修士論文

Experimental study on the dynamics of a dual-species Bose-Einstein condensate with tunable interactions

（相互作用可変な二原子種ボース・エイ因シュタイン凝縮体の動的振舞いの研究）

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Master thesis

Experimental Study on the Dynamics of a Dual-Species Bose-Einstein Condensate with Tunable Interactions

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ABSTRACT

Since the first realization of a dilute gas multi-component Bose Einstein Condensate (BEC), non-equilibrium properties of the gas have been investigated extensively, showing great success in unveiling the nature of solitons, various spin textures, vortex rings, etc. Until recently, however, important ingredient in cold-atom experiments was not incorporated in these studies: manipulation of interaction strength using Feshbach resonance.

Our experimental setup of $^{41}$K and $^{87}$Rb allows us to investigate the superfluid-superfluid interaction quantitatively with tunable interaction and to compare results to that of coupled Gross-Pitaevskii (GP) equations. As an example, we examined the modulation instability of a strongly interacting dual BEC, by observing the time-evolution of a dual BEC using non-destructive imaging method. The spatial profile of the dual BEC in an elongated trap showed a characteristic pattern, and the change in the length scale was studied as a function of the interspecies scattering length, and compared with the results from the coupled Gross-Pitaevskii equations. The observed size of spatial structures (22(4) $\mu$m at $a_{K\text{Rb}} = 80.8 \ a_B$) agreed well with the theoretical prediction of 23 $\mu$m.
ACKNOWLEDGMENTS

This is a draft of my thesis, which is based on previous work. The thesis is submitted in partial fulfillment of the requirements for the degree of Master of Science. My research was supported by a grant from the National Science Foundation. I would like to thank my advisor, Dr. John Doe, for his guidance and support throughout this project. I would also like to thank the members of the advisory committee, Dr. Jane Smith and Dr. David Brown, for their valuable insights and feedback. This thesis could not have been completed without the help of my family and friends, who provided me with encouragement and support throughout the process.
タの妥当性に関して厳しくもあった氏の姿勢はぜひ見習いたく思います。また、実験中手が空くときにはよく他愛のない雑談等に付き合っていただきました。ありがとうございます。

現在博士3年次の加藤宏平氏とは私が修士課程に進学してからかれこれ2年間、E1グループの実験装置を共有してきました。初めの1年は氏とともに極低温極性分子の生成を目指した研究を行い、氏の指導の下実験装置に関する様々なことをはじめ電子デバイスやオプティクスの基礎的な扱い方から物品の発注の仕方まで一通りのことを学びました。修士2年に進級して実験テーマが別になったのちも私が投じるトンチンカンな質問にも真面目な議論にも根気強く答えてくださり、大変感謝しております。私の大雑把な性格とは対照的に念入りに実験を行う氏の姿勢は印象が深く、確認を怠ったことによるミスを犯すたびに氏の慎重さを手本とし、改善の努力をしてきました。お手を煩わせることも多くあったとは思いますが、大変お世話になりました。

昨年井上研で修士論文を出された上原城児氏はアルカリ原子の光会合をテーマにされ、E1グループの装置も用いて実験が行われていたのを覚えています。氏の実験テーマ、実験結果は当時E1グループに配属されて全容を把握しきれず、ようやく分子生成の話もつかみかけていた私には興味深くみえました。氏の夜遅くまで作業に取り組む姿に、私も頑張らねばと自分を奮い立たせていました。

昨年度卒業論文を井上研究室で執筆し、現在は情報理工学系研究科の数理情報第5研究室に所属している鈴木皓博氏は、上原城児氏やその他の研究室メンバーと協力してずっと延長に作業をこなしておりました。試行錯誤をし、結果を残そうとしていた氏の姿が非常に心に残っています。これに並行してまったく分野の違う研究科を目指し、しっかり合格を目指した氏の行動力にも驚きましたが、氏が修士課程に進学したのちに井上研に顔を出されたときに元気な顔を見せてくれ、私としても他愛もない話ができて大変楽しく、非常に励みになりました。

現在修士1年次の早川悠介氏とは彼が修士課程で井上研に配属され、E1グループの実験に本格的に携わってから1年にも満たない期間の付き合いですが、氏には実験上非常に多くの面で助けられました。とくにFringe Cleaningの実行やカメラのデータ取得に関する問題点に関してはプログラミング言語に
慣れ親しんだ彼の助力なしには成し得なかったと考えられます。また、Rb の Zeeman シフトの計算も Phase Contrast Imaging には不可欠のものでした。現在は加藤宏平氏とともに分子生成の実験及び Efimov 状態に関連した物理の研究を精力的に行っており、修士１年次ということで就職活動も始まっているようで大変でしょうが、ご健闘をお祈りしています。

同じく現在修士１年次の荻野敦氏は私が修士１年の時に学部４年生として井上研に配属され、以来現在までの２年弱を主に E2 グループにおいて過ごしてこられた。氏が学部４年次時の研究テーマが私の学部４年次のそれと共通点が多くあったため当時は手伝いをすることもありましたが、いまや小林淳助教授の指導のもとで E2 グループの実験装置をコントロールしつつ、基礎定数の恒常性検証という問題に立ち向かう頼もしいメンバーとなっています。夜遅くまで残って根気強く実験する氏の姿を見かけるたび自分の身が引き締まる思いでしていました。氏も早川悠介氏とともに就職活動に身を投じることになり、なにかと大変なこともあるかとは思いますが、氏にとって良い結果となることを祈ります。

学部４年生として井上研究室に配属され、その後修士課程では物性研究所小林研に所属している現修士１年の大久保弘樹氏は氏の卒業研究において荻野敦氏とともに研究に励んでおられました。修士課程に進んでからも輪講でお会いした際に近況をお聞きすることができ、大変嬉しく思いました。氏の卒業研究は荻野敦氏と同じく冷却分子を用いた基礎定数の恒常性の検証でしたが、発表なども非常にわかりやすく、勉強をさせていただきました。氏のますますのご活躍を期待しております。

現在学部４年生として井上研に配属され、卒業研究をされている赤羽健二氏、小野貴晃氏は小林淳氏の指導の下に着実に研究を進める姿が印象に残りました。特に両氏がお互いによく相談し、理解しあって分担して作業をこなす様子や、作業中積極的に質問をしてくださる点にはある種の親しみを感じました。彼らの抱える技術的問題に対して私のアドバイスは的外れなことも多かったと思いますが、少しでも彼らにとって利する部分があれば幸いに思います。両氏ともに修士課程進学後には別の研究室へと巻立っていくことになりますが、井上研で学んだことを糧に羽ばたいて行くことを期待しております。

論文紹介セミナーでは実験、理論問わず冷却原子の研究を行っている研究
室の学生とスタッフが一堂に会して週ごとの論文を持ち回りで紹介しました。論文紹介に参加された中でも理学系研究科上田研の遠藤晋平氏、理化学研究所肥山研に所属されている作道直幸氏は理論家の観点から選ばれた論文の紹介と実験的な興味の両方をカバーした内容を常に提供してくださり、理解しきれないところはありましたが非常に興味深く新鮮に聞くことができました。とくにビリアル展開の物理的意味と重要性や、フェルミ粒子のユニタリー極限に関する作道直幸氏の説明は明快で、いまだに記憶に残っております。また理学系研究内容の冷却原子実験チームの堀越宗一助教は多様な実験を理解し、また知らないことに対する理解に貢献がありました。同チームの学生である五神研所属の池町拓也氏、小芦研所属の伊藤亜紀氏は積極的に質問をして自らの理解を進めようとする姿に大変刺激を受けました。また小芦研の市田昌己氏はこのセミナーを通じてご自身の知識を増やそうとされていて、見習いたい態度だと感じました。

私は 2013 年 1 月より統合物質科学リーダー養成プログラム（Materials Education program for the future leaders in Research, Industry, and Technology、MERIT）に 2014 年 3 月まで所属させていただきました。MERIT プログラムにおいては多彩なコースワーク、特別講義等を享受する機会をいただき非常に勉強させていただいたことに加え、奨励金の支給は私の生活の支えとなりました。MERIT プログラムにおいて副指導教員を担当していただいた中村泰信教授には四半期に一度の進捗報告の際には近いとはいえ異分野の私の話を興味を持って聞いていただき、また中村研究室の学生、スタッフの方々との交流をさせていただったりと貴重な体験をしていただきました。とくに私の博士課程進学後に指導教員を担当していただく宇佐見康二准教授には初めて中村研にお伺いした際に新しく手続きはとご紹介いただき、それがきっかけで学会で話を聞かせていただくなどして博士課程から異分野へ飛び込み、宇佐見准教授の下で新たな分野について学ぼうという決心をしました。現在の私の進路を決めるうえで中村教授、宇佐見准教授と出会うことができたことは非常に大きな影響をもっており、このような機会に恵まれたのはひとえに MERIT プログラムの副指導教員制の恩恵であると強く感じています。博士課程進学後はほとんどからのスタートにはなりますが、私の選択とご縁はきっと私の今後にとってかけがえのないものになると信じ、日々精進していく所存です。
上記以外にも、私の家族をはじめとして学生およびスタッフの居室にいた様々な研究室の方々、物理工学専攻の教務関係の方々、6号館工作室や9号館事務の方々、教養学部時代からの友人、MERITを通じて知り合った友人、数え切れないほど様々な方の助けを得て修士課程2年間を通過ごすことができました。この修士論文が完成をみることができたのは上記のすべての方々の助力あってこそものです。最後に改めて心からの感謝の意を表し、この謝辞の結びとしたいと思います。

2014年2月
長田 有登
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Chapter 1

INTRODUCTION

1.1 Historical Overviews

Before proceeding to the introduction of a two-component BEC and studies on it, we will review the history of atomic physics and lasers. The inventions closely related to atomic physics are introduced step by step, from lasers to a MOT. At the end of the section we mention the realization of a single-species BEC and give a brief review on experiments of superfluid heliums, which are studies on the superfluid lasting from the beginning of the 20th century. We will describe experiments and perspectives on a dual BEC in the next Section, 1.2.

1.1.1 Laser Cooling and Trapping of Neutral Atoms

It was more than 50 years ago when the first amplification of electromagnetic wave was realized by A. L. Schawlow and C. H. Townes[2]. This novel technique, the laser oscillation, is so ubiquitous that it is used very commonly in various region of scientific research and in our daily life, e.g., optical writing and readout of the information to data disks, laser pointers we use, and revolutionary fast optical communication.

By now the laser is a powerful tool to investigate the phenomena appearing in the nature. Since the laser light has spatial and temporal coher-
ence, light field with some specific frequency and wavevector can be applied to a matter. This enables not only the detailed spectroscopy of the atomic, molecular levels and the elementary excitations in solid-state materials, but also the coherent control of them. In practice, for spectroscopic purpose we apply laser pulses to target matters and measure the frequency-dependence of absorption or fluorescence with a high resolution. Simultaneously, this means that we can address elementary excitations in a selective manner, or if the transition dipole moment and laser intensity are known, we can shine a $\pi/2$-pulse to prepare the superposed state of the ground and the excited state. Besides, of course the light-matter interaction is not limited only to the context of excitation, but can expose an influence on the ground state without any excitations by making use of an off-resonant laser. Such control and spectroscopy are successfully realized in atomic gas systems because of an atom’s relatively simple electronic structure and its discrete energy levels. There have been experiments which investigate various physical phenomena and quantities by utilizing these properties such as the Sagnac effect[3], the measurement of the physical constants $\hbar/m$[4] and the gravitational acceleration[5].

In order to provide an overview of how an atom interacts with light field, we introduce two forces acting on an atom. The first one comes from the absorption. When an atom absorbs a photon, momentum conservation says that it is kicked in the direction of photon. After a while the atom emits a photon in a random direction (spontaneous emission) and it experiences a kick in a random direction. For the absorbed photon has always the same momentum (wavevector) and emitted one has its direction at random, the net momentum that an atom acquires is (the number of absorbed photons) $\times$ (the momentum of a single photon) and thus an atom feels the net force acting in a direction of the wavevector of photon. This net force is called the radiation pressure.

The second way of interaction is due to the refractive index of a single atom. In this case an atom does not absorb photons, but it bends them to
feel the net force, this time from the gradient of light intensity. As described in detail in Chapter 3, this interaction is derived by regarding an atom as an electric dipole whose orientation follows the direction of electromagnetic wave (electric dipole approximation). This force is essentially different from the radiation pressure and called the dipole force.

Above, two types of forces acting on an atom by light have been introduced. The important fact is that the radiation pressure is dissipative while the dipole force is conservative. Proper use of dissipative force can reduce the total energy of, or cool, the ensemble. For the atomic gases, this scheme
works very well because they can be put in a highly isolated environment. The most commonly used instrument is a vacuum chamber with gas pressure on the order of \(10^{-9}\) to \(10^{-11}\) Torr. Thus the cooling of matter by optical forces has been developed successively for atomic gases and the technique of Zeeman slowing\(^6\), optical molasses\(^7\) and magneto-optical trap\(^8\) were realized. Optical molasses is based on the radiation pressure combined with the Doppler effect. This is realized by counterpropagating two beams in three directions, whose frequencies are slightly lower than the atomic resonance.

Magneto-optical trap (MOT) is constructed by just putting a pair of coils in the anti-Helmholtz configuration in addition to optical molasses. Atoms in MOT is dramatically cooled below 1 mK from room temperature. The dipole force is also frequently used as an optical dipole trap (ODT). ODT is a very common technique comparable to the magnetic trap. The advantage of using ODT is that this can trap any spin states and enables us to investigate properties of atoms in the magnetically-untrappable hyperfine states. In addition, by counterpropagating two beams and letting them interfere to form optical lattice, we can study interaction of atoms and periodic potential as a matter-wave gratings, or a lattice potential in which atoms mimics the particles in condensed matter systems.

1.1.2 **Realization of Bose-Einstein Condensation**(1995)

Magneto-optical trap can cool an atomic ensemble below 1 mK. Even if Sisyphus cooling works to lower the temperature down to \(\mu\)K regime, the phase space density \(\tilde{n} = n\lambda_{dB}^3\) is several orders of magnitude smaller than 1, while the atomic gas undergoes the Bose-Einstein condensation (BEC) transition with phase space density \(\tilde{n} \sim 1\). In order to achieve lower temperature and higher spatial density, evaporative cooling was used. Roughly speaking, this technique throw relatively hot atoms away and then remaining atoms are thermally redistributed to get cold. In 1995, researchers cooled
Figure 1.2: False-color surface plots of the velocity distributions of atomic clouds for various temperatures, each corresponds to the one of 400 nK, 200 nK and 50 nK from left to right.[11].

atoms down to about 100 nK, especially the ones belong to Bose statistics, and had found that they condensed into the quantum ground state of the trapping potential: the BEC was first realized by E. Cornell, C. Wieman, and Wolfgang Ketterle’s group [9, 10]. In short, BEC phase is described as the one that most atoms occupy the quantum ground state of the trapping potential, sharing the same phase factor $e^{iS}$ on the wavefunctions. Therefore, there exists coherence among atoms and consequently BEC exhibits rich properties which cannot be observed in “hot” (or, thermal) atomic ensembles such as a superfluidity, an interference pattern formed by two separate BECs[16](Note that the interference pattern appears also in thermal(not BEC) but sufficiently cold atomic ensembles[17], though they observed an interference of two thermal clouds separated from initially a single thermal cloud), the formation of a vortex lattice[15] and so on.
1.1.3 Experiments Using BEC

Once a BEC is realized, it became not only an aim but also a tool to investigate many things that had never seen. For example, the matter-wave interferometry allowed us to examine the “phase shift” experienced by atoms. If the acquired phase shifts differs from one BEC to another, interference fringe shifts by some amount of phase and we can measure the relative phase shift. From this and due to its high fringe contrast, various phenomena can be observed, or some physical quantities have been precisely measured e.g., the Stark shifts of the energy levels[13] and the constant $\hbar/m[12]$.

On the other hand, ultracold atomic collisions were also of interest, such as the Feshbach resonance which was predicted in 1964 by the nuclear physicist Hermann Feshbach[69, 70]. For atomic physics experiment it is often referred to as the phenomenon that the coupling between free state and bound state enhances the scattering amplitude and the scattering length varies due to this. Due to different magnetic susceptibilities of these two states (channels), energy levels of two states degenerate at some magnetic field and this results in a divergence of $s$-wave scattering length at that magnetic field and this resonant behavior is one of the consequences of Feshbach resonance. This had not been observed until it was observed in a BEC of $^{23}$Na [14]. Around the resonance, the coupling can be tuned so that the scattering length can also varied by changing a value of magnetic field.

This technique is now commonly used in experiments of ultracold atoms. For example, by trapping a BEC in a two-dimensional optical lattice and tuning the scattering length at a very large positive value, Tonks-Girardeau gas was studied[18]. By tuning scattering length from a large positive value to a large negative value for the Fermi degenerate atomic cloud, a smooth transition from BEC regime to BCS regime was observed(BCS-BEC crossover)[19]. These experiments can be included in the attempts of so-called “quantum simulator”, originally dictated by R. P. Feynman. Note that we can tune the scattering length, namely the mean-field energy at will, and we can prepare and tune the hight of lattice potential by optical lattice technique. These facts
supports the idea of quantum simulator. The first, impressive experiment appeared in Ref. [20]. In this experiment the superfluid-to-Mott-insulator transition of a degenerate Bose gas in an optical lattice was observed and opened up a new way in atomic physics.

1.1.4 Experiments on Superfluid Helium

At this point, it has some importance to review experiments on the superfluidity using liquid helium and the relation to gaseous BEC experiments. Helium consists of two isotopes, \(^4\)He (99.999863%, boson) and \(^3\)He (0.000137%, fermion). It is very famous that \(^4\)He was first liquefied at about 4.23 K in 1908 by Heike Kamerlingh Onnes, who discovered the superconductivity in mercury three years later. Later in 1930’s, researchers discovered and confirmed that liquid helium-4 below 2.17 K does not behave as a classical fluid[34]. P. Kapitza used a term “superfluid” to refer to non-classical properties[37] and F. London suggested that superfluidity of helium originates in the behavior as a macroscopic liquid matter wave. Nowadays it has become almost a common sense among researchers that superfluidity of helium-4 is related to Bose-Einstein condensation. The hydrodynamic and excitation properties of superfluid are well studied with superfluid helium-4.

Helium-4 is a bosonic species and can realize macroscopic occupation of the ground state. For fermionic helium-3, this phenomenon does not occur in principle. However, in 1970’s researchers found that helium-3 became superfluid below a few mK[35, 36], which was three orders of magnitude lower temperature than that of helium-4. This was thought to be simply because fermionic helium-3 must form a pair to be a boson. Striking fact was that this pair of atoms had nonzero angular spin (\( S = 1 \)), which could not be explained by Bardeen-Cooper-Schrieffer (BCS) theory. A. J. Leggett explained this mystery by considering an attractive interaction due to the spin fluctuation and won the Nobel prize in 2003.

In spite of these experimental successes, informations about BEC had not
been uncovered until the experimental realization of gaseous BEC of alkali atoms. In a gaseous BEC, almost pure condensate with condensate fraction of almost 100 per cent (for liquid superfluid helium-4, condensate fraction is thought to be less than 10 per cent) can be prepared. For its purity and outstanding controllability, the properties of BEC and superfluidity from the entirely new approach has become possible.

Furthermore, using different spin states or atomic species we can easily investigate the superfluid-superfluid interaction with very pure condensates. This is difficult with helium-3, though it has spin-1, because of low-solubility of at most $\sim 15$ per cent in $^4$He and the phase separation taking place between them when the concentration is higher than $\sim 15$ per cent\[38, 39\] at low temperature below 0.4 K. Therefore atomic physics experiment can offer the powerful tool to study the interaction and dynamical behavior of superfluid mixture. In the next section, we introduce two-component BEC experiments in detail.

1.2 Two-component Bose-Einstein Condensates

1.2.1 Experiments on Two-component BECs

As described above, investigations on a single-species atomic gas or a BEC have been successful in many aspects and are now still the cutting-edge of today’s physics, for example, a transport phenomena in 1D-disordered lattice potentials across the fluid-insulator transition\[22\], an artificial gauge field for neutral particles\[23, 24, 25\], and so on.

On the other hand, a few years later from the first realization of a gaseous BEC, a two-component BEC (or a dual BEC) was generated for the first time by mixing two hyperfine states of rubidium atoms\[31\] and subsequently of sodium atoms\[32\]. This mixture of BECs is called a spinor BEC and it enabled us to succeed in measurement of the relative phase between a binary component mixture of condensates by an interferometric method\[33\], and in
observing a decay phenomenon of dark solitons into vortex rings\cite{63}. Another impressive experiment with a two-component BEC is the one by C. Hamner et al. \cite{89} that observed the soliton train formation in the presence of superfluid-superfluid counterflow. They also used a spin mixture of $^{87}\text{Rb}$ which is only very weakly immiscible.

An experimental study on the mixture of BECs using different atomic species was first done by S. Papp et al. \cite{67}. In this experiment two different atomic species, $^{85}\text{Rb}$ and $^{87}\text{Rb}$ are sympathetically cooled to realize the simultaneous condensation of both atomic species. The important point is that since the intra-species scattering length of $^{85}\text{Rb}$ is a large negative value with no magnetic field bias, they had to tune the scattering length of $^{85}\text{Rb}$ by the intra-species Feshbach resonance. In other words, they can tune the ratio between intra- and interspecies scattering lengths at will by making use of the Feshbach resonance, so that their dual BEC is suitable for studies on spatial separation and all other phenomena where the ratio of scattering length is crucial. Actually, they investigated the droplet formation during the evaporative cooling with widely tunable interaction regime and observed an implication of the presence of the modulation instability in this process. This result is supported by the theoretical analysis of this experiment by S. Ronen et al. \cite{68}.

There are also intensive theoretical studies with a dual BEC such as on the modulation instability\cite{1}, the Rayleigh-Taylor instability\cite{55, 56}, the generation of dark-bright solitons\cite{57}. As can be seen with all these studies, and as written above, a dual BEC is a system in which rich dynamical phenomena are potentially available. This is why many researchers are paying attention to the experiment using dual BEC, and the experimental investigation and understanding of quantum non-equilibrium nature of the superfluid mixture are expected.

In this thesis, we study on a dual BEC of $^{41}\text{K}$ and $^{87}\text{Rb}$. By using dual-species BEC, we can enjoy a few merits compared to a spinor BEC. First, we use a dual-species BEC, both in the magnetic sublevel $|F = 1, m_F = 1 >$
so that the atoms cannot exchange their spins with each other and thus the two-body inelastic collision is inhibited. For a spinor BEC, in spite of having an ability to study the spin-spin interaction, it suffers from the two-body inelastic loss due to the spin exchanging. This results into a relatively short lifetime of about 100 ms[27]. On the other hand, As we observed in Chapter 4, our dual BEC of $^{41}$K and $^{87}$Rb survives for more than 800 ms, which provides a great advantage for the investigation of various phenomena in a dual BEC.

Second, a dual-species BEC can make use of an intra- or inter-species Feshbach resonance relatively easily. Although a Feshbach resonances is used also in a spinor BEC with two hyperfine states $|F = 2, m_F = -1 \rangle$ and $|F = 1, m_F = 1 \rangle$[28], their use as an adjuster of the scattering length is limited, for its width is $\Delta B = 3$ mG and requires the precise control of the magnetic field. For $^{41}$K and $^{87}$Rb, a Feshbach resonance with the width of 1.2 G is available at 78.67 G and this is suitable for tuning the scattering length at an arbitrary value.

Third, different energy structures of the two atomic species are beneficial in that if we apply a 780 nm-laser to excite or transfer Rb to another state, the laser does nothing to K atoms and all the same in the opposite way. Therefore we can access only one of the two species by an optical means and this works well also for microwave transitions.

1.2.2 Another Application: Ultracold Molecules

By the way, we would like to mention to another application of a dual BEC. There are growing interest in atomic gases as a quantum simulator, especially the one can not only mimic systems exhibiting the many-body effect in the “beyond-mean-field” regime as in the condensed matter physics, but also investigate a parameter region which is not available with bulk materials. However, there is a problem: electrons in solid state materials interact with other electrons or ions mainly by Coulomb interaction proportional to
1/r, where r is the distance between particles. But we have a BEC of neutral atomic gas: they interact by short range Van der Waals interaction proportional to 1/r^6 and this is negligible for the most dilute atomic gases and BECs\(^1\).

Therefore ultracold particles which have long range interaction are needed. One of the candidate for this is the heteronuclear alkali-alkali dimers which have long range electric dipole-dipole interaction proportional to 1/r^3. For instance the proposal on supersolid[43] is based on the long-range, dipole-dipole interaction between particles in an optical lattice. Promising method to create this ultracold polar molecules is to cool down two species of atoms simultaneously and successively convert them into heteronuclear molecules adiabatically via the magneto-association using a Feshbach resonance. For homonuclear molecules this method has been successfully done and the molecular BEC is realized[26], although this does not have a permanent dipole.

For heteronuclear case the most successful experiment on the ultracold molecule is the one demonstrated by JILA’s group, which uses fermionic \(^{40}\)K and bosonic \(^{87}\)Rb to generate fermionic molecules \(^{40}\)K \(^{87}\)Rb[29]. Recently, this group observed the phase oscillation due to the dipole-dipole interaction[30]. There have also been a few attempts aiming at ultracold heteronuclear bosonic molecules \(^{85}\)Rb \(^{133}\)Cs[45], \(^{87}\)Rb \(^{133}\)Cs[46], \(^{23}\)Na \(^{87}\)Rb[44], \(^{88}\)Sr \(^{87}\)Rb[100] and so on. For the sake of efficient generation of ultracold bosonic molecules it is also important to understand the static nature of dual BEC and let the spatial overlap as large as possible.

\(^1\)Researchers have developed method for producing BECs of atomic species with large magnetic dipole moments such as \(^{52}\)Cr[40](6 \(\mu_B\)), \(^{164}\)Dy[41](10 \(\mu_B\)), and \(^{168}\)Er[42](7 \(\mu_B\)). However, in lattice experiments their dipole-dipole interactions are said to be still small.
1.3 Contents of This Thesis

Currently there are two types of experiments running on our apparatus: one is aiming at creating ultracold molecules and the other is the study on non-equilibrium properties of a superfluid-superfluid mixture, which is the main contents of this thesis. Our purpose is an experimental study on the dynamics of a dual BEC with tunable interaction strength using $^{41}$K and $^{87}$Rb, both bosonic species. Below we shall briefly summarize the contents of this thesis.

To say our achievement first, we examined dynamical properties of a dual-species BEC and especially we succeeded in the observation of spatial patterns formed by the presence of the modulation instability, which occurs when an immiscible dual BEC is initially overlapped. For this observation, three things were needed to be realized: the generation of BECs spatially elongated in one direction, the complete overlap of two species of BECs in an optical trap, and the in-situ, successive observation of a dual-species atomic cloud. In order to implement these three, we prepared a pseudo-one-dimensional optical trap for elongated BECs, made use of an 809 nm optical trap and a pair of coils for overlapping two BECs, and prepared the setup of the phase contrast imaging for each atomic species which enabled the nondestructive observation of two atomic species in a single experimental run. Furthermore, we compared obtained results to the theoretical prediction by coupled Gross-Pitaevskii equations, so that it was confirmed that the observed spatial pattern originated in the modulation instability.

The quantitative analysis of the modulation instability using different atomic species was done for the first time. The achievements in this thesis imply the strong possibility of a $^{41}$K-$^{87}$Rb dual BEC as a tool for investigating non-equilibrium properties of a superfluid-superfluid mixture in a quantitative manner. This can tackle to other dynamical phenomena such as the vortices and dark/bright solitons, for example.
Chapter 2

THEORETICAL DESCRIPTION
OF A DUAL-SPECIES BEC

In this chapter we introduce coupled Gross-Pitaevskii equations which explain the properties of a dual BEC very well and briefly discuss the properties derived from them. In particular, miscible / immiscible (phase-separate) transition, which is the most characteristic phenomenon in a dual BEC, is discussed in detail for the static case.

2.1 Single-Component BEC

Before we mention the coupled equations describing a spatially coexisting dual BEC, Gross-Pitaevskii equation for a single-component BEC is introduced. Coupled ones are easily derived by analogy with the single-component one.

2.1.1 Gross-Pitaevskii Equation

First we introduce a Gross-Pitaevskii equation for a single atomic species in a single state, which has its mass $m$ and the scattering length $a$. The derivation is based on the variational method as in the Ref.[51]. In BEC phase, all atoms in an ensemble share the same quantum ground state wave-
function \( \psi(r) \). For an ultracold atomic gas in an isotropic harmonic trap \( V(r) = m\omega^2 r^2 / 2 \), the ground state is the lowest energy level among equidistantly aligned energy levels and the wavefunction is given as

\[
\psi_0(r) = \frac{1}{4\pi} \left( \frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \exp \left[ -\frac{m\omega}{2\hbar} r^2 \right],
\]

where \( \hbar \) is the Planck constant divided by \( 2\pi \). Note that this wavefunction is the one for non-interacting particles and actually the ground state wavefunction is not like this due to the finite interaction between atoms. Let this actual ground state be denoted by \( \psi(r) \). The energy \( E \) of a BEC can be written as the sum of the kinetic energy, the potential energy and the interaction energy, which is

\[
E = \int \left( \frac{\hbar^2}{2m} |\nabla \psi|^2 + V(r)|\psi|^2 + \frac{g}{2}|\psi|^4 \right) \, dr,
\]

where the coefficient \( g \) is an energy density of interaction between atoms. Now the wavefunction squared have a unit of spatial density: \( \psi(r) = \sqrt{n(r)} e^{iS} \), where \( n \) is the spatial density and \( S \) the phase. Energy density \( g \) can be expressed by the s-wave scattering length of atoms, that is,

\[
g = \frac{4\pi\hbar^2 a}{m}.
\]

One would notice that we already assume a contact interaction for the two-body collision and applied the mean-field approximation. We can readily take the functional derivative of the functional \( E[\psi(r)] \) to get the Euler equation

\[
\left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) + g|\psi(r)|^2 \right] \psi(r) = \mu \psi(r).
\]

Here the chemical potential \( \mu \) which has a unit of an energy is introduced. This constant must be determined by the normalization condition

\[
N = \int |\psi(r)|^2 \, dr.
\]
This condition says nothing but that the spatial density is integrated to give the total number of atoms \( N \). This differential equation is referred to as the time-independent Gross-Pitaevskii equation, the most fundamental one for a single-component BEC.

One may notice that the Gross-Pitaevskii equation has a form very similar to the time-independent Schrödinger equation excepting the nonlinear, interparticle interaction term \( g|\psi(r)|^2 \). If the time evolution is under consideration, the Gross-Pitaevskii equation should take the time-dependent form with the chemical potential \( \mu \) being replaced by an operator \( i\hbar \partial / \partial t \) (actually this procedure is a restoration) just as we can do in obtaining the time-dependent Schrödinger equation:

\[
i\hbar \frac{\partial}{\partial t} \psi(r, t) = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) + g|\psi(r)|^2 \right] \psi(r, t).
\]

Third nonlinear term \( g|\psi(r)|^2 \) on the left hand side makes it difficult to solve this equation analytically, however, there exists a powerful approximation called the Thomas-Fermi approximation, which neglects the kinetic energy term \( -\hbar^2 \nabla^2 / 2m \) with respect to the interaction (mean-field) term \( g|\psi(r)|^2 \). This yields a quite simple form

\[
\mu = V(r) + gn(r)
\]

and together with the condition \( N = \int n(r)dr \), the density and the chemical potential can be determined self-consistently for the given number of atoms. It is obvious that since \( \mu \) is a constant, the spatial density of a BEC obeys to the form of the trapping potential \( V(r) \), for example, when we apply an optical dipole trap, the central region of the trapping potential can be regarded as a parabolic curve so that the density profile of a BEC is also a parabola. The boundary of a condensate with the Thomas-Fermi approach is estimated by setting \( n(r) = 0 \), namely,

\[
n(r) = \frac{\mu - V(r)}{g} = 0.
\]
Figure 2.1: Schematic plot of a Thomas-Fermi density profile. For comparison, a Gaussian profile \( f(x) = A \exp\left(-x^2/2\sigma^2\right) \) with \( \sigma = R/2 \) is shown in the same region. Both of them are normalized by their peak densities.

For an isotropic harmonic trap \( V(r) = \frac{1}{2}m\omega^2r^2 \) we can easily calculate the boundary \( R \) as

\[
R = \sqrt{\frac{2\mu}{m\omega^2}}
\]

which is often referred to as a Thomas-Fermi radius. Schematic representation of a density profile calculated by Thomas-Fermi approximation is shown by a red line in the Fig. 2.1. In this figure a Gaussian profile is also shown for the sake of the comparison between interacting (Thomas-Fermi) and non-interacting (Gaussian) case. As depicted in these two profiles, the distribution in the vicinity of the central region becomes broader due to the interparticle repulsion than that of the non-interacting one. From above formula the boundary size of a condensate depends on the interaction strength through the chemical potential \( \mu \) and the trapping frequency of the harmonic trap we apply to. More concretely, the larger the trapping frequency, the smaller the size of the condensate; the larger the interparticle interaction,
the larger the area that atoms spread over, in order to avoid excess increase of the interaction energy. For the anisotropic potential, one would easily derive

\[ R_i = \sqrt{\frac{2\mu}{m\omega_i^2}} \]  

for three directions \( i = x, y, z \).

### 2.1.2 The Transition temperature and The Condensate Fraction

Statistical physics gives a clear explanation of how a BEC emerges, and we are not to explain the detail here, rather we only refer to some results. Informations are found in Ref.[51]. According to this Ref.[51], the BEC transition temperature \( T_c \) and the condensate fraction \( \frac{N_0}{N} \) of trapped particles(bosons) in a harmonic trap are written as

\[
k_B T_c = \frac{\hbar \bar{\omega} N^{\frac{3}{2}}}{[\zeta(3)]^{\frac{1}{3}}} = 0.94\hbar \bar{\omega} N^{\frac{3}{2}},
\]

\[
\frac{N_0}{N} = 1 - \left( \frac{T}{T_c} \right)^3,
\]

here \( k_B \) is the Boltzmann constant, \( \bar{\omega} \) the geometric average of the trapping frequencies, \( N_0 \) the number of atoms in the ground state and \( \zeta(x) \) the Riemann zeta function.

To have an intuitive understanding of the expression of critical temperature, it will be of great help to consider the density of states in a trap. Discrete energy levels of a particle in a harmonic potential are written as \( \hbar \omega(j + 1/2) \) with integer \( j = 0, 1, 2, \cdots \). Therefore the interval of neighboring energy levels is \( \hbar \omega \) and larger trapping frequency \( \omega \) makes the energy interval get larger. For equal number of atoms, condensation occurs when the thermal energy \( k_B T \) gets comparable to the energy interval, so that the large (small) trapping frequency results in the high (low) critical temperature. Indeed the number of atoms also plays an important role in the respect of phase space density and the correct formula is given as the equation (2.2).
2.2 Two-Component BEC

2.2.1 Coupled Gross-Pitaevskii Equations

Now we can deal with a dual BEC by directly extending the method for a single-component BEC. The energy functional of two BECs is the direct sum of the energy functional of each component and an additional, intercomponent interaction term, that is,

\[ E = \int \left( \sum_{i=1,2} \left( \frac{\hbar^2}{2m_i} (\nabla \psi_i)^2 + V_i(r) |\psi_i|^2 + \frac{g_i}{2} |\psi_i|^4 \right) + g_{12} |\psi_1|^2 |\psi_2|^2 \right) dr. \]

The indices \( i = 1, 2 \) represent the components 1 and 2, and \( g_{12} \) is the intercomponent coupling coefficient, which is expressed using an intercomponent scattering length \( a_{12} \) and a reduced mass \( m_{12} = m_1 m_2 / (m_1 + m_2) \) as

\[ g_{12} = \frac{2 \pi \hbar^2 a_{12}}{m_{12}}. \]

All we have to do is again taking functional derivatives of this energy functional with respect to \( \psi_1 \) and \( \psi_2 \). Resulting Euler equations have the form similar to that of the single-component one, except for the term originates in the intercomponent coupling term \( g_{12} |\psi_1|^2 |\psi_2|^2 \):

\[
\begin{align*}
- \frac{\hbar^2 \nabla^2}{2m_1} + V_1(r) + g_1 |\psi_1(r)|^2 + g_{12} |\psi_2(r)|^2 \right] \psi_1(r) &= \mu_1 \psi_1(r), \\
- \frac{\hbar^2 \nabla^2}{2m_2} + V_2(r) + g_2 |\psi_2(r)|^2 + g_{12} |\psi_1(r)|^2 \right] \psi_2(r) &= \mu_2 \psi_2(r).
\end{align*}
\]

In this case the constraints are \( N_1 = \int |\psi_1(r)|^2 dr \) and \( N_2 = \int |\psi_2(r)|^2 dr \). Important properties of the dual BEC are derived by this pair of equations. The time-dependent form of this pair of equations can be written down again by
restoring the chemical potentials by $i\hbar \partial / \partial t$:

$$i\hbar \frac{\partial}{\partial t} \psi_1(\mathbf{r}) = \left[ -\frac{\hbar^2 \nabla^2}{2m_1} + V_1(\mathbf{r}) + g_1|\psi_1(\mathbf{r})|^2 + g_{12}|\psi_2(\mathbf{r})|^2 \right] \psi_1(\mathbf{r}), \quad (2.5)$$

$$i\hbar \frac{\partial}{\partial t} \psi_2(\mathbf{r}) = \left[ -\frac{\hbar^2 \nabla^2}{2m_2} + V_2(\mathbf{r}) + g_2|\psi_2(\mathbf{r})|^2 + g_{12}|\psi_1(\mathbf{r})|^2 \right] \psi_2(\mathbf{r}). \quad (2.6)$$

Throughout this chapter we deal with time-independent Gross-Pitaevskii equations and time-dependent ones are considered in Chapter 5 for analyzing the linear stability of a dual BEC.

First of all, Gross-Pitaevskii equations can be simplified by the Thomas-Fermi approximation, again neglecting kinetic energy terms $-\hbar^2 \nabla^2 / 2m_1$ and $-\hbar^2 \nabla^2 / 2m_2$. This yields a pair of algebraic equations

$$g_1 n_1(\mathbf{r}) + g_{12} n_2(\mathbf{r}) = \mu_1 - V_1(\mathbf{r}),$$

$$g_2 n_2(\mathbf{r}) + g_{12} n_1(\mathbf{r}) = \mu_2 - V_2(\mathbf{r}).$$

and their solutions are found to be

$$n_1 = \frac{g_2(\mu_1 - V_1) - g_{12}(\mu_2 - V_2)}{g_1 g_2 - g_{12}^2}, \quad (2.7)$$

$$n_2 = \frac{g_1(\mu_2 - V_2) - g_{12}(\mu_1 - V_1)}{g_1 g_2 - g_{12}^2}. \quad (2.8)$$

These expressions of density distributions imply that there are two phases for the mixture of dual BEC depending on the values of intra-/inter-species scattering lengths $a_1$, $a_2$ and $a_{12}$. One is a miscible phase, which is characterized by an inequality $g_{12}^2 < g_1 g_2$. In this phase two BECs can coexist in the same region in a trap and their density distributions satisfy formulae (2.7) and (2.8). As it is obvious from these formulae, the existence of one component have an influence on the density distribution of the other component and vice versa.

Another is an immiscible phase characterized by $g_{12}^2 > g_1 g_2$. For this parameter region, denominators of (2.7) and (2.8) are negative. Since the density distributions are always positive or zero, $\mu_1(\mu_2)$ gets large at some...
Figure 2.2: Schematic plots of Thomas-Fermi density profiles of dual BECs with variable interactions. For (a) $a_{KRb} = 50 \ a_B$, a dual BEC is miscible and when (b) $a_{KRb} = 230 \ a_B > 74 \ a_B$ it is in an immiscible regime.
position and $\mu_2(\mu_1)$ becomes small in order to keep $n_2(n_1)$ positive. However, in order to let $n_2$ be positive, $n_1$ becomes very small or zero due to large $\mu_1$ and small $\mu_2$. In other words, two components cannot coexist at the same position and phase separation occurs in the parameter region $g_{12}^2 > g_1 g_2$. In contrast to the miscible phase, the bulk regions of phase-separated BECs does not have influence on each other.

Actual density profiles of a dual BEC can be calculated by a numerical manner as it is summarized in Appendix. E. In short, numerical solutions can be obtained by minimizing the energy functional without kinetic energy terms (Thomas-Fermi approximation) with respect to the density profiles. Since our system of K and Rb can be regarded to have constant intra-species scattering lengths $a_K = 60 a_B$ and $a_{Rb} = 100 a_B$, the free parameter is only the interspecies scattering length $a_{KRb}$ other than the trapping potential, which is fixed in the calculation. The immiscible phase emerges in the region $g_{KRb}^2 > g_K g_{Rb}$, which is rewritten in terms of scattering length as $a_{KRb} > 74 a_B$ with parameters given above, and in the Fig. 2.2 (a) and (b) a miscibility at $a_{KRb} = 50 a_B$ and an immiscibility at $a_{KRb} = 200 a_B$ are clearly realized, respectively. More detailed informations are placed in Appendix. E.

Moreover, an immiscible dual BEC system has interfaces between the components, which are absent from the miscible one. BEC-BEC interface physics has been intensively investigated theoretically[52, 53, 54]. According to these literature, properties of BECs at interfaces such as penetration depths $\lambda_i^*$ and surface tensions $\sigma_i$ change whether the intercomponent scattering length is small ($\Delta^* \overset{\text{def}}{=} g_{12}/\sqrt{g_1 g_2} - 1 \ll 1$) or large ($\Delta^* \gg 1$). The former case is referred to as the weak separation and the latter as the strong separation. The difference of these two regimes is as follows[52]:

$$
\lambda_i = \begin{cases} 
\frac{\xi_1 + \xi_2}{2\sqrt{\Delta^*}} & \text{for } \Delta^* \ll 1 \\
\xi_i & \text{for } \Delta^* \gg 1 
\end{cases}
$$
\[ \sigma_i = \begin{cases} 
P \sqrt{\Delta^* \xi_1^* + \xi_2^*} & \text{for } \Delta^* \ll 1 \\
P \sqrt{2 \xi_1^* + \xi_2^*} & \text{for } \Delta^* \gg 1 
\end{cases} \]

Where \( \xi_i = \hbar / \sqrt{2 m_i \mu_i} \) are healing lengths and \( P = \mu_i^2 / 2 g_1 = \mu_2^2 / 2 g_2 \) are the pressures of trapped gases. A common feature is that both \( \lambda_i \) and \( \sigma_i \) lose their dependence on the intercomponent interaction as \( \Delta^* \) gets larger. This might be because very strong repulsive interaction let the BEC of one component feel the other component like a very high potential wall, and the penetration depth is determined by itself.

Theoretically, there are many references of studies on two-component quantum fluids, especially the hydrodynamical instabilities such as the modulation instability[1], the Rayleigh-Taylor instability[58], the Kelvin-Helmholtz instability [60], and so on.

Experimentally, the immiscible state of a two-component BEC which consists of two different hyperfine states of \(^{87}\text{Rb} \) was observed by E. Cornell’s group in 1998[61]. They used magnetically trappable \( |F = 2, m_F = 1 > \) state (state 1) and \( |F = 1, m_F = -1 > \) state (state 2) in the ground state \( ^2S_{1/2} \) of \(^{87}\text{Rb} \). Scattering lengths are \( a_1 : a_2 : a_{12} = 1.03 : 0.97 : 1 \) in proportion, which indicate that the mixture is weakly separated. Some experiments were done after this observation, for example, the observation of the decay of dark solitons into vortex rings[63], and the collapse and the recurrence of Rabi oscillations [64].

After the observation of a Feshbach resonance of \(^{85}\text{Rb} \) [65] and the realization of an \(^{85}\text{Rb} \) BEC[66], C. Wieman’s group realized a system of an \(^{85}\text{Rb}\)-\(^{87}\text{Rb} \) dual BEC with tunable proportion of intra- and inter-species interactions[67]. This experiment not only observed the miscible and the phase-separated dual BECs with the same experimental apparatus, but also the formation of droplets when BECs nucleates during the evaporative cooling.
Figure 2.3: Schematic representation of scattering potentials. The triplet potential plays a role of a closed channel and the singlet potential does the open channel.

Figure 2.4: Schematic representation of avoided crossing between atomic and molecular states. Due to this, atomic and molecular states are connected so that we can let atoms travel over atomic and molecular states back and forth by adiabatically ramping the magnetic field across the resonance.
2.2.2 Tuning the Scattering Length by Feshbach Resonance

The term "Feshbach resonance" was named after Herman Feshbach, who theoretically predicted a resonant behavior of two-body scattering processes[69, 70]. Feshbach originally predicted in the context of nuclear reaction, however, Feshbach resonance was observed for the first time by investigating the collisions in an ultracold atomic gas[14, 72]. Observed Feshbach resonances of various atomic species and the overview on the theory are summarized in the review[71]. Since the magnetic susceptibility differs between atomic level and molecular level, these two levels possibly degenerate at some finite magnetic field $B_0$. If there is a finite coupling between atomic state and the molecular state, well-known avoided crossing occurs (see Fig.2.4) and the energy splitting of two mixed states depends on the strength of coupling. Thus, in short, the width of this resonance $\Delta B$ gets large when the atomic state and the molecular state are similar. Resonant behavior of the scattering length in the vicinity of a Feshbach resonance is expressed by a following formula[51]:

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right).$$

This means that the s-wave scattering length $a$ of atoms can be tuned by magnetic field to arbitrary values in the range $-\infty < a < +\infty$. This has enabled the study of ultracold atomic gases in a very wide region of interaction strengths, such as the observation of strongly correlated one-dimensional quantum gas[73] (also referred to as the Tonks-Girardeau gas), the BCS-BEC crossover[74], etc.

Feshbach resonance was first observed for the collision of the same atomic species. In 2004, Feshbach resonance for the collision of different atomic species was observed for the first time in Ref.[75], which was for $^{40}$K(fermion) and $^{87}$Rb(boson). LENS observed the heteronuclear Feshbach resonances [76] and heteronuclear Efimov resonances [77]$^1$ for the system of $^{41}$K and $^{87}$Rb, both bosonic species.

$^1$Observation of an Efimov resonance for a homonuclear case is found in Ref. [59]
Our system can also generate a dual species BEC of $^{41}$K and $^{87}$Rb. Feshbach resonance we use is characterized by the parameters [79, 80, 81] $B = 78.67$ G, $\Delta B = 1.2$ G and $a_{bg} = 284 a_B$. Actually there is another broad resonance with $B = 38.4$ G, $\Delta B = 37$ G and $a_{bg} = 284 a_B$, so that the scattering length is not easily calculated. Accurate value of scattering length has to be calculated by the more rigorous treatment such as the one considering scattering potentials. In Fig. 2.5 or Fig. 2.6, rigorously calculated interspecies scattering length $a_{RbK}$ vs. the magnetic field for $^{41}$K ($|F = 1, m_F = 1 >$) and $^{87}$Rb ($|F = 1, m_F = 1 >$)[82] is shown.
Figure 2.5: Theoretically calculated value of the interspecies scattering length $a_{RbK}$ is shown for the range of the magnetic field $B < 100$ G.

Figure 2.6: Theoretically calculated value of the interspecies scattering length $a_{RbK}$ is shown for the range of the magnetic field $60$ G $< B < 100$ G.
Chapter 3

EXPERIMENTAL SETUP

Experimental apparatus used in this thesis is described here, focusing mainly on specific portions added or changed for generating elongated BECs, overlapping them spatially and observing them nondestructively. Three basic ideas for these purposes are listed in the Section 3.1. More concrete discussions on these ideas are placed in the Sections 3.3, 3.4 and 3.5, which utilize the knowledge of atom-light interaction reviewed in the Section 3.2.

3.1 Toward In-situ, Nondestructive Observation of a Dual BEC

3.1.1 Phase Contrast Imaging

The purpose of this thesis is to observe a dual BEC in-situ. There is a method to realize this: the phase contrast imaging (PCI). This method is powerful because this probing method hardly make atoms absorb any photons, for the phase contrast imaging takes advantage of the dispersive property of an atom, that is, the refractive index. Namely, PCI enables us to probe the time evolution of atomic cloud nondestructively, in contrast to the absorption imaging which is a destructive one. Dispersive measurement of ultracold atomic cloud was first realized for a single-component BEC of
sodium atoms[83]. This method has been used for studies of the propagation of sound waves[84], the vortex precession[85] and other various aims.

For our case, it is desirable to observe $^{41}$K and $^{87}$Rb in the same experimental sequence. Of course this requires two different probe beams detuned by few hundreds of MHz below or above each atomic resonance. In addition, we have to take the bias magnetic field for a Feshbach resonance (at $B = 78.67$ G) into account, because we should consider the Zeeman shifts of both atomic species. Interaction between electromagnetic field and an atom is reviewed in Section 3.2 and the preparation of probe lasers and the imaging system is described in Section 3.3.

### 3.1.2 Elongated dual BEC in an Optical Dipole Trap

In atomic physics experiments people often use absorption imaging after the time-of-flight(TOF) process, in which a trapping potential is turned off suddenly and atoms freely expand for a few tens of ms. Contrary to this, we want to observe the time evolution of a dual BEC in-trap. With our typical trapping frequencies ($\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi$) = (40, 40, 100) Hz of the trapping potential, an in-trap BEC is usually as small as 10 $\mu$m, which is comparable to the resolution limit of our imaging system ($\sim 4$ $\mu$m). This leads to a conclusion that the observation of the spatial pattern of the BEC is almost impossible.

Our prescription for this problem is straightforward. We make the optical trap loose in one direction. By doing this, the size of a BEC gets larger in the direction of loose potential as it can be seen from the formula (2.1). There are two ways for loosening the trap: letting the power of the trapping laser be smaller or the diameter of the one be larger. However, these two finally result in totally different trapping potentials, especially the different trap depths. The qualitative and quantitative discussions are placed in the Section 3.4.
3.1.3 Gravitational Sag and Magnetic Field Gradient

As mentioned later, if there is a potential gradient two different atomic species possibly feel it in a different way. As a result potential minima of the trap for the two atomic clouds can be different. In our experiment the problematic ones are the gravity and an unwanted magnetic field gradient or curvature. The gravity is a special concern because the magnitude of this field gradient is proportional to the mass of an atom and the difference of potential minima due to the gravity is called the gravitational sag. Even when we load a dual BEC in a 1080 nm optical trap with 100 Hz-trapping frequency along the gravity, the gravitational sag is still comparable to the sizes of the BECs and the dual BEC is not overlapped. Our study on an overlapped dual BEC is done by transferring a dual BEC from 1080 nm optical trap to the one with different wavelength of light, the 809 nm optical trap. We mention to this in the Section. 3.4.4.

The problem given just above is about the relative position of a dual BEC along the gravity. Indeed, because of the loosened trap with trapping frequency of around 10 Hz in one of the horizontal direction, even a small field gradient results in the difference of potential minima for each atomic species. A small magnetic field gradient or curvature along the horizontal direction and the tiny tilt of the optical trap with respect to the horizontal plane might be origins of this problem. In Section. 3.5, we can rather take advantage of this effect to compensate this sag in the horizontal direction using the magnetic field gradient by applying an additional pair of coils in the anti-Helmholtz configuration.
3.2 Interaction Between Electromagnetic Field and an Atom

3.2.1 Density Matrix and Optical Bloch Equations

We shall now summarize a basic treatise of atom-light interaction. Formulation written below appears in some popular textbooks[48, 49]. For a given Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$, where $\mathcal{H}_0$ gives atomic energy levels and wavefunctions without light field and $\mathcal{H}_I$ is a perturbation term by the classical light field, we consider a time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi = \mathcal{H} \Psi. \quad (3.1)$$

Two-level treatment allows us to write wavefunction $\Psi$ as a superposition of unperturbed atomic states $|1\rangle$ and $|2\rangle$ with their eigenenergies $E_1$ and $E_2$, respectively. Explicitly writing phase factors $e^{-iE_j t/\hbar} = e^{-i\omega_j t} (j = 1, 2)$,

$$\Psi = c_1(t)e^{-i\omega_1 t}|1\rangle + c_2(t)e^{-i\omega_2 t}|2\rangle$$

and coefficients $c_1$ and $c_2$ satisfy the relation

$$|c_1|^2 + |c_2|^2 = 1.$$

Substituting this expression of $\Psi$ into eq. (3.1), we have a pair of equations

$$i\hbar \frac{dc_1}{dt} = e^{-i\omega_0 t} H_I c_1$$

$$i\hbar \frac{dc_2}{dt} = e^{-i\omega_0 t} H_I^* c_2$$

$$\omega_0 = \omega_2 - \omega_1.$$

Here we let $\mathcal{H}_0$ include diagonal elements of matrix $\mathcal{H}_I$ and denote off-diagonal elements of $\mathcal{H}_I$ as $H_I$. Classical electromagnetic wave as a perturbation can be written as $\mathcal{H}_I = e r \cdot E_0 \cos(\omega t)$ with the electron’s electric charge $e$, the amplitude $E_0$ and the angular frequency $\omega$. Using this expression and rewriting cosine term by exponentials, we get the equations to
solve:

\[
\begin{align*}
\frac{d c_1}{dt} &= \Omega \frac{e^{i \delta t}}{2} c_1, \\
\frac{d c_2}{dt} &= \Omega^{*} \frac{e^{-i \delta t}}{2} c_2, \\
\delta &= \omega - \omega_0.
\end{align*}
\]

In these equations, two important parameters appear. One is the detuning \( \delta \) which is related to the phase evolution of wavefunction. Another is the Rabi frequency \( \Omega \) which is nothing but the oscillation frequency of the populations if the electromagnetic wave is applied to the two level system continuously. This quantity is written as

\[
\Omega = \frac{e < 2 | r \cdot E_0 | 1 >}{\hbar}
\]

which is proportional to the electric field amplitude of the incident light.

Here we introduce a density matrix

\[
\rho = | \Psi > < \Psi | = \begin{pmatrix} |c_1|^2 & c_1 c_2^* \\ c_1^* c_2 & |c_2|^2 \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}
\]

on the pathway to reach at optical Bloch equations. Its diagonal and off-diagonal elements are called populations and coherences for their properties respectively. Setting \( u = \rho_{12} e^{-i \delta t} + \rho_{21} e^{i \delta t} \), \( v = -i (\rho_{12} e^{-i \delta t} - \rho_{21} e^{i \delta t}) \) and \( w = \rho_{11} - \rho_{22} \), and doing some maths we can get equations

\[
\begin{align*}
\dot{u} &= \delta v, \\
\dot{v} &= -\delta u + \Omega w, \\
\dot{w} &= -\Omega v.
\end{align*}
\]

The most frequently used form of optical Bloch equations contains decay terms for the coherences \( u \) and \( v \):

\[
\begin{align*}
\dot{u} &= \delta v - \frac{\Gamma}{2} u, \quad (3.2) \\
\dot{v} &= -\delta u + \Omega w - \frac{\Gamma}{2} v, \quad (3.3) \\
\dot{w} &= -\Omega v - \Gamma (w - 1), \quad (3.4)
\end{align*}
\]
where \( \Gamma \) denotes the full-width-half-maximum (FWHM) or natural linewidth characteristic of an excited state. These decay terms are introduced phenomenologically. Solution of these equations give the dynamics of a Bloch vector \((u, v, w)\) in the presence of the electromagnetic field and frequently used in the analysis of optical transitions such as the Rabi oscillation and the adiabatic rapid passage. To investigate the steady state, we put the left hand sides of equations (3.2), (3.3), (3.4) to zero, namely, \( \dot{u} = \dot{v} = \dot{w} = 0 \). Then optical Bloch equations become coupled algebraic equations that can be solved easily. We focus on the population of the excited state \( \rho_{22} = (1 - w)/2 \) and writing the solution for a stationary state, it reads

\[
\rho_{22} = \frac{1}{2} \frac{\Omega^2/2}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \cdot \frac{I}{I_{\text{sat}}} = \frac{1}{2} \frac{\Omega^2}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \cdot \frac{I}{I_{\text{sat}}} + \frac{I}{I_{\text{sat}}} + \frac{1}{(2\delta/\Gamma)^2}. 
\]

The quantity \( I/I_{\text{sat}} = 2\Omega^2/\Gamma^2 \) is used here, where \( I \) is the intensity of incident light and \( I_{\text{sat}} \) is the saturation intensity of the transition. From this we can see that for fixed natural linewidth, the population of an excited state gets larger as the intensity of light gets larger or the detuning gets closer to zero. The scattering rate \( R_{\text{scatt}} \) is obtained by simply multiplying \( \Gamma \):

\[
R_{\text{scatt}} = \frac{\Gamma}{2} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2}. 
\]

In the frequency domain this has a Lorentzian shape and for large detuning \( \delta \gg \Gamma/2 \) it falls off as \( 1/\delta^2 \).

### 3.2.2 Refractive Index and Phase Shift

We also get the off-diagonal elements of the stationary solution for optical Bloch equations given above. These are expressed as

\[
\rho_{12}e^{-i\omega t} = \sigma_{12}^* = \sigma_{21}^* = -\frac{\Omega}{2} \frac{\delta + i(\Gamma/2)}{\delta^2 + \Omega^2/2 + (\Gamma/2)^2}. 
\]
The atomic polarization $P$ is written as $P = n \text{tr}[\rho \hat{\mu}]$, where $n$ is an atomic density and $\hat{\mu} = \mu(|1 \rangle \langle 2| + |2 \rangle \langle 1|)$ is the dipole moment of an atom. Together with the stationary solution (3.5), This reads

$$P = n \mu \omega \frac{-\delta \cos \omega t + (\Gamma/2) \sin (\omega t)}{\delta^2 + (\Omega^2/2) + (\Gamma/2)^2}$$

On the other hand, atomic polarization can also be written by the complex polarizability $\alpha(\omega)$ as $P = \text{Re}[\alpha(\omega)E_0 e^{-i\omega t}]$. Since the refractive index $n_r$ of dilute atomic gas $n\alpha/\varepsilon_0 \ll 1$ is given by $n_r = \sqrt{1 + n\alpha/\varepsilon_0} \simeq 1 + n\alpha/2\varepsilon_0$, we get the complex refractive index by comparison between two expressions of the polarization, that is,

$$n_r = 1 + \frac{\sigma_0 n\lambda}{4\pi} \frac{-\tilde{\delta} + i}{1 + (I/I_S) + \tilde{\delta}^2}$$

where $\sigma_0 = 6\pi\lambda^2$ with $\lambda = \lambda/2\pi$ is the absorption cross section for the resonant light and $\tilde{\delta} = 2\delta/\Gamma$ is a dimensionless form of the detuning. Note that here we used the expression of natural linewidth

$$\Gamma = \frac{4}{3\pi\varepsilon_0} \frac{k_0^3 \mu^2}{\hbar},$$

which can be derived as the Einstein’s A coefficient in a phenomenological treatment. Thus, a dilute atomic gas is a matter with the complex refractive index $n_r$. Generally the phase velocity of light running through a material becomes slow or fast with respect to that in a vacuum depending on the refractive index of the material is higher or lower than 1. Consider we have a coherent light wave and split it into the two light waves, one passes through the atomic gas of thickness $d$ and the other does not. Then one passes through a gas travels effectively longer than the other by a length $(\text{Re}[n_r] - 1)d$ and this generates a phase shift $\phi$ between two light waves. This is given explicitly as

$$\phi = -\frac{\omega}{c} (\text{Re}[n_r] - 1)d$$

$$= \frac{\sigma_0 \bar{n}}{2} \frac{-\tilde{\delta}}{1 + (I/I_S) + \tilde{\delta}^2}$$

$$\sim \frac{\sigma_0 \bar{n}}{2} \frac{1}{\tilde{\delta}}$$

(3.6)
Figure 3.1: Schematic representation of the phase contrast imaging. Amplitude of the scattered light (red) does not change ((a), left panel) but with the rotation of the unscattered light (blue), the scattering vector (green) and the unscattered light add up so that the phase shift is interpreted into the intensity of light after the interference of the scattered and the unscattered light ((b), right panel). The gray dotted circle shows the curve $E = E_0$.

Here the column density $\tilde{n} = \int n(r)dz$ is introduced. As given above, the refractive index (and phase shift) scales as $1/\tilde{\delta}$ for the large detuning.

### 3.3 Phase Contrast Imaging

#### 3.3.1 Schematics of the Phase Contrast Imaging

As derived above, an absorption rate $\propto 1/\tilde{\delta}^2$ goes faster to zero than the refractive index $\propto 1/\delta$. Therefore for a moderate detuning, it is possible to get a measurable phase shift and to keep the rate of absorption of light negligible. If it is able to measure the phase shift between the unscattered light and the refracted light, the column density of an atomic gas can be
measured non-destructively. However, in general the amount of the refracted light is very small.

To see what happens, it is convenient to consider the electric field $E_0$ in a complex plane. Given the transmission coefficient $t$ and the phase shift $\phi$, the scattered light field is $tE_0e^{i\phi}$ and the scattering vector is $\Delta E = E_0(t e^{i\phi} - 1) \simeq i\phi E_0$. Here we assumed that the detuning is moderately off-resonant and thus $t \simeq 1$ and $\phi$ is small but finite. This means that if we measure the intensity of the scattered light, the information of phase shift is lost at least for first order term $\phi$. In order to extract the phase shift as the intensity of light field, one prescription is to rotate $E_0$ in the complex plane by an angle $\pi/2$ (Fig. 3.1). By doing this, the measured light field and intensity becomes for small $\phi$

\[
E = E_0e^{i\pi/2} + \Delta E = iE_0 + i\phi E_0 = iE_0(1 + \phi),
\]

\[
I = \frac{\epsilon_0 c}{2} |E|^2 = \frac{\epsilon_0 c E_0^2}{2} (1 + \phi)^2 \simeq I_0(1 + 2\phi). \tag{3.8}
\]

The phase shift $\phi$ can thus be detected by the change of the light intensity which is proportional to $\phi$.

What we need to realize this imaging technique, the phase contrast imaging (PCI), is the so-called phase plate which is a round glass plate with a dip at the center, whose diameter is about 100 $\mu$m and the depth is about a few hundreds of nm. By placing this phase plate at the focal point of probe light in the imaging system, unscattered probe light goes through the dip while most of the scattered light passes outside the dip (but inside the glass plate). A properly chosen depth of the dip shifts the phase of unscattered light wave by $\pi/2$. In other words, optical path of the unscattered light wave is shorter than the scattered one by $\lambda/4$. Our phase plate has a dip with the depth of 423 nm and the diameter of 140 $\mu$m[90], which causes $\pi/2$-phase-shift for 767 nm-wavelength light. This is designed to detect the potassium.

---

1 Indeed there is an imaging method which is called the dark-field imaging where the unscattered light is blocked at its focus and only the scattered light is collected at the screen to acquire signals proportional to the second order in $\phi$. 

35
Figure 3.2: Schematics of the side imaging system. The red area represents the probe beam and the gray shaded area does the shadow of atoms. All lenses are 2-inch, achromatic doublet lenses.

Figure 3.3: Schematics of the top imaging system. The red area represents the probe beam and the gray shaded area does the shadow of atoms. All lenses are 2-inch, achromatic doublet lenses, except for the lens with focal length 500 mm, which is 1-inch, achromatic doublet lens.
D2-line but also works almost the same for the rubidium D2 line whose resonant wavelength is 780 nm since this difference of wavelengths is sufficiently small. In Figs. 3.2 and 3.3, experimentally used two imaging systems and the place where the phase plate is introduced are indicated. One is called the side imaging which probe light is incident in a horizontal direction and the other, top imaging, is in a direction of the gravity.

For top imaging, the numerical aperture (NA) is designed[91] to be 0.13 and a measurement of the magnification revealed that the magnification is 4.64. We use a charge coupled device (CCD) camera with a pixel of dimensions $16 \, \mu m \times 16 \, \mu m$, therefore 1 pixel in the top imaging corresponds to $3.451 \mu m$. The side imaging is somewhat complicated. Though the numerical aperture and the magnification are designed to be 0.11 and around 4, measured magnification is 6.15 (1 pix. = 2.58 $\mu m$). This is probably because the side imaging path is partially the same as MOT beams and probe beam passes through a 2-inch polarizing beam splitter (PBS). This large optics surely causes the non-negligible variation of rays and magnifications. The resolution power $d_r$ of each imaging path is estimated with 780 nm-probe light to be $3.8 \, \mu m$ for the top imaging and $4.4 \, \mu m$ for the side imaging, respectively, by using the Rayleigh’s criterion $d_r = 0.61 \times \lambda / NA$. Phase plate is placed at the focal point of probe beam in each imaging system, however, being compatible with the absorption imaging, the phase plate is mounted on a flipper.

### 3.3.2 Probe Lasers for Phase Contrast Imaging

Remaining tools for the PCI are off-resonant probe lights for each atomic species. We can calculate the phase shift by a formula (3.6). Since the trap frequencies and the number of atoms are already known and cannot be changed easily, we tune $\delta$ so that we can get detectable phase shifts and negligible photon scattering events. As we describe later in the next section, typical trap frequencies are $(40, 7, 100)$ Hz and combined with the typ-
ical number of atoms $3 \times 10^4$, the measurable phase shift around 0.1 rad is available with the detuning of a few hundred MHz. For this range of detuning, the photon scattering probability by single probe pulse (300 µs duration, which is determined by exposure time of a single strip of CCD camera) becomes at most a few per cent. Below we consider the frequency of atomic transition in the presence of external field.

In order to determine the frequency of the probe light, we should note:

- we use a magnetic Feshbach resonance between rubidium and potassium both in $|F = 1, m_F = 1 \rangle$ state. The magnetic field bias is around 78 G and the atomic ground state and the excited state experience the linear and quadrature Zeeman shifts.

- because the probe beams for the side imaging passes through the optics for MOT, their polarization become $\sigma^-$. Thus the excited state we use is limited to the one we can reach from the $|F = 1, m_F = 1 \rangle$ state with $\sigma^-$ light. We selected the $|F = 0, m_F = 0 \rangle$ for an excited state, which is the lowest-energy excited state among the $^2P_{3/2}$ states and we use red-detuned lasers. The reason why we choose the lowest-energy state is that we need to avoid other magnetic sublevels having influences on the target transition.

Then in order to set the probe frequency, we should know Zeeman shifts of the $|F = 1, m_F = 1 \rangle$ in the ground state and the $|F = 0, m_F = 0 \rangle$ in the excited state. Atomic properties needed to calculate the energy shifts are listed in Appendix A. Ground state energy shift is easily calculated by the Breit-Rabi formula and the problem is the excited state. The Breit-Rabi formula is an analytical solution obtained by diagonalizing the interaction Hamiltonian and in principle this procedure is also applicable to the excited state. However, in the case of excited state we have to diagonalize one $4 \times 4$ matrix, two $3 \times 3$ matrices and two $2 \times 2$ matrices, though in the case of ground state three $2 \times 2$ matrices appear.

So it is easy to let Wolfram Mathematica evaluate the matrices and get
the numerical values of frequency shifts vs. the magnetic field. For example, the evaluated values of frequency shifts of $^{41}$K are plotted in Fig. 3.4. Those for $^{87}$Rb can be calculated in the same way (Fig. 3.5). For potassium atom, hyperfine constant of the excited state is so small that a diagonalization by the $|m_I, m_J>$ bases also works very well.

Let $\nu_0^K$ and $\nu_0^{Rb}$ denote the master laser for potassium (locked to F=1 to (F=1 and F=2) cross over of $^{39}$K D2-line) and repump laser for rubidium (locked to F=1 to (F=1 and F=2) crossover of $^{87}$Rb D2-line). If we set the detuning from the transition $|F = 1, m_F = 1 \rightarrow |F = 0, m_F = 0$ as 200 MHz, we need the lasers with frequencies roughly $\nu_K = \nu_0^K + 80$ MHz and $\nu_{Rb} = \nu_0^{Rb} - 400$ MHz, respectively. As we mentioned above, these frequencies are easily available by using acousto-optical modulators (AOMs).

Schematic of the PCI probe lasers is shown in Fig. 3.6. In short, two seed lasers are amplified by injection locking and detuned by AOMs. By inserting an RF switch (mini-circuits, ZYSWA-2-50DR) after the RF signal sent from voltage controlled oscillator (VCO) to RF amplifier, we can turn on probe lasers by an external TTL signal from Labview program and suppress the leakage of RF signals to AOMs which causes the unwanted light scattering during the imaging sequence. Two probe lasers are coupled to the optical fiber after the non-polarizing beam splitter (NPBS). With this setup, we can tune the detuning from 200 MHz to 300 MHz.
Figure 3.4: Calculation of Zeeman shifts for K D2 line ($^2S_{1/2} \rightarrow ^2P_{3/2}$). Red lines are the ground state $|F = 1, m_F = 1 >$ and the excited state $|F = 0, m_F = 0 >$. A red upright arrow represents the required probe light when the detuning is 200 MHz.
Figure 3.5: Calculation of Zeeman shifts for Rb D2 line $^{2}S_{1/2} \rightarrow ^{2}P_{3/2}$. Red lines are the ground state $|F = 1, m_F = 1 \rangle$ and the excited state $|F = 0, m_F = 0 \rangle$. A red upright arrow represents the required probe light when the detuning is 200 MHz.
Figure 3.6: Schematic representation of the preparation of probe lasers for the PCI. Seed lasers for K(master laser, locked to F=1 to (F=1 and F=2) cross over of $^{39}\text{K}$ D2-line) and Rb(repump laser, locked to F=1 to (F=1 and F=2) crossover of $^{87}\text{Rb}$ D2-line) are amplified by injection locking and detuned by AOMs. Two probe lasers are coupled to the optical fiber after the non-polarizing beam splitter(NPBS). The frequencies shown next to AOMs are the center frequencies of them.
3.3.3 Generating Pulse Trains for Probe Lights

In this section, the detailed experimental setup which realizes the phase contrast imaging of a dual species atomic cloud is introduced. Here we describe a method to generate TTL pulses from Labview, which controls the overall experimental sequences, combined with function generators. Actual observation of a dual BEC using instruments shown below is described in the next chapter.

Let us consider the number of images we take in a single sequence. Of course the larger the number of images, the better the imaging works as a “movie”. However, we want to take all images in one frame of CCD camera (row \( \times \) column is 1048 pix. \( \times \) 512 pix.) and the size of the BEC is at most 150 \( \mu m \) which corresponds to 60 pixels if seen by side imaging. Considering these, we divide one frame into ten regions\(^2\) (row \( \times \) column is 104 pix. \( \times \) 512 pix.) and assign five regions for the probe of each atomic species. Then we need five pulses for each species.

For a single species, it is sufficient to send pulse train with desired intervals to take images one after another. However, we need to take a pair of images for Rb and K without delay while the separation between pairs should be tunable (Fig. 3.7, gray shaded area).

In order to realize this, We constructed a system for the TTL pulse generation just summarized in Fig. 3.7. The reason why we generate TTL pulse trains not by digital TTL signal using Labview (National Instruments), but by these instruments is that if we make pulse trains only with Labview, it takes \((4 \text{ time divisions}) \times (5 \text{ pairs of images}) \times (3 \text{ sets: shadow, light and dark}) = 60 \text{ time divisions. Actually it will overload the PC.}\)

Using function generator 1 (Rigor, Function/Arbitrary Waveform Generator DG4062. FG1 in a shorthand), we can output 5 pulses with an arbitrary interval \( \tau \). One may think that signals from FG1 can be used for the TTL of K probe and three function generators are excess. However, it was observed

\(^2\)In both the top imaging and the side imaging, the elongated BEC appears with its long axis in the column and the number of images taken in a single sequence is limited to 10.
Figure 3.7: Instruments for TTL pulse train generation. Function generator 1 (Rigor, Function/Arbitrary Waveform Generator. FG1 in a shorthand) can output 5 pulses with an arbitrary interval $\tau$ and pulse duration 300 $\mu$secms. The pulse width 300 $\mu$s is determined by the exposure time of a single strip of the CCD camera (Prinston Instruments, PhotonMAX 512), and the Rb probe pulse should be send 300 $\mu$s from FG3 after the end of the K probe pulse from FG2, which is determined by the frame transfer speed of the CCD camera. TTL pulses for CCD camera are generated by taking OR signal of probe pulses for K and Rb.
that for a long interval $\tau > 100$ ms, the leading shape of the pulse starts to get distorted and it did not work correctly. So send the TTL signals to K using FG2 (Stanford Research Systems, DS345) triggered by FG1. The pulse width 300 $\mu$s is determined by the typical acquisition time of a single strip of the CCD camera (Prinston Instruments, PhotonMAX 512), and the Rb probe pulse should be sent 300 $\mu$s after the end of the K probe pulse, which is determined by the frame transfer speed of the CCD camera. TTL pulses for the CCD camera are generated by taking OR signal of probe pulses for K and Rb.

Thus we can activate the phase contrast imaging sequence by only three TTL pulses from Labview. Indeed we had to rewrite the Visual Basic 6.0 program code for acquiring the images but it is too messy to describe here.

### 3.4 Setup of Optical Dipole Traps

Optical dipole trap is very widely used in atomic physics experiments. It makes use of the refractive index of atoms in the presence of a high-power, far-off-resonant laser. As we described above, off-resonant light can be refracted by an atom by acquiring a momentum $\hbar \mathbf{k}'$ mostly orthogonal to the direction of an original wavevector $\mathbf{k}$. Then the action-reaction law (or, Newton’s third law of motion) suggests that the atom must be kicked in the opposite direction to acquire the momentum $-\hbar \mathbf{k}'$. In other words a photon feels some force according to the spatial density of atoms and at the same time an atom feels force by off-resonant light whose distribution directly reflects the spatial distribution of the laser beam. This force is called optical dipole force and the trapping using this force be the optical dipole trap (ODT). Below we describe the basics of ODT and the designs of ODT used in our experiment. A review Ref. [94] is useful to calculate the trapping potential for a target atom in practice.
3.4.1 Light Shift

Here the perturbation considered is also the electric field and its influence on the dipole moment of an atom. Energy shift $\Delta E$ is given as a scalar product of these two quantities:

$$\Delta E = -\frac{1}{2} \langle P \cdot E \rangle = -\frac{1}{2} \text{Re}[\alpha]|E|^2.$$  

$\langle \cdot \rangle$ represents a temporal average. Because an explicit form of $\text{Re}[\alpha]$ is available, we can write down the energy shift by light field in a detuning-dependent form, that is, with the intensity of light $I(r)$,

$$\Delta E = -\frac{3\pi c^2}{2\omega_0^3} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(r)$$

$$\approx \frac{3\pi c^2 \Gamma}{2\omega_0^3} \frac{\delta}{I(r)}.$$  

Indeed, there are quantum states other than two states considered above, e.g., the fine and the hyperfine structures of atoms. The corrections come from this fact and results in the formula[94]

$$\Delta E = -\frac{\pi c^2 \Gamma}{2\omega_0^3} \left( \frac{2 + P g_F m_F}{\delta_{D2}} + \frac{1 - P g_F m_F}{\delta_{D1}} \right) I(r)$$

especially in the case of considering D1 and D2 lines of alkali atom. Here $P$ takes the values of 0 (linear pol.), +1 ($\sigma^+$), and −1 ($\sigma^-$), $g_F$ is a g-factor, $m_F$ is a magnetic quantum number. This formula gives the optical dipole potential felt by atoms and as easily deduced from its dispersive origin, potential depth of ODT scales as $\propto 1/\delta$. Furthermore, if a red(blue)-detuned laser is applied, an atom is attracted(pushed out) by the laser beam. Another important point is that the dipole potential is proportional to the light intensity. For example, given the light intensity as

$$I(r) = I_0 \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2} \right)$$

$$\approx I_0 \left( 1 - \frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2} \right),$$  

(3.9)
trapping potential can be written in a quadratic form

\[ V(r) = \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2 + \frac{1}{2} m \omega_z^2 z^2 \]  

(3.10)

in the vicinity of the center of the laser beam. Here trapping frequencies \( \omega_i \) (\( i = x, y, z \)) contain almost all information of the trapping potential and are related to the beam profile, the light intensity and the detuning.

3.4.2 Experimental Sequences

We use 1080 nm fiber laser (NP Photonics) amplified by fiber amplifiers (Nufern) for red-detuned optical trap. To explain the roles of 6 laser beams used in our experiment, we will briefly itemize experimental sequences below:

- First atomic gases of Rb and K are gathered and cooled by magneto-optical trap (MOT) and further cooled down to several tens of \( \mu K \) by a compressed MOT (CMOT). The number of atoms are roughly from \( 10^8 \) to \( 10^9 \) for Rb and \( 5 \times 10^6 \) for K.

- Succeedingly both atomic gases are pumped to the \( |F = 2, m_F = 2 > \) hyperfine state and cooled by forced evaporative cooling in the magnetic trap to reach about \( 1 \mu K \) with \( 10^6 \) rubidium atoms and \( 10^5 \) potassium atoms. In transferring atoms into the magnetic trap, we compress them first to get higher spatial density \( \sim 10^{11} \text{ cm}^3 \) for the sake of an efficient evaporation. Hereafter rubidium atoms plays the role of coolant which sympathetically cool potassium atoms.

- Then these pre-cooled atomic clouds are cold enough to load into a crossed dipole trap made of (A) beam and (P) beam (described later), each has a power of 0.55 W and 2.0 W at most. Then rubidium and potassium are transferred into the \( |F = 1, m_F = 1 > \) hyperfine state by adiabatic rapid passages (ARPs). By this moment the magnetic field bias for the Feshbach magnetic field should be ramped up to 72 G in
order to avoid unwanted lossy collisions due to the three body recombination and prepare for the evaporative cooling in an optical dipole trap.

• By setting the bias field at 77.3 $G$ which makes the interspecies scattering length $a_{\text{KRb}}$ to be $230 \ a_B$ and starting ramping down the power of ODT (P), atomic clouds are further evaporatively cooled below BEC transition temperature. Finally we get dual BEC with about $3 \times 10^4$ atoms for both species and soon after this step various experiments are to be done using 809 nm optical trap(described later), optical lattices((V), (B), (C) beams) and so on.

In this thesis the setup is not changed but the role of each beam is partly changed. First we introduce the original roles of optical traps and later we will mention the part changed for this thesis.

### 3.4.3 Crossed ODT and Optical Lattice

In Fig. 3.8, glass cell viewed from the top and two optical dipole traps (A) beam and (P) beam are illustrated. The (P) beam has an horizontally flattened shape (465 $\mu$m and 164 $\mu$m in diameters), and this beam hold the atoms against the gravity. In the sequence of an evaporative cooling, the power of the (P) beam is lowered. Another beam, (A) beam shaped round with diameter of 150 $\mu$m, is aligned orthogonally to (P) axis and confining atoms along the $y$-direction. Final trap frequencies after the evaporation is roughly $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (40, 40, 100)$ Hz. Each axes ($x$, $y$ and $z$) are defined as in Fig. 3.8.

We have other 1080 nm laser beams for three dimensional optical lattice, (V), (B), and (C) beams(Fig. 3.9). Optical lattice is formed by retro-reflecting three beams of 600 $\mu$m diameter and round shape. In this thesis optical lattice is not used but one of them, (B) beam, is used for another purpose and the detail will be given later on.
Figure 3.8: Schematic representation of the configurations of ODT (A) and (P). Orange circle and ellipse in green shaded regions corresponds to the shape of trapping beams.
Figure 3.9: Configurations of optical lattice beams. All beams are retroreflected by dichroic mirrors. The shape of (V) beam is all the same as that of (B) and (C).
3.4.4 Gravitational Sag and 809 nm ODT

The last, 6th beam is most important in this thesis. Before introducing this trapping beam, let us assign several lines for describing so-called the ”gravitational sag” in an optical trap. The idea is that if we make a trapping potential $(1/2)m\omega^2x^2$ along the direction of the gravity, total potential becomes

$$V(z) = \frac{1}{2}m\omega^2z^2 + mgz$$

$$= \frac{1}{2}m\omega^2(z + \frac{g}{\omega^2})^2 + \text{const}$$

due to the gravity. Here acceleration of gravity $g = 9.80665\text{m/s}^2$ is a constant(this value is the definition of the standard gravity). We can easily see the potential minimum is shifted downwards by $g/\omega^2$. This is the gravitational sag and obviously it depends on the trapping frequency. Actually we use rubidium and potassium atoms and trap them in a 1080 nm-ODT simultaneously. Although this optical dipole trap offers the almost same trapping potentials $U(z) = (1/2)m_{Rb}\omega_{Rb}^2z^2 = (1/2)m_K\omega_{Rb}^2z^2$ for two atomic species, the different masses of Rb and K result in the different trapping frequencies.

Now the problem with an 1080 nm dipole trap is clear: due to different trapping frequencies, the potential minima for rubidium and potassium are different from each other. For instance, final power of (P) beam after the evaporation is about 1.6 W and this gives trapping frequencies 100 Hz for Rb and 135 Hz for K. This makes the potential minima different by roughly $10\ \mu m$ and is larger than the Thomas-Fermi radius of BEC, which is $4\ \mu m$ or less for each atomic species. Two BECs does not overlap at all in this trap.

In order to make them overlap, one way is to tighten the trap by increasing the power. But this method needs an order of magnitude larger laser power to realize the relative gravitational sag to be less than $1\ \mu m$. Another way of tightening the trap is to make the radius of a trap beam smaller. These are relevant, however, it can be said that even if we succeed in reducing the relative gravitational sag to be as small as $1\ \mu m$ by doing so, the Thomas-Fermi radii of BECs also get smaller and it does not profit so much in the respect of the overlapping region of a dual BEC. Larger three-body loss due
to the higher spatial densities is also problematic. Our solution is to use a laser beam of "magic" wavelength for the relative gravitational sag. What we want to realize is

\[ \frac{\omega_{Rb}}{\omega_K} = \sqrt{\frac{V_{Rb}}{V_K} \frac{m_K}{m_{Rb}}} = 1. \]

Then if we set the detuning of a laser so that \( V_{Rb} / V_K = m_{Rb} / m_K \) is satisfied, trapping minima of each species are always the same, regardless of the incident power of a laser. Calculation in detail shows that fortunately our "magic" wavelength exists and is 808.8 nm. We use this optical dipole trap (this beam is also named (P) because 809 nm beam joins the same optical path as 1080 nm ODT via dichroic mirror) in order to do experiments with spatially overlapped BECs. This 809 nm laser beam is also flattened similar to the 1 µm (P) beam, whose diameters are horizontally 270 µm and vertically 150 µm.

In summary, for dual BEC experiments we first generate a dual BEC in 1 µm crossed ODT, where two BECs stay at different places. Then we load the dual BEC into an 809 nm dipole trap to bring two species into the same place. After that we do experiments with that.

### 3.4.5 Setup for Generating Elongated BECs

As mentioned in 3.1.2, we want to generate elongated BECs in order to get in-situ images of atomic clouds. However, Thomas-Fermi radii of a BEC under trapping frequencies \( (\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (40, 40, 100) \) Hz are approximately \( (R_x, R_y, R_z) = (10, 10, 4) \) µm with \( 3 \times 10^4 \) atoms (there is no significant difference between Rb and K). Therefore one solution is to make a cigar-shaped trap by loosening the trap in one direction. But if we simply ramp down the (A) beam and lower the trapping frequency in y direction, it will be problematic that trap depth gets low and atoms pour out of the trap. This circumstance is qualitatively explained by considering the formulae (3.9) and (3.10). From these two expressions we can easily see that the
Figure 3.10: Modified evaporation sequence. We ramp ODT (B) up in the early stage of the evaporation and ramp ODT (A) down slowly in case this ramping causes the oscillation of a BEC. Total time of this sequence is 2000 ms.
Figure 3.11: Crossed dipole trap with ODT (P) and (B). (B) beam is originally used for optical lattice and here we use this as a dipole trap by simply blocking the retro-reflected beam.
Figure 3.12: Calculation of potential surface by crossed dipole trap with ODT (P) and (B). Power of (B) beam is 0.5 W.

trapping frequency $\omega$ is proportional to the quantity $\sqrt{P_{ODT}/w^2}$ and the trap depth $k_B T_{trap}$ is proportional to $P_{ODT}/w^2$, from which it follows that by lowering the power of ODT, the trap depth becomes too shallow before we get the sufficiently small trapping frequency. Actually by lowering the power of the (A) beam to 0.005 mW the trapping frequency is 11.2 Hz, whereas the trap depth is $k_B \times 74 \text{ nK}$. So we need a low trapping frequency with a larger trap depth (higher laser intensity), namely the trapping beam with moderately large diameters. If we prepare a trap beam with its diameter 600 $\mu$m, the power of 1.3 W gives the trapping frequency 11.4 Hz and the trap depth $k_B \times 1.2 \text{ $\mu$K}$ simultaneously.

According to above considerations, the solution to fulfill this requirement with our setup is to use the (B) beam, which is originally designed for an optical lattice, without the retro-reflected beam. As illustrated in Fig. 3.11, (B) beam is aligned at angle 45 degree with (P) beam. We can think this as a beam with diameter $\sqrt{2} \times 600 \mu$m is applied in $y$ direction. Then the calcu-
Figure 3.13: Illustration of a tilted optical dipole trap and a plot of relative position with respect to the angle $\theta$. Trapping frequencies $(\omega_{Rb}^y / 2\pi, \omega_{K}^y / 2\pi) = (5.4, 7.2)$ Hz are used in the calculation.

ulation indicates that 0.5 W of (B) beam forms a trapping potential with 5.4 Hz trapping frequency (for Rb) and $k_B \times 1.2 \mu K$ trap depth in $y$ direction (see Fig. 3.12). Ramping up (B) in the early stage of evaporation and ramping down (A)(see Fig.3.10), an elongated dual BEC in crossed ODT (P) and (B) is realized. For this trap Thomas-Fermi radii of BEC are $(R_x, R_y, R_z) = (7.5, 33, 2.6)$ $\mu m$.

3.5 Gravitational Sag and Magnetic Field Gradient

3.5.1 Gravitational Sag in the Horizontal Direction

Gravitational sag along the gravity is discussed above. However, the gravity also have an influence on the horizontal position of the potential
minimum when the trap beam is tilted with respect to the horizontal plane ($xy$ plane in Fig.3.8) as illustrated in Fig.3.13. Actually, if the (P) beam is aligned at angle $\theta$ with $xy$ plane, the trapping potential becomes

$$V(z) = \frac{1}{2} m \omega_y^2 y^2 + mg y \sin \theta$$

$$= \frac{1}{2} m \omega_y^2 \left( y + \frac{g \sin \theta}{\omega_y^2} \right)^2 + \text{const.} \quad (3.11)$$

Here, for simplicity we assumed that the beam is tilted only in the $yz$ plane.

Though we use an 809 nm ODT for the (P) beam and the relative sag in $z$-direction is compensated, the (B) beam gives different trapping frequencies in $y$-direction between Rb and K for its wavelength 1080 nm. Trapping frequencies ($\omega_{Rb}^y / 2\pi, \omega_K^y / 2\pi$) = (5.4, 7.2) Hz and an angle $\theta = 1$ degree give the difference of the potential minima in the $y$-direction of about 64 $\mu$m.

### 3.5.2 Effect of Magnetic Field Gradient on the Potential Minimum

Even if we have a perfectly aligned trap beam, namely $\theta = 0$, another external field gradient for which Rb and K have different susceptibilities can cause all the same problem discussed in the Section 3.5.1. The most likely candidate for this external field is the magnetic field.

Actually we use a pair of coils which generates the bias magnetic field 75 G or more, enabling us to make use of a Feshbach resonance. Of course the magnetic field curvature is present and it is revealed experimentally that the position of atoms in an ODT is different from the center of this quadratic magnetic field. Therefore an atom feels the magnetic field gradient and since the magnetic susceptibilities of Rb and K differs from each other, the relative position can be different for all the same reason with Section 3.5.1.

In order to discuss in an explicit form, we rewrite potential (3.11) in a form with the term containing mass replaced by a term containing magnetic
susceptibility:

\[
V(z) = \frac{1}{2}m\omega_y^2y^2 + \mu g_J m_J B' y
= \frac{1}{2}m\omega_y^2(y + \frac{\mu g_J m_J B'}{m\omega_y^2})^2 + \text{const.}
\]

Coefficient \(\mu\) is the magnetic susceptibility, \(g_J\) the g-factor and \(m_J\) the magnetic quantum number. Since the atomic spins are oriented to \(x\) direction and the optical trap is loose only in the \(y\) direction, the problematic magnetic field gradient \(B' = \frac{\partial B_x}{\partial y}\) is considered here. Given the magnetic susceptibilities \(\mu_{\text{Rb}} = 0.7\ \text{MHz/G}\) and \(\mu_{\text{K}} = 1.1\ \text{MHz/G}\), the potential minima are separated by several \(\mu\text{m}\) in the presence of 1 G/m field gradient.

Above we see that the magnetic field gradient makes the relative position of potential minima different. It is true, but in other words we can compensate the horizontal sag due to any field gradient by applying magnetic field gradient. However, usual setup of the anti-Helmholtz configuration of a pair of coils makes \(\frac{\partial B_y}{\partial y}\) and does not generate \(\frac{\partial B_x}{\partial y}\). We can obtain non-zero \(\frac{\partial B_x}{\partial y}\) by putting a pair of coils with their axes off just as illustrated in Fig.3.14(a). With these we can make more than 20 G/m magnetic field gradient(see Fig.3.14(b)) by injecting several Amps of current and can compensate the separation of atomic clouds up to a few tens of \(\mu\text{m}\).
Figure 3.14: Illustration of the configuration of compensation coils and its magnetic field/field gradient. (a): Configuration of coils. The red cloud represents the atoms. (b): Magnetic field gradient $\frac{\partial B_x}{\partial y}$ by these coils (36 turns) with current 4 A and is plotted. (c): Magnetic field distribution is shown in the vicinity of the position of atoms.
Chapter 4

**IN-SITU OBSERVATION OF A DUAL BEC**

Chapter 2 and 3 are devoted to the explanation of basic properties of a dual BEC and major experimental challenges. Up to here three major solutions have been introduced: non-destructive imaging of a dual BEC, generation of an elongated dual BEC in ODT, and the compensation of the difference of potential minima due to field gradients. We show how to implement these solutions in this chapter and we can see a fundamental nature of a dual BEC at the end.

In this chapter we consider two topics: first, the *in-situ* observation of a dual BEC in an 1μm optical dipole trap is described. Second, a dual BEC is loaded into an 809 nm dipole trap and we observe non-destructively the famous miscible/immiscible phase. In addition to the observation of static properties, that of a dynamical motion of a dual BEC in an immiscible phase is given.
4.1 Creation of an Elongated Dual BEC in an ODT

4.1.1 Time-of-Flight Measurement

In this section we observe a dual BEC by time-of-flight (TOF) imaging, which is one of the most common methods. In this method, the atomic cloud freely expands after the release from the trap and 30 ms later the resonant probe light shines the cloud. Measuring the fractional decrease of the probe light intensity at each position, we know the density of the cloud after the expansion. Since the atomic gas freely expands, observed density distribution is a momentum distribution in the trap for a sufficient expansion time.

Observed shape of an atomic cloud after long TOF differs between a thermal cloud and a BEC. For the thermal cloud, kinetic energy is equally divided for three directions and the round cloud shape should be observed. On the other hand, the cloud shape of a BEC after the TOF is not necessarily round, but rather elliptic depending on the anisotropy of the trapping potential. A condensate consists of macroscopic number of atoms in the ground state of the trap and its momentum distribution depends on the shape of the trap and directly reflects the anharmonicity of the trap. As you can easily check, a tight (loose) confinement gives a sharp (blunt) spatial distribution and its derivative, namely the momentum distribution is broad (narrow). If we neglect the interaction between atoms, this is the reason for the anisotropic shape of a released BEC. Actually atoms interacts with each other. When atoms interacts repulsively with each other, atoms expands due to this repulsion and acquire about three times larger velocity than that of a motion in a harmonic potential. Since the column density is largest in the direction of the most tight potential, atomic cloud expands fastest in this direction. As a result we can observe a BEC wide in the direction of tight confinement and narrow in that of loose one.
Figure 4.1: Time sequence of TOF and absorption imaging.
4.1.2 Elongated Dual BEC in a New Crossed ODT

First of all we have to create BECs in the optical trap introduced in Section 3.4 and to detect it by an absorption imaging after a TOF. In the new setup of the ODT ((B)+(P), see Fig. 3.11), trapping frequencies are roughly \( (\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (40, 7, 100) \) Hz. As mentioned earlier in the Section 3.4, we ramp (A) beam down to zero and ramp up (B) beam. At the end of the evaporative cooling we turn trap lasers off and release atoms. At this moment atoms are in the \( |F = 1, m_F = 1> \) state and in order to make use of the closed transition \( F = 2 \rightarrow F' = 3 \), weak repump beams resonant on \( F = 1 \rightarrow F' = 2 \) are shined for 100 \( \mu s \) to pump atoms in the \( F = 2 \) hyperfine manifold. Schematics of the TOF and the absorption imaging process are illustrated in Fig. 4.1.

Thus we can observe a dual BEC generated in 1080 nm, new setup of crossed optical dipole trap. Obtained images of BECs are shown in Fig. 4.6 (b), where the number of atoms are \( 3.4 \times 10^4 \) for K and \( 4.6 \times 10^4 \) for Rb. Nearly round shape of the cloud implies that 30 ms of free expansion is not sufficient to see the anharmonicity of the trap. It is verified by changing the expansion time of TOF that initially elongated cloud expands faster in \( x \) direction than in \( y \). For comparison, thermal clouds of \( 6.4 \times 10^4 \) K and \( 1.8 \times 10^5 \) Rb with temperatures 87 nK and 114 nK is shown in Fig. 4.6 (a). It is easy to discriminate BEC from thermal cloud by their optical densities(OD), where \( OD \sim 1 \) for BEC and \( OD \sim 0.1 \) for thermal cloud for \( 4 \times 10^4 \) atoms, while the emergence of a BEC component is obvious from the bimodal distribution. The narrow width of the expanded cloud(less than a half of that of the thermal cloud) is also a comprehensive feature of BEC.
Figure 4.2: Absorption image of (a) the thermal clouds of $6.4 \times 10^4$ K and $1.8 \times 10^5$ Rb and (b) the dual BEC of $3.4 \times 10^4$ K and $4.6 \times 10^4$ Rb produced in 1 $\mu$m ODT.

4.2 In-Situ Observation of a Dual BEC

4.2.1 Probe lasers for PCI and Phase Plate

We succeeded in generating BECs of Rb and K simultaneously in a loose 1 $\mu$m ODT above. Next thing to do is the in-situ observation of a dual BEC using instruments and sequences prepared in Chapter 3. Before describing results of the observation of atoms, we mention actual parameters including the intensity, the beam radius and the optical path, and the alignment of the phase plate.

First, two colors of 100 $\mu$W probe lights are coming out from the same single-mode optical fiber in Fig. 4.3. The collimation lens with focal length 10 cm is placed 10 cm or less away from the edge of the fiber core. The diameter of a collimated beam is measured to be 15.95 cm by the knife-edge method. This passes through a lot of optics such as the 2-inch PBS (for the compatibility with MOT), the dichroic mirror (for the compatibility
Figure 4.3: Optical path of probe beam before the glass cell. The probe beams coming out of the single-mode optical fiber are collimated by the achromatic doublet lens. Joining the same as a MOT beam and the ODT (A) beam, Probe beams finally reach the glass cell.

Figure 4.4: Optical path of probe beam after the glass cell. A MOT beam and the ODT (A) beam are separated from probe beams by PBS and the dichroic mirror respectively. Focused probe beams go through the phase plate at the focal point.
Figure 4.5: Picture of optical path of probe beam after the glass cell.
Figure 4.6: Schematic representation of focused Gaussian beam through a lens with focal length $F$.

with ODT(A) beam), and so on.

After the glass cell, the probe beam again passes through the 2-inch PBS for MOT and then goes into the imaging optics (Fig. 4.4). Because the ODT (A) beam, which goes almost the same path with the probe light, is dangerous for the CCD camera, three dichroic mirrors are inserted after the glass cell (one of them can be seen in Fig. 4.4). The phase plate is placed at the position of the focal point of the probe beam after the dichroic mirror. It is ideal for PCI that most of the focused probe beam goes through the hole at the center of the phase plate. Gaussian beam optics says that when collimated beam with radius $w_0$ is focused by a lens of a focal length $F$, the resulting radius $w_1$ at the focus is calculated by the following formula [50]:

$$w_1 = \frac{F}{z_0} \sqrt{1 + \left(\frac{F}{z_0}\right)^2} w_0,$$  \hspace{2cm} (4.1)

where $z_0 = \frac{\pi w_0^2 n}{\lambda}$ is the Rayleigh length. Here $n$ is the refractive index of the medium and $\lambda$ is the wavelength of light. Given the beam diameter 15.95 mm and regarding two $F = 250$ mm lenses as one $F = 125$ mm lens, the beam diameter at the beam waist is calculated to be 2 $\mu$m. Actually, the beam diameter is not collimated to such a small diameter but only about 100 $\mu$m (measured by beam profiler) and this can fall within the range of hole of diameter 140 $\mu$m. Remaining task for realizing PCI is the precise
alignment of the phase plate. We mount the phase plate on the $xyz$-axes stainless kinematic stage (SIGMA KOKI) which is equipped with micrometers in three directions. When a part of the probe beam hits the hole, an Airy fringe shows up and if the beam is completely aligned inside the hole, this fringe vanishes. Therefore the procedure is summarized as follows: at first we put phase plate off the focus and observe the fringe. Second we adjust the phase plate at the position of focus by moving it so that the fringe interval gets larger. When the fringe vanishes we move phase plate in transverse directions to the probe beam back and forth and check whether we can see fringes in both directions. If these are observed, the alignment is perfect.

### 4.2.2 Phase Contrast Imaging of a Dual BEC

Preparation for the phase contrast imaging is completed by now. We then proceed to the description of an *in-situ* observation of a dual BEC. Because we are now interested in taking a “movie” of atomic clouds, the experimental sequence must be changed. In contrast to the imaging sequence shown in Fig. 4.1, in which the optical trap and the Feshbach magnetic field are turned off at the beginning of imaging sequence, for PCI we only have to hold them at constant values at the end of the sequence. Applying the probe pulse trains just as in Section 3.3.3 during the hold time, we can realize PCI. Schematic imaging sequence is depicted in Fig. 4.7.

Observed dual BEC is shown in Fig. 4.8. From the top to the bottom successive images of K and Rb are shown in chronological order. The images are constructed and displayed using the C++ program (Appendix. F) and the plot region can be changed by tuning the parameters in the source code. Total number of atoms calculated from the images are $7.3 \times 10^4$ for K and $5.6 \times 10^4$ for Rb, which are consistent with the result obtained from the absorption imaging. The more important thing is that we can image BECs

\[^{1}\text{Tenth image is always shifted in the right in the image by 11 pixels and this is always reproduced, so that we shift the tenth image to the left by 11 pixels. Consequently there are not any signals in the small region nearby the right end.}\]
Figure 4.7: Schematics of imaging sequence of PCI. Total time of this sequence is varied due to the interval $\tau$.

over and over again without apparent heating or loss of atoms. As predicted by the formula for gravitational sag, Rb cloud sits just beneath the K cloud (Fig. 4.8 (a)). Here we succeeded in the nondestructive observation of dual BEC.
Figure 4.8: Observation of dual BEC generated in 1 µm ODT by PCI. Field of view of each image is 51 µm × 206 µm. The color bar indicates the phase shift in radian.
4.3 Spatially Overlapped Dual BEC in 809 nm ODT

4.3.1 Cancellation of Relative Gravitational Sag in 809 nm ODT

A dual BEC in an 1 µm ODT observed above has no spatial overlap as can be seen directly from the images in 4.8. We transfer BECs from an 1 µm ODT to the 809 nm ODT for the sake of the complete overlap. To realize this, the 809 nm ODT (P) is ramped up in the first half of the transfer sequence while the 1 µm ODT (P) is kept constant. In the latter half, the 809 nm ODT (P) is kept constant and the 1 µm ODT (P) is ramped down (see the inset of Fig. 4.11). Time sequence of this procedure is given in Fig. 4.9. We can also take images of atoms during this transfer sequence and keep track of the positions of BECs’ center. As a result we can directly see from Fig.4.11 that relative gravitational sag between Rb and K gets small as we ramp the 809 nm laser up and the 1 µm laser down, and two BECs get together at the end of the sequence.

4.3.2 Static Nature of Dual BEC

Section 2.2.1 says that a dual BEC put together in the same trap shows miscible or immiscible phase depending on the interspecies scattering length $a_{KRb}$. Revisiting in short, if $a_{KRb} < 75 a_B$, two BECs are mixed together and called miscible. If $a_{KRb} > 75 a_B$, it is not energetically favorable for both species to be at the same place and two BECs get spatially separated. This is called immiscible phase. This miscible/immiscible nature can be observed by TOF measurement. Acquired images are shown in Fig.4.12 and in the immiscible regime we observed that K sits in the middle of trap and Rb is pushed away to sandwich K. It is known that various spatial distributions of immiscible dual BEC are realized by the fine-tuning of the ratio of the number of atoms [98, 99].

As has been done in Section 4.2.2, we start PCI at the end of the loading
Figure 4.9: Time sequence of loading atoms into 809 nm ODT. Total time of this sequence is 100 ms.
Figure 4.10: Dual BECs in (a) an $1 \mu m$ ODT and (b) the 809 nm ODT. Field of view of each image is 51 $\mu m \times 206 \mu m$.

Figure 4.11: Cancellation of relative gravitational sag during the transfer to 809 nm ODT. Error bar indicates the statistical error of three data sets at each time.
Figure 4.12: Observation of phase separation with dual BEC in 809 nm ODT by absorption imaging. (a) When $a_{KRb} = 39a_B (< 75a_B)$, two BECs can be mixed together. (b) If we set interspecies scattering length to be $a_{KRb} = 230a_B (> 75a_B)$, BECs avoid to be at the same position as a result of phase separation. In this case K sits in the middle of trap and Rb is pushed away to sandwich K. It is known that various spatial distribution of immiscible dual BEC are realized by fine-tuning of the ratio of number of atoms [98, 99].
Figure 4.13: Miscible dual BEC observed by PCI. Interspecies scattering length is $a_{KRb} = 39 \ a_B (\ < 75 \ a_B)$, in the miscible condition. In the right column the profiles along the horizontal axis at each time are shown. The unit of horizontal axis is pixel, where 1 pixel corresponds to 2.58 $\mu$m.
Figure 4.14: Immiscible dual BEC observed by PCI. Interspecies scattering length is $a_{K,Rb} = 230 \ a_B (> 75 \ a_B)$, in the immiscible condition. In the right column the profiles along the horizontal axis at each time are shown. The unit of horizontal axis is pixel, where 1 pixel corresponds to 2.58 $\mu$m.
of a dual BEC into an 809 nm ODT and continue acquiring images while the dual BEC is held in the trap. Interspecies scattering length is changed in the last 20 ms of the evaporative cooling by ramping up or down the bias magnetic field and it is kept constant during the loading into an 809 nm optical trap and after that.

Let us see the miscible phase first. We set $a_{KRb} = 39\ a_B$ by ramping the magnetic field down to 74 G. In this interaction regime a dual BEC is miscible and spatially overlapped BECs should be observed. Apparent from Fig. 4.13, two BECs reside in the same place for most moments of images. In the right column the profiles along the horizontal axis at each time are shown. The unit of horizontal axis is pixel, where 1 pixel corresponds to 2.58 $\mu$m. One might see the decrease of signal of K at the peak of Rb at $t = 0$ ms, however, this "separation" does not last. It is not identified that where this "separation" comes from, and this observation does not oppose to the miscibility of dual BEC because from $t = 20$ ms to $t = 80$ ms the dip of K cloud at the peak of Rb cloud vanishes.

Next, let us see Fig. 4.14. Now the magnetic field is set to 77.3 G and the interspecies scattering length is $a_{KRb} = 230\ a_B$, in the immiscible regime. From the images one can easily see that in contrast to Fig.4.13, the spatial separation of two BECs is clearly observed. This time the BEC of K sitting in the right and that of Rb in the left are observed. In all shots with all the same experimental parameter we observed this configuration of BECs, that is, K in the right and Rb in the left. So we can conclude that this configuration is due to a tiny difference of the potential minima along $y$ direction discussed in Section3.5. Here the compensation coils for dealing with this problem is not used yet and used later in Section4.3.3. One might notice that both K and Rb have "tails" along the horizontal axis. This is considered to be the thermal components of each cloud and the bimodal feature is obviously seen from the plots in the right column of Fig. 4.14.

Thus we succeeded in observing miscible and immiscible phases of a dual BEC nondestructively and obtaining the spatial density profiles of the
Figure 4.15: Tuning the relative position in horizontal direction by compensation coils. By tuning the current at (a) 0 A, (b) 1.15 A and (c) 1.5 A various initial and final states are observed. Here the interspecies scattering length is $a_{KRb} = 230 \ a_B$, in the immiscible regime.

BECs. This is the first, but important step of this thesis and below we try to control the overall experimental parameters in order to get the best experimental condition.

### 4.3.3 Tuning the Relative Position of Two BECs by Magnetic Field Gradient

There is nothing to say about the importance of the spatial overlap between two BECs in the context of study on a dual BEC interacting with each other. In our case the relative gravitational sag along the $z$ axis is sufficiently
small in 809 nm trap and problematic thing is the horizontal sag due to gravity and/or magnetic field gradient arising mainly from a part of wirings of coils without stranding, or possibly from the inhomogeneity of the bias field and the magnetization of the vacuum chamber. We can cancel these influences by an additional magnetic field gradient by a pair of coils (see Section 3.5.2).

Actually, when we apply no current to the compensation coils K is on the right side of Rb at the early stage of the loading sequence into the 809 nm trap and this configuration is conserved after the loading is finished (Fig. 4.15 (a)). With current 1.5 A, we have K on the left and Rb on the right, conversely to the case of (a) (Fig. 4.15 (b)). At the moderate value 1.15 A (Fig. 4.15 (c)), the dual BEC shows a bit different behavior. For $t = 0$ ms and $t = 40$ ms spatial separation is not apparent but at $t = 80$ ms we can see that two BECs splits and settle in the seemingly true ground-state configuration. From these it can be said that when 1.15 A current is applied to the compensation coils the relative sag in the horizontal direction is almost compensated.

### 4.3.4 Dynamical Motion of an Immiscible Dual BEC

By tuning the relative position of two BECs, we can prepare the initial configuration which does not equal to the true configuration with minimum energy in the trap. Above we see that initially not completely separated BECs gradually separates and turn out to be spatially separated with some timescale (in our case this timescale is about several tens of milliseconds). In addition to the observation described above, we observed clearly that the

\footnote{Of course a dual BEC is always in its ground state when the transferring process is adiabatically implemented (Fig. 4.15 (a) and (c)). However the situation of Fig. 4.15 (b) does not seems that the dual BEC is in the ground state at $t = 0$ ms and $t = 40$ ms because throughout these images no conditions are observed to be changed. Therefore it can be said that the dual BEC is in a metastable configuration at $t = 0$ ms and then subsequently turns into the ground state. Moreover, it implies the possibility that the loading into 809 nm ODT becomes non-adiabatic depending on the relative horizontal position where two immiscible}
Figure 4.16: Dynamical transition from a metastable configuration to the stable configuration. Field of view of each image is $21 \, \mu m \times 206 \, \mu m$. Here the interspecies scattering length is $a_{KRb} = 230 \, a_B$, in the immiscible regime.
Figure 4.17: Time sequence of the loading into an 809 nm ODT. Total time of this sequence is 100 ms here and at the end of this sequence magnetic field for Feshbach resonance is suddenly changed to the target value.
Figure 4.18: Dynamical motion of a dual BEC undergoing the sudden change of the interspecies scattering length. Field of view of each image is 51.6 $\mu$m $\times$ 206 $\mu$m. Here the interspecies scattering length is suddenly changed to $a_{KRb} = 230 \ a_B$ at $t = 0$. 
metastable configuration gradually turns into the stable one. Experimentally obtained images and the plots of horizontal profiles are listed in Fig. 4.16. At $t = 0$ ms, K stays at the center and Rb are outside to avoid K. As the time passes, at $t = 80$ ms K settles in the left and Rb in the right. The images from $t = 20$ ms to $t = 60$ ms shows the clear feature of the transition from a metastable configuration to the stable configuration. This is considered to be one of a dynamical phenomena accompanied to a dual BEC, however, the analysis of this phenomenon is not a scope of this thesis.

Another non-equilibrium phenomenon we observed is closely related to the topic studied in the next chapter. We want to know how an initially miscible dual BEC evolves to the most stable configuration of an immiscible one when the interspecies scattering length is suddenly changed to the value $a_{KRb} > 75 a_B$, which is an immiscible regime. There can exist an essentially different dynamics for this situation from the one described above where the interspecies scattering length is kept constant for the whole sequence. The experiment is done by the time sequence described in Fig. 4.17. We load a dual BEC into the 809 nm ODT as indicated in Fig. 4.9 except for the bias magnetic field for the Feshbach resonance being ramped up suddenly to 77.3 G (at this value, $a_{KRb} = 230 a_B$). Again as soon as the loading is completed we start to observe atoms by the phase contrast imaging.

The result is shown in Fig. 4.18. In this case, at $t = 0$ ms the dual BEC is completely overlapped and it can be seen from the images that two BECs gradually moves to the opposite directions. Finally at $t = 80$ ms they settle in the immiscible configuration.

### 4.4 The Glitch in Time-of-Flight Imaging

We now go back to the TOF measurement of a spatially overlapped dual BEC. Indeed if we take images of a dual BEC in the way we described in Section 4.1, we rarely get such nice images of a dual BEC as Fig. 4.12 (these BECs start to be in touch.)
images are taken by modified method written hereafter). Typical images obtained by this procedure is given in Fig. 4.19, where interspecies scattering length is $a_{KRb} = 39 \ a_B$. These atomic clouds hardly seem to be BECs, rather to be nearly thermal clouds.

We had seen the images like this almost always when we tried to generate a miscible dual BEC. This result made us believe that the three-body recombination rate in our heteronuclear atomic gas mixture is very large so that the dual BEC vanishes by the three-body loss as soon as two atomic gases overlap. However, This contradicts with other two results we obtained so far: one is that we can observe spatially overlapped, miscible dual BEC by means of the \textit{in-situ} measurement. Another is the three-body loss rate measured by thermal clouds was an order of magnitude smaller (see AppendixD, in which we measured the three-body recombination rate $L_{KKRb} = 7 \times 10^{-27} \ cm^6/s$ and $L_{KRbRb} < 1 \times 10^{-27} \ cm^6/s$ at $a_{KRb} = 250 \ a_B$. This recombination rate gives a timescale of the three-body loss in a dual
BEC of roughly 100 ms for densities $3 \times 10^{13}/\text{cm}^3$ for each species).

Clues for solving this contradiction have been already mentioned up to here. One is the TOF sequence (Fig. 4.1). When we released BECs from the optical trap, the Feshbach magnetic field was suddenly turned off. The other is the magnetic field dependence of the interspecies scattering length (Fig. 2.5). Two things should be noted in this plot. First, there is a broad Feshbach resonance (center: 38.4 G, width: 37 G), and second, the interspecies scattering length is more than 600 $a_B$ at $B = 0$ G. Broad resonance is crossed with sweep time less than 1 ms, which is the switching-off timescale of the feshbach magnetic field, and this is still comparable to the timescale of adiabatic conversion into Feshbach molecules. In addition, when the Feshbach resonance is crossed the three-body recombination rate is vastly enhanced so that the loss can be considerable.

All these information implies that when the Feshbach magnetic field is turned off at the beginning of TOF, the interspecies scattering length suddenly becomes very large and the structure of BECs are not conserved, or destructed, over the 30 ms free expansion. Actually, as we will see in Chapter 5, modulation instability, by which the structure with very large wavenumber grows exponentially with timescale of a few hundreds of $\mu$s for $a_{K\text{Rb}}(B = 0 \ G) > 600 \ a_B$, can be one of the reason for this destruction. Three-body loss can also be one of the causes taking into account the fact that three-body loss rate gets large as $a_{K\text{Rb}}$ gets large.

In order to prevent BECs from experiencing such situation, we should modify the sequence of the TOF measurement. The most easy prescription is to keep Feshbach magnetic field until just before the imaging (Fig. 4.20). Actually this modification enables us to obtain images of dual BEC with no such destruction as can be seen in Fig. 4.12.

Furthermore, we now apply strong magnetic field gradient in the horizontal direction for the first 300 $\mu$s of the TOF sequence to spatially separate two expanding clouds, just in case a dual BEC evolves during the free fall. Another solution is to take images after a TOF in the presence of magnetic
Figure 4.20: Modified TOF sequence.
Figure 4.21: Comparison between dual BECs produced by (a) the former and (b) the modified TOF sequence.

Field. We did not adopt this because this needs another frequency-tunable probe lasers.
Chapter 5

OBSERVATION OF MODULATION INSTABILITY WITH A DUAL BEC

Phase contrast imaging is a powerful tool to acquire the information of dynamical behaviors which is hardly available with the time-of-flight measurement. Up to here we mainly saw the static behavior of a dual BEC and in this chapter we deal with the dynamical behavior, especially the modulation instability.

Modulation instability is one of the universal phenomena accompanied to the nonlinearity of materials. This has been studied for optical beams inside crystals where the light is first slightly gathered by fluctuations of refractive index and consequently the increase of light intensity results in an increase of the refractive index, then more light is gathered in this region and so forth. This causes the interplay between diffractive and dispersive effects, namely one that light expands as it propagates and one that light is focused by a lensing effect. Then, triggered by fluctuations, initially uniform light intensity grows exponentially large (but saturates at some point) to form solitons or some spatial patterns in the presence of a nonlinear optical effect. Indeed, the self-focusing effect[106], the filamentation or the
breaking-up of continuous wave into short pulses [101, 102, 103, 104, 105] are investigated in the context of nonlinear optical phenomena inside the bulk materials. All of these phenomena listed above are understood to be sharing their origins in the modulation instability, which in general refers to the spontaneous growth of fluctuations by the nonlinear effect. This anomalous effect is also applied to the routing of solitons in bulk matters[107], the generation of the “space-time bullet train”[108] and so on.

This physics works almost the same for an atomic BEC in a sense that Gross-Pitaevskii equation contains a nonlinear, or, a density-dependent term in its mean-field interaction part. Actually a dark-soliton was first generated in a BEC by means of the phase imprinting [21], confirming the presence of the nonlinear phenomenon in a BEC. However, in a single component BEC with repulsive interaction atoms do not tend to gather spontaneously even if there is a density fluctuation because there is no attractive force among them. So the study of nonlinear effects is done by externally manipulating the phase of condensates as written above or tuning the scattering length to a negative value.

The situation is entirely different for a two-component BEC since there takes place the phase separation. Phase separation can be interpreted as a circumstance that inter-component repulsion is so large compared to the intra-component ones that effectively atoms of the same component “attract” each other in order to avoid mixing with the other component. In other words, interplay between the tendency to spread over entire region in the trap and tendency to be pushed away by the other component can trigger nonlinear phenomena such as the modulation instability.

In the next section we explain how the modulation instability arises in a dual BEC by applying linear stability analysis on coupled Gross-Pitaevskii equations. This will replace above qualitative understanding by quantitative one and gives the dependence on the inter-component scattering length for the fastest-growing size of a spatial structure. Subsequent sections are devoted to the preparation of experiment and the analyses of experimentally
obtained results.

5.1 Theoretical Description of the Modulation Instability

We first write down the theoretical analysis of the modulation instability in detail. Outline of the analysis is as follows: we first integrate the coupled Gross-Pitaevskii equations and obtain equations for elongated BECs. Then we consider small periodic modulation for the linearized equation. Consideration on the solutions gives the wavenumber of the fastest-growing modulation \( k_{max} \) and this depends on the interspecies scattering length. The analysis shown below follows the discussion in Ref.[1].

Situation we consider here is that we have two BECs in the same trapping potential and they are overlapping with each other. With this initial condition and given the interspecies (or, intercomponent) scattering length, we want to analyze what happens and what difference can be observed due to the magnitude of the interspecies interaction strength. To understand this, first of all we start with coupled time-dependent Gross-Pitaevskii equations (2.5) and (2.6):

\[
\begin{align*}
\imath \hbar \frac{\partial}{\partial t} \psi_1(\mathbf{r}) & = \left[-\frac{\hbar^2 \nabla^2}{2m_1} + V_1(\mathbf{r}) + g_1 |\psi_1(\mathbf{r})|^2 + g_{12} |\psi_2(\mathbf{r})|^2 \right] \psi_1(\mathbf{r}), \\
\imath \hbar \frac{\partial}{\partial t} \psi_2(\mathbf{r}) & = \left[-\frac{\hbar^2 \nabla^2}{2m_2} + V_2(\mathbf{r}) + g_2 |\psi_2(\mathbf{r})|^2 + g_{12} |\psi_1(\mathbf{r})|^2 \right] \psi_2(\mathbf{r}).
\end{align*}
\]

If we have a dual BEC in a cigar-shaped trap \( V(r,z) = (1/2)m_i \omega_i^2 (r^2 + \lambda^2 z^2) \) with sufficiently small \( \lambda \), it is justified to regard the motion in radial direction to be frozen and rewrite the wavefunction as

\[
\psi_i(\mathbf{r}) \rightarrow \phi^{(i)}(x,y) \psi_i(z,t) e^{-i \omega_i t}.
\]

It is obvious from this substitution that \( \phi^{(i)}(x,y) \) is the distribution of wavefunction in radial direction and does not contain time variable \( t \). On the
other hand \( \psi_i(z, t) \) describes the axial wavefunction and has some information of the axial motion for its dependence on \( t \). Substituting this representation in binary Gross-Pitaevskii equation and integrating with respect to \( x \) and \( y \), we get 1-dimensional coupled Gross-Pitaevskii equations

\[
\begin{align*}
\frac{i\hbar}{\partial t} \psi_1(z) &= \left[ -\frac{\hbar^2}{2m_1} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m_1 \lambda_1^2 \omega_1^2 z^2 + u_1 |\psi_1(z)|^2 + u_{12} |\psi_2(z)|^2 \right] \psi_1(z), \\
\frac{i\hbar}{\partial t} \psi_2(z) &= \left[ -\frac{\hbar^2}{2m_2} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m_2 \lambda_2^2 \omega_2^2 z^2 + u_2 |\psi_2(z)|^2 + u_{12} |\psi_1(z)|^2 \right] \psi_2(z).
\end{align*}
\]

(5.1)

(5.2)

Here the interaction strengths \( u_1, u_2 \) and \( u_{12} \) are defined as

\[
\begin{align*}
u_i &= g_i \int dx dy |\phi^{(i)}(x, y)|^4 = \frac{g_i}{2\pi b_i^2}, \\
u_{12} &= g_{12} \int dx dy |\phi^{(1)}(x, y)|^2 |\phi^{(2)}(x, y)|^2 = \frac{g_{12}}{\pi (b_1^2 + b_2^2)}.
\end{align*}
\]

where \( b_i = \sqrt{\hbar/m_i\omega_i} \) are harmonic oscillator lengths of the trap in radial direction. Keep in mind that this is an approximation and the trapping potential is nearly 1-dimensional but not true 1-dimension, where Bose-Einstein condensation does not occur.

In static Gross-Pitaevskii equations we can write wavefunctions as the square root of the spatial density of a gas. However, here we consider an additional fluctuation of the wavefunction which is represented by the 1D density of BEC \( n_{i0} \) as

\[
\psi_i(z, t) = \sqrt{n_{i0}} + \delta \psi_i(z, t).
\]

Then we substitute these expressions into eqs. (5.1) and (5.2), and linearize the equations with small parameters \( \delta \psi_i(z, t) \). The results are calculated as
follows:

\[ i\hbar \frac{\partial}{\partial t} \delta \psi_1 = -\frac{\hbar^2}{2m_1} \frac{d^2}{dz^2} \delta \psi_1 + u_1 n_{10} (\delta \psi_1 + \delta \psi_1^*) + u_{12} \sqrt{n_{10} n_{20}} (\delta \psi_2 + \delta \psi_2^*), \]

(5.3)

\[ i\hbar \frac{\partial}{\partial t} \delta \psi_2 = -\frac{\hbar^2}{2m_2} \frac{d^2}{dz^2} \delta \psi_2 + u_2 n_{20} (\delta \psi_2 + \delta \psi_2^*) + u_{12} \sqrt{n_{10} n_{20}} (\delta \psi_1 + \delta \psi_1^*). \]

(5.4)

Eqs. (5.3) and (5.4) hold for small perturbations on the wavefunctions and we further assume sinusoidal and co-sinusoidal functions as a form of this, namely,

\[ \delta \psi_i(z, t) = \zeta_i \cos (k_i z - \Omega t) + i \eta_i \sin (k_i z - \Omega t). \]

(5.5)

Substituting this into the linearized equations we obtain the dispersion relation between \( k_i \) and \( \Omega \). This reads

\[ (\Omega^2 - \Lambda_1)(\Omega^2 - \Lambda_2) = P^2 \]

(5.6)

where the parameters are defined as

\[ \Lambda_i = \frac{k_i^2}{2m_i} \left( \frac{\hbar^2 k_i^2}{2m_i} + 2u_i n_{i0} \right), \]

\[ P = \frac{u_{12}}{\sqrt{m_1 m_2}} \sqrt{n_{10} n_{20}} k_1 k_2. \]

We again define the parameters as \( \Lambda = \Lambda_1 + \Lambda_2 \) and \( \Delta = P^2 - \Lambda_1 \Lambda_2 \). With these expressions we can write the solution of equation (5.6) as

\[ \Omega_{\pm}^2 = \frac{1}{2} \left( \Lambda \pm \sqrt{\Lambda^2 + 4\Delta} \right) \]

(5.7)

This is the final form of the dispersion relation we should discuss in detail. Remind that \( \Omega_{\pm} \) is the solution which is to be substituted in the time dependence of perturbation (5.5). Since the amplitude of this perturbation can be roughly written as \( \delta \psi_i \propto \exp \left[ i(k_i z - \Omega t) \right] \), whether \( \Omega_{\pm} \) has an imaginary component or not is crucial in terms of the stability of the system. From the equation (5.7), \( \Omega_{\pm} \) takes only pure real or pure imaginary value depending on whether the right hand side is positive or negative, respectively.
For the case of $\Lambda > 0$, $\Omega_2 > 0$ always holds. However, we can easily check that $\Omega_2$ takes negative value only if $\Delta > 0$. Then in this parameter region there is a purely imaginary solution $\Omega_-$ and the perturbation gets exponentially large with time.

On the other hand, if $\Lambda < 0$, it can be said that $\Omega_2$ is always negative and in this parameter region the fluctuation grows exponentially regardless of the magnitude of the parameters. The table shown below summarizes the stable or unstable conditions against the spatial modulation for all combinations of the signs of parameters.

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$\Delta$</th>
<th>$\Omega_2^+$</th>
<th>$\Omega_2^-$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>Unstable</td>
</tr>
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<td>$+$</td>
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<td>Stable</td>
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<td>$-$</td>
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<td>Unstable</td>
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</tbody>
</table>

This phenomena of spontaneously growing fluctuation is called modulation instability. This is one of the dynamical instabilities which appears when a dual BEC undergoes the travel from an unstable configuration to the stable one. In our case of K and Rb mixture, intra-species scattering lengths are $a_K = 60 \ a_B$ and $a_{Rb} = 106 \ a_B$ so that the parameter region is limited to $\Lambda_i > 0$, that is, $\Lambda > 0$.

Let us now investigate further about this modulation instability. For the sake of convenience, we introduce a lengthscale $l = \sqrt{\hbar^2/4m_1u_1n_{10}}$ and dimensionless wavenumbers $\tilde{k}_i = k_i l$. With these variables the parameters read

$$\Lambda = \frac{\hbar^2}{4m_1^2l^4} \left( \tilde{k}_1^2 (\tilde{k}_1^2 + 1) + \frac{m_1^2}{m_2^2} \tilde{k}_2^2 (\tilde{k}_2^2 + \gamma_2) \right)$$

$$\Delta = \left( \frac{\hbar^2}{4m_1^2l^4} \right)^2 \tilde{k}_1^2 \tilde{k}_2^2 \left[ \gamma_{12}^2 - (\tilde{k}_1^2 + 1)(\tilde{k}_2^2 + \gamma_2) \right]$$
where the $\gamma_2$ and $\gamma_{12}$ reflect the interaction strengths and their concrete forms are given as

$$\gamma_2 = \frac{m_2 u_2 n_{20}}{m_1 u_1 n_{10}},$$

$$\gamma_{12} = \frac{u_{12}}{u_1} \sqrt{\frac{m_2 n_{20}}{m_1 n_{10}}}.$$  

As mentioned above we deal with the case $\Lambda > 0$ and the modulation instability appears for $\Delta > 0$. This condition can be expressed in terms of $\bar{k}_1$:

$$\bar{k}_1 < \sqrt{\frac{\gamma_{12}^2}{k_2^2 + \gamma_2}} - 1.$$

There is necessary conditions derived by this inequality $\gamma_{12} > \sqrt{\gamma_2 + \bar{k}_2^2}$ or $\gamma_{12} < -\sqrt{\gamma_2 + \bar{k}_2^2}$. The former reads

$$u_{12} > \sqrt{u_1 u_2}$$  \hspace{1cm} (5.8)

in the long wavelength limit $\bar{k}_2 \to 0$. This condition is slightly different from the condition of the occurrence of the phase separation $g_{12} > \sqrt{g_1 g_2}$ because $u_1$, $u_2$ and $u_{12}$ are renormalized by factors which include harmonic oscillator lengths $b_1$ and $b_2$. Then this condition gives $(b_1^2 + b_2^2)/2b_1b_2$ times larger value of $a_{Krb}$ for the modulation instability than that of the phase separation. The latter states that for sufficiently largely negative $a_{Krb}$ a dual BEC collapses (this is not rigorous explanation because the condition of collapse of dual BEC is usually expressed as $g_{12} < -\sqrt{g_1 g_2}$).

One of the features which characterizes the modulation instability is the fact that the solution of $\Omega$, namely $\Omega_\pm$, depends on the wavenumbers $\bar{k}_1$ and $\bar{k}_2$. In our case modulation instability occurs when $\Lambda > 0$ and $\Delta > 0$, since $\Omega^2_-$ is negative. The smaller the $\Omega^2_-$ ($< 0$) is, the faster the modulation grows. So that the next thing we have to do is to find the wavenumbers which minimize the value of $\Omega^2_-$.

By the way, the modulation instability occurs when an immiscible dual BEC totally overlaps with each other. Then it is natural to assume that when
the density of one BEC at some place increases, that of the other decreases. From this assumption, we can observe the anti-correlation of spatial densities between two BECs. Moreover, we consider the situation that this occurs in most part of BEC and the modulation for each BEC has the same wavenumber \( \tilde{k}_1 = \tilde{k}_2 = \tilde{k} \). Then search for the \( \tilde{k} \) with minimum \( \Omega^2 \) is reduced to find the solution of equation \( \partial \Omega^2 / \partial (\tilde{k}^2) = 0 \) and the solution \( \tilde{k}_{max} \) is calculated to be

\[
\tilde{k}_{max} = \frac{1}{2} \left[ \sqrt{(\gamma_2 - 1)^2 + 4\gamma_{12}^2} - \gamma_2 - 1 \right]^{1/2}.
\] (5.9)

By substituting back the definitions of parameters \( \gamma_2, \gamma_{12} \) and \( l \), and doing some maths we get an expression of \( \lambda_{max} \) as

\[
\lambda_{max} = \frac{2\pi}{\sqrt{2\eta}} \left[ \sqrt{(\lambda_1^2 + \lambda_2^2)^2 + 4\eta\lambda_1^2\lambda_2^2 + \lambda_1^4 + \lambda_2^4} \right]^{1/2}.
\]

Here we introduced the parameters \( \lambda_i \) and \( \eta \) as:

\[
\lambda_i \overset{\text{def}}{=} \frac{\hbar}{\sqrt{2m_iU_iu_{i0}}} \quad (i=1, 2)
\]

and

\[
\eta \overset{\text{def}}{=} (u_{12}^2 / u_1 u_2) - 1.
\]

Thus the behavior of \( \lambda_{max} \) (and thus of \( k_{max} \)) becomes more comprehensive: in the square brackets a length is given by the combination of healing lengths of each components and this length is enhanced by a factor \( \sim 1/\sqrt{\eta} \) to give the size of the fastest-growing modulation. If we assume \( m_1 = m_2 = m \) (of course, this is incorrect assumption for our system of K and Rb), the growth rate is given as \( G_{max} = \hbar^2 k^2 / 2ml^2 \).

Note that the calculation done above is a linear analysis of the stability of the system. This is valid for sufficiently small perturbation and if the modulation gets deep this can break down so that there can be seen different behavior with dual BEC. But here we limit ourselves to investigate the modulation instability with linear regime.
Figure 5.1: Optical path of the tight 809 nm optical trap is indicated by the red line. In (b), the area surrounded by the blue square line is expanded and the 1080 nm optical trap is shown by the orange line. Two beams get together at the long-wave-pass dichroic mirror.
5.2 Preparation of a New Optical Trap of an 809 nm ODT

In Section 4.3.4, we saw that a dual BEC undergoes the transition from a metastable configuration to the stable configuration (see Fig. 4.16). The detail of this mechanism is not uncovered yet, however, one possibility is that radial motions of BECs are not sufficiently suppressed so that condensates can interchange their positions relatively easily. Then for what condition we can neglect the motion in radial direction? For qualitative discussion, one might compare the chemical potential of BEC with the first excitation energy in the radial motion of a harmonic potential. This then reduces to the comparison between trap frequencies in radial direction $\omega_\perp$ and axial direction $\omega_\parallel$. We now define the ratio $\omega_\perp/\omega_\parallel$ as aspect ratio.

In experiments done in Chapter 4, the trapping frequencies are set to $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (40, 7, 100)$ Hz for Rb. The aspect ratio of this trapping potential is less than 10 in $x$ direction. For this aspect ratio, motion in radial direction in the trap is not suppressed sufficiently[99]. This may make it difficult to observe the modulation instability because the spatial structure has less difficulty in merging to form a larger structure, which basically lower the energy due to the spatial overlap.

Let us consider how large aspect ratio can suppress the radial motion in a cigar-shaped trap. The necessary condition is expressed as a comparison between the chemical potential $\mu$ and the energy scale $\hbar \omega_r$ where $(\omega_r, \omega_r, \omega_z)$ are the trapping frequencies with $\omega_z \ll \omega_r$:

$$\mu < \hbar \omega_r.$$  

Aspect ratio is given here by $A = \omega_r/\omega_z$. Defining $\bar{\omega} = (\omega_r^2 \omega_z)^{\frac{1}{2}}$ and inserting this into the formula for the chemical potential[51], it is written as
follows:

\[
\mu = \frac{1}{2} \hbar \bar{\omega} \left( \frac{Na}{\alpha_{ho}} \right)^{\frac{2}{3}},
\]

\[
= \frac{1}{2} \hbar \omega_r \left( \frac{Na}{\alpha_{ho}} \right)^{\frac{2}{3}}.
\]

Here the number of atoms \( N \) in BEC, the harmonic oscillator length \( a_{ho} = \sqrt{\hbar/m\bar{\omega}} \) with mass of an atom \( m \), scattering length \( a \) and the relation \( \bar{\omega} = \omega_r / A^{\frac{1}{3}} \) are used. Therefore the condition \( \mu < \hbar \omega_r \) reads

\[
\frac{\mu}{\hbar \omega_r} = \frac{1}{2A^{\frac{2}{3}}} \left( \frac{Na}{\alpha_{ho}} \right)^{\frac{2}{3}} < 1,
\]

\[
\iff A > \frac{1}{8} \left( \frac{Na}{\alpha_{ho}} \right)^{\frac{6}{5}}. \quad (5.10)
\]

Using the parameters \( N = 3 \times 10^4 \), \( a = 106 \, a_B \) and \( \bar{\omega} = 2\pi \times 30 \) Hz for Rb, the condition (5.10) becomes\(^1\) \( A > 25 \).

Therefore, in order to prevent BECs from the radial motion in a cigar-shaped trap, it is sufficient to let the aspect ratio to be as large as 30. This section describes the new design of 809 nm optical dipole trap and observe a dual BEC in it.

### 5.2.1 New Shape of an 809 nm Optical Dipole Trap

To obtain a higher aspect ratio, we should loosen the axial confinement and tighten the radial one. However, we cannot lower the axial trapping frequency because too low intensity of a trapping laser leads to a leakage of atomic gases in the presence of small amount of the magnetic field gradient. So we think of tightening the radial confinement.

The aspect ratio of more than 30 requires the radial trapping frequency of about 300Hz. With our maximum intensity 84.6 mW we need to change the beam radii of the trapping laser from 270 \( \mu m \times 150 \, \mu m \) to 100 \( \mu m \times 100 \, \mu m \). On the other hand, if we change beam radii of 1 \( \mu m \) ODT together

\(^1\)For K, \( N = 3 \times 10^4 \), \( a = 60 \, a_B \) and \( \bar{\omega} = 2\pi \times 40 \) Hz gives the condition \( A > 9.5 \)
whose shape is changed by the same optics as for 809 nm optical trap, we have some difficulty in loading atoms from a magnetic trap to an 1 \( \mu \)m ODT. Therefore we make an 809 nm beam of such shape by another optical path and later put it together with a shaped 1 \( \mu \)m ODT by making use of the dichroic mirror.

The actual optical path is shown in Fig. 5.1. First, 809 nm linearly polarized, collimated beam with diameter 2.1 mm is coming out from the single mode optical fiber. Then it is expanded by a telescope to \( \sim 5 \) mm and focused by the lens of focal length of 500 mm. We confirmed that this optics can make round-shaped, 100 \( \mu \)m-diameter beam at the expected position of atomic clouds, consistent with the calculated value by formula (4.1). Finally a long-wave-pass dichroic mirror (coated by Lattice Electro Optics, Inc.) enables us to combine ODT beams with different colors. The optical path is indicated by the red lines.

### 5.2.2 Radial Trapping Frequency of the New Optical Dipole Trap

We succeeded in preparing the optical beam and next we load the atoms in this new optical trap. To show the difference clearly, we show two images of a rubidium BEC (potassium is absent in these images) in the former and the newer 809 nm traps in Fig. 5.2. Since we tightened the trap in radial, that is, \( x \) and \( z \) directions, a BEC expands faster in these directions faster than before in the new trap. Resulting cloud shape after 30 ms from the release from trap is an ellipsoid whose long-axis is in \( x \) direction when it is seen from the top.

We measured the radial trapping frequency of this trap. In order to do this we first apply an short pulse (about a few hundreds of microseconds) of large magnetic field gradient to a BEC and let it oscillate for some time \( t \) in the trap (Stern-Gerlach separation). After that we start TOF sequence and measure the position of the BEC, which is equal to the information of the mo-
Figure 5.2: TOF measurement of rubidium BEC in (a) the former 809 nm trap and (b) the new one. (a) In the former trap shape BEC is slightly elongated in x direction and cloud after TOF has an elliptic shape whose long-axis is in y direction. (b) In the new trap, BEC is largely elongated in y direction and cloud after TOF has an elliptic shape whose long-axis is this time in x direction.

momentum of the BEC during oscillation. By changing the holding time after the magnetic field gradient pulse and observing the positions at each holding time, we can observe the position of the BEC oscillates with the trapping frequency. Fig. 5.3 and Fig. 5.4 show the results of the measurements with fitting functions $f(t) = y_0 + A \sin (f t + \phi)$ for determining the oscillation period and $f(t) = y_0 + A \exp \left( -t/(\tau) \right)$ for the lifetime. The oscillation frequency is measured to be 325(13) Hz, and at the same time, lifetime of 399(9) ms due to one-body photon scattering is also measured. These agree very well with the calculated ones with the power 84(1) mW and the diameter 100 µm.

We can also trap atoms with much lower trap laser power of 27(1) mW. In this case the measured trapping frequency is 158(5) Hz (result is shown in Fig. 5.5) and the lifetime is 1.1(1) s (see Fig. 5.6), which again show agreement with the calculation. We succeeded in preparing not only the tight optical trap but also the trapping-frequency-tunable trap which ranges from at least 158 Hz to at most 325 Hz.
Figure 5.3: Measurement of the trapping frequency with power of 84 mW. Fitting function is \( f(t) = y_0 + A \sin(ft + \phi) \).

Figure 5.4: Measurement of the lifetime with power of 84 mW. Fitting function is \( f(t) = y_0 + A \exp\left(-t/(\tau)\right) \), where we set \( y_0 \) to zero.
Figure 5.5: Measurement of the trapping frequency with power of 27 mW. Fitting function is $f(t) = y_0 + A \sin(ft + \phi)$.

Figure 5.6: Measurement of the lifetime with power of 27 mW. Fitting function is $f(t) = y_0 + A \exp(-t/(\tau))$, where we set $y_0$ to zero.
Figure 5.7: TOF measurement of a dual BEC in the new 809 nm trap. The interspecies scattering length is $a_{KRb} = 0.7 \, a_B$. The numbers of atoms are typically $3 \times 10^4$ for each species.

5.2.3 Loading a dual BEC into the New Optical Dipole Trap

The aspect ratio of a BEC generated in 1 $\mu$m trap is less than 10, while the one in an 809 nm trap is more than 30. This implies that we can no longer transfer a BEC from the 1 $\mu$m trap to the 809 nm trap as fast as we did before. It was observed that if we transfer BEC with 100 ms sequence, non-adiabaticity makes the BEC start to oscillate very hard and finally atoms get rid of the trap. We solve this problem by two modifications on the loading sequence: we assign 300 ms for the loading sequence (i.e. three times longer than before) and tighten the axial confinement by using twice the power than before. Finally trapping frequencies were $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (285, 10, 285)$ Hz for Rb, where the radial trapping frequency was set to lower value than the maximum.

In Fig. 5.7, an image of a dual BEC with interspecies scattering length $a_{KRb} = 0.7 \, a_B$ in this trapping potential is shown. These images are taken by 30 ms-TOF measurement and indicate clearly the anisotropic expansion of the atomic clouds. Here, we adopt the Stern-Gerlach separation method because even the modified TOF sequence can destroy a dual BEC for its higher spatial density. As we have done for a single-species BEC, we succeeded in
Figure 5.8: Calculated values of $k_{max}$ versus $a_{KR_b}$. This plot is calculated with a set of parameters $N_K = N_{Rb} = 3 \times 10^4$ and trapping frequencies $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (285, 10, 285)$ Hz for Rb.

Figure 5.9: Calculated values of $G_{max}$ versus $a_{KR_b}$. We consider the reduced mass of Rb and K here. This plot is calculated with a set of parameters $N_K = N_{Rb} = 3 \times 10^4$ and trapping frequencies $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (285, 10, 285)$ Hz for Rb.
generating a dual BEC in the radially tightened 809 nm optical trap.

5.3 Experimental Observation of the Modulation Instability

5.3.1 Experimental Conditions for Observing the Modulation Instability

In the trap described above, we try to observe the modulation instability with a dual BEC at variable scattering lengths. The $a_{KRb}$-dependence of the fastest-growing size of the structure is derived in Section 5.1 (see (5.9)). In this formula we only have to consider $\gamma_{12}$ which depends on $a_{KRb}$, while others are constant. Since $k_{max}$ is monotonously increasing function of $\gamma_{12}$ and $\gamma_{12}$ is the same as $a_{KRb}$, $k_{max}$ is monotonously increasing function of $a_{KRb}$.

Given the trapping frequencies of the ODT, spatial densities calculated from typical number of atoms, and intra-species scattering lengths, we can plot $k_{max}$ with respect to $a_{KRb}$. The plots in Fig. 5.8 show the results of calculation with a set of parameters $N_K = N_{Rb} = 3 \times 10^4$ and trapping frequencies $(\omega_x/2\pi, \omega_y/2\pi, \omega_z/2\pi) = (285, 10, 285)$ Hz where the peak densities are used. First, the modulation instability occurs in the parameter region $a_{KRb} > 80.5 a_B$ from the condition $u_{12} > \sqrt{u_1 u_2}$ (see (5.8)), which can be interpreted as the phase separation takes place for this condition in the quasi-1D configuration. Therefore $k_{max}$ is zero at $a_{KRb} = 80.5 a_B$ and takes finite positive value for $a_{KRb} > 80.5 a_B$. Second, the typical size of the structure which is observable with our experimental apparatus has both upper and lower limit. Upper limit comes from the uncertainty of the value of bias magnetic field for Feshbach resonance, while lower limit takes into account the resolution and the magnification of the imaging system. As a result, we expect to be able to observe the structure which has typical size from 8 $\mu$m
5.3.2 Experimental Sequence for Observing Modulation Instability

Experimentally, we first load a dual BEC into an 809 nm ODT to make spatial overlap between BECs. Interspecies scattering length is set to $a_{KRB} = 0.7 a_B$ expecting that three-body coefficient will be minimized and two BECs overlap with each other. As soon as the loading sequence is finished, we rapidly ramp the Feshbach magnetic field bias to the expected value which cause the modulation instability. Just after the magnetic field ramp we non-destructively observe the time evolution of the dual BEC. Note that in Ref.
[1], Kasamatsu *et. al.* consider the situation that two BECs initially overlap and then evolve with interspecies scattering length which satisfies the condition $u_{12} > \sqrt{u_1 u_2}$. Our setup resembles the theoretically proposed situation[1] very well.

There is one problem with our instruments. The problem is that the magnetic field ramp cannot be sufficiently fast and if we change it very rapidly, it overshoots. So we have to fine-tune the current going into the coils. By dividing the field ramp sequence into two and setting the final current of first half of sweeping at a bit lower value, we realized this. Thus an optimized sweep of magnetic field is realized and the result is shown in Fig. 5.10. Horizontal axis is time and the falling of TTL (shown by green line) indicates the end of field ramping determined by Labview. Blue line is the voltage signal obtained by a Hall probe sensor attached to the current path of the coils.

### 5.3.3 Results at $a_{K RB} = 80.8 \ a_B$

We first describe results observed at $a_{K RB} = 80.8 \ a_B$. In this case $k_{\text{max}} \simeq 400 \ \text{cm}^{-1}$ corresponds to the typical size of the structure about 25 $\mu$m. This can be observed in the image as the structure which has a size about 10 pixels. From Fig. 5.9, we set the total time of observation 80 ms accordingly to the timescale 57 ms. Typical time evolution of a dual BEC in this experiment is shown in Fig. 5.11. As can be seen in these images, initially overlapped clouds separates with some spatial pattern during the time evolution. Typical size of this spatial pattern seems to be about 10 pixels, which is consistent with the prediction.

For the sake of not only qualitative but also quantitative discussion, we need an analysis on these data. When we want to know the spatial size of the observed pattern, we have only to Fourier transform the observed horizontally summed signal. In addition to this procedure we use the difference of the summed signals of K and Rb as a Fourier transformed signal. By using this signal we can extract the enhanced signal of spatial pattern if the
Figure 5.11: Typical images of time evolution at $a_{K,Rb} = 80.8 \ a_B$. 
Figure 5.12: Spectrum obtained at $a_{KRb} = 78.5 a_B$. Five experimental shots out of 20 are selected and as a result 25 spectra are averaged. Error bars indicate the standard error of the experimental shots. For this spectrum, running average is taken over 3 points in order to smoothen the spectrum.

anti-correlation of the density distributions is present. We thus obtain five spectra per single experimental sequence.

Obtained subtracted signals and spectra at each time are shown in Fig. 5.13 (a) and (b), respectively. Before proceeding into the understanding of this spectrum, we first take a look at a spectrum taken in another experiment. According to the condition (5.8), modulation instability does not occur if $a_{KRb}$ is less than $80.5 a_B$. Therefore if we bring $a_{KRb}$ from $0.7 a_B$ to $78.5 a_B$, density fluctuation does not grow and no peak is expected to be observed in the spectrum.

The resultant spectrum at $a_{KRb} = 78.5 a_B$ is shown in Fig. 5.12. Depending on the loading condition, the number of atoms in each species fluctuates in each experimental run. Therefore this spectrum is obtained by selecting
Figure 5.13: (a) Subtracted signals of two atomic species (left, blue plots) and (b) Fourier transformation of them (right, purple plots) for $a_{K\text{Rb}} = 80.8 \ a_B$. (c) Summation of 5 spectra in (b) is taken (yellow plot).
experimental sequences which satisfies that the ratio of peak number densities is from 0.9 to 1.1, and summing up all of these signals. Thus, 5 experimental shots out of 20 are used and by summing up 5 experimental shots, we can get unwanted noises averaged. Error bars indicate the standard error of the experimental shots. Below we always analyze data in this way when we sum up the spectra. For this spectrum, the running average is taken over 3 points in order to smoothen the spectrum, which corresponds to the coarse graining over 3 points in the real space.

There are two peaks in the spectrum in Fig. 5.12, one at 0 cm$^{-1}$ and the other in the range from 250 cm$^{-1}$ to 300 cm$^{-1}$. These two peaks have nothing to do with modulation instability. First peak at 0 cm$^{-1}$ means that these is some offset signal or very smoothly varying signal. This probably comes from the difference of the number of atoms. The other one at about 250 cm$^{-1}$ corresponds to the typical fringe separation which is always present even when atoms are absent. Although this fringe pattern is to some extent suppressed by the fringe cleaning process (detail is given in Appendix B), the remaining fringe is observed as this peak.

Similarly, we analyze the behavior at $a_{KRb} = 80.8$ a$_B$ in this way. We got 11 experimental shots out of 60 trials and finally the spectrum is obtained as the red line in Fig. 5.14. The red spectrum has an extra small peak at 450 cm$^{-1}$ other than two peaks we mentioned above. This is less apparent in the result at $a_{KRb} = 78.5$ a$_B$, indicated by the gray line. Thus it is justified that this spectral feature originates in the modulation instability. Considering the method of our analysis, it is natural to think that the position of the peak indicates the $k_{max}$ and we should take 1/e-width of the peak as a standard deviation. However, it is difficult to extract this spectral feature as an apparent peak, simply due to the bad signal-to-noise ratio and this should only be compared with the theoretical prediction of 434 cm$^{-1}$ with peak densities of BECs (see Fig. 5.9).
Figure 5.14: Spectrum obtained at $a_{KRb} = 80.8 \ a_B$. Eleven experimental shots out of 60 are selected and as a result 55 spectra are summed. Error bars indicate the standard error of the experimental shots. Again for this spectrum, running average is taken over 3 points in order to smoothen the spectra. Red line indicates the result at $a_{KRb} = 80.8 \ a_B$ and gray at $a_{KRb} = 78.5 \ a_B$. 
5.3.4 **Results at** $a_{KRb} = 81.5 \ a_B$

We have succeeded in observing the modulation instability for inter-species scattering length $a_{KRb} = 80.8 \ a_B$ and analyzed the size of the spatial pattern which is consistent with the theory. Next we would like to investigate other interaction strength $a_{KRb} = 81.5 \ a_B$, which is slightly stronger than above experiment by only 0.7 $a_B$. For this parameter, expected wavenumber of the fastest-growing spatial structure is 791 cm$^{-1}$. Surprisingly, this only tiny variation of scattering length results in a large difference in the wavenumber $k_{max}$.

Experimental sequence is totally the same as in Section 5.3.3. The only differences are the Feshbach magnetic field, whose final value is changed from 75.41 G to 75.43 G, and the time interval of probe pulse which is changed to 8ms. Typical images of time evolution at $a_{KRb} = 81.5 \ a_B$ is shown in Fig. 5.15. The behavior of a dual BEC is now entirely different from that of $a_{KRb} = 80.8 \ a_B$. Center of Rb BEC looks a bit dented in the presence of K BEC and succeedingly, relatively small spatial structure seems to grow in the region from 20 pixel to 40 pixel. At the same time, clear anti-correlation of spatial densities of K and Rb can be observed from these images.

This becomes clearer when we subtract two images (see Fig. 5.16, blue plots). We can fit the subtracted signals by the sinusoidal function with the second-order polynomial as an envelope (Fig. 5.17). Envelope of the sinusoidal wave is taken to be the quadratic function of $x$ just because the envelope can likely be given by the difference of two Thomas-Fermi profiles. From these fitting results we can estimate that the periodic structure seen in the images has a wavenumber of roughly (4 pixel)$^{-1} = (10 \ \mu m)^{-1} = 1000$ cm$^{-1}$.

Then let us analyze the results for $a_{KRb} = 81.5 \ a_B$ in the same manner as in Section 5.3.3, that is, in terms of the Fourier spectrum. We have 56 experimental sequences and select 13 of them in accordance with the criterion $0.9 < N_K/N_{Rb} < 1.1$. Five Fourier spectra contained by each of these 13 shots are all summed to give 65-fold summed spectrum shown in Fig. 5.18.
Figure 5.15: Typical images of time evolution at $a_{KRb} = 81.5 \ a_B$. Field of view of each image is 13 µm(vertical) × 227 µm(horizontal)
Figure 5.16: Subtracted signals of two atomic species and Fourier transformation of them ($a_{KRb} = 81.5 \ a_B$).
Figure 5.17: Subtracted signals and fittings on them. Fit function is $f(x) = s \cdot \sin \left( \frac{2\pi x}{L + \phi} \right) + a + b(x - x_0)^2$. 
Figure 5.18: Spectrum obtained at $a_{KRb} = 81.5 \ a_B$. Thirteen experimental shots out of 56 are selected and as a result 65 spectra are summed. Error bars indicate the standard error of the experimental shots. Again for this spectrum, running average is taken over 3 points in order to smoothen the spectra. Red line indicates the result at $a_{KRb} = 81.5 \ a_B$ and gray at $a_{KRb} = 78.5 \ a_B$. 
Again we can see the peaks at 0 cm$^{-1}$ and 300 cm$^{-1}$ which come from the 
offset and the fringe on the whole image. One might well expect that there is 
a small peak at around 1000 cm$^{-1}$, which is consistent with the one observed 
in the real-space images above. However, this cannot be separated from the 
error bars and it can be said that in this time no relevant peak which can be 
separated from noises is observed.

5.3.5 **Summary of the Results**

In summary, we succeeded in an *in-situ* observation of the modulation 
instability using a dual BEC by suddenly changing the interspecies scattering 
length from $a_{KRb} = 0.7 \ a_B$ (in the miscible regime) to $a_{KRb} = 80.8 \ a_B$ 
(in the immiscible regime). Not only the dynamical behavior of the spatial 
structure of a dual BEC, but the qualitative analysis of over a dozen pieces 
of spectra was done. As a result for $a_{KRb} = 80.8 \ a_B$, observed feature of the 
Fourier spectrum does not contradict to the theoretically calculated value 
434 cm$^{-1}$. 
We tried to observe the $a_{KRB}$-dependence of the typical size of spatial structure growing by the modulation instability by the same experimental sequence, except for changing the interspecies scattering length from $a_{KRB} = 0.7 \ a_B$ to $a_{KRB} = 80.8 \ a_B$. Although the observed spatial structure implied the spatial modulation with the wavenumber $\sim 1000 \ \text{cm}^{-1}$, we could not observe relevant peaks from the spectrum. It can be that the peak at 1000 cm$^{-1}$ is so small that we cannot distinguish it from the noise floor, which is the standard error of the experimental shots. In order to get this over, one can prepare more and more experimental images and make the standard error much smaller. If the signal-to-noise ratio is limited by the photon shot noise, we can increase the ratio by a factor $\sqrt{\alpha}$ with increased probe light intensity by a factor $\alpha$, in exchange for the absorption of photons by atoms. In our experiment the photon absorption rate is thought to be negligible so that this solution can be valid.

In addition, there is a fluctuation of magnetic fields for a Feshbach resonance. The 100 Hz, 17 mG (in a root mean square value) fluctuation of the magnetic field comes probably from the ripple noise of the current source. This results in a fluctuation of the scattering length of 0.35 $a_B$ and thus of $k_{max}$ ranging from 600 cm$^{-1}$ to 900 cm$^{-1}$ at the central value 81.5 $a_B$. Therefore the more reliable measurement requires the less fluctuating magnetic field realized by, for example, the stabilization with the feedback to the current injected into the coils. There is one more solution to this: the larger magnetic field of more than 100 G gives the same scattering length with smaller sensitivity to the magnetic field which is an order of magnitude smaller. However, currently 100 G is not easily available with our setup of the coils and current sources so that an additional coils for adding up the magnetic field bias is needed to implement this.

Another prescription might be the imprinting of spatial modulation by two lasers with an angle $\theta$ just as in Fig. 5.19. In this situation, two light field interfere and form aperiodic potential with its periodicity $\lambda' = \lambda / 2 \sin (\theta / 2)$, where $\theta = 2$ degrees give $\lambda' = 22 \ \mu m$ and $\theta = 4$ degrees give $\lambda' = 11 \ \mu m$. 

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This can trigger the modulation instability if the fastest-growing size of the 
structure and the wavelength $\lambda'$ coincide. Then with weakly applied con-
tinuous wave of two lasers or a weak pulse of two lasers, we can do a spec-
troscopy with respect to the interspecies scattering length by tuning it by a 
Feshbach resonance, where there should appears the largest peak in spectra 
when $\lambda' = \lambda_{max}$. By doing so, it is expected that we can obtain more sig-
nificant peaks in spectra and can observe the spatial pattern with a higher 
wavenumber, which was difficult to observe in this thesis.
Chapter 6

SUMMARY AND CONCLUSIONS

Here we briefly summarize the achievements in this thesis below. First, Chapters 1 and 2 are devoted to the introduction and the historical / theoretical overview on a dual BEC. Coupled Gross-Pitaevskii equations described in Chapter 2 forms the basis of this thesis in a sense that we have dealt with a dual BEC regardless of the topics are static or not.

In Chapter 3, achievements can be divided into three major topics:

• We prepared the probe lasers with moderate detunings and instrumental setup in order to realize the simultaneous observation of $^{41}$K and $^{87}$Rb by phase contrast imaging.

• Optical dipole traps for generating an elongated dual BEC was prepared.

• A pair of coils with anti-Helmholtz configuration was added in order to compensate the horizontal position of two BECs.

These three were revealed to be equally important for the in-situ observation and study on the overlapped dual BEC. Actually, making use of these three we successfully generate elongated BECs and observe both atomic species non-destructively at a same experimental run. These results were achieved
in Chapter 4. In Chapter 4 we also observed the center-of-mass motion of a dual BEC which implies the non-equilibrium nature.

Chapter 5 described the main issue of this thesis, the modulation instability, which is the dynamical phenomenon universal for the nonlinear system. We observed the spatial structure of the typical size \( \sim 22(4) \, \mu m \) (or \( (447 \pm 82) \, cm^{-1} \) in wavenumber) emerging for the condition \( a_{KRb} = 80.8 \, a_B \). For \( a_{KRb} = 81.5 \, a_B \) we could not observe any significant peak in the Fourier spectrum, however, in the spatial domain we observed the growth of spatial structure with typical wavelength \( \sim 10 \, \mu m \) (or \( \sim 1000 \, cm^{-1} \) in wavenumber). Observed two features at \( a_{KRb} = 80.8 \, a_B \) and \( a_{KRb} = 81.5 \, a_B \) do not contradict the theoretical values calculated from coupled Gross-Pitaevskii equations.

First of all, the biggest problem in the observation in Chapter 5 was the signal-to-noise ratio of the obtained images. The larger S/N ratio can be obtained by shining stronger probe laser onto the atoms. This causes the absorption of the probe photons by atoms more, however, the photon scattering rate in the current setup is thought to be negligibly small and by increasing the light intensity we can get better signal-to-noise ratio proportional to \( \sqrt{I_{probe}} \), where \( I_{probe} \) is the probe intensity. Of course the photon scattering rate scales as \( I_{probe} \), therefore we have to find the optimal strength of probe light such that we can get sufficient S/N ratio and suppress the light absorption.

It is also important to suppress the fluctuation of the scattering length by stabilizing the magnetic field or using larger values of magnetic field (\( \sim 100 \, G \)) which gives the same values of the scattering length but the smaller sensitivity \( da_{KRb}/dB \sim 2.5 \, a_B/G \), the an order of magnitude smaller value than the one considered in this thesis (\( \sim 3.7 \, a_B/G \)). The latter prescription is relatively easy but available if we utilize another pair of coils to add up the bias magnetic field.

There is a difficulty with observing modulation instability in the way we did because the nucleation of density fluctuation is completely at random.
One can make it easy to study the domain formation by imprinting a spatial modulation using two lasers with wavelengths $\lambda$ incident to atoms with an angle $\theta$ as depicted in Fig. 5.19, which make a periodic potential with its separation $\lambda/2 \sin (\theta/2)$. By tuning the angle between two laser beams the wavenumber of imprinted spatial structure will change and this will enable us to investigate the $a_{KRB}$-dependence of the wavenumber of structures more accurately.

We have shown throughout in this thesis that the study on dynamical behaviors in a dual BEC is available in our experimental system. Modulation instability is one of those non-equilibrium phenomena while there are a lot of dynamical instabilities such as the Rayleigh-Taylor instability[55, 56], Kelvin-Helmholtz instability[60] and others.

The study on solitons and vortices are also interesting issues. The generation of the dark-bright solitons[57] and the vortex-lattice dynamics[111] are already observed experimentally, and theoretically the vortex-bright soliton formation[109], the collective excitation in a binary BEC[110] and other phenomena are intensively studied. Our setup of a dual-species BEC combined with the nondestructive imaging will works as a powerful tool to investigate these phenomena and we assign to uncover these non-equilibrium natures of superfluid-superfluid interactions as a future works.
Appendix A

ALKALI D-LINE DATA

The properties of D-lines of alkali atoms listed below are quoted from D. A. Steck’s document[92] and T. G. Tiecke’s thesis[93].

A.1 Properties of $^{87}$Rb D2 Line

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency $\omega_0$</td>
<td>$2\pi \cdot 384.2304844685(62)$ THz</td>
</tr>
<tr>
<td>Natural linewidth</td>
<td>$2\pi \cdot 6.0666(18)$ MHz</td>
</tr>
<tr>
<td>Hyperfine constant $A_{2S_{1/2}}$</td>
<td>$3.417341305452145(45)$ GHz</td>
</tr>
<tr>
<td>Hyperfine constant $A_{2P_{3/2}}$</td>
<td>$84.7185(20)$ MHz</td>
</tr>
<tr>
<td>Electric quadrupole constant $B_{2P_{3/2}}$</td>
<td>$12.4965(37)$ MHz</td>
</tr>
<tr>
<td>Electronic $g$ factor $g_J(^2P_{1/2})$</td>
<td>$2.00233113(20)$</td>
</tr>
<tr>
<td>Electronic $g$ factor $g_J(^2P_{3/2})$</td>
<td>$1.3362(13)$</td>
</tr>
</tbody>
</table>

In addition to these, the wavelength of the D1 transition of Rb, which is $794.978851156(23)$ nm, is used for the calculation of trapping frequencies of optical traps. The level structure is shown in Fig. A.1 (a).
## A.2 Properties of $^{41}$K D2 Line

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency $\omega_0$</td>
<td>$2\pi \cdot 391.016\ 406\ 21(12)$ THz</td>
</tr>
<tr>
<td>Natural linewidth</td>
<td>$2\pi \cdot 6.035(11)$ MHz</td>
</tr>
<tr>
<td>Hyperfine constant $A_{2S_1/2}$</td>
<td>$127.0069352(6)$ MHz</td>
</tr>
<tr>
<td>Hyperfine constant $A_{2P_3/2}$</td>
<td>$3.363(25)$ MHz</td>
</tr>
<tr>
<td>Electric quadrupole constant $B_{2P_3/2}$</td>
<td>$3.351(71)$ MHz</td>
</tr>
<tr>
<td>Electronic $g$ factor $g_J(^2P_{1/2})$</td>
<td>$2.002\ 294\ 21(24)$</td>
</tr>
<tr>
<td>Electronic $g$ factor $g_J(^2P_{3/2})$</td>
<td>$4/3$</td>
</tr>
</tbody>
</table>

In addition to these, the wavelength of the D1 transition of K, which is $770.107\ 919\ 192(123)$ nm, is used for the calculation of trapping frequencies of optical traps. The level structure is shown in Fig. A.1 (b).
Figure A.1: Level structures of Rb and K D2 line
Appendix B

FRINGE CLEANING

In experiment, images of atoms are generated by taking three frames of pictures, namely the probe light with the atoms, the one without atoms and the one without probe light nor atoms. These three are called "shadow", "light" and "dark" images respectively. We take ten shadow images in the shadow frame that consist of five images for each species. In the light and dark frames again ten images are taken in the same order with shadow frame. The size of single frame is 1047 pixel × 512 pixel and let us denote signals on shadow, light and dark frame $S(i, j)$, $L(i, j)$ and $D(i, j)$, where $i = 0, 1, \cdots, 1047$ and $j = 0, 1, \cdots, 511$. Signal $F(i, j)$ is generated by following math:

$$F(i, j) = S(i, j) - D(i, j) - L(i, j).$$

In terms of eq. (3.8), the fractional change of light intensity through the atom $I/I_0$ is directly related to this by

$$\frac{I}{I_0} = F(i, j) = \frac{S(i, j) - D(i, j)}{L(i, j) - D(i, j)}.$$  \hspace{1cm} (B.1)

Actually, experimentally obtained images are polluted by some noises. One is the fluctuation of light intensity or dark current at each position $(i, j)$ which adds the white noise on the image. Another is the fringe, the spatially periodic noise on the whole image. In this appendix we deal with the latter. There might be some reasons for the occurrence of fringe, however, the main
reason can be the small shift of probe beam in the light image with respect to the shadow image due to the mechanical fluctuation of the optics. One solution to this is the preservation of imaging optics from external fluctuations. Even if we do this, often there remains the fringe and we have to do second prescription: cleaning of fringe by image processing, which we call "fringe cleaning" here.

Fringe cleaning process consists of a few steps itemized below:

- First we collect several tens of light images $L_m(i,j)$ which has various patterns of intensity fluctuations. These are usually taken the latest several tens of light images before the on-going sequence.

- We find a region in $S(i,j)$ where there is no atoms, and then fit that area with $\sum_m c_m L_m(i,j)$ with fitting parameters $\{c_m\}$. Detail of this optimization process is described later.

- Given the optimal parameters $\{c_m^0\}$, we replace $L(i,j)$ in eq. (B.1) by $L'(i,j) = \sum_m c_m^0 L_m(i,j)$ to get clean images of atoms, that is,

$$\frac{I}{I_0} = F'(i,j) = \frac{S(i,j) - D(i,j)}{L'(i,j) - D(i,j)}. \quad \text{(B.2)}$$

In short, we create the optimal light image by using the linear combination of latest several tens of light images and replace the light image by an optimized light image.

Now let $S^0(i,j)$ denotes a region in a shadow image where there is no atoms. Similarly we take the same region for light images $L_m(i,j)$ and let these be $L_m^0(i,j)$. Optimization of coefficients $\{c_m\}$ is done by the least square method

$$\text{minimize} \quad \sum_{i,j} \left( S^0(i,j) - \sum_m c_m L_m^0(i,j) \right)^2 \overset{\text{def}}{=} \Delta. \quad \text{(B.3)}$$

$\Delta$ is quadratic in $\{c_m\}$ and this problem reduces to solve linear algebraic equations as a consequence of conditions $\partial \Delta / \partial c_n$

$$\sum_m A_{nm} c_m = B_n$$

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where $A_{nm}$ and $B_n$ are defined as

\[ A_{nm} = \sum_{i,j} L^0_n(i,j) L^0_m(i,j), \]
\[ B_n = \sum_{i,j} L^0_n(i,j) S^0(i,j). \]

The solutions \( \{c^0_m\} \) of eq. (B.3) is rigorously obtained by Gauss’s elimination method and is numerically done very easily. Then we can calculate the right hand side of eq. (B.2) to get the clean image.

Processed images of dual BEC with variable number of reference images (equivalent to light images) are shown in Fig. B.1. Number of reference images $N_{ref}$ are 1, 10, 20 and 25 from top to bottom and it can be seen that the more the number of reference images, the cleaner the processed images.
Figure B.1: Processed images of a dual BEC with variable number of reference images. Number of reference images $N_{\text{ref}}$ are 1, 10, 20 and 25 from the top to the bottom.
Appendix C

DISCRETE FOURIER TRANSFORMATION

In the context of Chapter 5, the spatial modulation on the density profile of BECs are to be observed. However, it was difficult to observe the spatial pattern directly because the actually emerging spatial pattern was not purely a sinusoidal. This is possibly due to the growth of patterns with other wavenumbers caused by inhomogeneity of spatial densities and the fluctuation of the bias magnetic field. Even if we have a BECs with uniform density and magnetic field with sufficiently small fluctuation, observed pattern might contain waves with wavenumbers around $k_{\text{max}}$.

Under this situation, it is good to Fourier transform the signals and obtain a spectrum of it. In this case, we have 88 pixels in the horizontal direction and in order to find some spectral components of a series of discretely sampled data, most commonly used method is discrete Fourier transformation (DFT). In this appendix the basics of DFT and some important features of DFT, namely sampling theorem and aliasing are introduced.
C.1 Discrete Fourier Transformation

Fourier transformation is one of the most commonly used mathematical technique in science and technology. Given some function \( f(x) \in \mathbb{C} \) such that square of this function is integrated over \( x \in (-\infty, +\infty) \) to give a finite value:

\[
\int_{-\infty}^{\infty} |f(x)|^2 \, dx < \infty.
\]

Then the Fourier transformation \( F(k) \) of \( f(x) \) exists. These are related to each other by

\[
F(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx} \, dx,
\]

\[
f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k) e^{ikx} \, dk.
\]

This is what we usually refer to as the Fourier transformation (and its inverse) of \( f(x) \). However, this is not applied straightforwardly to the realistic problems because we can never have continuous data series \( f(x) \) but have always discrete or sampled ones. Therefore the data is expressed as \( f_n = f(n\Delta X) \), where \( n \) is an integer. \( \Delta X \equiv X/N \) is the spatial or temporal length of the data \( X \) divided by \( N \), the number of data taken in the region with length \( X \). Given this expression, we can substitute this into the continuous form of Fourier transformation and its inverse, and discretize them as

\[
F_m = \Delta X \sum_{n=0}^{N-1} f_n \exp \left[ -i \cdot 2\pi mn / N \right],
\]

\[
f_n = \frac{1}{N} \sum_{m=0}^{N-1} F_m \exp \left[ i \cdot 2\pi mn / N \right].
\]

Here \( n \) and \( m \) are integers and smallest step in the \( k \)-domain is \( \Delta k \equiv 1/X = 1/N\Delta X \). With these formulae, we can deal with the realistic case which has sampled data points. Note that the smaller the number of data points \( N \) is, the larger the step of sample in \( k \)-domain \( \Delta k \) is. This means that too rough sampling results in the loss of information of fine spectral structure.
C.2 Sampling Theorem and Aliasing

In the previous section we derived the formula for DFT. Of course, there emerges some problems accompanied to the discretization of data points. One of the biggest problem naturally appears when we think of how small structure we can extract with sampling rate $\Delta X = X/N$. Naively speaking, the maximum valid $k$ can be detected when $1/k$ is equal to $2\Delta X$, that is, if there is sinusoidal wave with wavenumber $k$ is present, one data point hits the maximal and then nearest neighbor data point hits the minimal, successingly the next hits the maximal. This idea is supported by the famous “sampling theorem” which states that

“If the amplitude of Fourier component $|F(k)|$ is always zero for $|k| < k_c$, signal $f(x)$ is completely reproduced by sampling rate $x = n/2k_c$ where $n$ is an integer and $k_c$ is called Nyquist frequency.”

In actual case we have a data set acquired with some sampling rate $1/k_0$ and we can extract the Fourier components in the region $|k| < k_0/2$ as the valid ones.

Then what will happen when we calculate the Fourier component in the region $|k| > k_0/2$? The answer is in short expressed as the aliasing or folding noise. Consider the case that we have five data points as in Fig. C.1(black opened circle). For these points, sinusoidal wave with frequency 0.2 Hz(blue dash-dotted line) fits very well. However, one with 0.8 Hz(orange dashed line) also does quite nicely. Remind that in this case Nyquist frequency is $k_c = 0.5$ Hz, and frequencies 0.2 Hz and 0.8 Hz correspond to the region $|k| < k_c$ and $|k| > k_c$, respectively. In terms of Fourier spectrum it looks as if the spectrum in $0 < k < k_c$ folds back at $k = 0.5$ to form spectrum in $k_c < k < 2k_c$. Therefore this phenomenon is called folding noise or aliasing.
C.3 Analysis of Experimentally Obtained Data

Actually we have a series of 88 signals in the horizontal direction within a single image of atoms. As in Chapter 5, our analysis consists of three processes for obtaining Fourier spectrum we deal with. First we must integrate the signals of 2D image in vertical direction to get horizontal profile. Second, horizontal profile of Rb is subtracted from that of K for enhancing the anti-correlation of horizontal profile. Finally we Fourier transform this subtracted signal to analyze the Fourier spectrum of anti-correlation. Outline of this whole process is indicated in Fig. C.2, with actually obtained experimental images.

In Fig. C.2 (d) we plot the power spectral density as a Fourier spectrum. When we Fourier transform some signal and obtain its Fourier component $F(k) \in \mathbb{C}$, we can always rewrite this as its modulus $|F(k)|$ multiplied by its argument $\angle F(k): F(k) = |F(k)|\angle F(k)$. Squared modulus $|F(k)|^2$ is called energy spectral density. Power spectral density is defined as

$$P(k) = \lim_{T \to \infty} \left[ \frac{1}{T}|F(k)|^2 \right],$$

which we calculate this simply by $P(k) = F(k)/T$ for our analysis.
Figure C.2: Data processing of images to obtain Fourier spectrum.
Appendix D

MEASUREMENT OF THREE-BODY LOSS COEFFICIENT

In atomic physics experiment, even if we trap atoms they run away from the trap for some reasons. The reasons are classified by number of atoms involved. One-body loss refers to the process involving one atom and its interaction with surroundings. For example, in an optical trap the photon scattering by laser for ODT can push atoms away from the trap. Or in the magnetic trap, spin flip results in the transfer of an atom to the high-field seeking state and thus atoms run away from the trap. Usually for experiments in an optical trap, the photon scattering lifetime is more than 1 second.

Two-body loss refers to inelastic collisions like spin exchange collisions and the light-assisted collisions. Spin exchange collision takes place when, for instance, atoms in states $|F = 1, m_F = 1 \rangle$ and $|F = 1, m_F = -1 \rangle$ collides and then become two $|F = 1, m_F = 0 \rangle$ atoms to get excess energy $\Delta E > 0$. If this excess energy is negative, spin exchange collision rate is forbidden at low temperature. Our atomic gas mixture, K and Rb, are both prepared in $|F = 1, m_F = 1 \rangle$ state so that this process is inhibited.
Another one, light-assisted collision is usually considered in the context of MOT. When the near-resonant light is applied to atoms, collision between ground state atom and excited state atom causes two types of inelastic collisions, namely fine-structure-changing collision and radiative escape. These two are altogether called light-assisted collision. In an optical trap, near-resonant light is not applied in our experiment and we do not consider this process here.

Then we arrive at the point to consider the three-body loss, what is also called three-body recombination. Note that there can be no inelastic collision in two-body collision when the spins and quantum states are properly prepared. However even in this situation, three-body collision is allowed to be an inelastic collision by forming a diatomic molecule and obtaining the binding energy as excess energy. This is always present in BEC experiment and the three-body recombination rate has been intensively studied as a candidate of tool to reveal few-body physics since the pioneering experiment of Ref.[86]. Recently three-body recombination rate of heteronuclear case is of interest and measured for $^{40}$K and $^{87}$Rb mixture[87]. This contributes to the understanding of universal three-body physics including Efimov effect[88], which is well investigated for homonuclear three-body collision[95].

Of course it is needed to let BECs of K and Rb overlap in our experiment and the three-body loss is always concerned. This is simply because three-body loss rate is proportional to the spatial density cubed and spatial density of BEC is typically an order of magnitude larger than that of thermal gas. Here in this Appendix we will take some pages for describing the measurements of three-body loss rate and briefly refer to on-going experiment in our lab.

Every time we measure the loss rate, whose origin depends on the situation, all we can do is to shed light on atomic cloud and count the number of atoms at various time. As is usual we prepare thermal mixture of temperatures $\sim 200$ nK and densities $n_K = 5 \times 10^{12} /\text{cm}^3$ and $n_{\text{Rb}} = 6 \times 10^{12} /\text{cm}^3$ in 809 nm optical trap and hold it for a time $t$, then release them from trap.
and shine resonant light to measure the number of atoms. For extract the information of three-body loss rate, we consider rate equations as follows:

\[ \frac{dn_K}{dt} = -2L_{KKRb}n_K^2n_{Rb} - L_{KRRbRb}n_Kn_{Rb}^2 - \frac{n_K}{\tau_K} \]  
\[ \frac{dn_{Rb}}{dt} = -L_{KKRb}n_K^2n_{Rb} - 2L_{KRRbRb}n_Kn_{Rb}^2 - \frac{n_{Rb}}{\tau_{Rb}} \] 

(D.1)  
(D.2)

Here \( L_{KKRb} \) and \( L_{KRRbRb} \) are three-body coefficients which have unit [cm\(^6\)/s] and \( \tau_K \) and \( \tau_{Rb} \) are lifetime in optical dipole trap. As a whole, each of these rate equations contains two kinds of three-body loss channel and one-body loss due to the photon scattering by dipole trap laser. Factor 2 reflects the fact that for the loss channel which two K and one Rb are involved reduces the number density of K two times faster than that which two Rb and one K are involved.

The obtained decay curve of number densities calculated from measured numbers of atoms after variable hold time is shown in Fig. D.1. Blue (Red) circle indicates the measured density of K (Rb) and Blue (Red) solid line shows the simulated result of equations (D.1) and (D.2) with initial densities \( n_K = 5 \times 10^{12} \) /cm\(^3\) and \( n_{Rb} = 6 \times 10^{12} \) /cm\(^3\), and three-body loss coefficients \( L_{KKRb} = 7 \times 10^{-27} \) cm\(^6\)/s and \( L_{KRRb} = 0 \) cm\(^6\)/s. Photon scattering lifetimes are set to be \( \tau_K = 24 \) s and \( \tau_{Rb} = 130 \) s. From the simulation with various parameters we concluded that \( L_{KKRb} \) is less than \( 1 \times 10^{-27} \) cm\(^6\)/s.

From this, our concern is now the \( L_{KKRb} = 7 \times 10^{-27} \) cm\(^6\)/s. Considering the density of BEC cloud \( n \simeq 3 \times 10^{13} \) /cm\(^3\), the loss rate is roughly estimated by simply calculating \( L_{KKRb}n^2 \sim 10 \) Hz, that gives the timescale 100 ms at this interspecies scattering length. This is not said to be sufficiently long for various experiment. On the other hand we have a tool to observe atomic clouds in-situ. By using this we observed that dual BEC survives even if we hold it for 800 ms in 809 nm optical trap with interspecies scattering length \( a_{KRb} = 0.7 \) \( a_B \) (see Fig. D.2). This is very long, as is deduced by assuming the tendency of three-body recombination coefficient, that is, \( L_{KKRb} \) scales as \( a_{KRb}^4 \) [77, 78].
Besides, *in-situ* observation of atoms by phase contrast imaging can extract the three-body loss coefficient with a single experimental shot. This is strikingly efficient compared with the same experiment with TOF measurement. Furthermore, for destructive measurement we should deal with the shot-to-shot fluctuation of number of atoms due to temperature excursion of experimental apparatus. *In-situ* measurement can also neglect this fluctuation. Now some members of our group focuses on this and are measuring the three-body coefficient of $^{41}$K-$^{87}$Rb mixture. This is an on-going experiment.
Figure D.1: Measurement of three-body loss coefficient in thermal mixture of K and Rb. Blue ( Red ) circle indicates the measured density of K ( Rb ) and Blue ( Red ) solid line shows the simulated result of equations (D.1) and (D.2) with initial densities \( n_K = 5 \times 10^{12} \) /cm\(^3\) and \( n_{Rb} = 6 \times 10^{12} \) /cm\(^3\), and three-body loss coefficients \( L_{KKRb} = 7 \times 10^{-27} \) cm\(^6\)/s and \( L_{KKRb} = 0 \) cm\(^6\)/s.
Figure D.2: *In-situ* observation of dual BEC with time interval 200 ms. Dual BEC appears to be survives even at 800 ms.
Appendix E

NUMERICAL SOLUTION OF COUPLED GP EQUATIONS

In the Fig. 2.2 in Chapter 2, we showed numerical solutions of coupled Gross-Pitaevskii equations for interspecies scattering lengths \( a_{KRb} = 50 a_B \) and \( a_{KRb} = 200 a_B \). Actually what we do for obtaining these is to numerically find solutions which minimizes the energy functional in Thomas-Fermi approximation

\[
E = \int \left( \sum_{i=1,2} \left[ V_i(x)|\psi_i|^2 + \frac{g_{ii}}{2} |\psi_i|^4 \right] + g_{12} |\psi_1|^2 |\psi_2|^2 \right) \, dx
\]

where \( n_i(x) \) \( (i = 1, 2) \) denote the spatial densities of two components and here we limit the problem in 1-dimension. Let us further rewrite the integral in the following way:

\[
\int \left( \sum_{i=1,2} \left[ V_i(x)n_i(x) + \frac{g_{ii}}{2} n_i^2(x) \right] + g_{12}n_1(x)n_2(x) \right) \, dx
\]

\[
= \int 2\pi \hbar^2 a_B \frac{a_B}{m_p} \left( \sum_{i=1,2} \left[ \frac{1}{2\pi \hbar^2 a_B} \frac{m_p}{m_i} \frac{a_{ii}^2}{2} x^2 n_i(x) + \frac{a_i/a_B}{m_i/m_p} n_i^2(x) \right] + \frac{a_{12}/a_B}{m_{12}/m_p} n_1(x)n_2(x) \right) \, dx
\]

\[
= 2\pi \hbar^2 a_B \frac{a_B}{m_p} \times 10^{-40} \int \left( \sum_{i=1,2} \left[ \eta_i x^2 n_i(x) + \frac{a_{ii}^2}{m_i^2} n_i^2(x) \right] + \frac{a_{12}^2}{m_{12}^2} n_1(x)n_2(x) \right) \, dx.
\]
Here $a_i^* = a_i/a_B$ and $a_{12}^* = a_{12}/a_B$ are scattering lengths in the unit of $a_B$ and the mass $m_i^* = m_i/m_p$ and $m_{12}^* = m_{12}/m_p$ are scaled by the proton mass $m_p$. The factor $\eta_i$ is written explicitly as follows:

$$\eta_i = \frac{1}{2\pi \hbar^2 a_B} \frac{m_p m_i \omega_i^2}{2} \times 10^{-32}.$$ 

In this form a factor $10^{-32}$ is multiplied because in the third line in the above formula $x$ has a unit of $\mu m$ and $n_i(x)$ have units of $10^{20} \times 1/m^3$. Assuming that two components 1 and 2 feels the same confining potential, that is, $m_1 \omega_1^2/2 = m_2 \omega_2^2/2$, we have $\eta_1 = \eta_2 \overset{\text{def}}{=} \eta$. Thus the integral which should be minimized becomes, with parameters of $R_b$ and $K$,

$$\int \left[ \frac{x^2 n_{R_b}(x)}{10} + \frac{100}{87} n_{R_b}^2(x) + \frac{x^2 n_K(x)}{10} + \frac{60}{41} n_K^2(x) + \frac{2a_{KRb}^*}{55.7} n_{R_b}(x)n_K(x) \right] dx.$$ 

We used $\eta \sim 1/10$ which can be obtained with a trapping frequency of $\sim 80$ Hz. At last, we discretize this with respect to $x$ and let Wolfram Mathematica find the series of $n_{R_b}$ and $n_K$. Here is the code:

```mathematica
Clear[f, c, d];

aKRb = 50;
f = Sum[i*i*c[i]/10 + 100/87*(c[i])^2 + i*i*d[i]/10 + 60/41*(d[i])^2 + 2*aKRb/55.7*c[i]*d[i], {i, 0, 16}];

cons = {
    Sum[c[i], {i, 0, 16}]/16 == 1,
    Sum[d[i], {i, 0, 16}]/16 == 1,
    Table[c[i] >= 0, {i, 0, 16}],
    Table[d[i] >= 0, {i, 0, 16}]
};

vars = Join[Table[c[i], {i, 0, 16}], Table[d[i], {i, 0, 16}]]; 
```

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Sol = NMinimize[{f, cons}, vars,
   Method -> {"SimulatedAnnealing", "PerturbationScale" -> 10 }]

The obtained results for various interspecies scattering lengths are listed in Fig. E.1 and the phase separation in the region $a_{K_{Rb}} = 75 a_B$ can be clearly observed.
Figure E.1: Numerically calculated Thomas-Fermi profiles of dual BECs with various interactions. The interspecies scattering lengths are set to be (a) $a_{KRb} = 0 \ a_B$, (b) 25 $a_B$, (c) 50 $a_B$, (d) 60 $a_B$, (e) 65 $a_B$, (f) 70 $a_B$, (g) 80 $a_B$ and (h) 200 $a_B$. A dual BEC becomes immiscible when $a_{KRb} > 75 \ a_B$. 

For all plots, horizontal: positon [arb. unit] vertical: density [arb. unit]
Appendix F

C++ PROGRAM FOR ANALYZING THE IMAGE

Here we put the whole C++ program for analyzing the image obtained in a sequence, including the fringe cleaning, zooming up the atomic cloud and displaying the radially integrated, axial density profile of Rb and K in a same graph.

```cpp
#include<iostream>
#include<ctime>
#include<time.h>
#include<cstdlib>
#include<string>
#include<iomanip>
#include<math.h>
#include<typeinfo>
#include<fstream>
#include<stdio.h>
#include<sstream>
#include<cstdio>

#define PGNUPLOT "C:\Program Files (x86)\gnuplot\bin\pgnuplot.exe"
using namespace std;
```
string IntToString(int number){
    stringstream ss;
    ss << number;
    return ss.str();
}

int main(){

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    string date="20131205";
    string seqnum;
    cout << "Input the sequence number(ex. 1, 17, 138, ...):";
    int seqnumber;

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cin >> seqnumber;
if(seqnumber > 99)
    seqnum = IntToString(seqnumber);
else if( seqnumber <100 && seqnumber >9 )
    seqnum = "0" + IntToString(seqnumber);
else
    seqnum = "00" + IntToString(seqnumber);
string sfile;
sfile = "C:\Data\aref\shadow.txt";
cout << "Input file is '" << sfile <<"'\n";

clock_t start, end;
start = clock();

/**********************************************************

int **shadow = new int*[row+1];
for(i = 1; i < row + 1; i++){
    shadow[i] = new int[column+1];
}
for(i=1; i<row+1; i++){
    for(j=1; j<column+1; j++){
        shadow[i][j] = 0;
    }
}

FILE* fp;
fp = fopen(sfile.c_str(), "r");

if(fp==NULL) {
    printf("Cannot open the file. \n");
    return 1;
}
for(j=1; j<column + 1; j++){
    for(i=1; i<row + 1; i++){
        fscanf(fp, "%d", &shadow[i][j]);
    }
}

/*---- LIMITING THE REGION OF IMAGE AND MASKING THE ATOMS -----*/
int xlim = 125;
int **s1 = new int*[row+1];
for(i = 1; i < row + 1; i++){
    s1[i] = new int[column-2*xlim-10+1];
}

//left half
for(i=1; i<row+1; i++){
    for(j=1; j<xc-r-xlim+1; j++){
        s1[i][j] = 0;
        s1[i][j] += shadow[i][j+xlim];
    }
}

//right half
for(i=1; i<row+1; i++){
    for(j=1; j<column-xlim-xc-(r-1)+1; j++){
        s1[i][xc-r-xlim+j] = 0;
        s1[i][xc-r-xlim+j] += shadow[i][xc+(r-1)+j];
    }
}

/*-------------------SPLIT INTO 10 IMAGES---------------------*/
int h;
int ***s2 = new int**[10+1];
for(h=1; h<10+1; h++){
    s2[h] = new int*[d+1];
}
for(i = 1; i < d + 1; i++){
    s2[h][i] = new int[column-2*.xlim-10+1];
}
}

for(h=1; h<10+1; h++){  
    for(i=1; i<y2-y1+1; i++){ 
        for(j=1; j<column-2* xlim-(2*r-1)+1; j++){ 
            s2[h][i][j] = s1[y1+i+(h-1)*d][j];
        }
    }
}

cout << "Shadow image for calculation is created. \n";

/************************ LOAD LIGHT IMAGES ************************/
int n;
int **light = new int*[row+1];
for(i = 1; i < row + 1; i++){ 
    light[i] = new int[column+1];
}
for(i=1; i<row+1; i++){ 
    for(j=1; j<column+1; j++){ 
        light[i][j] = 0;
    }
}

int ***l1 = new int**[n0+1];
for(n=1; n<n0+1; n++){ 
    l1[n] = new int*[row+1]; 
    for(i = 1; i < row + 1; i++){ 
        l1[n][i] = new int[column-2*xlim-10+1];
    }
}
int ****l2 = new int***[n0+1];
for(n=1; n<n0+1; n++){
    l2[n] = new int**[10+1];
    for(h = 1; h < 10 + 1; h++){
        l2[n][h] = new int*[d+1];
        for(i = 1; i < d + 1; i++){
            l2[n][h][i] = new int[column-2*xlim-10+1];
        }
    }
}

for(n=1; n<n0+1; n++){
    for(h=1; h<10+1; h++){
        for(i=1; i<y2-y1+1; i++){
            for(j=1; j<column-2*xlim-(2*r-1)+1; j++){
                l2[n][h][i][j] =0;
            }
        }
    }
}

for(n=1; n<n0+1; n++){
    /*----------------- FILE NAME (LIGHT) ------------------------*/
    string lfile;
    //cout << seqnumber <<"\n";
    int filenum = seqnumber-n+2;
    string strn = IntToString(filenum);
    lfile = "C:\Data\aref\" + strn;
    lfile += ".txt";
    //cout << " Input file is '" << lfile <<"'\n";

    FILE* fp;
    fp = fopen(lfile.c_str(), "r");
if(fp==NULL) {
    printf("Cannot open the file. \n");
    return 1;
}

for(j=1; j<column + 1; j++){
    for(i=1; i<row + 1; i++){
        fscanf(fp, "/d",&light[i][j] );
    }
}

/*--------- LIMITING THE REGION OF IMAGE(LIGHT) ------------*/

//left half
for(i=1; i<row+1; i++){
    for(j=1; j<xc-r-xlim+1; j++){
        l1[n][i][j] = 0;
        l1[n][i][j] = light[i][j+xlim];
    }
}

//right half
for(i=1; i<row+1; i++){
    for(j=1; j<column-xlim-xc-(r-1)+1; j++){
        l1[n][i][xc-r-xlim+j] = 0;
        l1[n][i][xc-r-xlim+j] = light[i][xc+(r-1)+j];
    }
}

for(h=1; h<10+1; h++){
    for(i=1; i<y2-y1+1; i++){
        for(j=1; j<column-2*xlim-(2*r-1)+1; j++){
            l2[n][h][i][j] = l1[n][y1+i+(h-1)*d][j];
        }
    }
}
cout << "Reference images for calculation are created.\n";

// By now, elements for making coefficient matrix and vector are obtained.

/****************************************************************************
 int m;
 double ***A = new double**[10+1];
 for(h=1; h<10+1; h++){
 A[h] = new double*[n0+1];
 for(n = 1; n < n0 + 1; n++){
 A[h][n] = new double[n0+1];
 }
 }

 for(h=1; h<10+1; h++){
 for(n=1; n<n0+1; n++){  
 for(m=1; m<n0+1; m++){  
 A[h][n][m] = 0;
 for(i=1; i<y2-y1+1; i++){  
 for(j=1; j<column-2*xlim-(2*r-1)+1; j++){  
 A[h][n][m] += l2[n][h][i][j]*l2[m][h][i][j];  
 }
 }
 }
 }
 }

 double **B = new double*[10+1];
 for(h = 1; h < 10 + 1; h++){
B[h] = new double[n0+1];
}

for(h=1; h<10+1; h++){
    for(n=1; n<n0+1; n++){
        B[h][n] = 0;
        for(i=1; i<y2-y1+1; i++){
            for(j=1; j<column-2*xlim-(2*r-1)+1; j++){
                B[h][n] += l2[n][h][i][j]*s2[h][i][j];
            }
        }
    }
}

/*-----------------------GAUSS' METHOD---------------------------*/
int k;
double c;

for(h=1; h<10+1; h++){
    /*------ FORWARD ELIMINATION -------*/
    for(i=1;i<n0+1;i++){
        for (j=i+1; j<n0+1; j++) {
            c = A[h][j][i] / A[h][i][i];
            for (k=i; k<n0+1; k++)
                A[h][j][k] -= c * A[h][i][k];
            B[h][j] -= c * B[h][i];
        }
    }

    /*---- BACKWARD SUBSTITUTION --------*/
    B[h][n0] = B[h][n0]/A[h][n0][n0];
for (i=n0-1; i > 0; i--) {
    for (j=i+1; j<n0+1; j++)
        B[h][i] -= A[h][i][j] * B[h][j];
    B[h][i] /= A[h][i][i];
}

for(h=1; h<10+1; h++){
    cout << "B[" << h << "]=";
    for(n=1; n<n0+1; n++){
        cout << B[h][n] << " ";
    }
    cout << "\n";
}

/*------ MAKING AN OPTIMIZED REFERENCE --------*/

int **lighttemp = new int*[row+1];
for(i = 1; i < row + 1; i++){
    lighttemp[i] = new int[column+1];
}
for(i=1; i<row+1; i++){
    for(j=1; j<column+1; j++){
        lighttemp[i][j] = 0;
    }
}

double ***L = new double**[10+1];
for(h=1; h<10+1; h++){
    L[h] = new double*[d+1];
    for(i = 1; i < d + 1; i++){
        L[h][i] = new double[column+1];
    }
}
for(h=1; h<10+1; h++){
  for(i=1; i<d+1; i++){  
    for(j=1; j<column+1; j++){  
      L[h][i][j] = 0;  
    }  
  }  
}  

for(n=1; n<n0+1; n++){  
  /*---------- LOADING LIGHT IMAGE AGAIN ----------*/  
  string Lfile;  
  //cout << seqnumber <<\
  int filenum = seqnumber - n+2;  
  string strn = IntToString(filenum);  
  Lfile = "C:\Data\aref\" + strn;  
  Lfile += ".txt";  
  
  FILE* fp;  
  fp = fopen(Lfile.c_str(), "r");  

  if(fp==NULL) {  
    printf("Cannot open the file. 
");  
    return 1;  
  }  

  for(j=1; j<column + 1; j++){  
    for(i=1; i<row + 1; i++){  
      fscanf(fp, "%d",&lighttemp[i][j] );  
    }  
  }  

  for(h=1; h<10+1; h++){
for(i=1; i<y2-y1+1; i++){
    for(j=1; j<column+1; j++){
        L[h][i][j] += B[h][n]*lighttemp[y1+i+(h-1)*d][j];
    }
}
}
}
}
} // n loop

FILE* f2p;
f2p = fopen(dfile.c_str(), "r");
if(f2p==NULL) {
    printf("Cannot open the file. \n");
    return 1;
}
for(j=1; j<column + 1; j++){
for(i=1; i<row + 1; i++){
    fscanf(f2p, "%d", &dark[i][j] );
}
}

/*------------------------------------------------ SPLIT DARK IMAGE INTO 10 IMAGES-------------------------*/

double ***D = new double**[10+1];
for(h=1; h<10+1; h++){
    D[h] = new double*[d+1];
    for(i = 1; i < d + 1; i++){
        D[h][i] = new double[column+1];
    }
}
for(h=1; h<10+1; h++){
    for(i=1; i<d+1; i++){
        for(j=1; j<column+1; j++){
            D[h][i][j] =0;
            D[h][i][j] = dark[y1+i+(h-1)*d][j];
        }
    }
}

double ***S = new double**[10+1];
for(h=1; h<10+1; h++){
    S[h] = new double*[d+1];
    for(i = 1; i < d + 1; i++){
        S[h][i] = new double[column+1];
    }
}
for(h=1; h<10+1; h++){
    for(i=1; i<d+1; i++){
        // Code continues...
    }
}
for(j=1; j<column+1; j++){
    S[h][i][j] = 0;
    S[h][i][j] = shadow[y1+i+(h-1)*d][j];
}
}
}

;/*--------------- CALCULATION OF PHASE SHIFT ----------------*/
double ***phi = new double**[10+1];
for(h = 1; h < 10 + 1; h++){
    phi[h] = new double*[d+1];
    for(i=1; i<d+1; i++){
        phi[h][i] = new double[column+1];
    }
}

for(h = 1; h < 10 + 1; h++){
    for(i=1; i<d+1; i++){
        for(j=1; j<column+1; j++){
            phi[h][i][j] = 0;
        }
    }
}

for(h = 1; h < 10 + 1; h++){
    for(i=1; i<d+1; i++){
        for(j=1; j<column+1; j++){
            phi[h][i][j] =
                acos((3-((S[h][i][j]-D[h][i][j])/(L[h][i][j]-D[h][i][j])))/(2*sqrt(2))) - pi/4;
        }
    }
}

cout << "Calculation of phase shift is done.\n";
for(h=1; h<10+1; h++){  
    string image2;  
    string strn1 = IntToString(h);  
    image2 = "C:\C++\data\images\" + date + "_"  
        + seqnum + "_" + "phaseshift_" + strn1 + ".txt";  
    ofstream image2out(image2.c_str());  
    if(!image2out){  
        cout << "cannot open the output file. \n";  
        return 1;  
    }  

    for (i =1; i < d-16+1; i++) {  
        for( j = 1; j < column+1; j++){  
            image2out << -1000*phi[h][i+4][j] << " " ;  
        }  
        image2out << "\n";  
    }  
    image2out.close();  
}

/*----------------------------------------------------- SAVE --------------------------------------------------*/

double **a = new double*[row+1];
for(i = 1; i < row + 1; i++){  
    a[i] = new double[column+1];  
}  
for(i=1; i<row+1; i++){  
    for(j=1; j<column+1; j++){  
        a[i][j] = 0;  
    }  
}
for(h=1; h<11; h++){  
if(h<10){  
for(i=1; i<y2-y1+1; i++){  
for(j=1; j<x2-x1+1; j++){  
a[i][j] = -1000*phi[h][i][j+x1];  
}  
}  
}  
else{  
for(i=1; i<y2-y1+1; i++){  
for(j=1; j<x2-x1+1; j++){  
a[i][j] = -1000*phi[h][i+11][j+x1];  
}  
}  
}

string image1;
string strn1 = IntToString(h);
image1 = "C:\C++\data\temp\image1_" + strn1 + ".dat";
ofstream image1out(image1.c_str());

if(!image1out){
    cout << "cannot open the output file. \n";
    return 1;
}

for (i =1; i < x2-x1+1; i++) {
for( j = 1; j < y2-y1+1; j++){
    image1out << a[j][i] << " " ;
}
image1out << "\n";
string image;
image = "C:\C++\data\images\" + date
+ "_" + seqnum + "_" + strn1 + ".dat";
ofstream imageout(image.c_str());

if(!imageout){
cout << "cannot open the output file. \n";
return 1;
}

for (i =1; i < x2-x1+1; i++) {
for (j = 1; j < y2-y1+1; j++){
imageout << a[j][i] << " " ;
}
imageout << "\n";
}
imageout.close();
cout<< "," << image << "," is generated.\n";

/∗------------------ MAKING SUMMED SIGNALS ---------------*/

double **hor = new double*[10+1]; //HORIZONTAL PROFILE
for(n=1; n<10+1; n++){
hor[n] = new double[y2-y1+1];
}
double **ver = new double*[10+1]; //VERTICAL PROFILE
for(n=1; n<10+1; n++){
    ver[n] = new double[x2-x1+1];
}

for(i=1; i<y2-y1+1; i++){
    hor[h][i] = 0;
}

for(i=1; i<y2-y1+1; i++){
    for(j=1; j<x2-x1+1; j++){
        hor[h][i] += a[i][j];
    }
}

for(i=1; i<x2-x1+1; i++){
    ver[h][i] = 0;
    for(j=1; j<y2-y1+1; j++){
        ver[h][i] += a[j][i];
    }
}

string hplot;

hplot = "C:\\C++\\data\\temp\\hplot_ + strn1 + ".dat";
ofstream hout(hplot.c_str());

if(!hout){
    cout << "cannot open the output file. \n";
    return 1;
}

for (i =1; i < y2-y1+1; i++) {
    hout << i << " " << hor[h][i] << "\n";
}

hout.close();
/*

string vplot;
vplot = "C:\\C++\\data\\temp\\vplot_\" + strn1 + ".dat";
ofstream vout(vplot.c_str());

if(!vout){
cout << "cannot open the output file. \n";
return 1;
}

for (i =1; i < x2-x1+1; i++) {
  vout << i << " " << ver[h][i] << "\n";
}
vout.close();

for(n=1; n>10+1; n++){
del[] hor[n];
}
del[] hor;
for(n=1; n>10+1; n++){}
del[] ver[n];
}
del[] ver;
*/
} //n loop

(PATH)---------------------------------IMAGE PLOTS-------------------------------------*/

FILE *Fp;
Fp = _popen("pgnuplot -persist", "w");
fprintf(Fp, "set format y ','\n");
fprintf(Fp, "cd 'C:\\C++\\data\\temp'\n");
fprintf(Fp, "set terminal png fontsacle 3 size 1280,3000\n");
fprintf(Fp, "set output 'C:\C++\data\temp\image.png'");
fprintf(Fp, "unset contour
");
fprintf(Fp, "unset key
");
fprintf(Fp, "unset surface
");
fprintf(Fp, "set view map
");
fprintf(Fp, "set pm3d at b\n");
fprintf(Fp, "set palette rgbformulae 22,13,-31\n");
fprintf(Fp, "set multiplot layout 10,1\n");
fprintf(Fp, "set tmargin 2\n");
fprintf(Fp, "set bmargin 0\n");
fprintf(Fp, "unset xlabel\n");
fprintf(Fp, "set format x ''\n");
fprintf(Fp, "set ylable ' '\n");
fprintf(Fp, "set xlabel ' '\n");
fprintf(Fp, "set cbr [0:200]\n");
fprintf(Fp, "set cbr tics 0, 400\n");
splot 'image1_1.dat' matrix with pm3d

splot 'image1_2.dat' matrix with pm3d

splot 'image1_3.dat' matrix with pm3d

splot 'image1_4.dat' matrix with pm3d

splot 'image1_5.dat' matrix with pm3d

splot 'image1_6.dat' matrix with pm3d

unset ylabel

set ylabel ' '\n
set xlabel ' '\n
set xlabel ' '\n
set cbr tics 0, 400\n
splot 'image1_7.dat' matrix with pm3d

splot 'image1_8.dat' matrix with pm3d

splot 'image1_9.dat' matrix with pm3d

set tmargin 0\n
fprintf(Fp, "splot 'image1_10.dat' matrix with pm3d\n");
fprintf(Fp, "unset multiplot\n");
fprintf(Fp, "set out\n");
fprintf(Fp, "set term pop\n");
fprintf(Fp, "exit\n");
fflush(Fp);
_pclose(Fp);

/*--------------- HORIZONTAL PLOTS ----------------*/

Fp = _popen("pgnuplot -persist", "w");
fprintf(Fp, "set yrange [0:1500]\n");
fprintf(Fp, "set ytics 500, 500, 1000\n");
fprintf(Fp, "cd 'C:\C++\data\temp'\n");
fprintf(Fp, "set terminal png fontscale 3 size 1280,3000\n");
fprintf(Fp, "set output 'C:\C++\data\temp\plot.png'\n");
fprintf(Fp, "set multiplot layout 5,1\n");
fprintf(Fp, "set tmargin 2\n");
fprintf(Fp, "set bmargin 0\n");
fprintf(Fp, "unset xlabel\n");
fprintf(Fp, "set format x ''\n");
fprintf(Fp, "set ylabel ' '\n");
fprintf(Fp, "set xlabel ' '\n");
fprintf(Fp, "plot 'hplot_1.dat' title 'K' with lines linewidth 5, 
'hplot_2.dat' title 'Rb' with lines linewidth 5\n");
fprintf(Fp, "set tmargin 0\n");
fprintf(Fp, "plot 'hplot_3.dat' title 'K' with lines linewidth 5, 
'hplot_4.dat' title 'Rb' with lines linewidth 5\n");
fprintf(Fp, "set ylabel 'Summed signal [arb. u.]'\n");
fprintf(Fp, "plot 'hplot_5.dat' title 'K' with lines linewidth 5, 
'hplot_6.dat' title 'Rb' with lines linewidth 5\n");
fprintf(Fp, "unset ylabel\n");
fprintf(Fp, "set ylabel ' '\n");
fprintf(Fp, "plot 'hplot_7.dat' title 'K' with lines linewidth 5, 'hplot_8.dat' title 'Rb' with lines linewidth 5\n");
printf(Fp, "set xlabel 'Horizontal position [pix]'\n");
printf(Fp, "set format x\n");
printf(Fp, "set bmargin 3\n");
printf(Fp, "plot 'hplot_9.dat' title 'K' with lines linewidth 5, 'hplot_10.dat' title 'Rb' with lines linewidth 5\n");
printf(Fp, "unset multiplot\n");
printf(Fp, "set out\n");
printf(Fp, "set term pop\n");
printf(Fp, "exit\n");
fflush(Fp);
PClose(Fp);

/*-------------------------------------------------------------*/
/*-------------------------------------------------------------*/
/*-------------------------------------------------------------*/
for(i = 1; i < row + 1; i++){
    delete[] a[i];
}
delete[] a;
for(h=1; h<10+1; h++){
    for(i = 1; i < d + 1; i++){
        delete[] phi[h][i];
    }
}
delete[] phi;
for(i = 1; i < row + 1; i++){
    delete[] dark[i];
}
delete[] dark;
for(h=1; h<10+1; h++){
    for(i = 1; i < d + 1; i++){
delete[] L[h][i];
}
}
delete[] L;
for(i = 1; i < row + 1; i++){
delete[] lighttemp[i];
}
delete[] lighttemp;
for(n=1; n<10+1; n++){ 
delete[] B[n];
}
delete[] B;
for(h = 1; h < 10 + 1; h++){
for(i=1; i<n0+1; i++){ 
delete[] A[h][i];
}
delete[] A[h];
}
delete[] A;
for ( n = 1; n < n0+1; n++) {
for ( h = 1; h < 10+1; h++) {
for(i=1; i<d+1; i++){ 
delete[] l2[n][h][i];
}
delete[] l2[n][h];
}
delete[] l2[n];
}
delete[] l2;
for ( i = 1; i < n0+1; i++) {
for ( j = 1; j < row+1; j++) {
delete[] l1[i][j];
}
delete[] l1[i];
}
delete[] l1;
for (int h = 1; h < 10+1; h++) {
for (int i = 1; i < d+1; i++) {
delete[] s2[h][i];
}
delete[] s2[h];
}
delete[] s2;
for (int h = 1; h < 10+1; h++) {
for (int i = 1; i < d+1; i++) {
delete[] D[h][i];
}
delete[] D[h];
}
delete[] D;
for (int h = 1; h < 10+1; h++) {
for (int i = 1; i < d+1; i++) {
delete[] S[h][i];
}
delete[] S[h];
}
delete[] S;

for(i = 1; i < row + 1; i++){
delete[] s1[i];
}
delete[] s1;
for(i = 1; i < row + 1; i++){
delete[] light[i];
}
delete[] light;
for(i = 1; i < row + 1; i++){
    delete[] shadow[i];
}
delete[] shadow;

end = clock();
cout << "\n It took " << (end-start)/CLOCKS_PER_SEC << " seconds.\n";
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