$

$$H_2 = \frac{t}{t_0} + \frac{Q_2}{r} \qquad H_{2'} = 1 + \frac{Q_{2'}}{r} \qquad H_5 = 1 + \frac{Q_5}{r}\,, \qquad H_{5'} = 1 + \frac{Q_{5'}}{r}$$

- 4種のブレーンのうち，どれか 1 つのみ時間依存性をもつことが可能

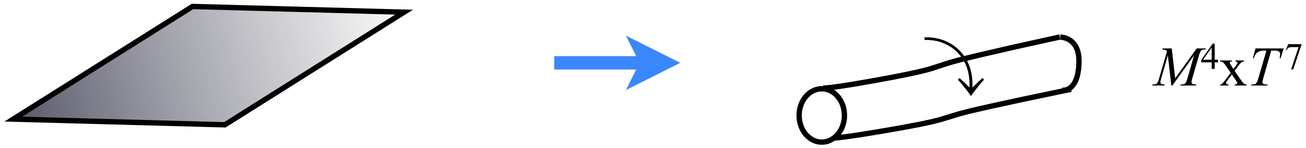
- Q_i=0とすると, 11Dは*Kasner*宇宙(空間一様性を保つ真空解)

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + (\tau/\tau_0)^{-1}(\mathrm{d}y_4^2 + \mathrm{d}y_6^2) + (\tau/\tau_0)^{1/2}\mathrm{d}x_8^2 \,. \qquad \tau \propto t^{2/3}$$

4D black hole from intersecting branes

ブレーン方向を丸めて，4Dへコンパクト化

$$S=\frac{1}{2\kappa_{11}^2}\int \mathrm{d}^{11}x\sqrt{-g_{11}}(R_{11}+\cdots)\quad\longrightarrow\quad S=\frac{1}{2\kappa_4^2}\int \mathrm{d}^4x\sqrt{-g_4}(R_4+\cdots)$$



4D Einstein frame metric from *dynamical* M2/M2/M5/M5

$$\mathrm{d}s_4^2=-\Xi\mathrm{d}t^2+\Xi^{-1}\left(\mathrm{d}r^2+r^2\mathrm{d}\Omega_2^2\right),\qquad \Xi=\left(H_TH_SH_{S'}H_{S''}\right)^{-1/2}$$

$$H_T=\frac{t}{t_0}+\frac{Q_T}{r},\quad H_S=1+\frac{Q_S}{r},\quad H_{S'}=1+\frac{Q_{S'}}{r},\quad H_{S''}=1+\frac{Q_{S''}}{r}.$$

- 解は時間依存+空間的非一様

- ダイナミカルブラックホールを表していると期待できる

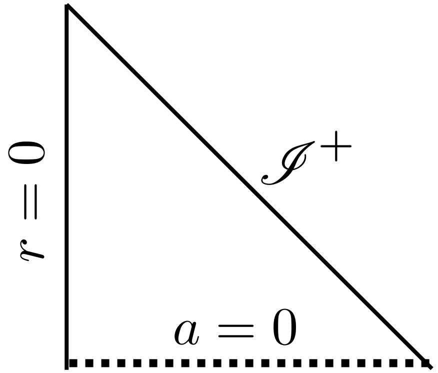
Dynamical black hole in FRW universe?

$$ds_4^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega_2^2\right), \qquad \Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r}\right)\left(1 + \frac{Q_S}{r}\right)\left(1 + \frac{Q_{S'}}{r}\right)\left(1 + \frac{Q_{S''}}{r}\right)\right]^{-1/2}$$

- asymptotically ($r \to \infty$) tends to $P=\rho$ FRW universe

$$ds_4^2 = -d\bar{t}^2 + a^2(dr^2 + r^2 d\Omega^2)$$

$$\bar{t} \propto t^{3/4} \qquad a = (\bar{t}/\bar{t}_0)^{1/3}$$



- reduces to AdS$_2$ x S^2 as $r \to 0$ with t :fixed

$$ds_4^2 = -\frac{r^2}{\bar{Q}^2}dt^2 + \frac{\bar{Q}^2}{r^2}\left(dr^2 + r^2 d\Omega_2^2\right), \qquad \bar{Q} = (Q_T Q_S Q_{S'} Q_{S''})^{1/4}$$

typical near-horizon geometry of extremal BH

Kunduri-Lucietti-Reall 2007

Dynamical black hole in FRW universe?

$$ds_4^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega_2^2\right) , \qquad \Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r}\right)\left(1 + \frac{Q_S}{r}\right)\left(1 + \frac{Q_{S'}}{r}\right)\left(1 + \frac{Q_{S''}}{r}\right)\right]^{-1/2}$$

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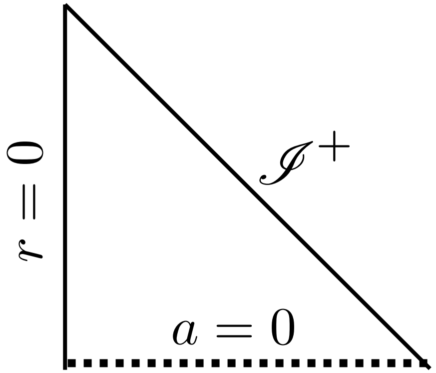
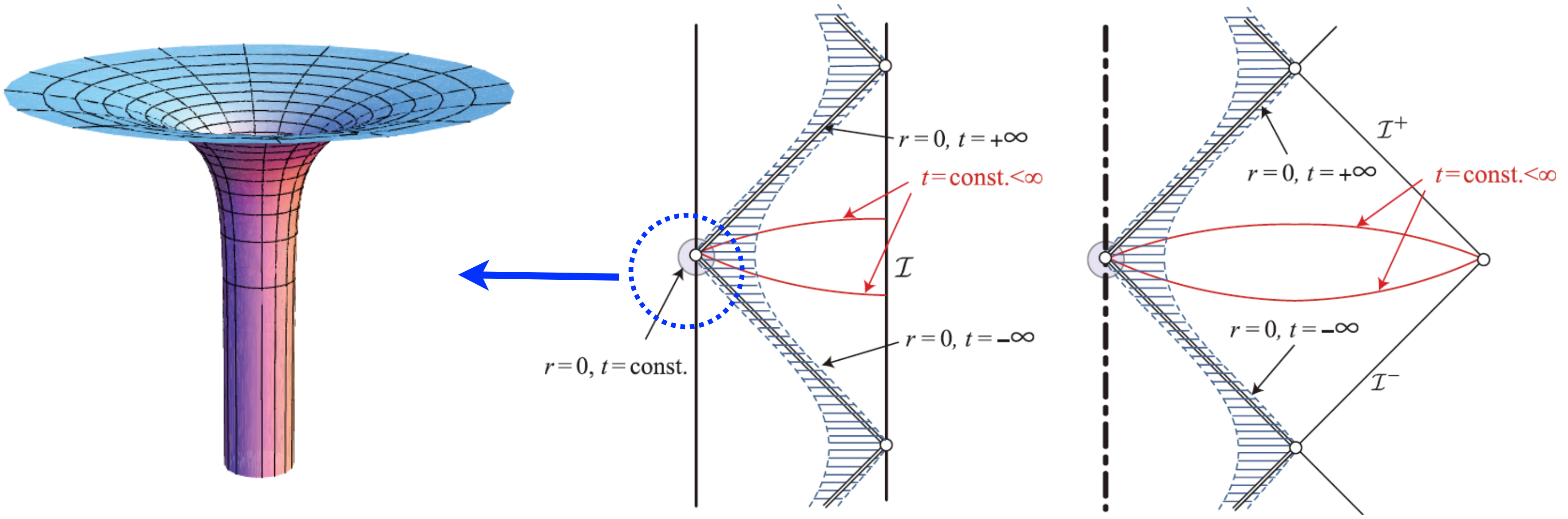
$$\bar{t} \propto t^{3/4} \qquad a = (\bar{t}/\bar{t}_0)^{1/3}$$



- reduces to $AdS_2 \times S^2$ as $r \to 0$ with t :fixed

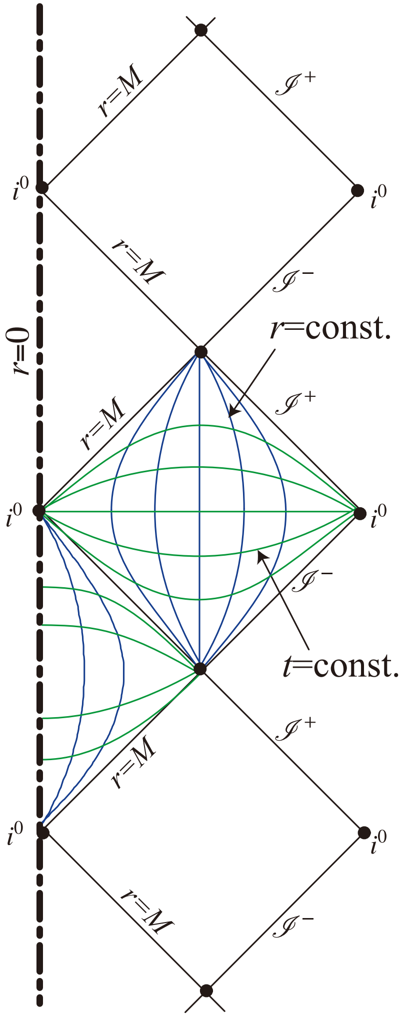


Extremal black hole in FRW universe?

Naive Picture

extreme RN

(r~0)

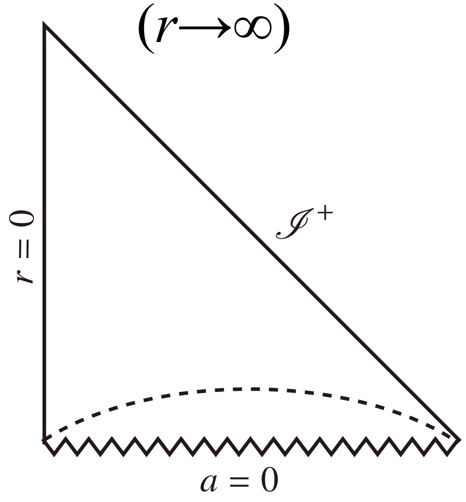


$$\mathrm{d}s_4^2 = -\Xi \mathrm{d}t^2 + \Xi^{-1}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2\right),$$

$$\Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r}\right)\left(1 + \frac{Q_S}{r}\right)\left(1 + \frac{Q_{S'}}{r}\right)\left(1 + \frac{Q_{S''}}{r}\right)\right]^{-1/2}$$

P=ρ FRW

(r⟶∞)



Naive Picture

extreme RN

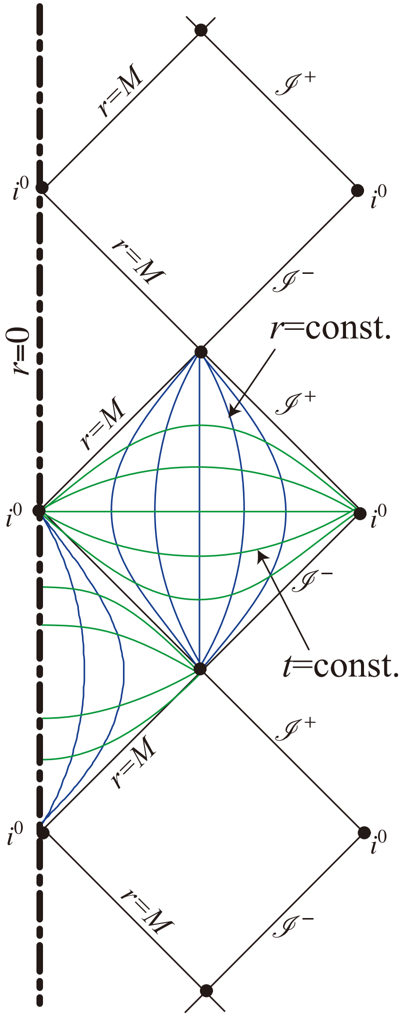
(r~0)

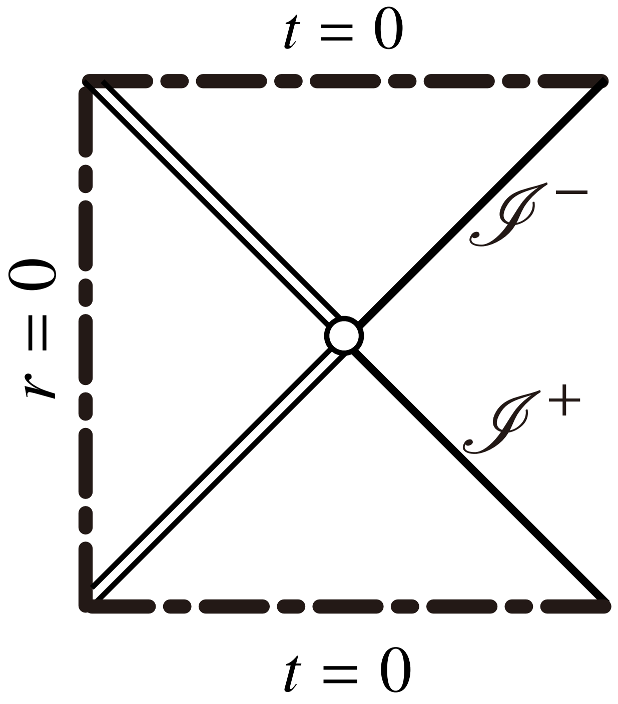
$$ds_4^2 = -\Xi \mathrm{d}t^2 + \Xi^{-1}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2\right),$$

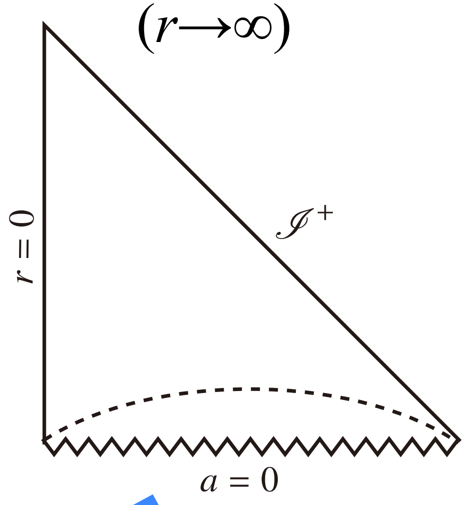
$$\Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r}\right)\left(1 + \frac{Q_S}{r}\right)\left(1 + \frac{Q_{S'}}{r}\right)\left(1 + \frac{Q_{S''}}{r}\right)\right]^{-1/2}$$

P=ρ FRW

(r⟶∞)



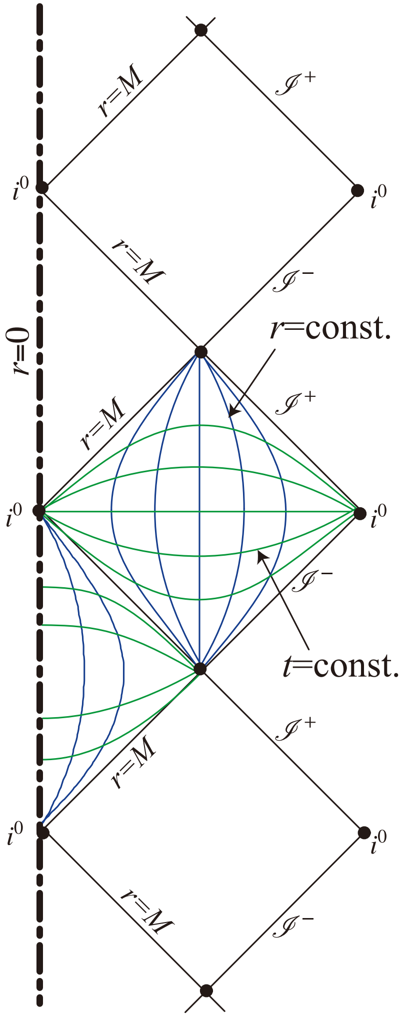




Naive Picture

extreme RN

(r~0)

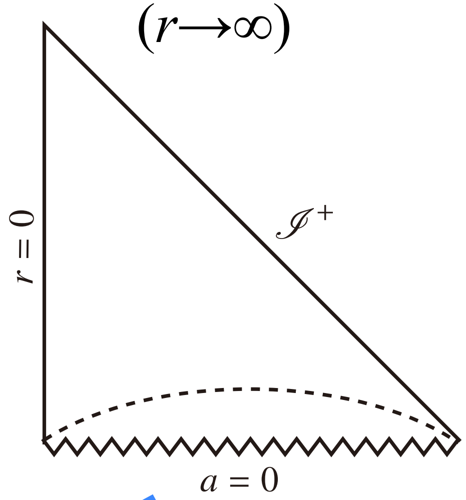


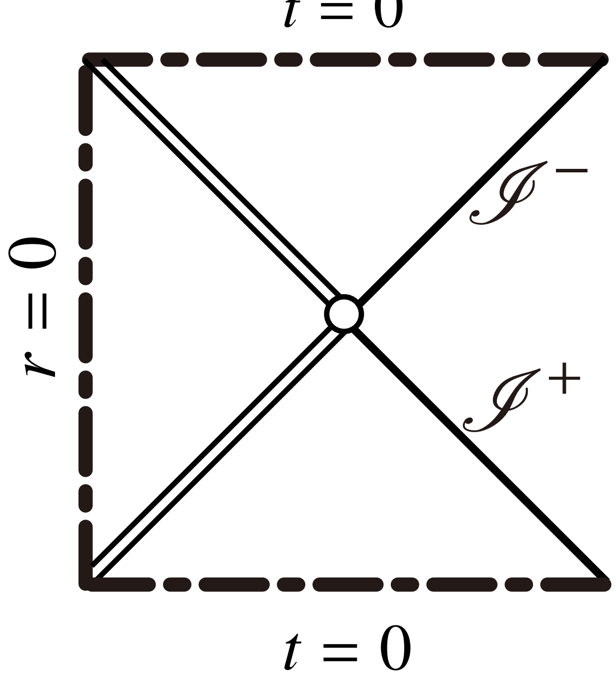
$$ds_4^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega_2^2\right) ,$$

$$\Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r}\right)\left(1 + \frac{Q_S}{r}\right)\left(1 + \frac{Q_{S'}}{r}\right)\left(1 + \frac{Q_{S''}}{r}\right)\right]^{-1/2}$$

P=ρ FRW

(r⟶∞)





Is this rough estimate indeed true?

Naive Picture

extreme RN

(r~0)

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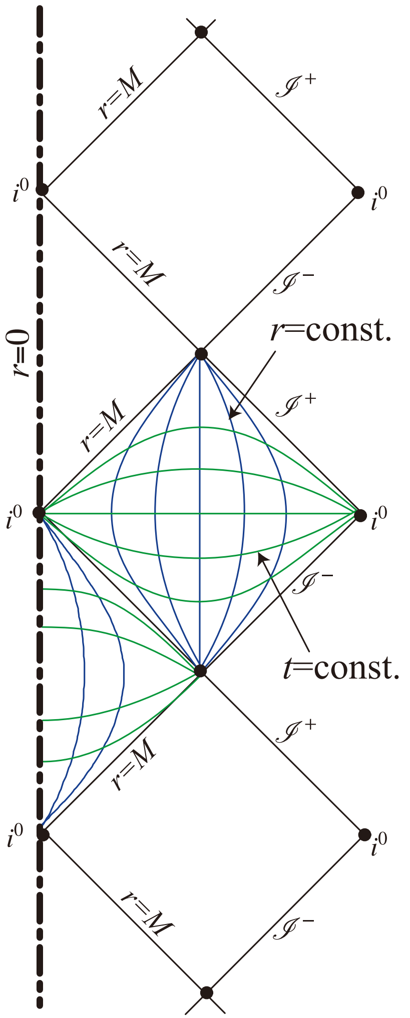
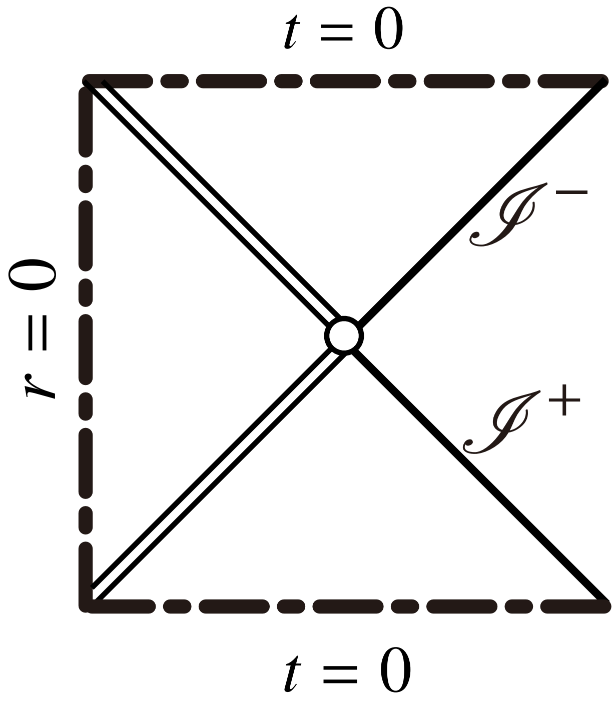
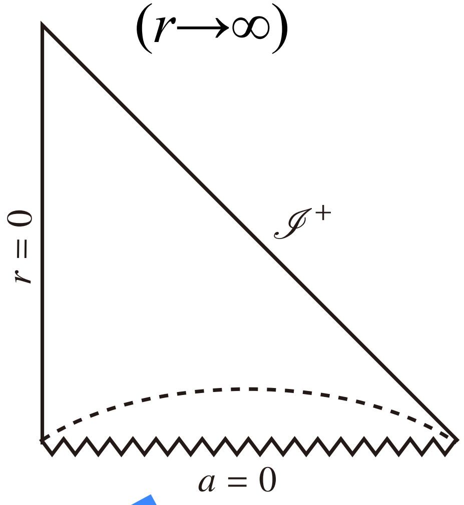






Is this rough estimate indeed true?

NO.

Our goal

“ダイナミカルブラックホール”の時空構造を知りたい

- 時空特異点

  Singularity is naked?

- 捕捉領域

  Black hole tends to attract ⟷ Universe itself is expanding

- 事象の地平線

  Event horizon exists? Singularity is covered?

■ For simplicity, we assume

$$\begin{cases} Q_2 = Q_{2'} = Q_5 = Q_{5'} \equiv Q \ (> 0) \\ t_0 > 0 \ \text{:膨張宇宙} \end{cases}$$

$$ds_4^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega_2^2\right),$$

characterized by $(Q,\ t_0)$

$$\Xi = \left(H_T H_S^3\right)^{-1/2} \qquad H_T = \frac{t}{t_0} + \frac{Q}{r}, \qquad H_S = 1 + \frac{Q}{r}.$$

Contents

Matter fields

Einstein-Maxwell(x2)-dilaton系 (Q_i≡Qのとき)

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{16\pi}\sum_A e^{\lambda_A\kappa\Phi}(F^{(A)}_{\mu\nu})^2\right]$$

A=T, S,S',S''

- 時間依存ブレーンのみが異なる結合定数

$$\lambda_T = \sqrt{6}\,, \quad \lambda_S \equiv \lambda_{S'} \equiv \lambda_{S''} = -\sqrt{6}/3.$$

- Maxwell場とdilatonがソース

$$\kappa\Phi = \frac{\sqrt{6}}{4}\ln\left(\frac{H_T}{H_S}\right)$$:massless scalar → "P=ρ universe"

$$A^{(T)} = \sqrt{2\pi}\kappa^{-1}H_T^{-1}\mathrm{d}t\,, \qquad A^{(S)} = A^{(S')} = A^{(S'')} = \sqrt{2\pi}\kappa^{-1}H_S^{-1}\mathrm{d}t\,,$$

$$H_T = \frac{t}{t_0} + \frac{Q}{r}\,,$$

$$H_S = 1 + \frac{Q}{r}\,,$$

- QはMaxwell電荷 $$\frac{Q}{\sqrt{G}} = \frac{1}{4\pi}\int_S e^{\lambda_A\kappa\Phi}F^{(A)}_{\mu\nu}\mathrm{d}S^{\mu\nu}\,,$$

- 優勢エネルギー条件を満足 $T_{\mu\nu}v^\mu u^\nu \geq 0$

Singularities

時空特異点

- すべての曲率不変量 ($R_{abcd}R^{abcd}$, $C_{abcd}C^{abcd}$, Ψ_2 etc)が発散

$$t = t_s(r) := -Qt_0/r \Leftrightarrow H_T = 0$$

$$r = -Q \Leftrightarrow H_S = 0$$

$$\mathrm{d}s_4^2 = -\Xi \mathrm{d}t^2 + \Xi^{-1}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2\right),$$

$$\Xi = \left(H_T H_S^3\right)^{-1/2}$$

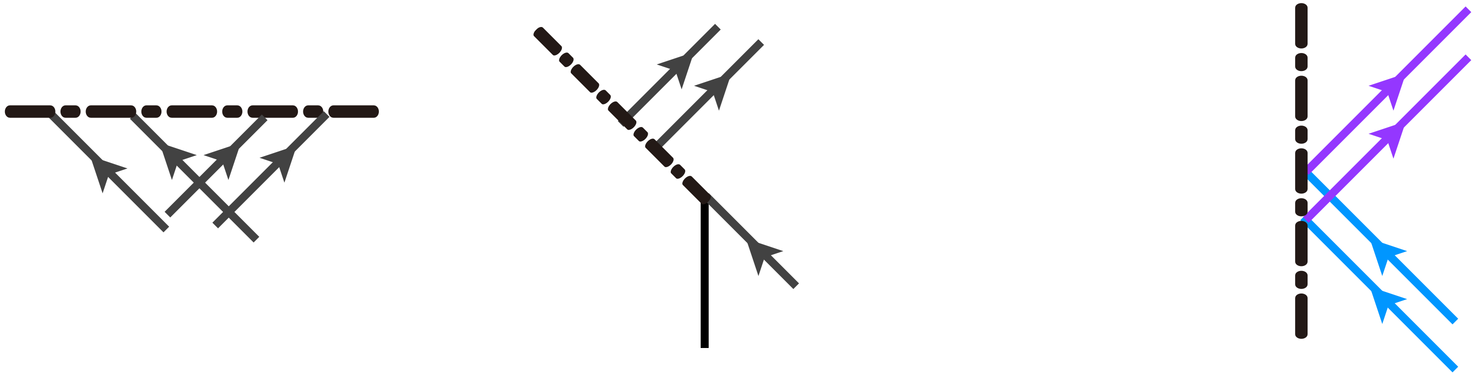
$$H_T = \frac{t}{t_0} + \frac{Q}{r}, \quad H_S = 1 + \frac{Q}{r},$$

- これらの特異点は *timelike & central* (面積半径0)

there exist *an infinite number of*

c.f. Christdoulou 1984

$\Big\{$ *ingoing null geodesics* terminating into singularities
outgoing null geodesics emanating from singularities



$$k^\mu = \left(\frac{\partial}{\partial \lambda}\right)^\mu$$

Singularities

○すべての曲率不変量は

t=0で有限 → t=0 はBig-bang 特異点ではない(t<0 に接続可能)

r=0で有限 → 地平線の候補 (static braneではr=0が地平線)

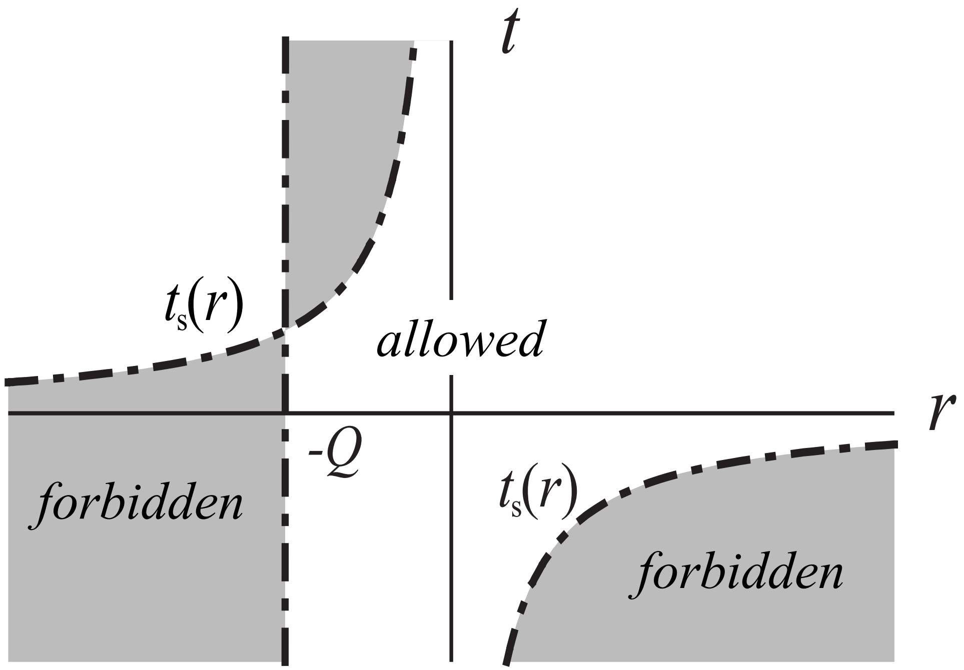
(degenerate null surface)

- 許される座標範囲は　$H_T H_S^3 > 0$

$$\Xi = \left(H_T H_S^3\right)^{-1/2}$$

$$H_T = \frac{t}{t_0} + \frac{Q}{r} \equiv \frac{t - t_s(r)}{t_0}$$

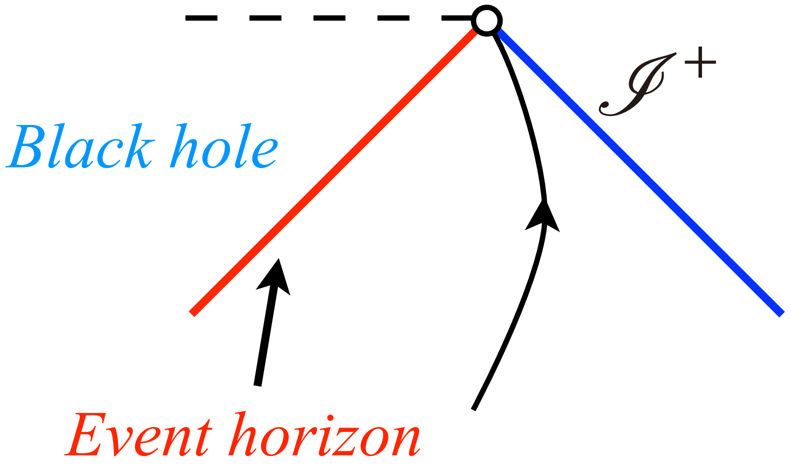
$$H_S = 1 + \frac{Q}{r}$$



How to find event horizon

N.B　事象の地平線は大域的概念

地平線上の各点は局所的に
なんら他の点と区別はない



Strategy

(i) 捕捉領域を調べる(局所的構造)

(ii) 地平線の候補を探す

(iii) “地平線近傍”の幾何を解析

(iv) 測地線を数値的に解いて本当にEHか否か確認

⇒ we can sketch the Penrose diagram

Trapped surface

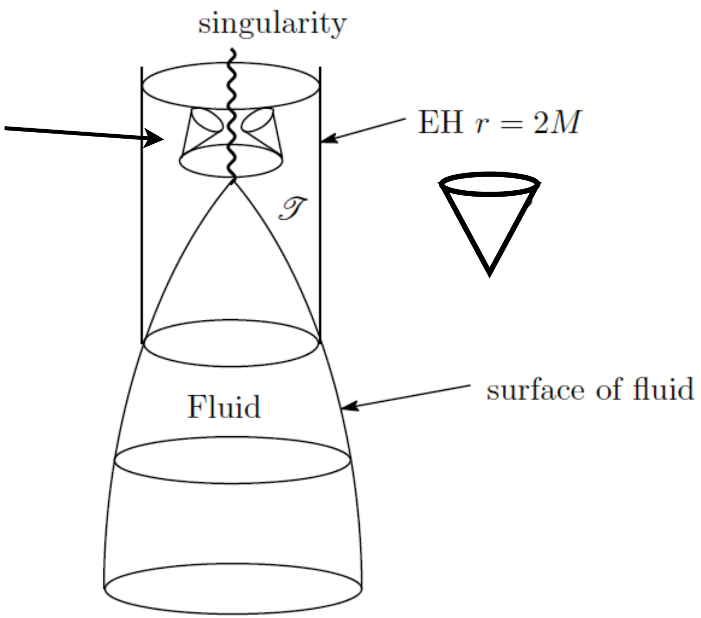
○ 捕捉領域／みかけの地平線 (*à la* Penrose) *Penrose 1967*

- a two-dimensional *compact* surface on which ougoing null rays have negative expansion $\theta_+ < 0$ in asymptotically flat spacetimes

- causally disconnected from $\mathscr{I}^+$ if asymptotically flat *Hawking 1971*

→ *trapped regions must be contained within BH region*

trapped region



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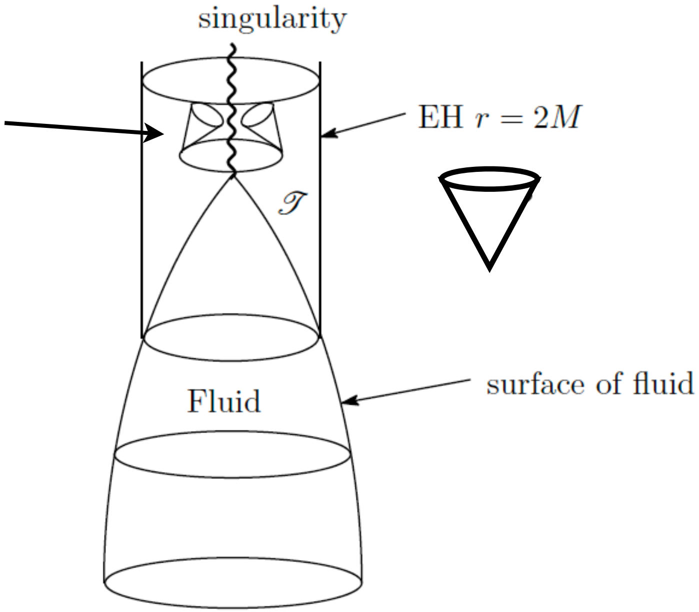


Trapped region characterizes strong gravity

Trapped regions in spherical symmetry

○ 捕捉領域／捕捉地平線 (*à la* Hayward)　　*Hayward 1993*

- a 2D *compact* surface on which $\theta_+\theta_- > 0$

→ Schwarzchild BH ($\theta_+ <0$, $\theta_-<0$), WHの内部 ($\theta_+ >0$, $\theta_->0$) はtrapped

- a 3D surface foliated by marginal surface $\theta_+\theta_- =0$ is called a *trapping horizon*

○ 球対称時空における捕捉領域

線素は次のように書ける:　$ds^2 = g_{AB}(y)\mathrm{d}y^A\mathrm{d}y^B + R^2(y)\mathrm{d}\Omega_2^2 .$

$g^{AB} = -2l_+^{(A}l_-^{B)}$,　　$R(y)$: 面積半径($Area = 4\pi R^2$)

$$\begin{aligned}\theta_\pm &:= (g^{ab} + 2l_+^{(a}l_-^{b)})\nabla_a l_{\pm b} \\ &= 2l_\pm^a\nabla_a(\ln R)\end{aligned}$$

:null normalに沿った面積変化率

→ $\theta_+\theta_- = -2R^{-2}(\nabla R)^2$

trapped: $\theta_+\theta_- >0$　⇔　$(\nabla R)^2<0$

⇔　R=const. is spacelike

Future trapping horizons

(i) black hole type (future $\theta_+ = 0$)

ex. 光的ダストの重力崩壊

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right)dv^2 + 2dvdr + r^2 d\Omega_2^2 \,.$$

$$M(v) = \begin{cases} 0 \ (v<0)\text{: flat space} \\ m(v) \ (0<v<v_0)\text{: Vaidya} \\ \qquad T_{ab} = \dfrac{M'(v)}{4\pi r^2} l_{(-)a} l_{(-)b} \,, \\ M \equiv m(v_0) \ (v_0<v)\text{: Schwarzschild} \end{cases}$$

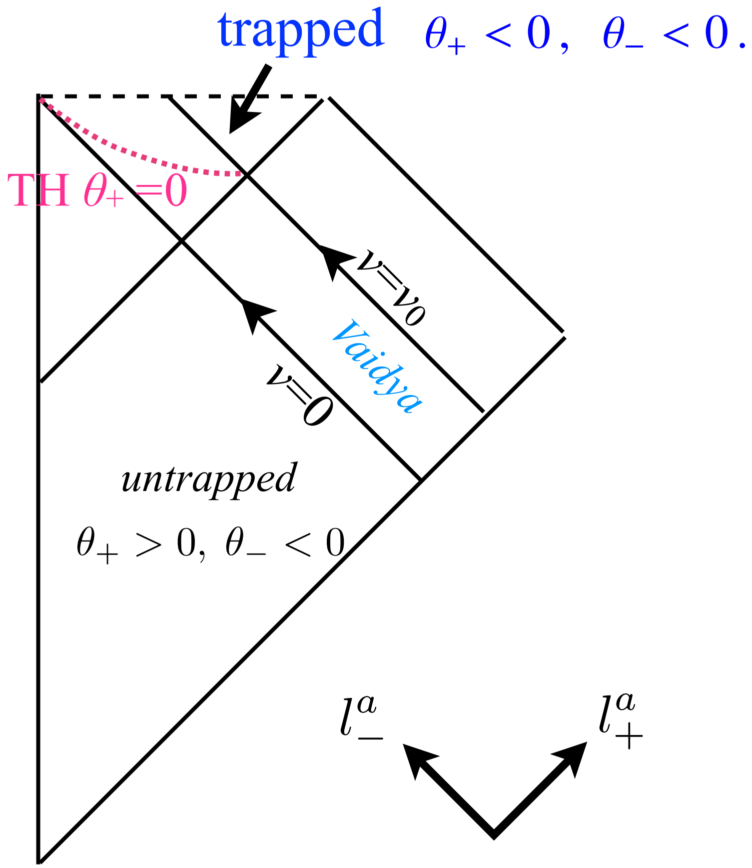
$$l_+^a = \left(\frac{\partial}{\partial v}\right)^a + \frac{1-2M(v)/r}{2}\left(\frac{\partial}{\partial r}\right)^a \,, \quad l_-^a = -\left(\frac{\partial}{\partial r}\right)^a \,.$$

- Trapping horizon occurs at $r=2M(v)$

$$\theta_+ = \frac{1}{r}\left(1 - \frac{2M(v)}{r}\right) \,, \quad \theta_- = -\frac{2}{r} \,.$$

- Trapping horizon is spacelike

$$ds_{\rm TH} = ds^2|_{r=2M(v)} = 2M'(v)dv^2 > 0 \,.$$



Past trapping horizons

(ii) cosmological/white hole type (past θ_- =0)

ex. P=ρのFriedmann宇宙

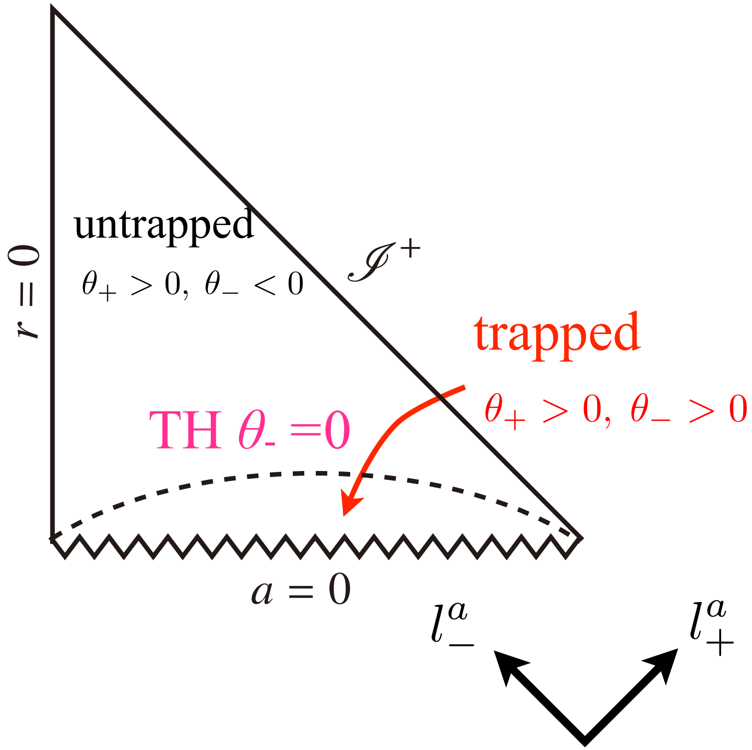
$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega_2^2)\,.$$

$$P = \rho = \frac{3}{8\pi}H^2 \qquad a = (t/t_0)^{1/3}$$

H=a'/a: Hubble パラメータ

R=ar: 面積半径

$$l_\pm^a = \left(\frac{\partial}{\partial t}\right)^a \pm \frac{1}{a}\left(\frac{\partial}{\partial r}\right)^a$$



- Trapping horizon occurs at R=1/H (Hubble horizon)

$$\theta_\pm = 2\,(H \pm 1/R)\,.$$

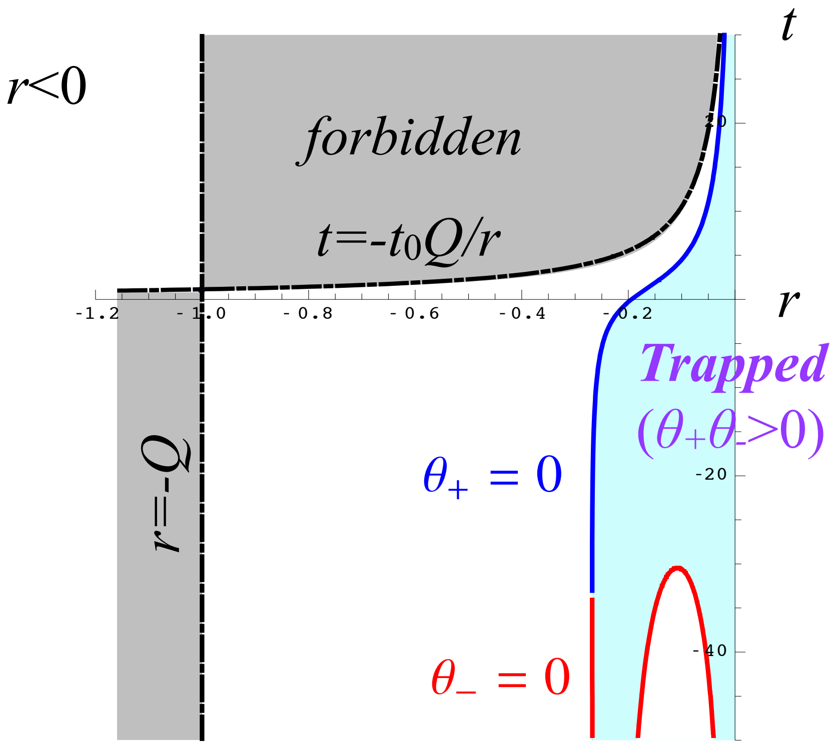
- Trapping horizon is spacelike

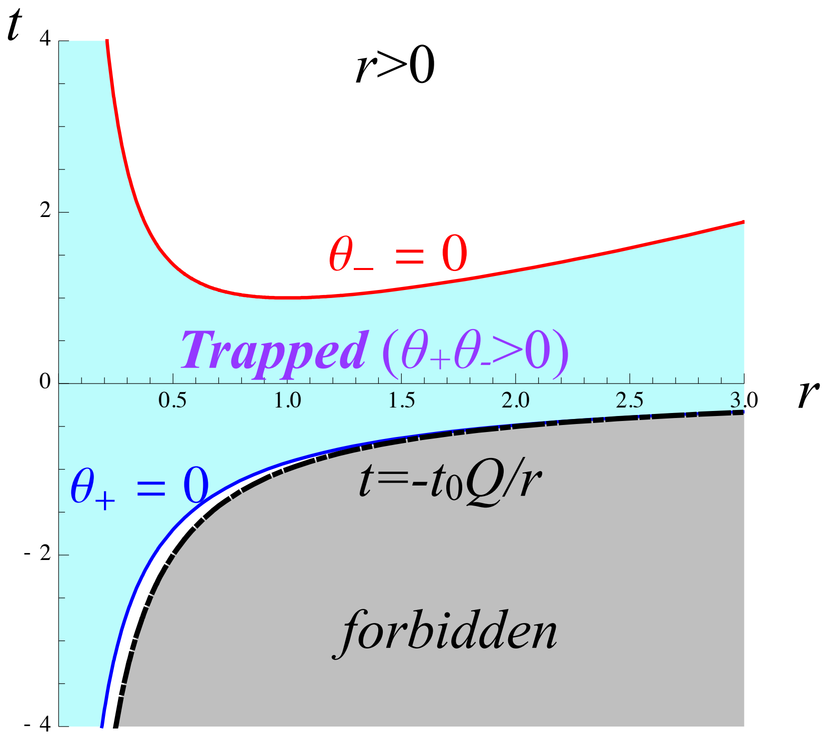
$$ds_{\mathrm{TH}}^2 = ds^2|_{\dot{a}r=1} = 3dt^2 > 0\,.$$

Trapping horizons

Future and past trapping horizons occur at

$$\theta_{\mp} = 0 \quad \Leftrightarrow \quad t_{\mathrm{TH}}^{(\pm)} = \frac{r^2}{2t_0(H_S+3)^2}\left[H_S^5 - \frac{6t_0^2 Q}{r^3}(H_S+3) \pm H_S^3\sqrt{H_S^4 + \frac{4t_0^2 Q}{r^3}(H_S+3)}\right]$$





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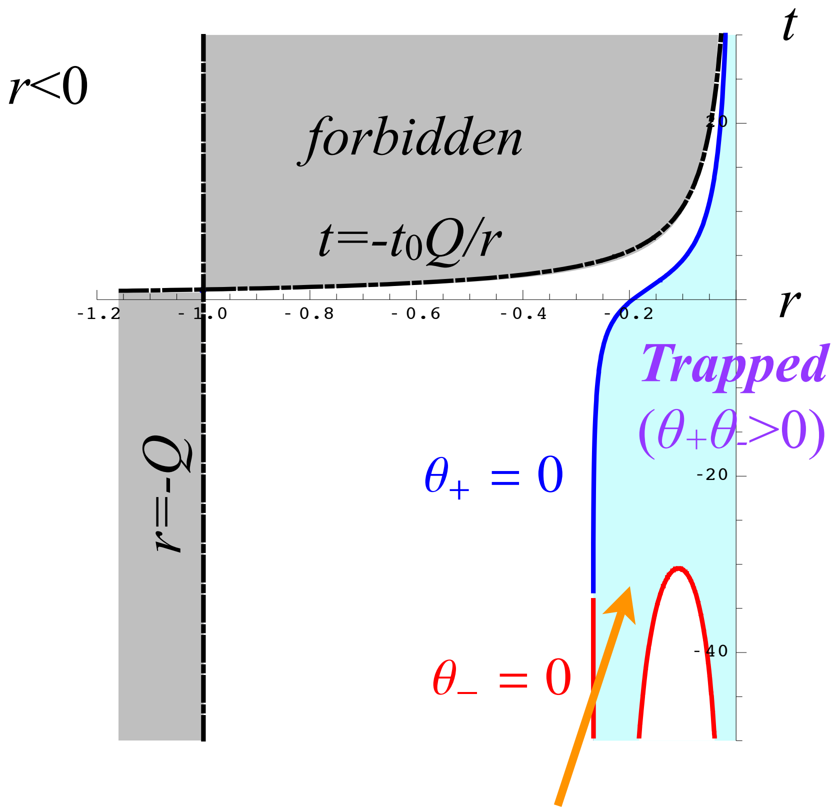
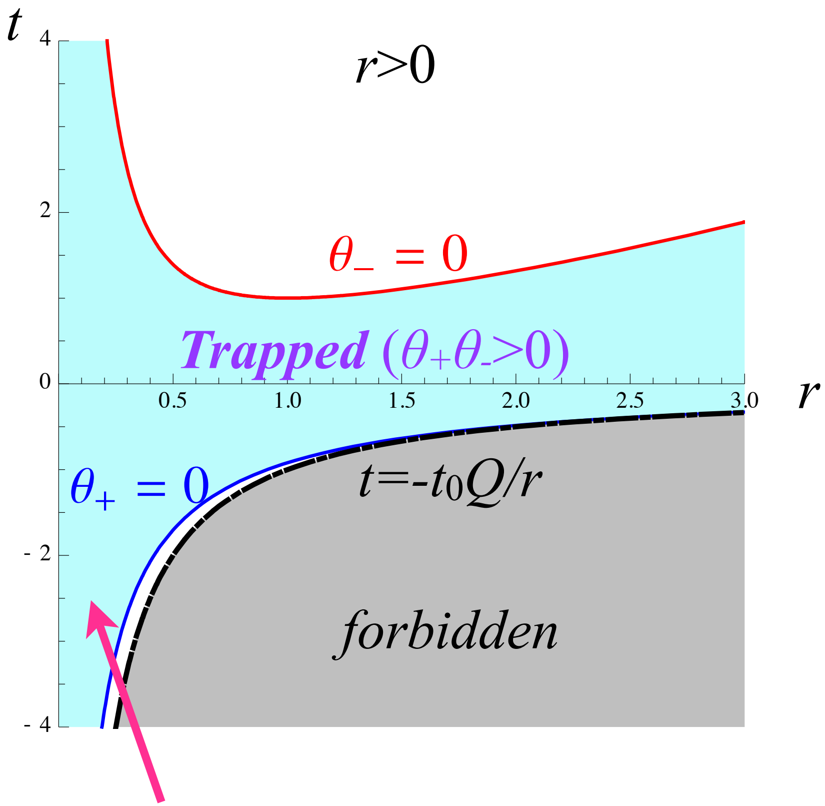




$\theta_+ < 0\,,\quad \theta_- < 0$

attractive due to `BH’

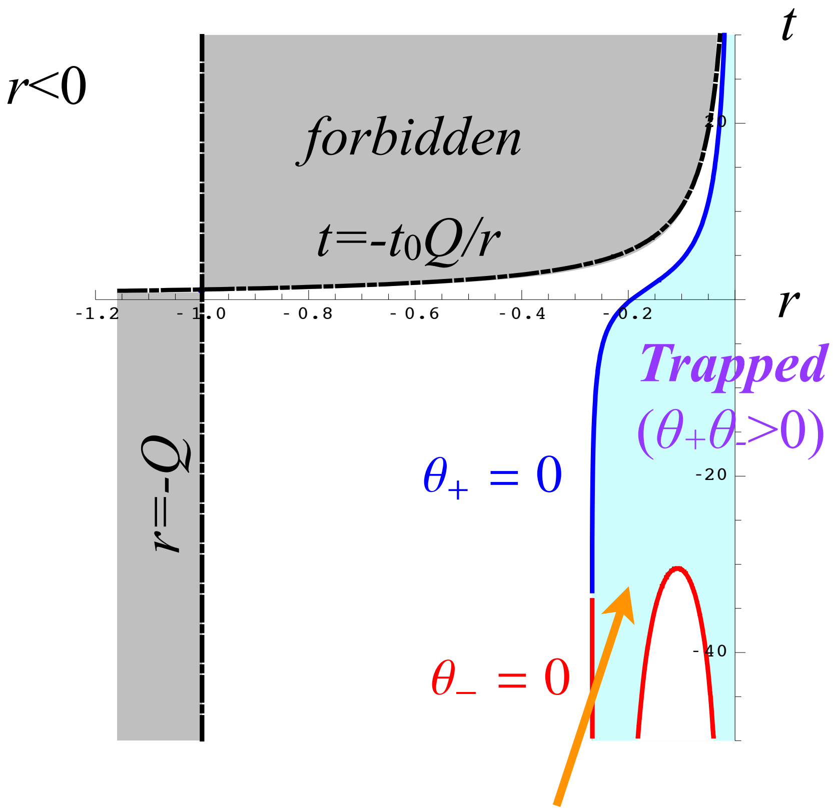
$\theta_+ > 0\,,\quad \theta_- > 0$

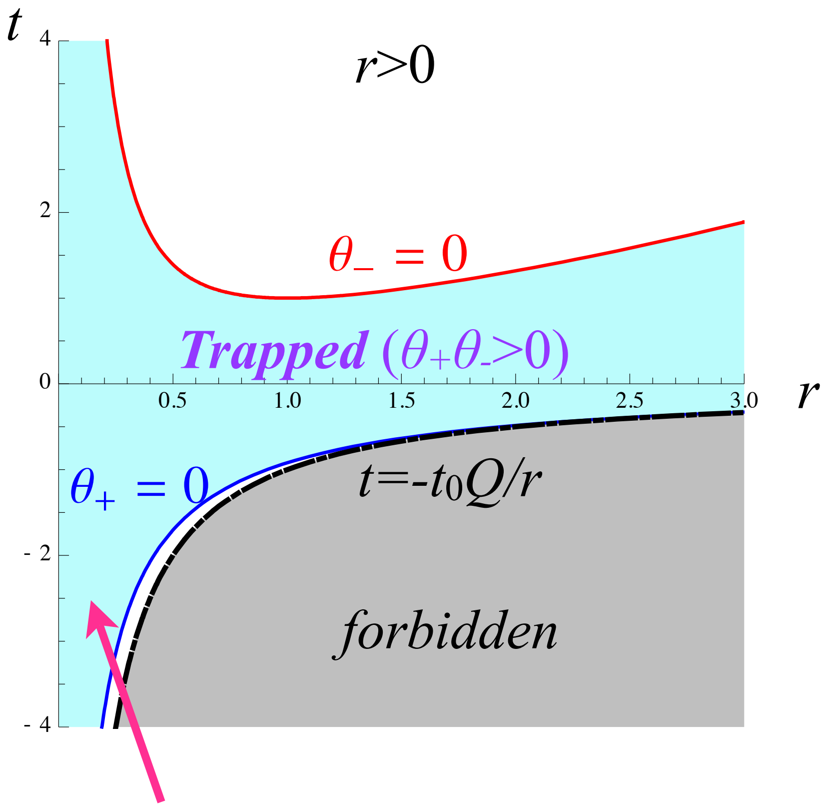
repulsive due to `expanding univ.’

Trapping horizons

Future and past trapping horizons occur at

$$\theta_\mp = 0 \quad \Leftrightarrow \quad t_{\rm TH}^{(\pm)} = \frac{r^2}{2t_0(H_S+3)^2}\left[H_S^5 - \frac{6t_0^2 Q}{r^3}(H_S+3) \pm H_S^3\sqrt{H_S^4 + \frac{4t_0^2 Q}{r^3}(H_S+3)}\right]$$





$\theta_+ < 0$, $\theta_- < 0$

attractive due to `BH’

$\theta_+ > 0$, $\theta_- > 0$

repulsive due to `expanding univ.’

捕捉地平線の$r\to 0$ 極限 が事象の地平線のlikely-candidate

Trapped surface

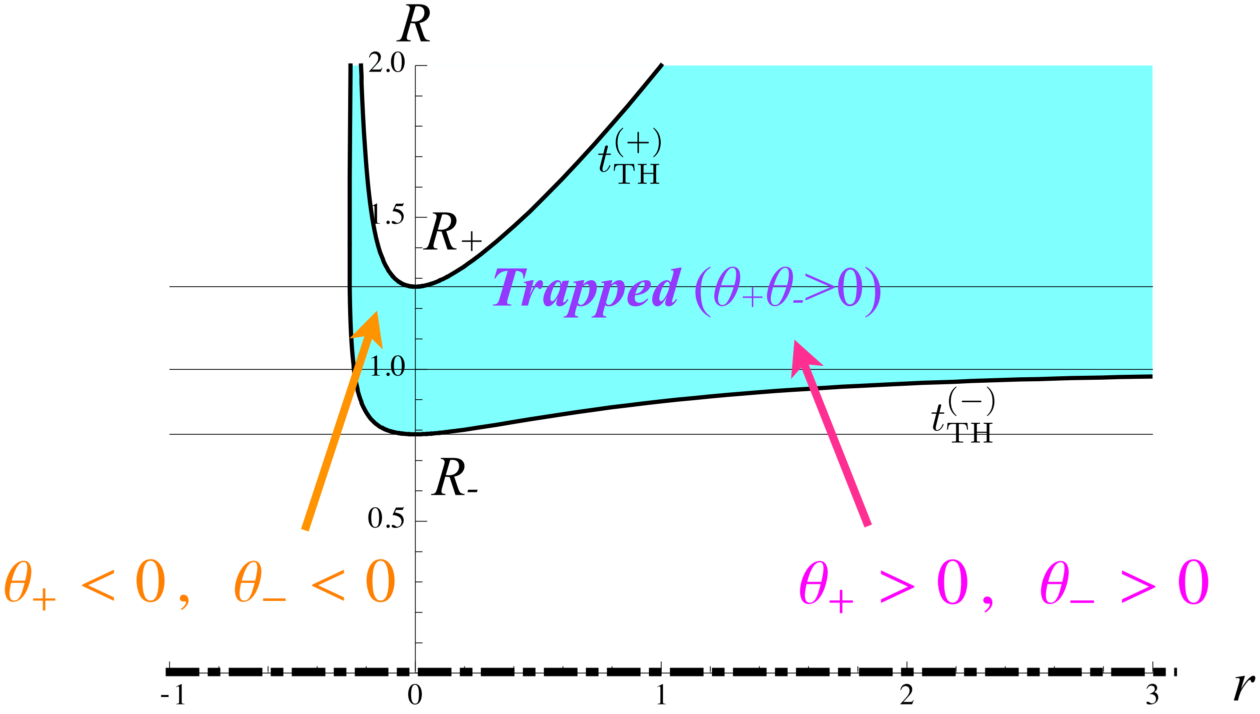
- 捕捉地平線の性質はr=0の面を境に変わる

→ 捕捉地平線の$r \to 0$ 極限 が事象の地平線のlikely-candidate

- 無限大の赤方(青方)変位面に対応 $t_{\rm TH}^{(\pm)} \to \pm \frac{c_\pm}{r} \to \infty$ as $r \to 0$

- 面積半径は一定値に漸近 $R_\pm = Q \left(\frac{\pm Q + \sqrt{Q^2 + 4t_0^2}}{2t_0} \right)^{1/2}$

$$R = \left[\left(\frac{tr}{t_0} + Q \right) (r + Q)^3 \right]^{1/4}$$



Near horizon geometry

- 捕捉地平線の“地平線近傍”　$t_{\rm TH}^{(\pm)} \to c_\pm / r\,,\quad r \to 0\,,$

スケーリング極限によりwell-defined な*near-horizon limit*

$$t \;\to\; t/\epsilon,\;\; r \;\to\; \epsilon r\,,\;\; \epsilon \;\to 0\,.$$

$$ds_{\rm NH}^2 = -(r/Q)^2 \left(1 + \frac{tr}{t_0 Q}\right)^{-1/2} dt^2 + (r/Q)^{-2}\left(1 + \frac{tr}{t_0 Q}\right)^{1/2} (dr^2 + r^2 d\Omega_2^2)$$

$$\xi^\mu = t\left(\frac{\partial}{\partial t}\right)^\mu - r\left(\frac{\partial}{\partial r}\right)^\mu\,,$$

$\mathscr{L}_\xi g_{\mu\nu}^{\rm NH} = D_\mu \xi_\nu + D_\nu \xi_\mu = 0\,,$　:Killingベクトル

$\xi_{[\mu} D_\nu \xi_{\rho]} = 0\,,$　:超曲面直交

$(t, r) \to (T, R)$

$$ds_{\rm NH}^2 = -\frac{f(R)}{R^2 Q^6} dT^2 + \frac{16 R^8}{f(R)} dR^2 + R^2 d\Omega_2^2\,.$$

$$f(R) = (R^4 - R_+^4)(R^4 - R_-^4),\quad \xi^\mu = \left(\frac{\partial}{\partial T}\right)^\mu$$

$R_\pm$をKilling地平線に持つ静的ブラックホール

$$R_\pm = Q\left(\frac{\pm Q + \sqrt{Q^2 + 4t_0^2}}{2t_0}\right)^{1/2}$$

Near horizon geometry

“horizon-candidate”は静的ブラックホールのKilling地平線で記述される

$$ds^2_{\rm NH} = -\bar{\Xi}dt^2 + \bar{\Xi}^{-1}\left(dr^2 + r^2 d\Omega_2^2\right) = -\frac{f(R)}{R^2Q^6}dT^2 + \frac{16R^8}{f(R)}dR^2 + R^2 d\Omega_2^2\,.$$

$$\bar{\Xi} = (r/Q)^{-2}\left(1 + \frac{tr}{t_0Q}\right)^{-1/2}\,,\quad f(R) = (R^4 - R_+^4)(R^4 - R_-^4)$$

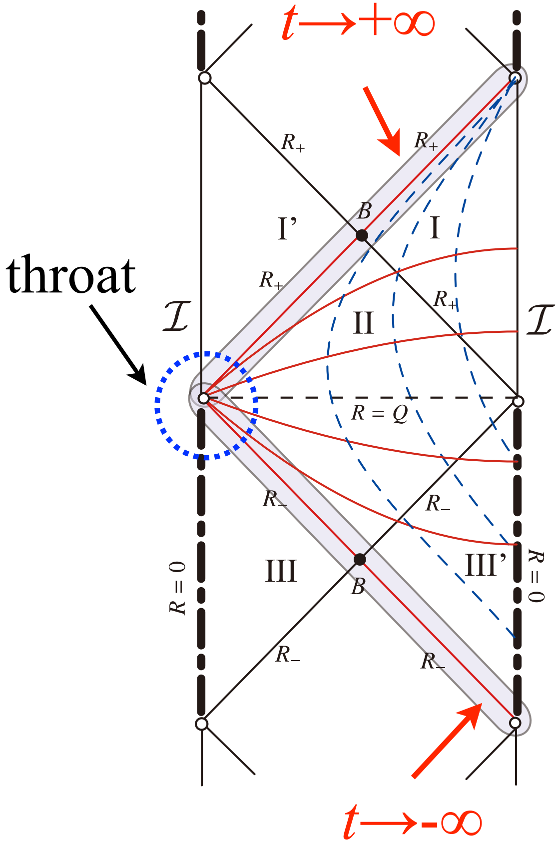
- $R_+ > R_-$ ⇒ 地平線は非縮退

$$Q = \sqrt{R_+R_-}\,,\qquad \frac{t_0}{Q} = \frac{R_+R_-}{R_+^2 - R_-^2}$$

  - ▶ 温度はノンゼロ

$$\kappa_\pm = \frac{\sqrt{Q^2 + 4t_0^2}}{\sqrt{Q^2 + 4t_0^2} \pm Q}$$

  - ▶ 流入するエネルギーなし



- Near-horizon計量の大域構造はRN-AdSと同じ

$t \longrightarrow \pm\infty$で$R_\pm$ ⇒ R_+はBHとWHの`外側’の地平線
R_-はBHとWHの`内側’の地平線

- t:有限，$r \rightarrow 0$とすればスロート(AdS$_2$xS^2)

N.B もとの時空でξ^μ がKillingとなるのは地平線上のみ　$(\mathscr{L}_\xi g)^\mu{}_\nu \overset{H}{=} 0\,,$

Global spacetime structure

- Outside the horizon $r > 0$

Global spacetime structure

Outside the horizon $r>0$

- $t=0$ is a regular slice

Global spacetime structure

Outside the horizon $r>0$

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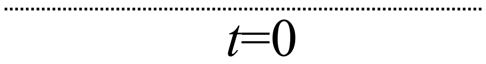
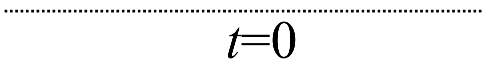


Global spacetime structure

Outside the horizon *r*>0

- *t*=0 is a regular slice

$$R = \left[\left(\frac{tr}{t_0} + Q\right)(r+Q)^3\right]^{1/4} \;\Rightarrow\; R \longrightarrow \infty \text{ as } r \longrightarrow \infty \text{ on } t \geq 0$$



Global spacetime structure

Outside the horizon $r>0$

- $t=0$ is a regular slice

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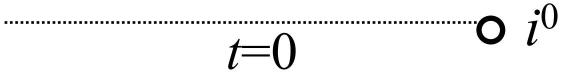

Global spacetime structure

Outside the horizon $r>0$

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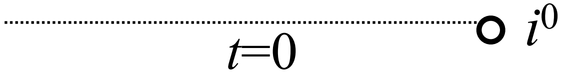


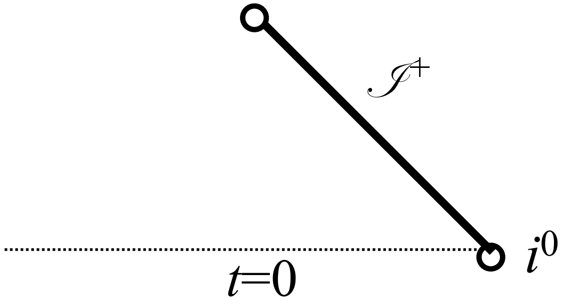
- For $t/t_0>0$, metric asymptotes to $P=\rho$ FRW as $r\to\infty$

Global spacetime structure

Outside the horizon $r>0$

- $t=0$ is a regular slice

$$R = \left[\left(\tfrac{tr}{t_0} + Q\right)(r+Q)^3\right]^{1/4} \quad \Rightarrow \quad R\to\infty \text{ as } r\to\infty \text{ on } t\geq 0$$



- For $t/t_0>0$, metric asymptotes to $P=\rho$ FRW as $r\to\infty$

Global spacetime structure

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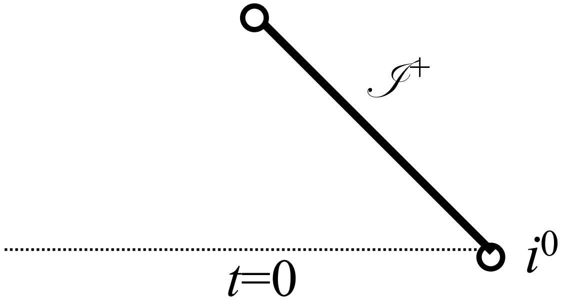

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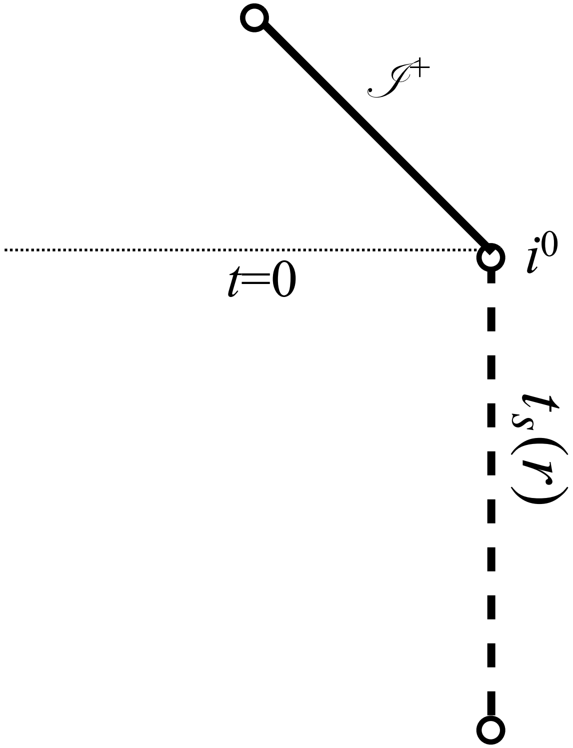

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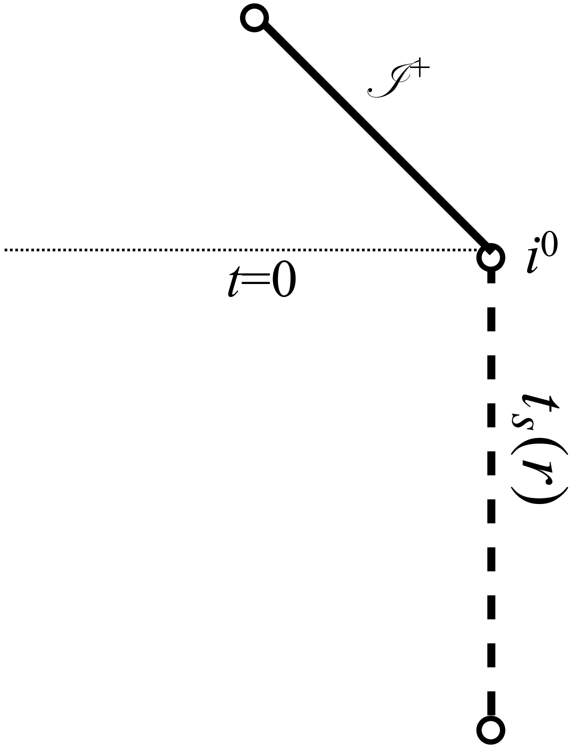

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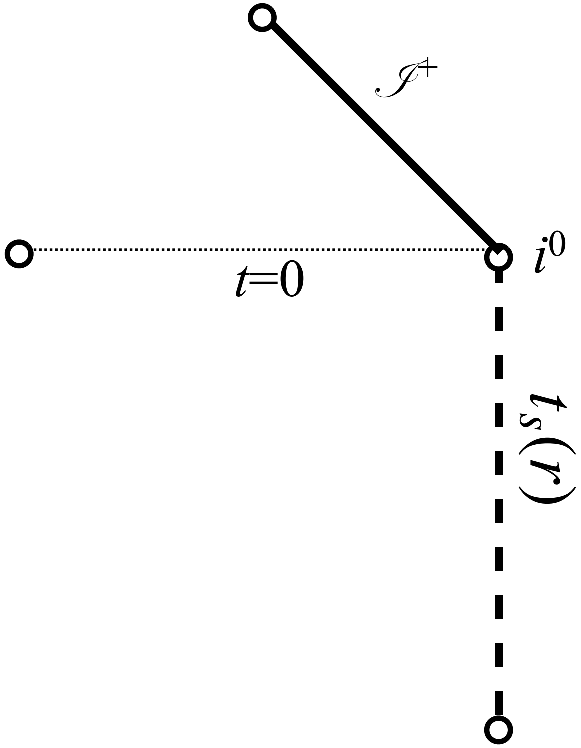

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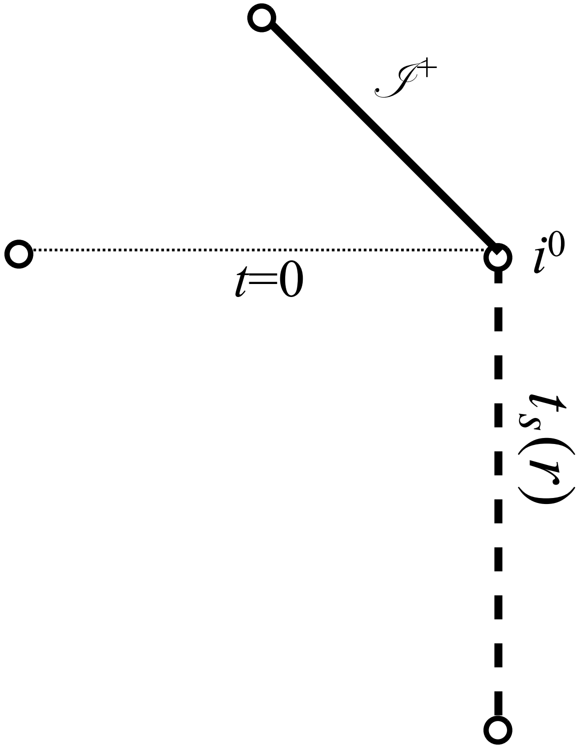

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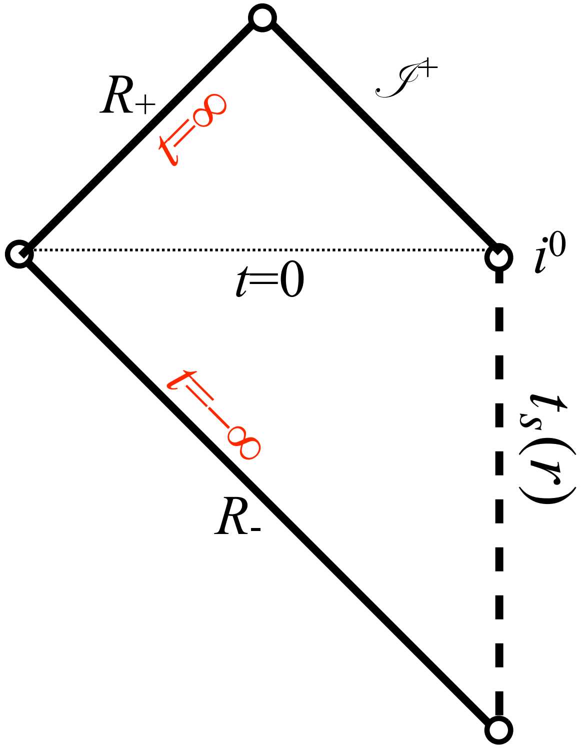

Global spacetime structure

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Global spacetime structure

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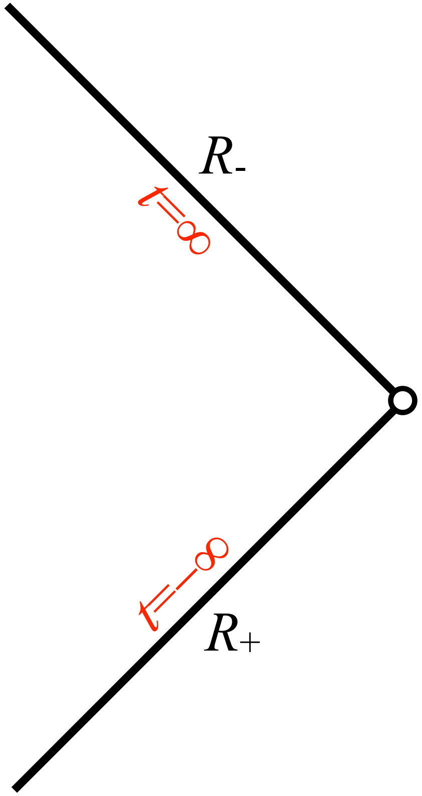

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- there exist Killing horizons $R_\pm$ as $r \to 0-$, $t \to \mp\infty$
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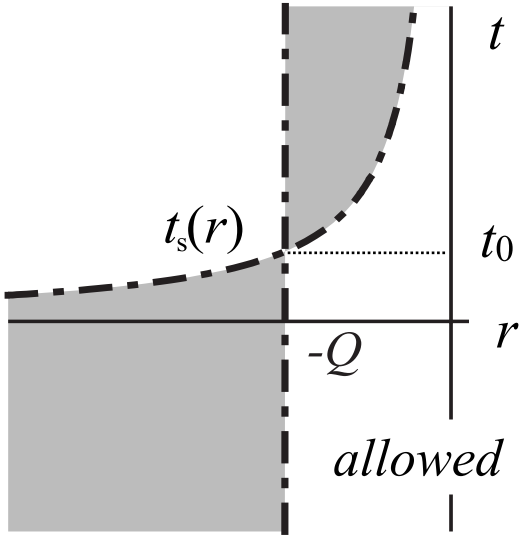

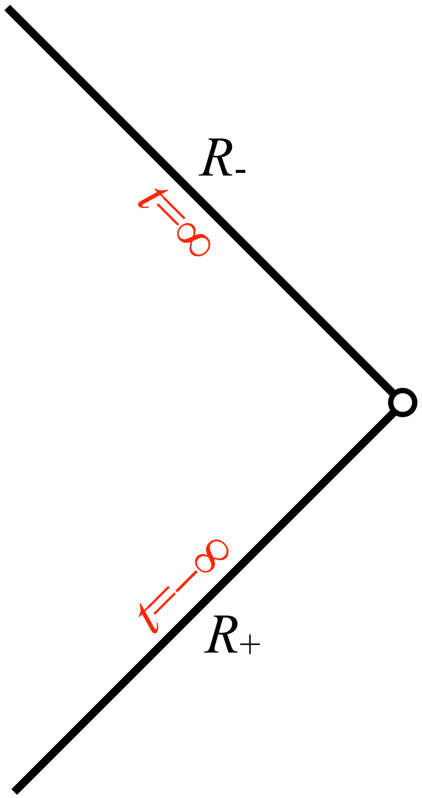

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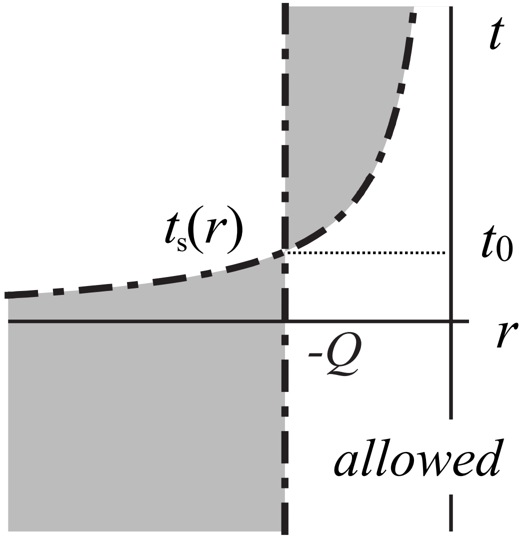

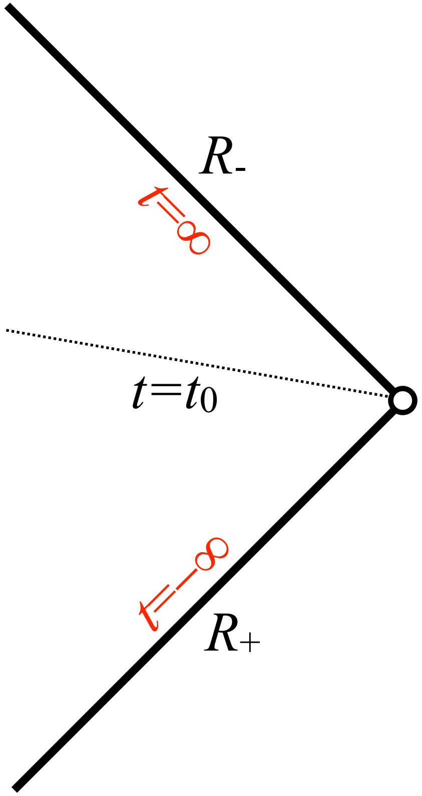

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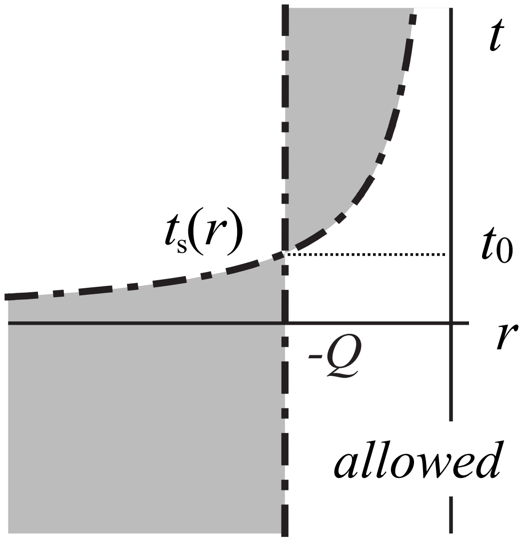

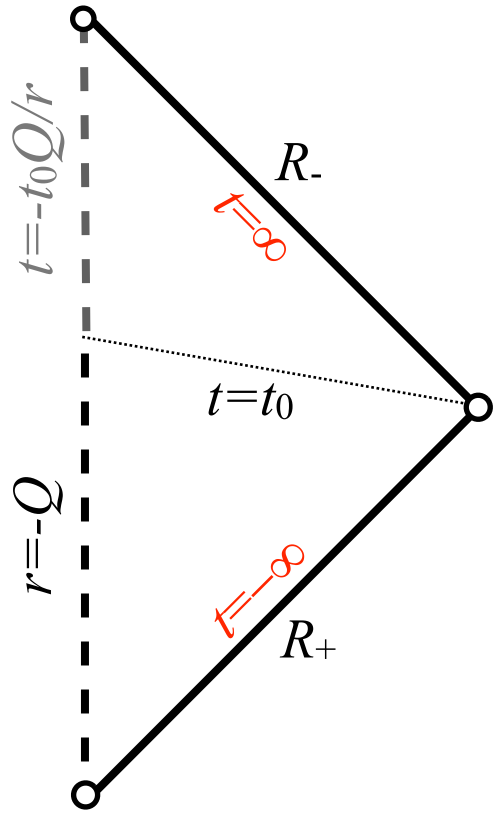

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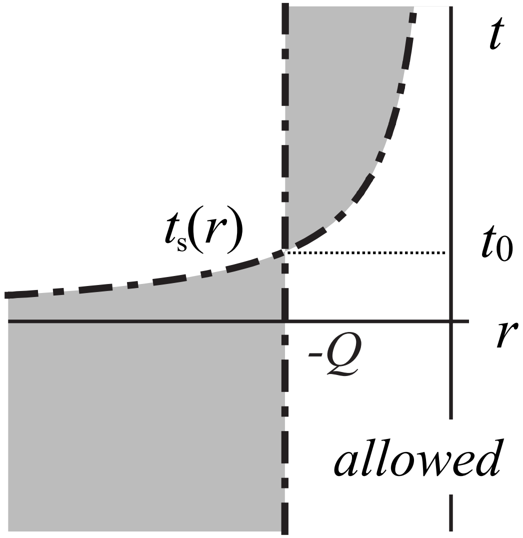
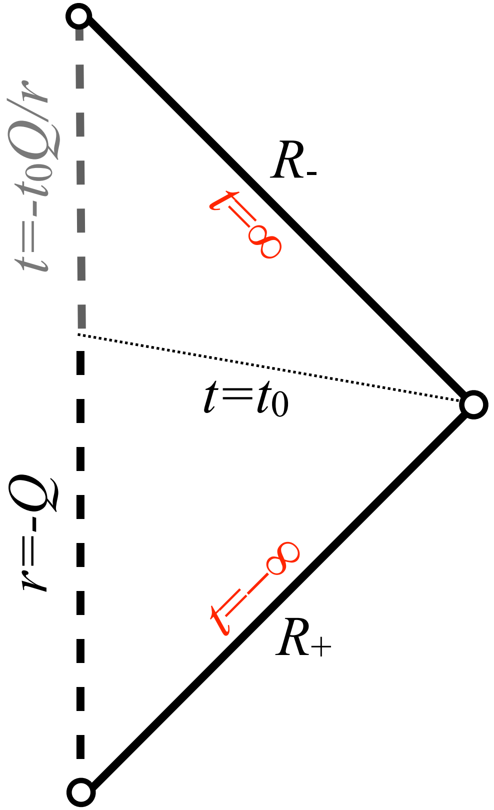




- $t \to -\infty$ *with* $r(<0)$: *fixed* is a past infinity

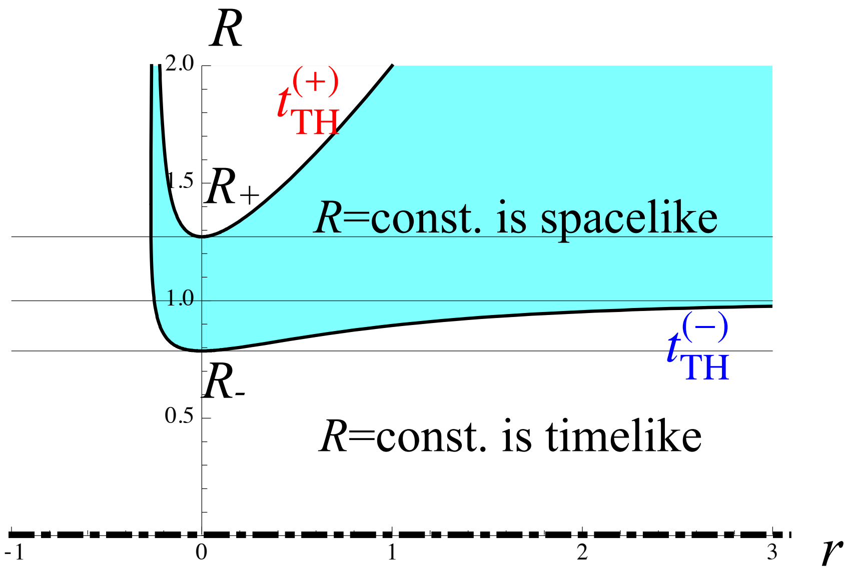
$$R = \left[\left(\frac{tr}{t_0} + Q\right)(r+Q)^3\right]^{1/4} \longrightarrow \infty$$

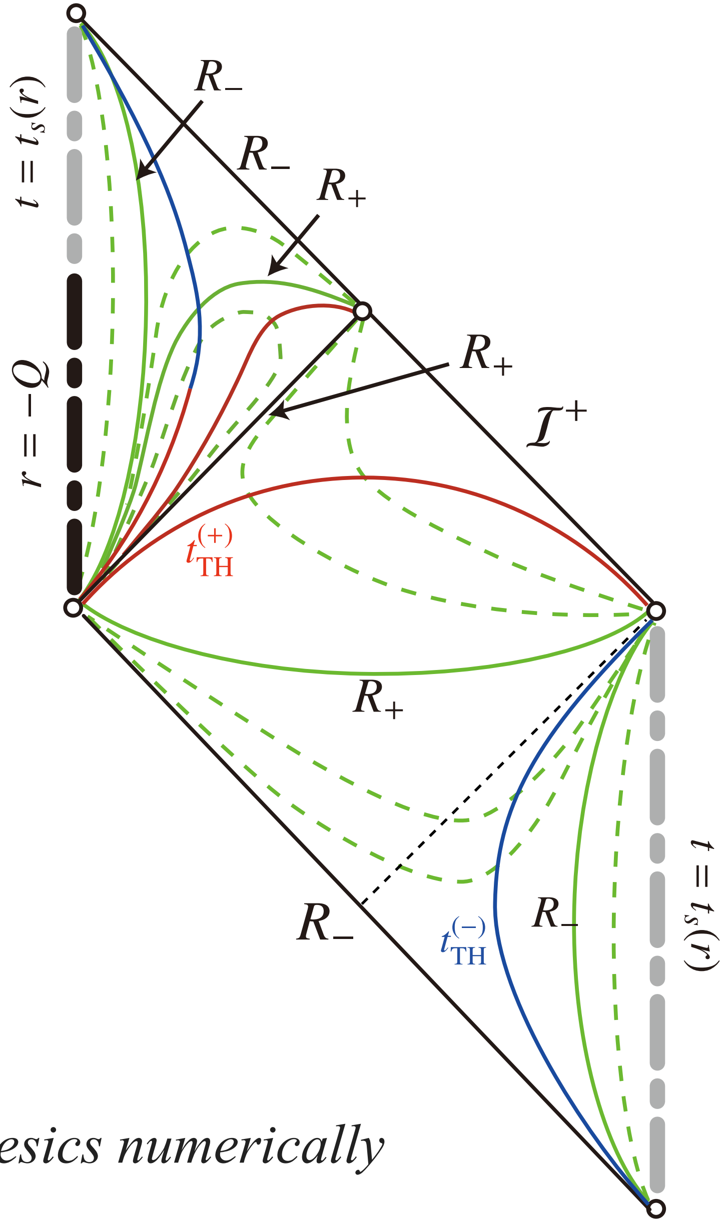
Global spacetime structure

$$\theta_+\theta_- = -2R^{-2}(\nabla R)^2$$

trapped: $\theta_+\theta_- > 0 \quad \Leftrightarrow \quad (\nabla R)^2 < 0$

$\Leftrightarrow \quad R$=const. is spacelike

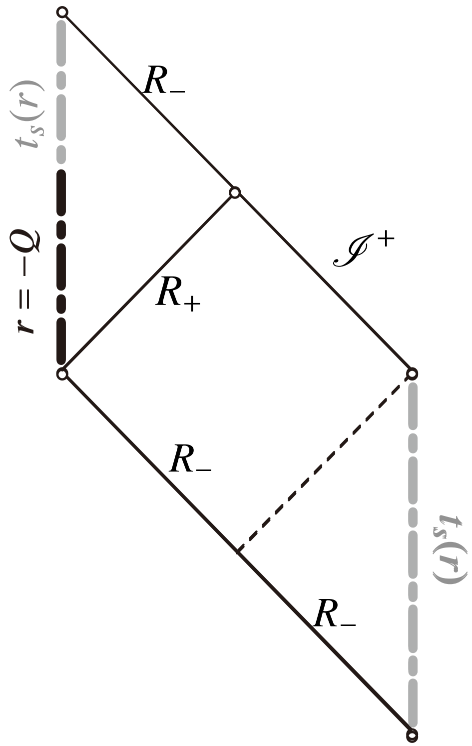




✦ *Consistency has been checked by solving geodesics numerically*

The solution indeed turns out to describe a BH in FRW cosmology

Extensions



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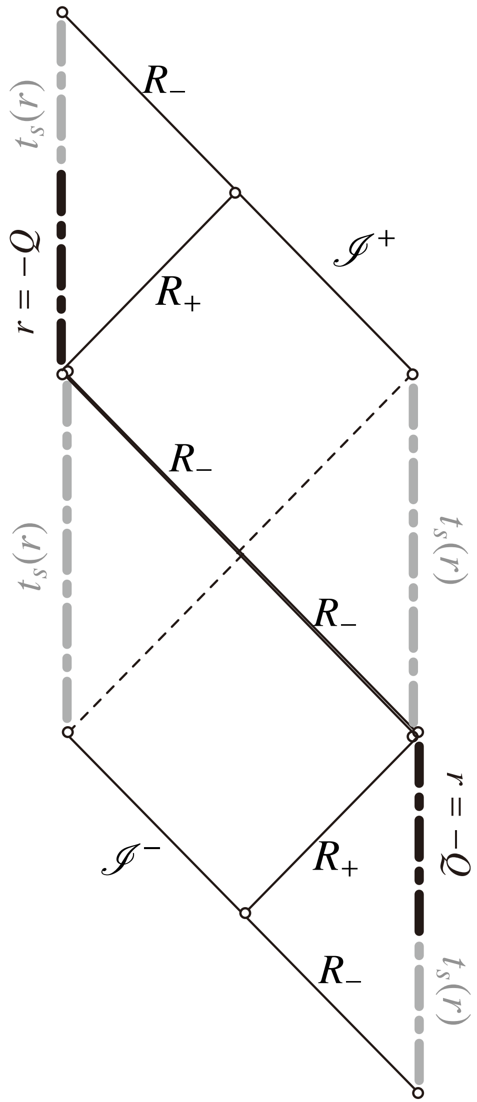

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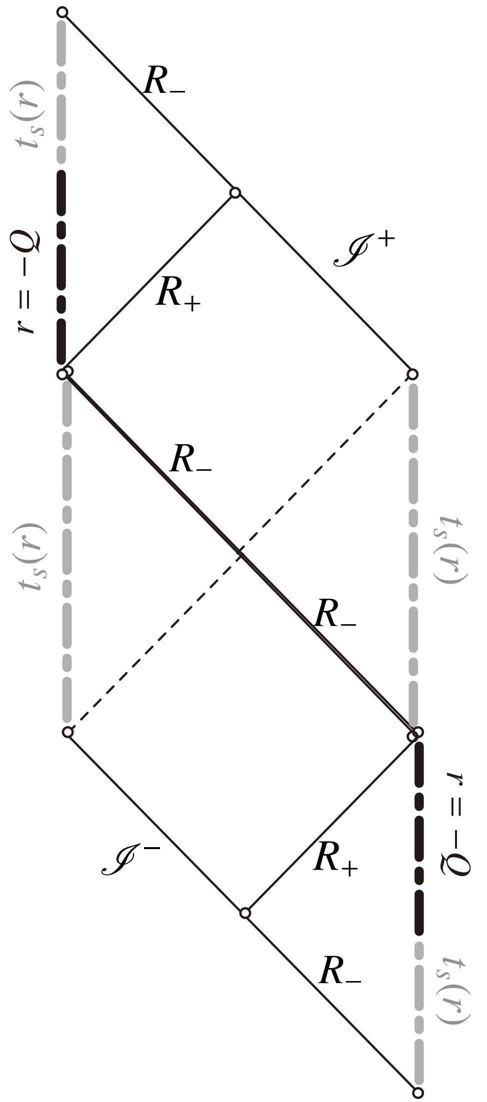
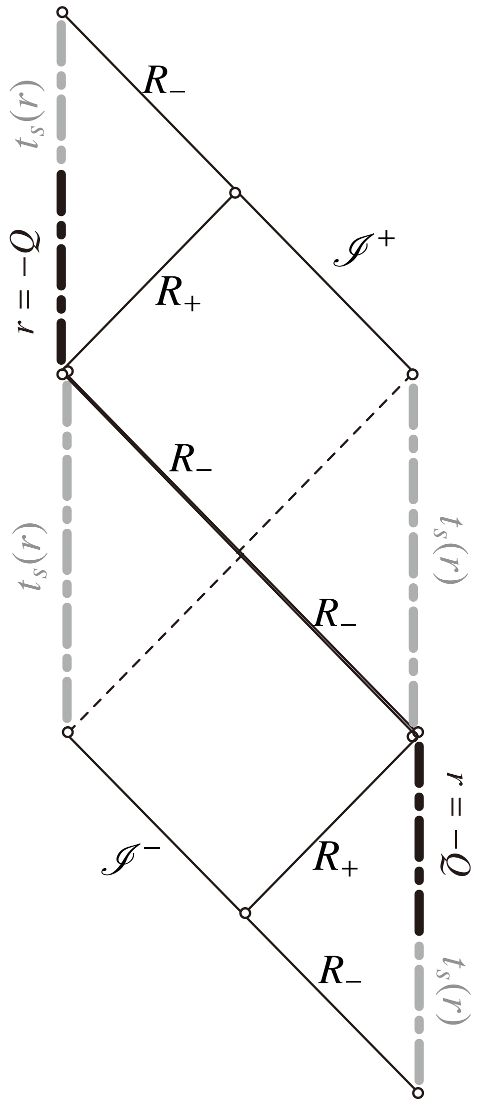


- Patched region corresponds to $t_0<0$

Extensions

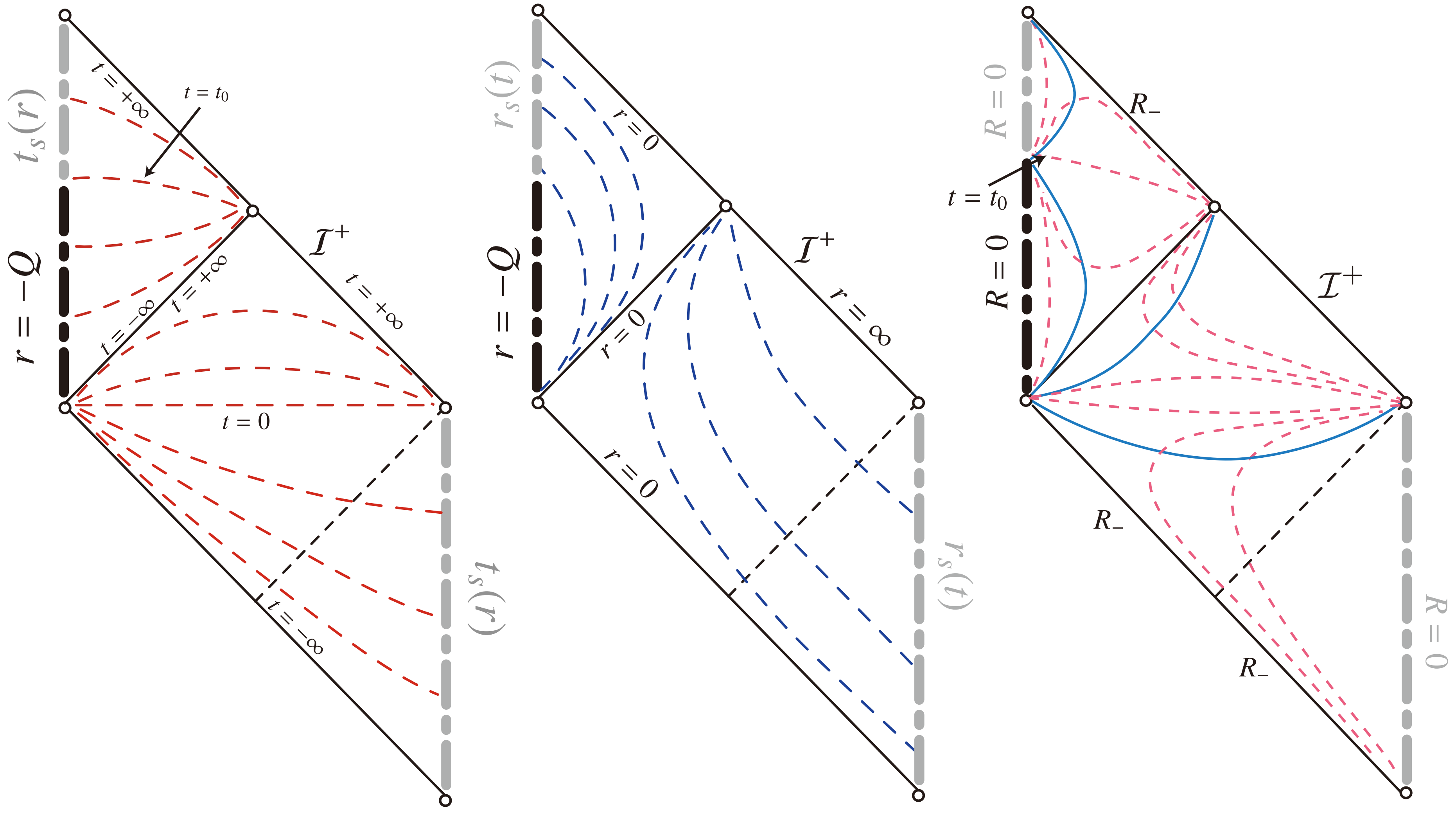


- Patched region corresponds to $t_0<0$

but...

extension is not unique due to nonanaliticity

Contours



t=const. r=const. Φ=const.

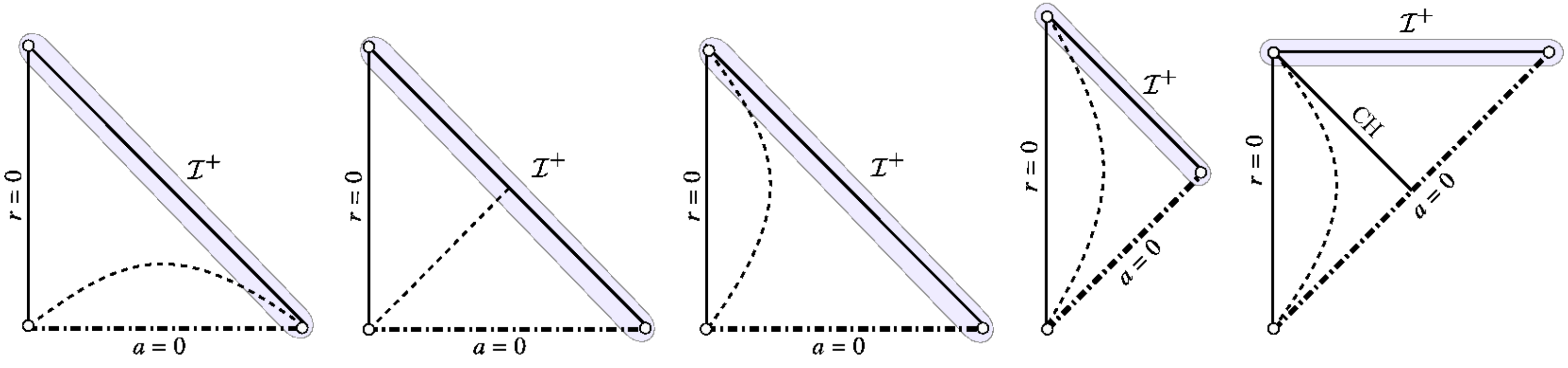
$(\nabla\Phi)^2<0$ at infinity

Extension to arbitrary power-law FRW

$$L = -\frac{1}{2}(\nabla\Phi)^2 , \qquad \rightarrow \qquad a \propto \bar{t}^{1/3}$$

$$L = -\frac{1}{2}(\nabla\Phi)^2 - V , \quad V = V_0 \exp\left(-\sqrt{\frac{2}{p}}\kappa\Phi\right) \qquad \rightarrow \qquad a \propto \bar{t}^{\,p}$$

Background: $ds_4^2 = -d\bar{t}^2 + a^2(dr^2 + r^2 d\Omega^2)$ $\quad a \propto \bar{t}^{\,p}$



(1) (2) (3) (4) (5)

$$\underbrace{p<1/2 \qquad p=1/2 \qquad 1/2<p<1}_{\text{decelerating}} \qquad p=1 \qquad \underset{\text{accelerating}}{1<p}$$

Black hole in power-law FRW universe

- Einstein-Maxwell-dilaton theory with a *Liouville potential*

Gibbons-Maeda 2009,
Maeda-M.N 2010

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}\left(\nabla_\mu \Phi\right)(\nabla^\mu \Phi) - V(\Phi) - \frac{1}{16\pi}\sum_{A=S,T} n_A e^{\lambda_A \kappa \Phi} F^{(A)}_{\mu\nu} F^{(A)\mu\nu}\right],$$

$$V(\Phi) = V_0 \exp(-\alpha\kappa\Phi) \qquad n_T + n_S = 4\,,\ \ \lambda_T = \alpha = \sqrt{\frac{2n_S}{n_T}}\,,\ \ \lambda_S = -\sqrt{\frac{2n_T}{n_S}}\,,$$

$$\mathrm{d}s^2 = -\Xi \mathrm{d}t^2 + \Xi^{-1}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2), \qquad \Xi = (H_T^{n_T} H_S^{n_S})^{-1/2}$$

$$H_T = \frac{t}{t_0} + \frac{Q_T}{r}, \quad H_S = 1 + \frac{Q_S}{r} \qquad t_0^2 = \frac{n_T(n_T - 1)}{4\kappa^2 V_0},$$

$(t_0, Q, p = n_T/n_S)$の3パラメータ族

n_T=1: Maeda-Ohta-Uzawa solution
n_T=4: M=Q RN-de Sitter solution

- 遠方で *power-law FRW universe* に漸近

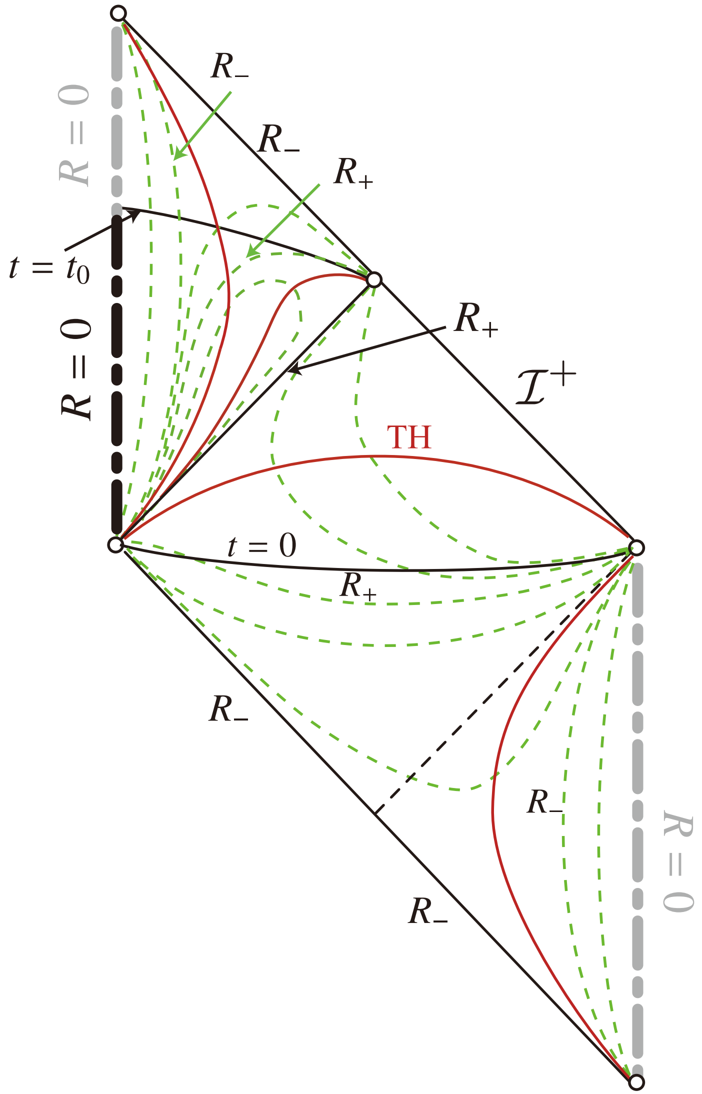
$$\mathrm{d}s_4^2 = -\mathrm{d}\bar{t}^2 + a^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2) \qquad a \propto \bar{t}^p, \ \ p = n_T/n_S$$

- 弱エネルギー条件 を満足　　$T_{\mu\nu}u^\mu u^\nu \geq 0$

- 事象の地平線は Killing地平線 と一致

Global structures

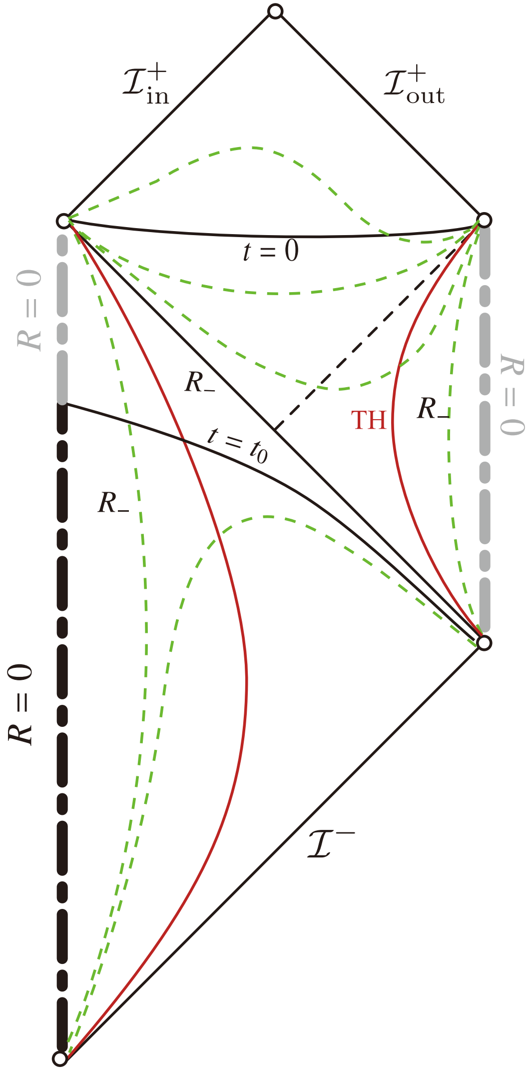
(I) decelerating universe: $p<1$



admits two horizons

(II) Milne universe: $p=1$

$a \propto \bar{t}^p$



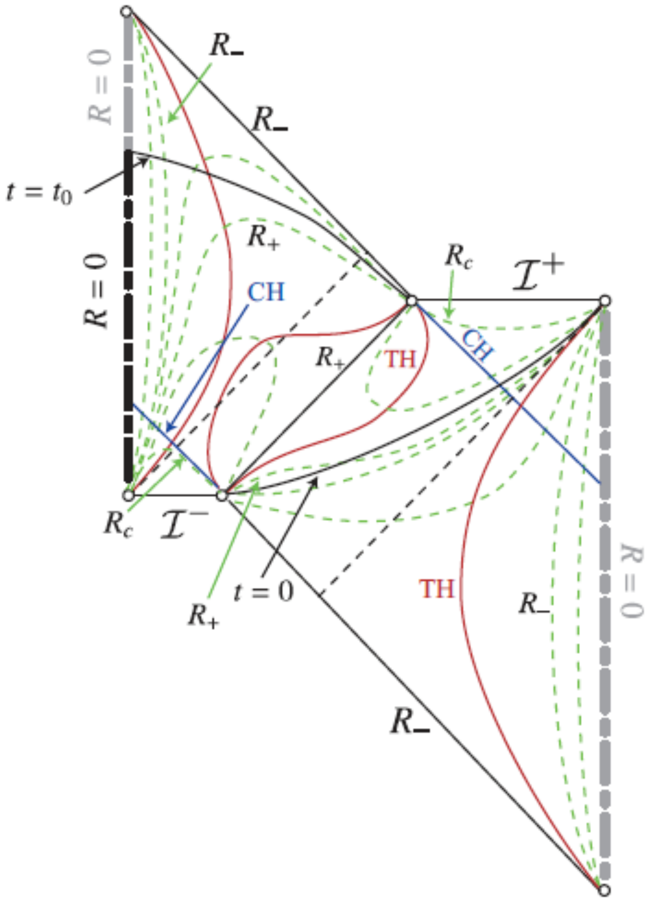
no event horizon

Global structures

(III) accelerating universe: $p>1$

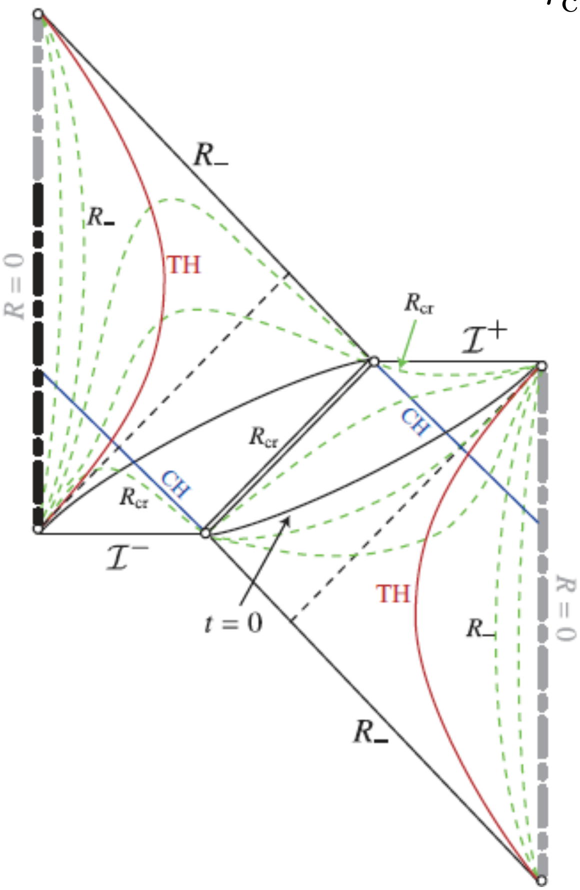
$$a \propto \bar{t}^p \qquad \tau := t_0/Q$$

$$\tau_{\rm cr} = \frac{1}{2} n_T^{n_T/2}(n_T - 2)^{1-n_T/2}$$



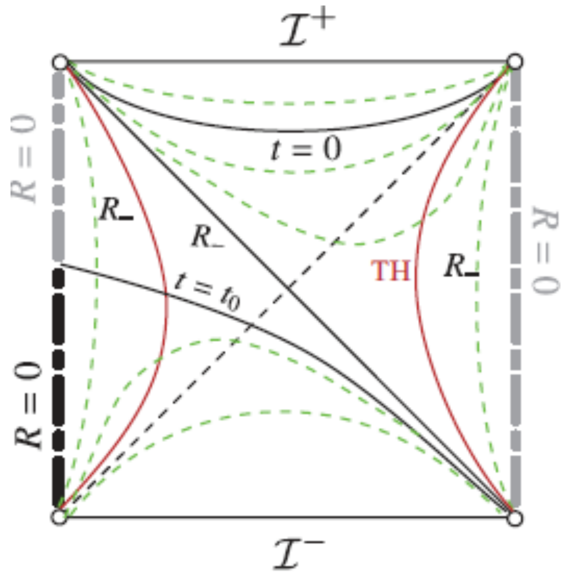
$\tau > \tau_{\rm cr}$

admits three horizons



$\tau = \tau_{\rm cr}$

admits two horizons (degenerate)



$\tau < \tau_{\rm cr}$

no event horizon

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- asymptotes to *FRW universe* $a \propto \bar{t}^p, \ p = n_T/n_S$

- satisfies suitable energy conditions

- additional symmetry appears at the event horizon (=Killing horizon)

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The solution describes an *equilibrium* BH in *dynamical* background

Further generalizations

- Higher-dimensional and/or rotating generalizations

$$ds^2 = -\Xi^2 \left(dt + \frac{j}{2r^2}\sigma_3^R \right)^2 + \Xi^{-1} \left(dr^2 + r^2 d\Omega_3^2 \right) , \qquad \Xi = \left[\left(\frac{t}{t_0} + \frac{Q_T}{r^2} \right) \left(1 + \frac{Q_S}{r^2} \right) \left(1 + \frac{Q_{S'}}{r^2} \right) \right]^{-1/3}$$

  - describes a BMPV black hole in FRW *Breckenrige et al 1996*
  - possesses CTCs around singularities ($g_{\psi\psi}$<0)

- Black hole thermodynamics

  - Can we define meaningful mass function in FRW universe?

$$\Psi_2 = -C_{abcd} l^a m^b n^c \bar{m}^d = -\frac{M(t)}{R^3} + O(1/r)$$

$$m_{\rm MS} - \frac{4}{3}\pi R^3 \rho^{(\Phi)} = M(t) + O(1/r)$$

$$\boxed{M(t) = \frac{Q}{4}\left(n_S a + \frac{n_T}{a^{4/n_T}} \right)} \quad ??$$

- Multiple generalizations *c.f. Kastor-Traschen 1993*

  - Multi-center metric is expected to describe *BH collisions* in FRW universe

$$H_T = -\frac{t}{t_0} + \sum_i \frac{Q_i}{|\mathbf{r} - \mathbf{r}_i|} , \quad H_S = 1 + \sum_i \frac{Q_i}{|\mathbf{r} - \mathbf{r}_i|}$$

Why superposition is possible?

Analogue of supersymmetric solutions

The solution inherits properties of *supersymmetric black holes*

- BPS solutions satisfy the `no force' condition *c.f. Majumdar-Papapetrou sol.*

  gravitational attractive force ⟷ electromagnetic repulsive force

- However, supergravity admits only *AdS vacua*

  e.g. Minimal gauged SUGRA coupled to U(1)^N vector fields with scalars

$$S = \frac{1}{2\kappa_5^2}\int\left[\left({}^5R+2\mathfrak{g}^2U\right)\star_5 1-\mathcal{G}_{AB}\mathrm{d}\phi^A\wedge\star_5\mathrm{d}\phi^B-G_{IJ}F^{(I)}\wedge\star_5F^{(J)}-\frac{1}{6}C_{IJK}A^{(I)}\wedge F^{(J)}\wedge F^{(K)}\right],$$

$U=C^{IJK}V_IV_JX_K>0$ $\quad$ $I, J,...=1,...,N$; $A,B,..=1,...,N$-1

C_{IJK} :intersection numbers of CY $\quad$ (N: Hodge number $h_{1,1}$ of CY)

$\mathfrak{g}$:(inverse) AdS radius

SUSY transformation

$$\delta\psi_\mu=\left[\mathcal{D}_\mu+\frac{i}{8}X_I\left(\gamma_\mu{}^{\nu\rho}-4\delta_\mu{}^\nu\gamma^\rho\right)F^{(I)}_{\nu\rho}+\frac{1}{2}\mathfrak{g}\gamma_\mu X^IV_I\right]\epsilon\,,$$

$$\delta\lambda_A=\left[\frac{3}{8}\gamma^{\mu\nu}F^{(I)}_{\mu\nu}\partial_AX_I-\frac{i}{2}\mathcal{G}_{AB}\gamma^\mu\partial_\mu\phi^B+\frac{3i}{2}\mathfrak{g}V_I\partial_AX^I\right]\epsilon\,,$$

Embedding into supergravity

Wick rotation ($\mathfrak{g} \to \mathrm{i}\lambda$) gives an *inverted potential* $V = 2\lambda^2 C^{IJK} V_I V_J X_K > 0$

"Fake supergravity"

Our 5D metric is a solution of fake supergravity with $C_{123} = 1$

$$\mathrm{d}s^5 = -(H_1 H_2 H_3)^{-2/3} \mathrm{d}t^2 + (H_1 H_2 H_3)^{1/3} h_{ij} \mathrm{d}x^i \mathrm{d}x^j$$

$$A^{(I)} = H_I^{-1}\,, \quad X^I = H_I^{-1} (H_1 H_2 H_3)^{1/3} \qquad h_{ij} : \text{hyper-Kähler space}$$

e.g. $H_1 = \dfrac{t}{t_0} + h_1(x)\,, \quad H_2 = \dfrac{t}{t_0} + h_2(x)\,, \quad H_3 = h_3(x)$ $\qquad V_1 = V_2 = (6\lambda t_0)^{-1}$, $V_3 = 0$

- *"Killing spinor"* equation is satisfied for

$$\mathrm{i}\gamma^0 \epsilon = \epsilon\,,$$

→ 1/2-"BPS" state

$$\epsilon = (H_1 H_2 H_3)^{-1/6} \epsilon_{\mathrm{HK}}\,,$$

$$\delta\psi_\mu = \left[\mathcal{D}_\mu + \frac{i}{8} X_I \left(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu \gamma^\rho\right) F^{(I)}_{\nu\rho} + \frac{1}{2} \mathrm{i}\lambda \gamma_\mu X^I V_I\right] \epsilon\,,$$

$$\delta\lambda_A = \left[\frac{3}{8} \gamma^{\mu\nu} F^{(I)}_{\mu\nu} \partial_A X_I - \frac{i}{2} \mathcal{G}_{AB} \gamma^\mu \partial_\mu \phi^B + \frac{3i}{2} (\mathrm{i}\lambda) V_I \partial_A X^I\right] \epsilon\,,$$

- 4D solution is obtainable via *Gibbons-Hawking space*

$$\mathrm{d}s^2_{\mathrm{GH}} = H^{-1} \left(\mathrm{d}x^5 + \chi\right)^2 + H \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j\,, \qquad \vec{\nabla} \times \chi = \vec{\nabla} H\,. \qquad \mathcal{L}_{\partial/\partial x^5} h_{GH} = 0$$

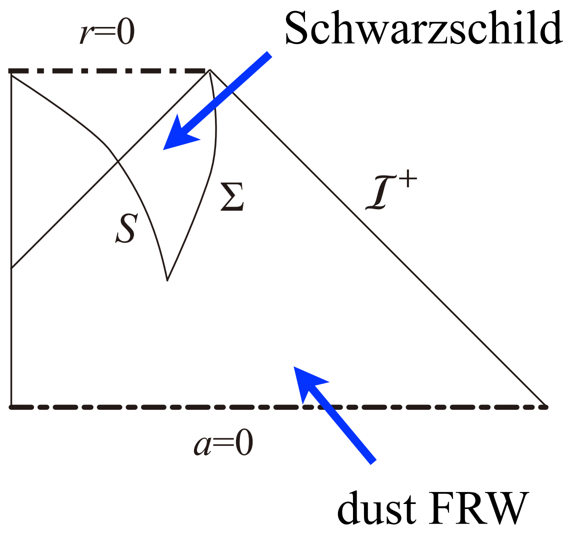
$$\mathcal{L}_{\partial/\partial x^5} g_{\mu\nu} = 0\,, \quad \longrightarrow \quad \mathrm{d}s^2_4 = -\Xi \mathrm{d}t^2 + \Xi^{-1} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j\,, \quad \Xi := (H H_1 H_2 H_3)^{-1/2}\,.$$

We expect *all* BPS solutions can be obtained using Killing spinors *M.N. in work*

Black holes in FRW universe

Black hole in “Swiss-Cheese Universe”

Einstein-Straus 1945



•glue Schwarzschild BH w/ FRW universe

$$ds^2 = -dt^2 + a^2(dr^2 + r^2 d\Omega_2^2)$$

$$ds^2 = -f(R)dT^2 + f^{-1}(R)dR^2 + R^2 d\Omega_2^2,$$

$$f(R) = 1 - 2M/R$$

•Israel’s junction condition at Σ: $R_\Sigma = ar_\Sigma$

$$M = \frac{4\pi}{3}(r_\Sigma a)^3 \rho_0 \, , \quad \rho = \rho_0/a^3$$

-Schwarzschild portion is static

-matters do not accrete onto the hole