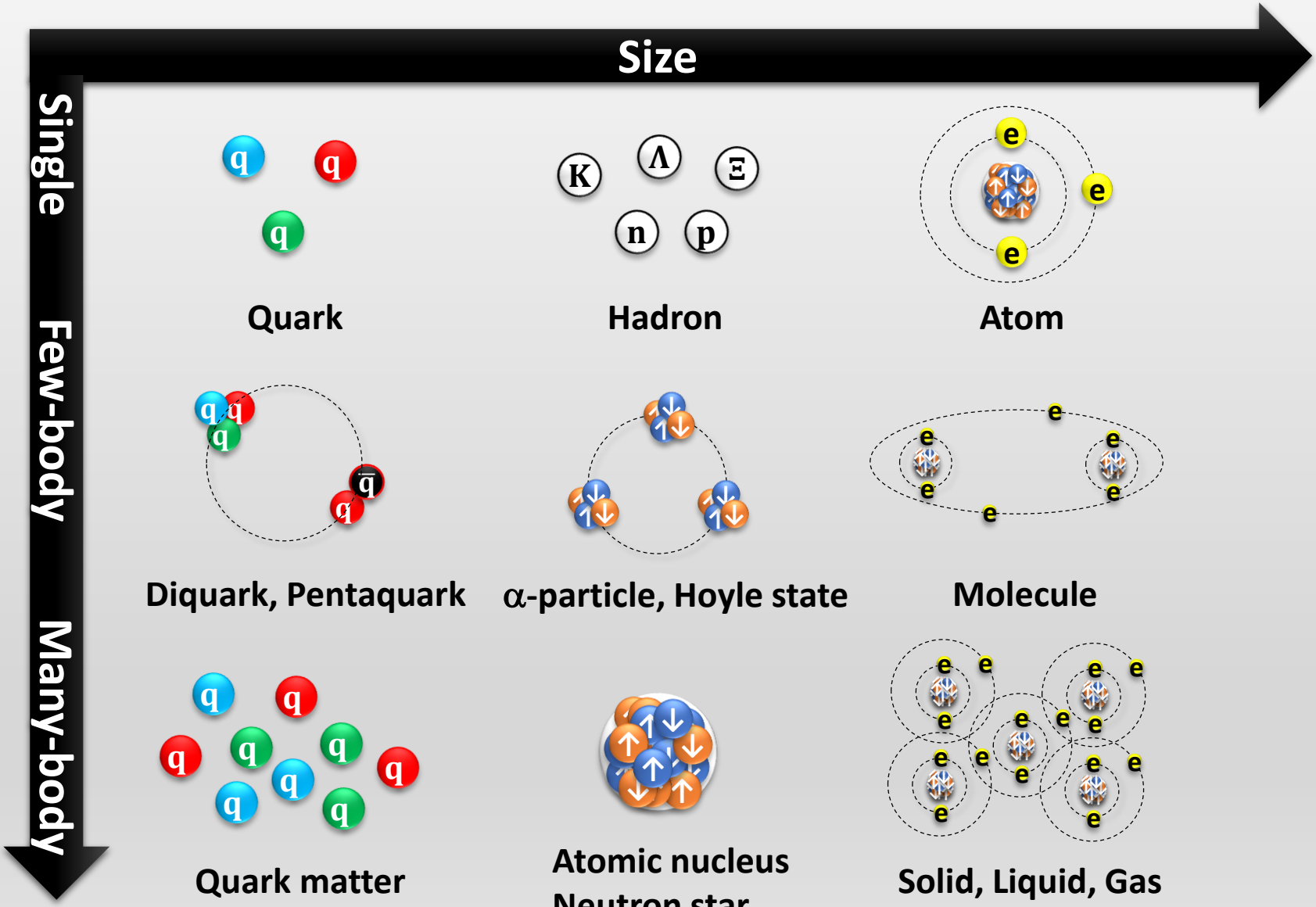
大阪市大NITEP
レーザー量子・原子核合同MONTHLY　MEETING：第2回

冷却フェルミ原子系で実現される少数系と多体系の概観

話題提供者：堀越　宗一

2019/07/16

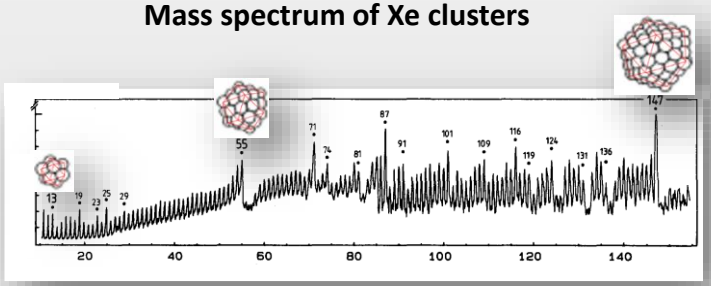
Background : Understanding fundamental physics underlying various quantum systems



Background : Understanding fundamental physics underlying various quantum systems

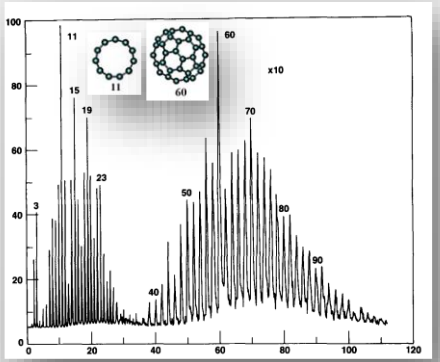
Cluster science : Evolution from Few-body to Many-body

Van der Waals force
Mass spectrum of Xe clusters



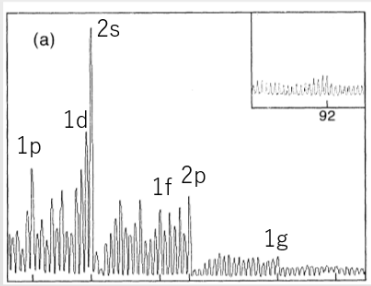
[J. Chern. Phys. 91, 5940 (1989)]

Covalent bond
Mass spectrum of C clusters

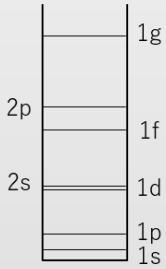


[J. Chem. Phys. 81, 3322 (1984)]

Metallic bond
Mass spectrum of Na clusters

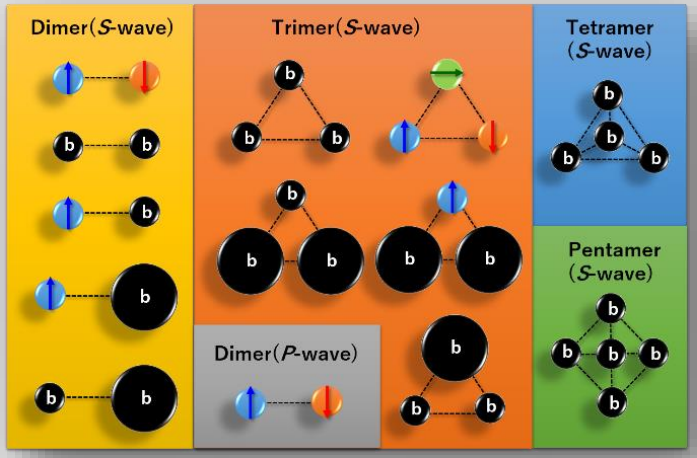


Orbits

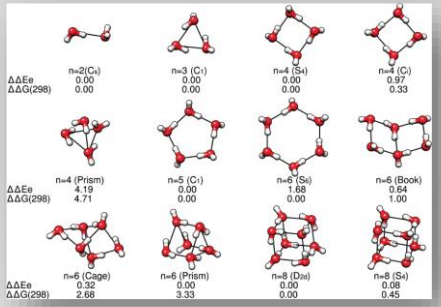


[W.D.Knight, Phys.Rev.Lett. 52, 2141 (1984)]

Van der Waals force
Bound state of cold atoms

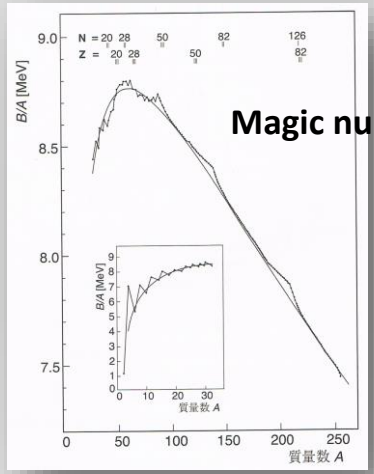


Hydrogen bond
Water clusters



[J. Phys. Chem. A, 114, 11725 (2010)]

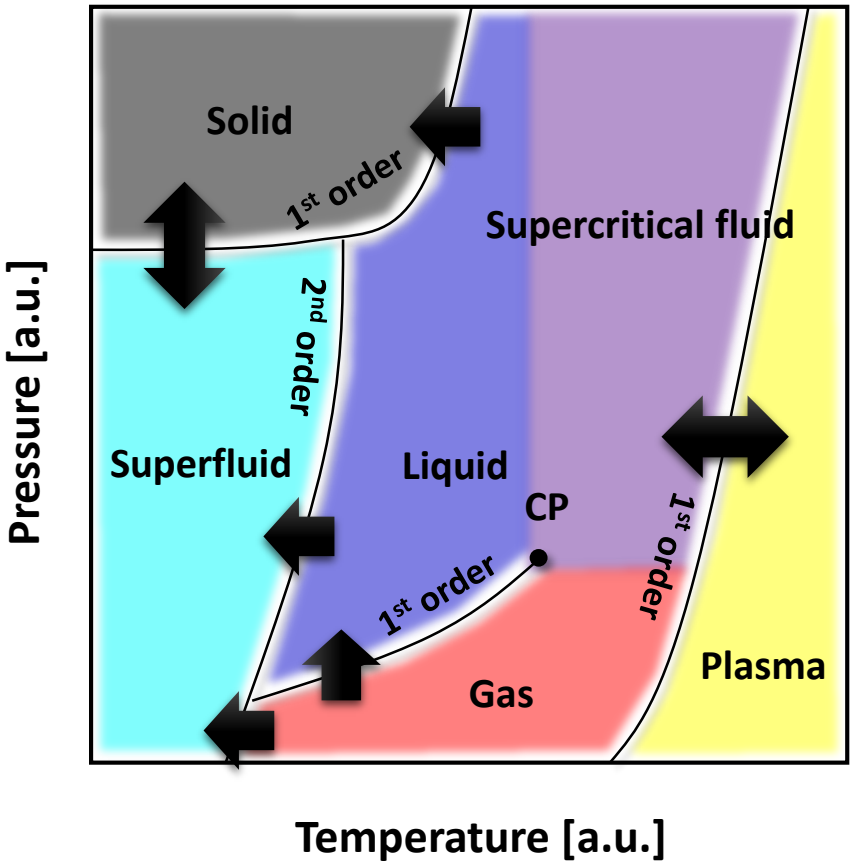
Nuclear force
Binding energy of nucleon



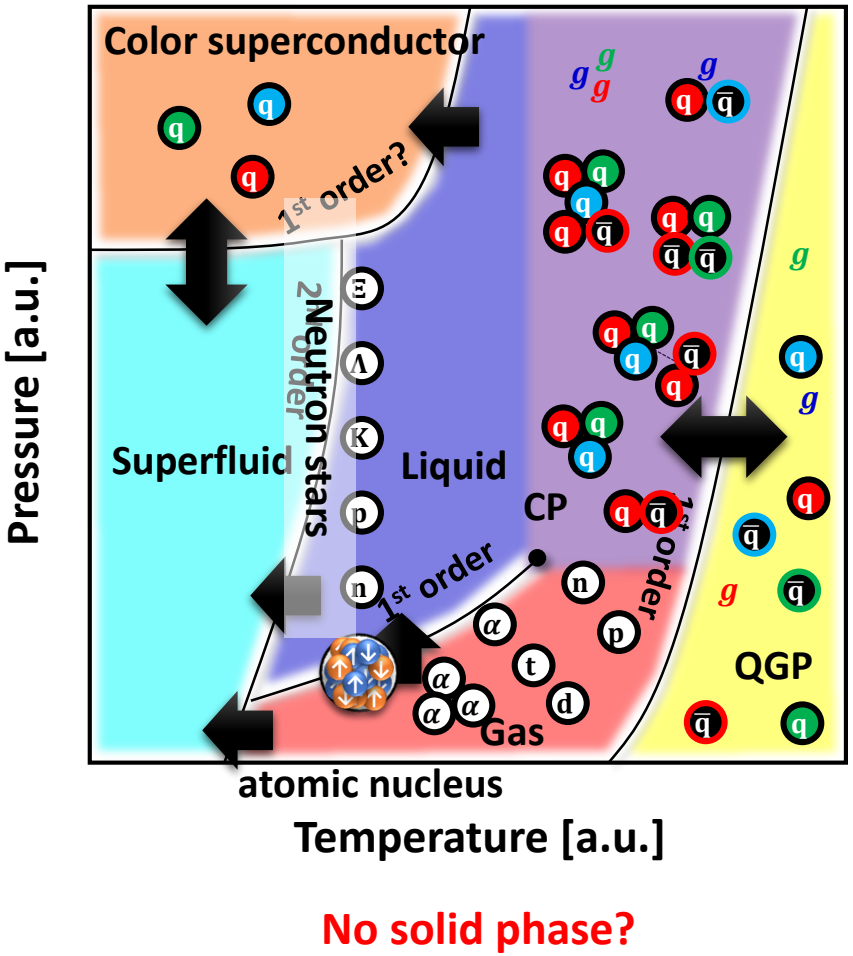
[Particles and Nuclei]

Clusters in the phase diagram

Helium3 atom (fermion)



Hadron



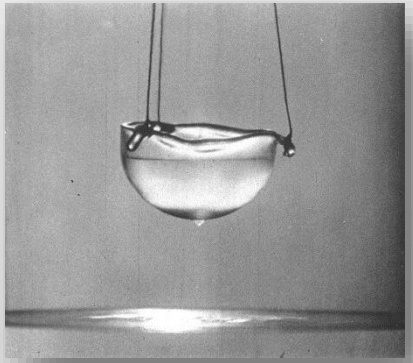
No solid phase?

<u>Background</u> : Understanding fundamental physics underlying various quantum systems

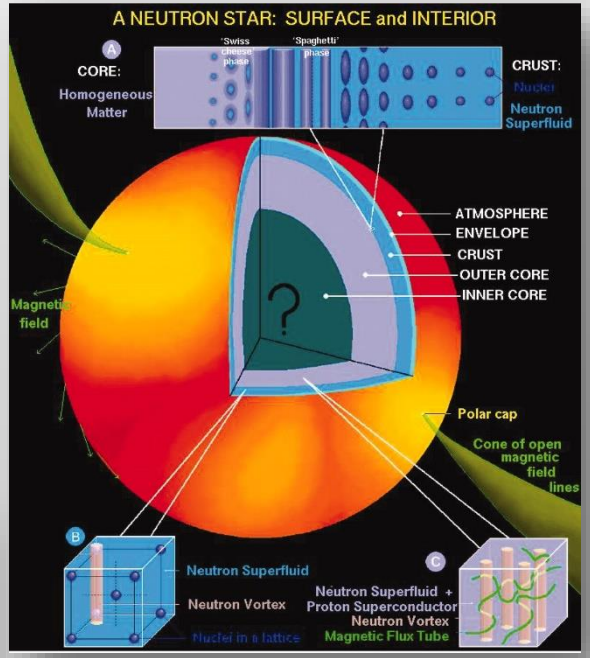
Bose-Einstein condensation (BEC)

Superconductor

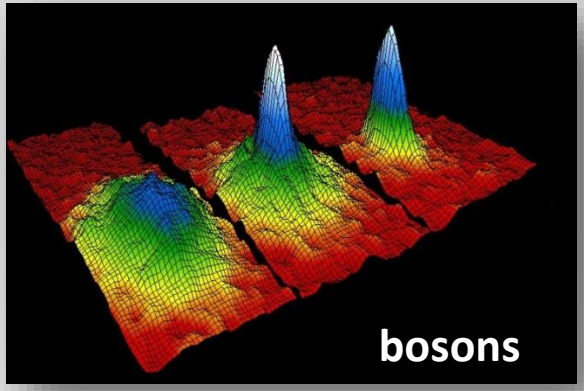
Liquid Helium

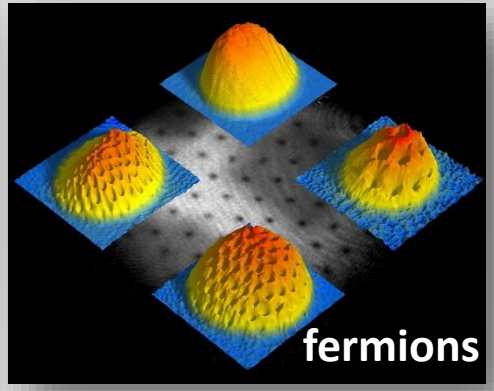


Neutron star



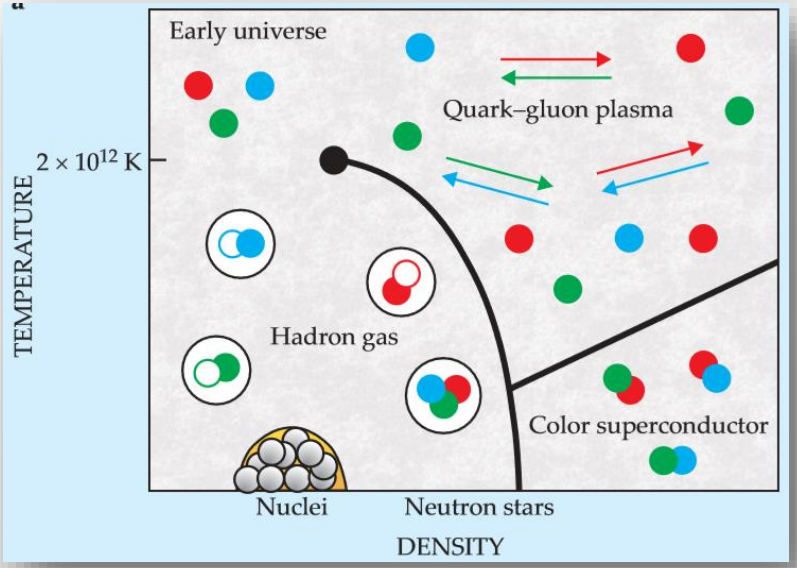
Ultracold atomic gas



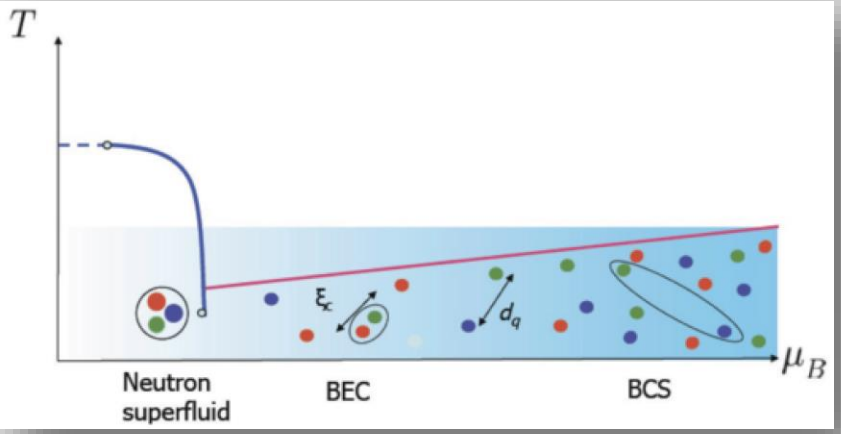


Phase diagram

Quark system

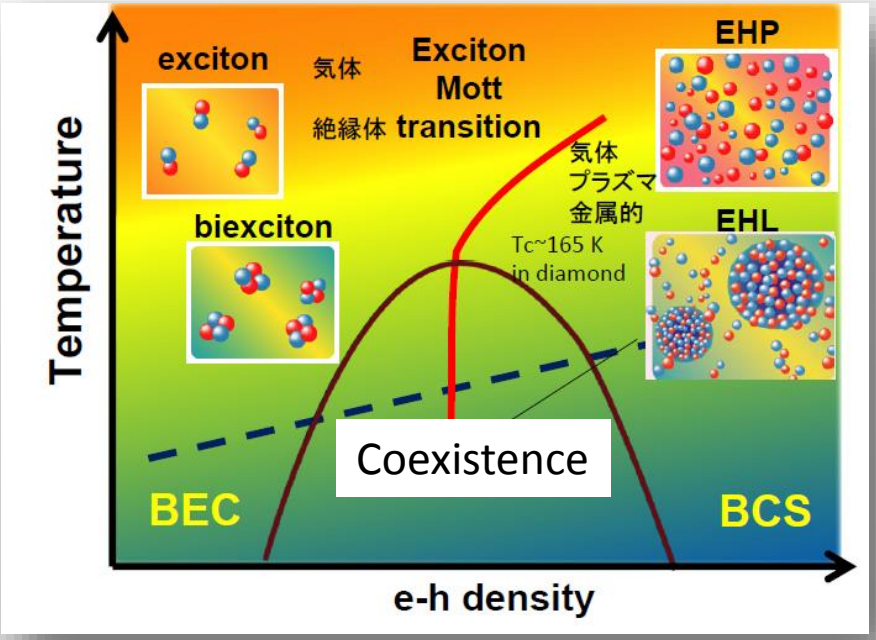


[Carlos A. R. Sá de Melo; *Physics Today* 2008, 61, 45-51]



Gordon Baym, Nuclear Physics A 956 (2016) 1–10

Electron-Hole system

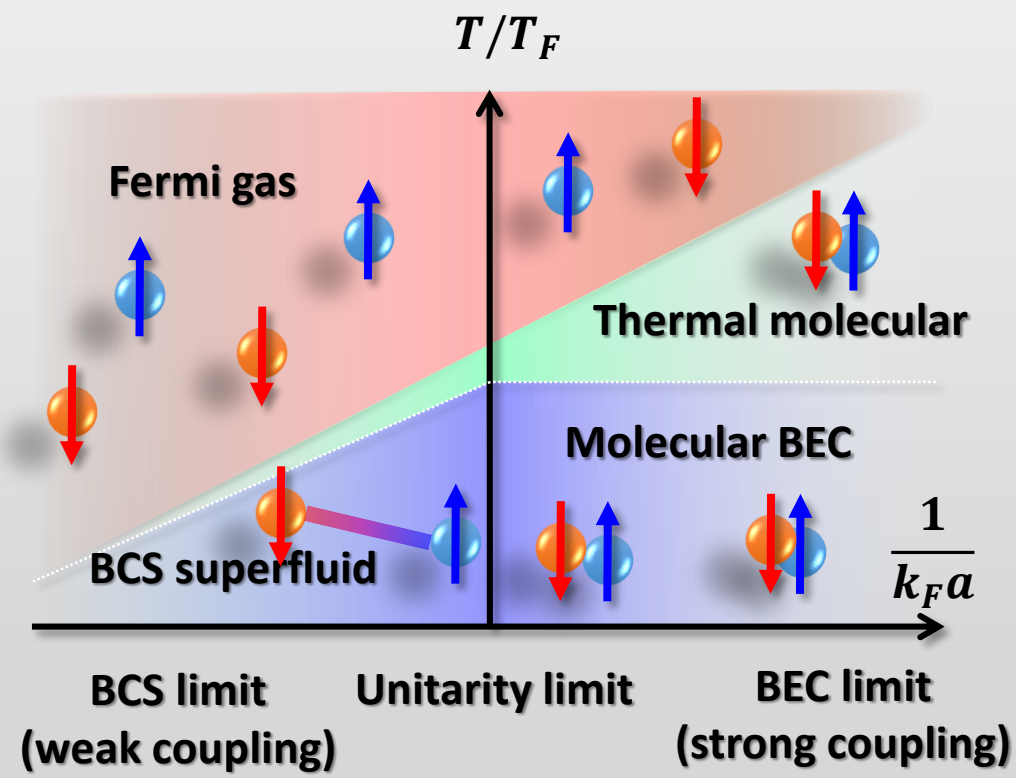


M. Omachi, From Neutron Star winter school 2016

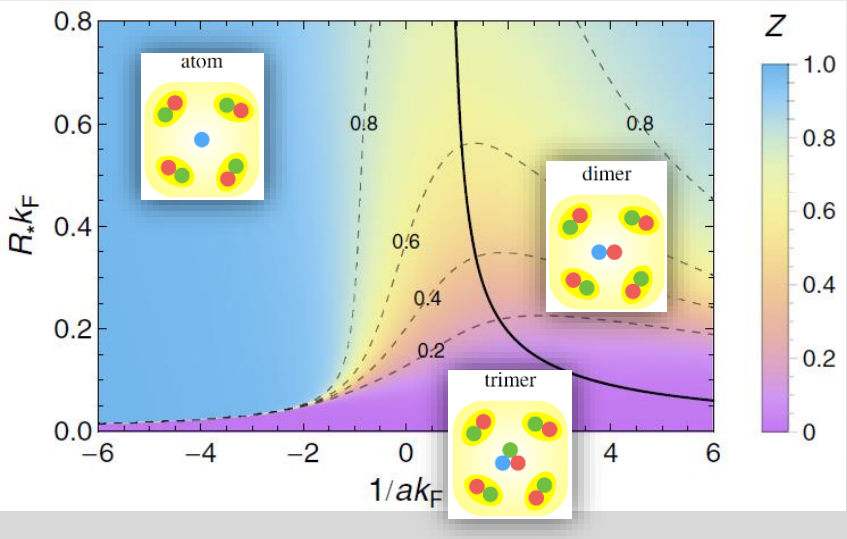
Phase diagram

Cold atom system

Spin-1/2 Fermi system



Impurities in a condensate spin-1/2 Fermi system



Yusuke Nishida, PRL 114, 115302 (2015)

BCS-BEC crossover

閾値近傍の普遍的物理

PRL **118,** 202501 (2017) PHYSICAL REVIEW LETTERS week ending 19 MAY 2017

Nuclear Physics Around the Unitarity Limit

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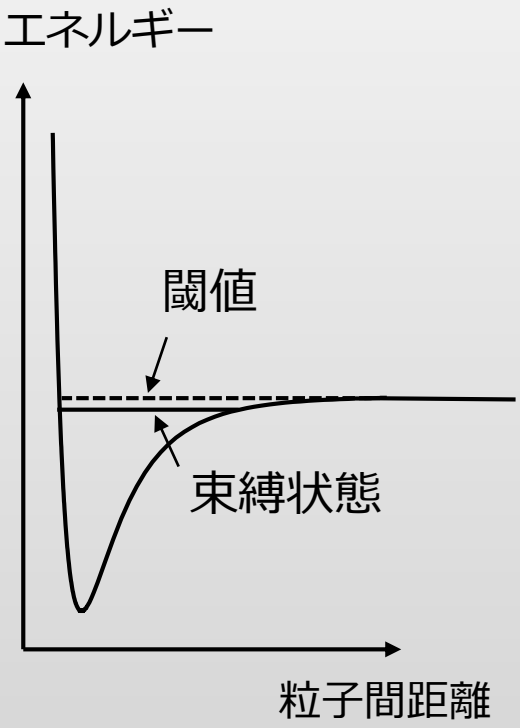
[6]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
(Received 18 July 2016; revised manuscript received 10 February 2017; published 15 May 2017)

We argue that many features of the structure of nuclei emerge from a strictly perturbative expansion around the unitarity limit, where the two-nucleon S waves have bound states at zero energy. In this limit, the gross features of states in the nuclear chart are correlated to only one dimensionful parameter, which is related to the breaking of scale invariance to a discrete scaling symmetry and set by the triton binding energy. Observables are moved to their physical values by small *perturbative* corrections, much like in descriptions of the fine structure of atomic spectra. We provide evidence in favor of the conjecture that light, and possibly heavier, nuclei are bound weakly enough to be insensitive to the details of the interactions but strongly enough to be insensitive to the exact size of the two-nucleon system.

DOI: 10.1103/PhysRevLett.118.202501

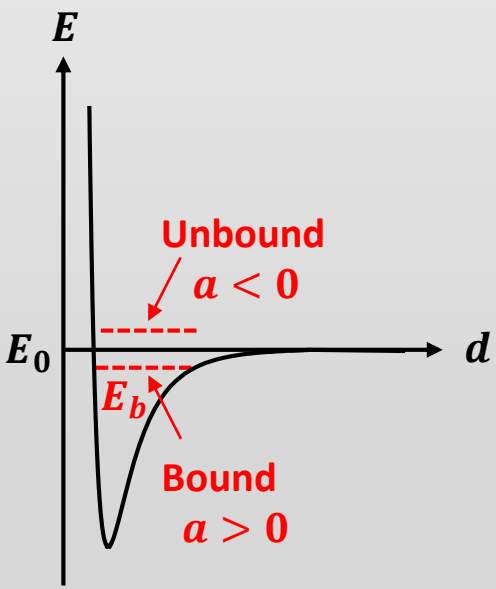


Background : Low energy scattering and loosely-bound state around the zero energy

Shape resonance

$E_0 \sim E_b$

- n-p (bound)
- n-n (unbound)

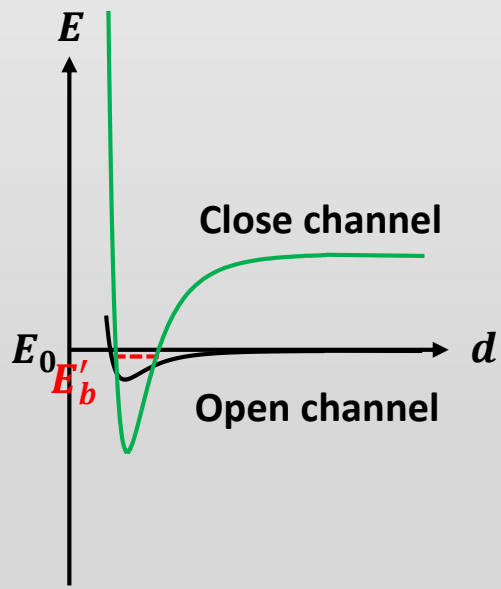


Resonance with own bound state

Feshbach resonance
Two channel model

$E_0 \sim E'_b$

- cold atoms
- Λ(1405)

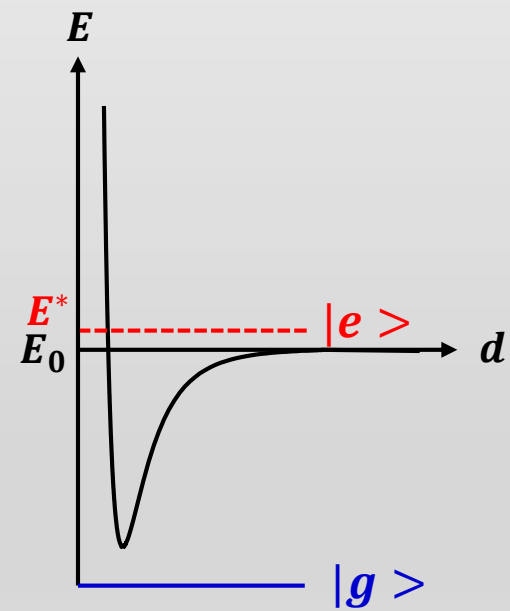


Resonance with a bound state in the other scattering channel

Ikeda threshold rule

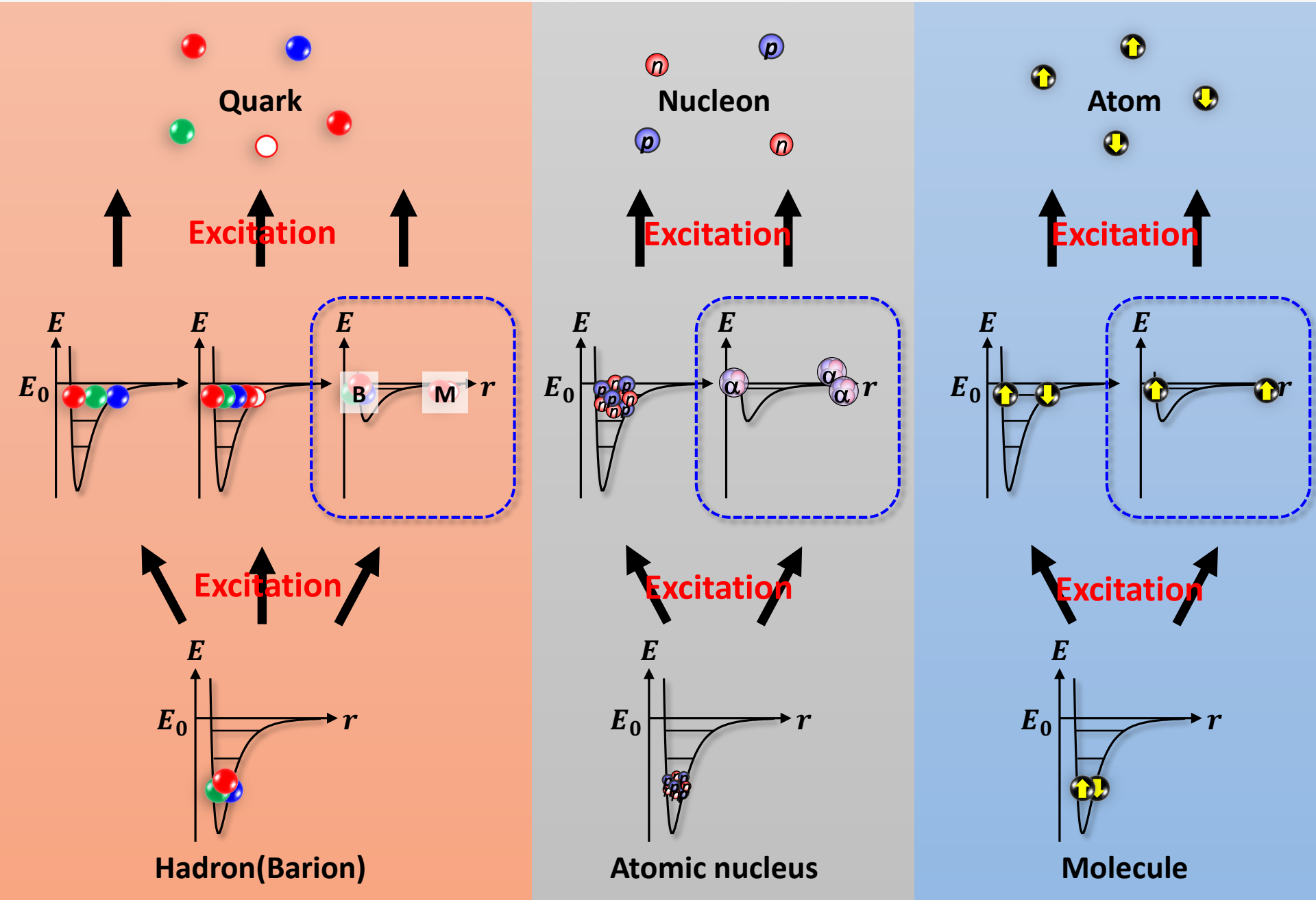
$E_0 \sim E^*$

- α-cluster



Resonance with an excited state of the composite particle

Background : Low energy scattering and loosely-bound



Histrical success examples

- In 1953, a **nuclear theorist**, **H. Feshbach**, studied control of scattering length

  ⟶ In 1998, the **Feshbach resonance** was demonstrated in **cold atom experiment**

- In 1970, a **nuclear theorist**, **V. Efimov**, predicted 3-body bound state in the condition of $a < 0$

  ⟶ In 2006, the **Efimov trimer** was confirmed in **cold atom experiment**

- In 1999, a **nuclear theorist**, **G. F. Bertsch**, posed the problem "How does a system of Fermi particles with infinite s-wave scattering length but vanishing interaction range behave?"

  ⟶ Since 2004, the **unitary regime** has been one of the hot topics in **cold atom experiment**

Background : Quantum systems at ultralow temperature

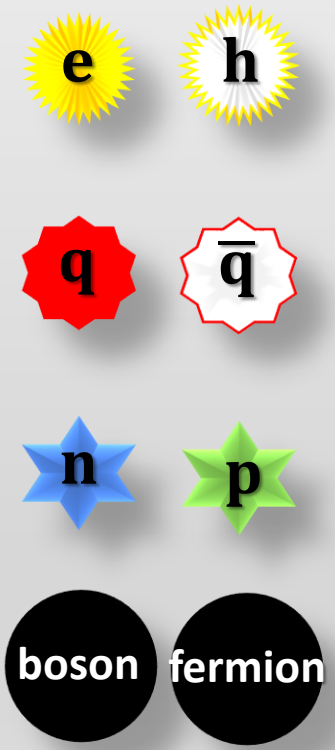
Various particle systems

Electron, Hole

Quark

Hadron

Atom

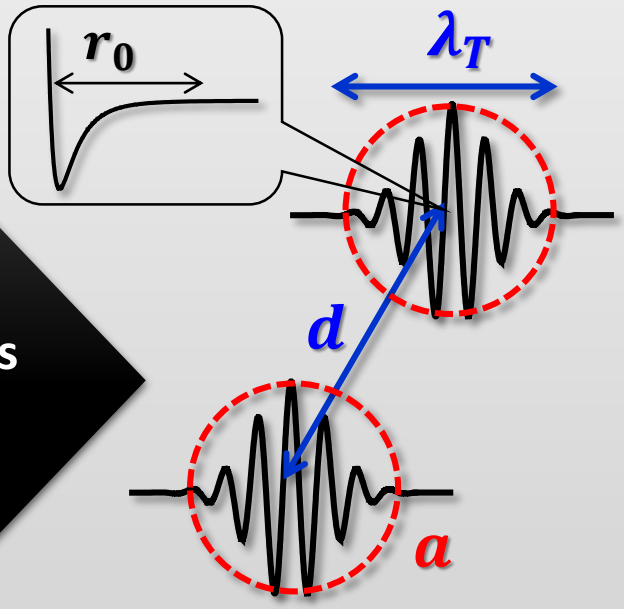


$$\frac{\lambda_T}{r_0}, \frac{d}{r_0} \gg 1$$

Details of particles are suppressed

$$\frac{a}{r_0} \gg 1$$

Similar quantum systems



Unitary regime

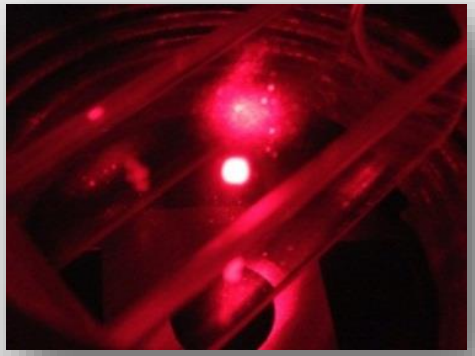
Cold atoms

Dilute *quantum* gases

Solid (300K)

Heating

Hot gas (600K)



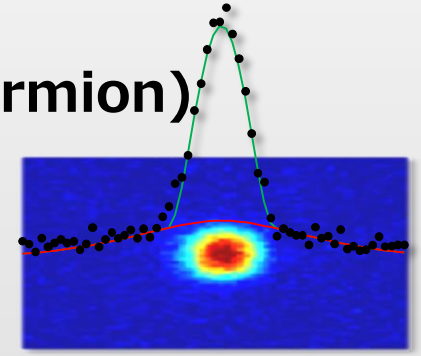
Cold gas (1mK)

Laser cooling

Evaporative cooling

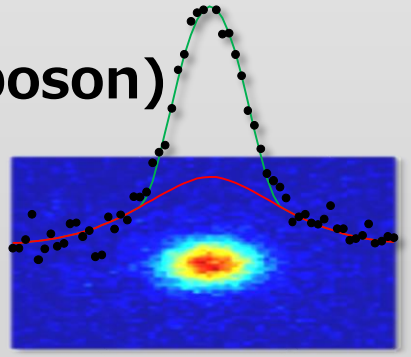
Momentum distribution

^{6}Li (fermion)



Fermi superfluid

^{7}Li (boson)



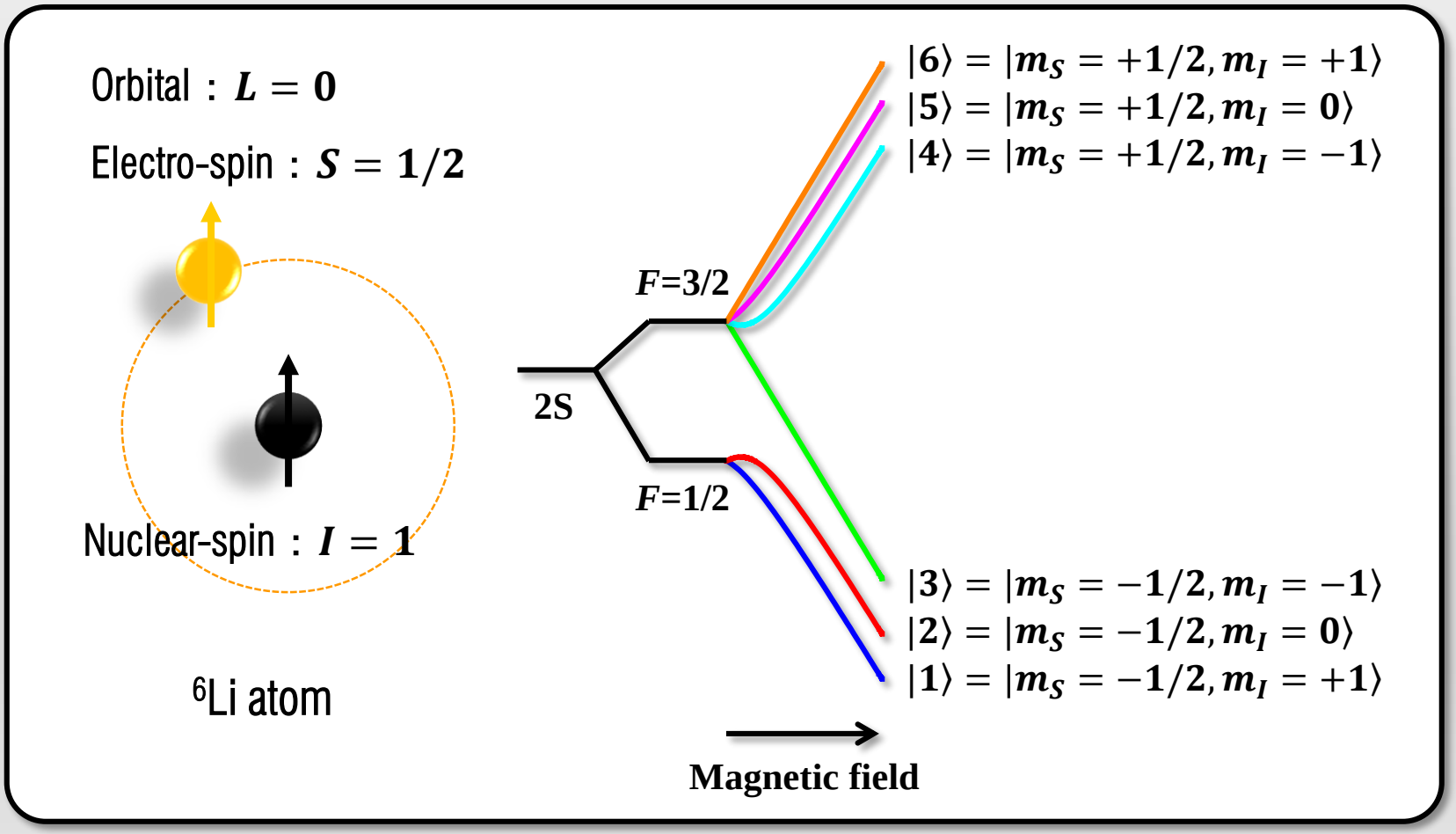
Bose-Einstein condensation

Quantum gas (100nK)

6Li atom

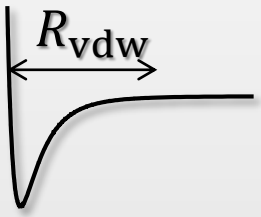
Internal degree of freedom of the atom

Lifetime of the states ≫ Experimental time (~30s)

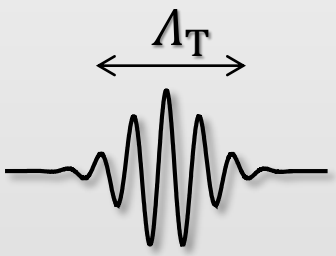


Length scales of our cold atomic system

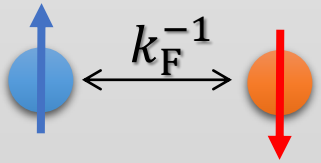
- Van der Waals length : $R_{\mathrm{vdw}} = \frac{1}{2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{1/4} = 1.7\mathrm{nm}$



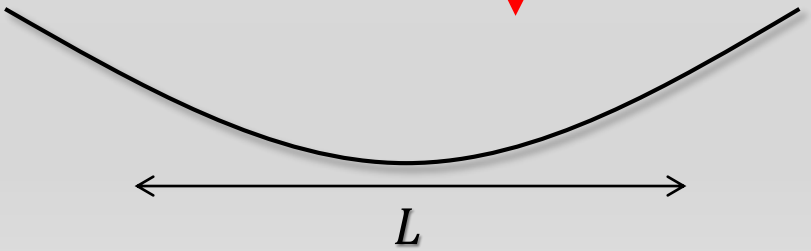
- Thermal length : $\Lambda_{\mathrm{T}} = \frac{\hbar}{\sqrt{2\pi m k_{\mathrm{B}} T}} \sim 100\mathrm{nm}@1\mu\mathrm{K}$



- Mean distance : $n^{-1/3} \sim k_{\mathrm{F}}^{-1} \sim 100\mathrm{nm}$



- Potential size : $L \sim 10\mu\mathrm{m}$



Length scales

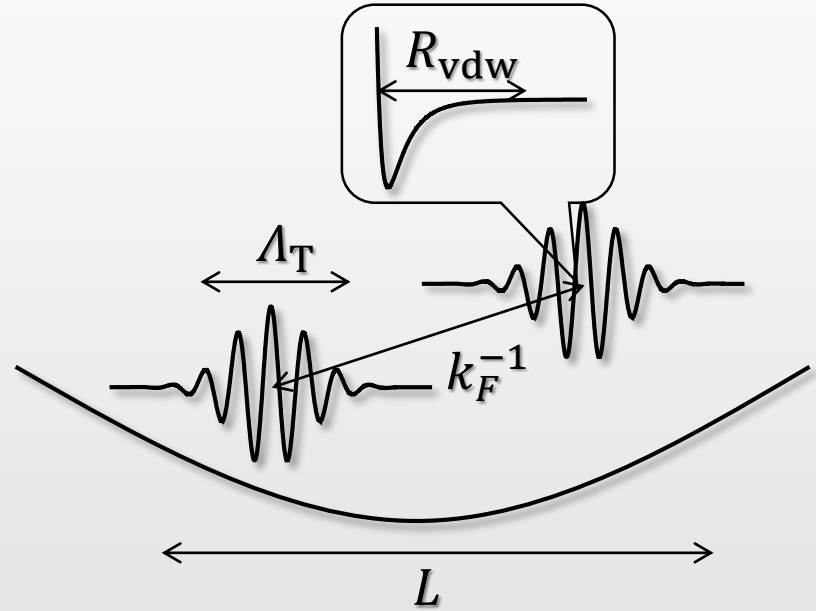
Locally homogeneous

Hierarchy : $R_{\mathrm{vdw}} \ll \Lambda_T, k_F^{-1} \ll L$

Low energy

Dilute

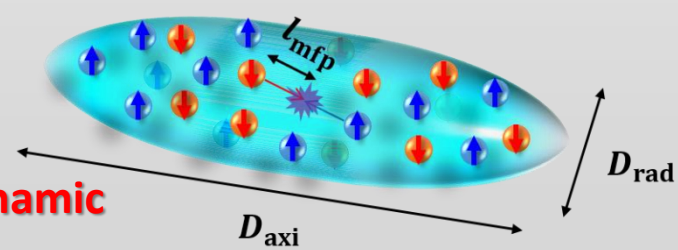
Short-range potential



interaction is basically s-wave

mean-free-path : $l_{\mathrm{mfp}} \sim v\Gamma_{\mathrm{col}}(n, a)$

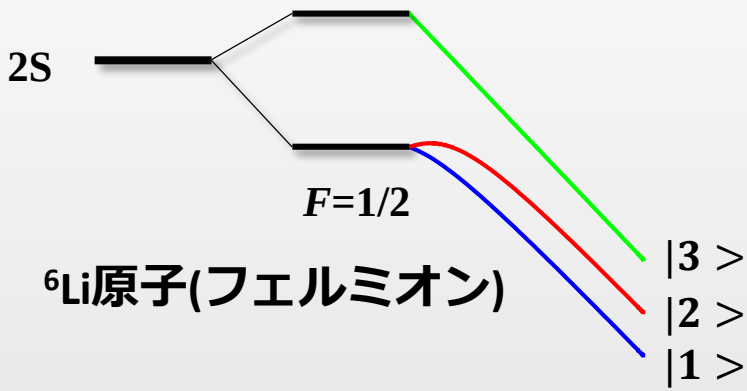
Hydrodynamicity : $0 < \frac{D_{\mathrm{axi}}}{l_{\mathrm{mfp}}}, \frac{D_{\mathrm{rad}}}{l_{\mathrm{mfp}}} < \infty$

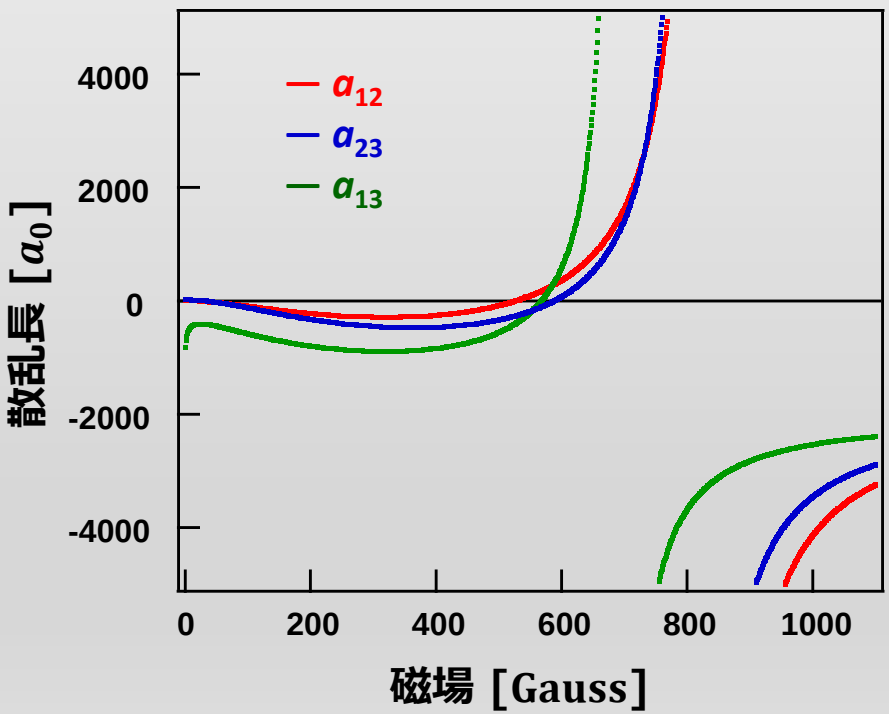


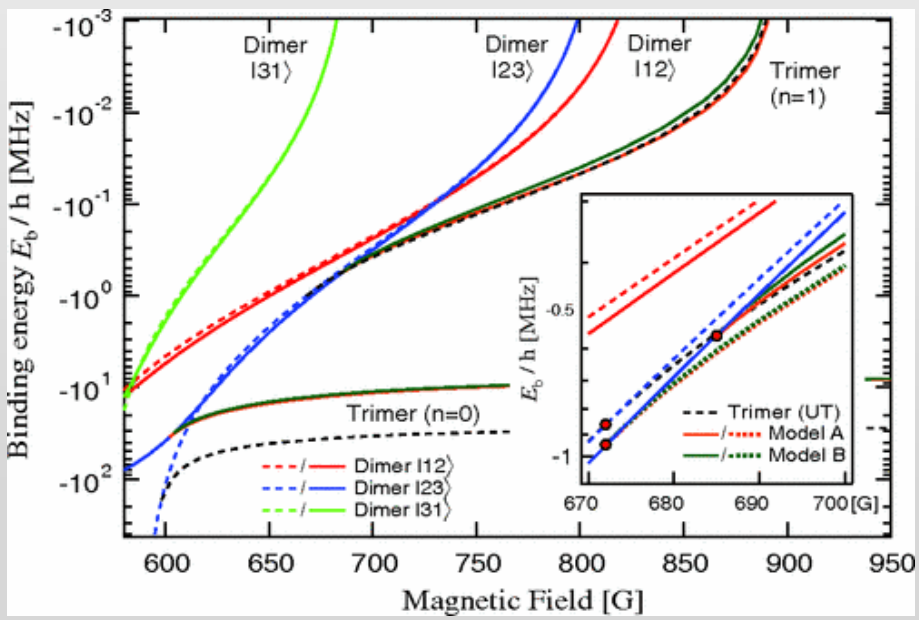
Ballistic ⟷ Hydrodynamic

Control knobs : $T, \mu, \delta n = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}, a, r_e, D, B, F, \delta m, \ldots$

フェッシュバッハ共鳴







[G. Zürn, Phys. Rev. Lett. **110**, 135301 (2013)]

[P. Naidon,, Phys. Rev. Lett. 105, 023201 (2010)]

位相シフトと散乱長（簡単に復習）

２粒子の相対運動のS.E. : $\left[\frac{1}{m}\left(p_r^2+\frac{\boldsymbol{L}^2}{r^2}\right)+V(r)\right]\Psi(r,\theta,\phi)=E\Psi(r,\theta,\phi)$

↓

角度方向と半径方向に変数分離 : $\Psi(r,\theta,\phi)=\psi_l(r)Y_l^m(\theta,\phi)$

半径方向のS.E. : $\left[\frac{\hbar^2}{m}\left(-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+\frac{l(l+1)}{r^2}\right)+V(r)\right]\psi_l(r)=E\psi_l(r)$

↓

ポテンシャルサイズより遠方での一般解 : $\psi_l(r)\cong\frac{C}{kr}\sin\left(kr+\eta_l-\frac{1}{2}l\pi\right)$, for $r\gg r_0$

ポテンシャルサイズより遠方でのs波散乱 : $\boxed{\psi(r)\cong\frac{C}{kr}\sin(kr+\eta_0)\text{, for } r\gg r_0}$

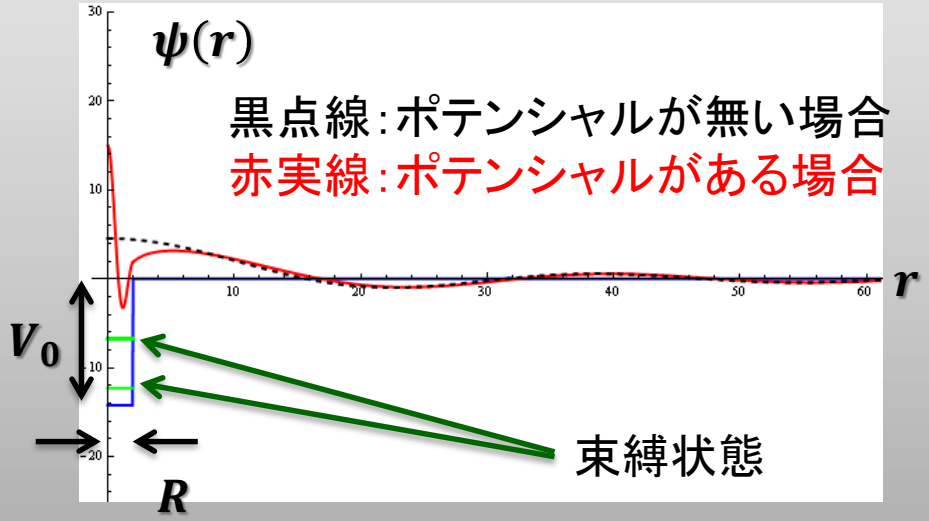
s波の散乱長、有効長と位相シフトとの関係 : $\boxed{\cot\eta_0=-\frac{1}{ak}+\frac{1}{2}r_{\mathrm{e}}k}$

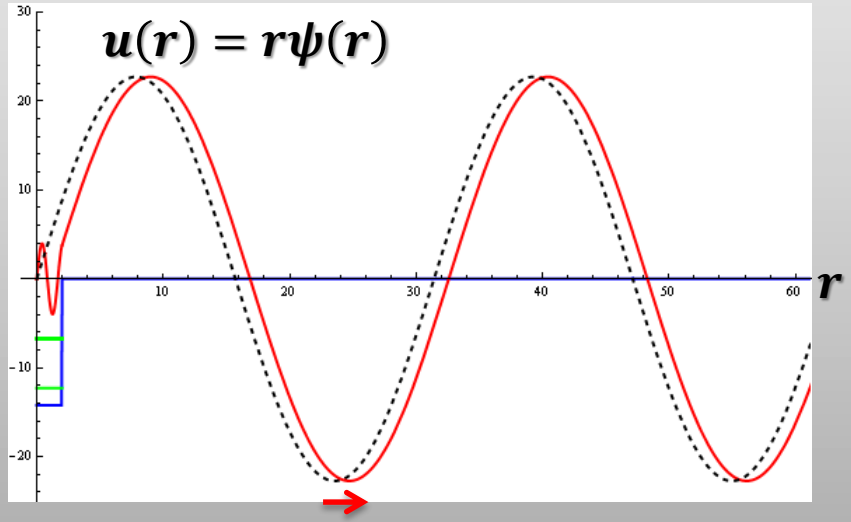
s波散乱長の変化　：　Shape resonance

井戸型ポテンシャル

３次元井戸型ポテンシャルの幅(R)と深さ(V_0)に対する散乱長の変化

$$\frac{a}{R} = 1 - \frac{\tan\left(\sqrt{V_0}R\right)}{\sqrt{V_0}R}$$

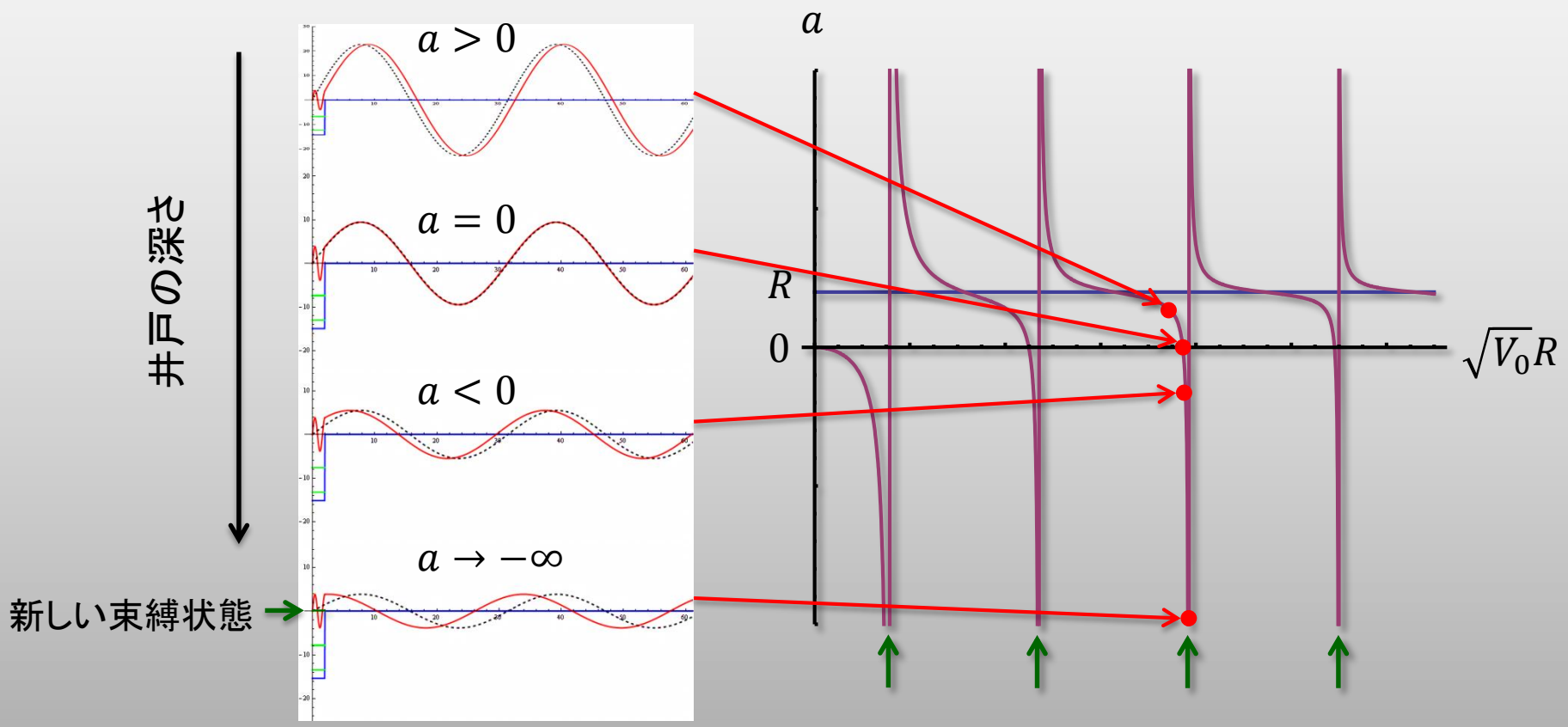




位相シフト：δ_s

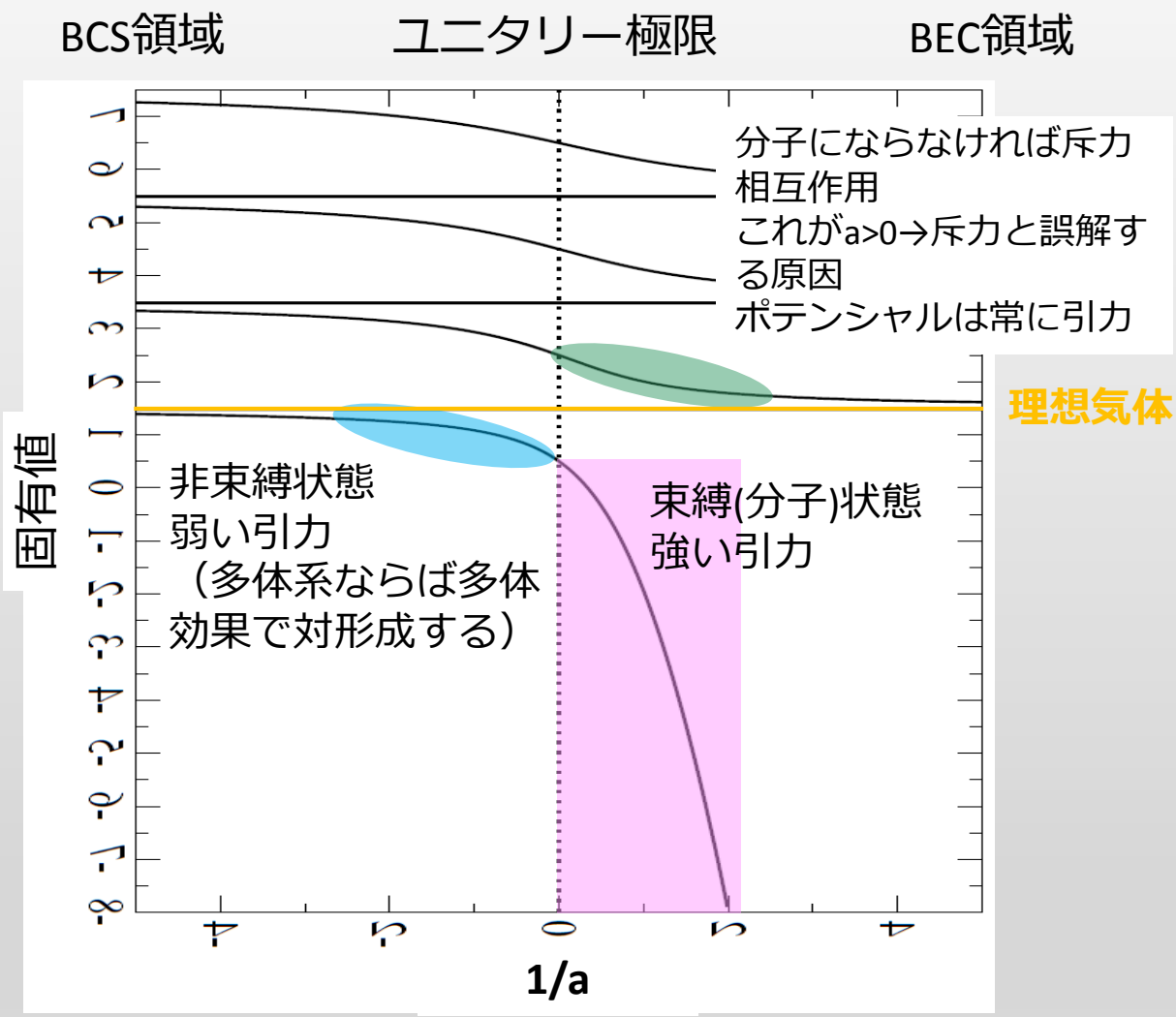
s波散乱長の変化　：　Shape resonance

入射エネルギーと束縛状態の共鳴散乱



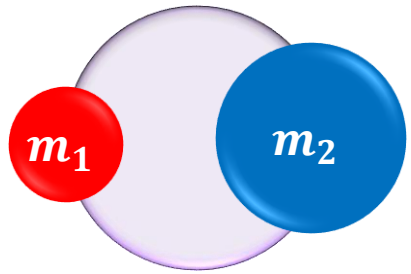
そうは言っても粒子間ポテンシャルの形は容易には変えられない

調和ポテンシャル中の二粒子の相互作用に依存した固有値



[Yvan Castin, arXiv:1103.2851v2 (2011)]

Two-body bound state determined by S-wave scattering length



Two particles can realize loosely bound state in vacuum for $a > 0$

Wave function of the bound state: $\psi_b(r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|) = \dfrac{1}{\sqrt{2\pi a}} \dfrac{\exp\left(-\dfrac{r}{a}\right)}{r}$

Radius: $\langle r \rangle = \displaystyle\int_0^\infty 4\pi r^2 r |\psi_b(r)|^2 \, dr = \frac{a}{2}$

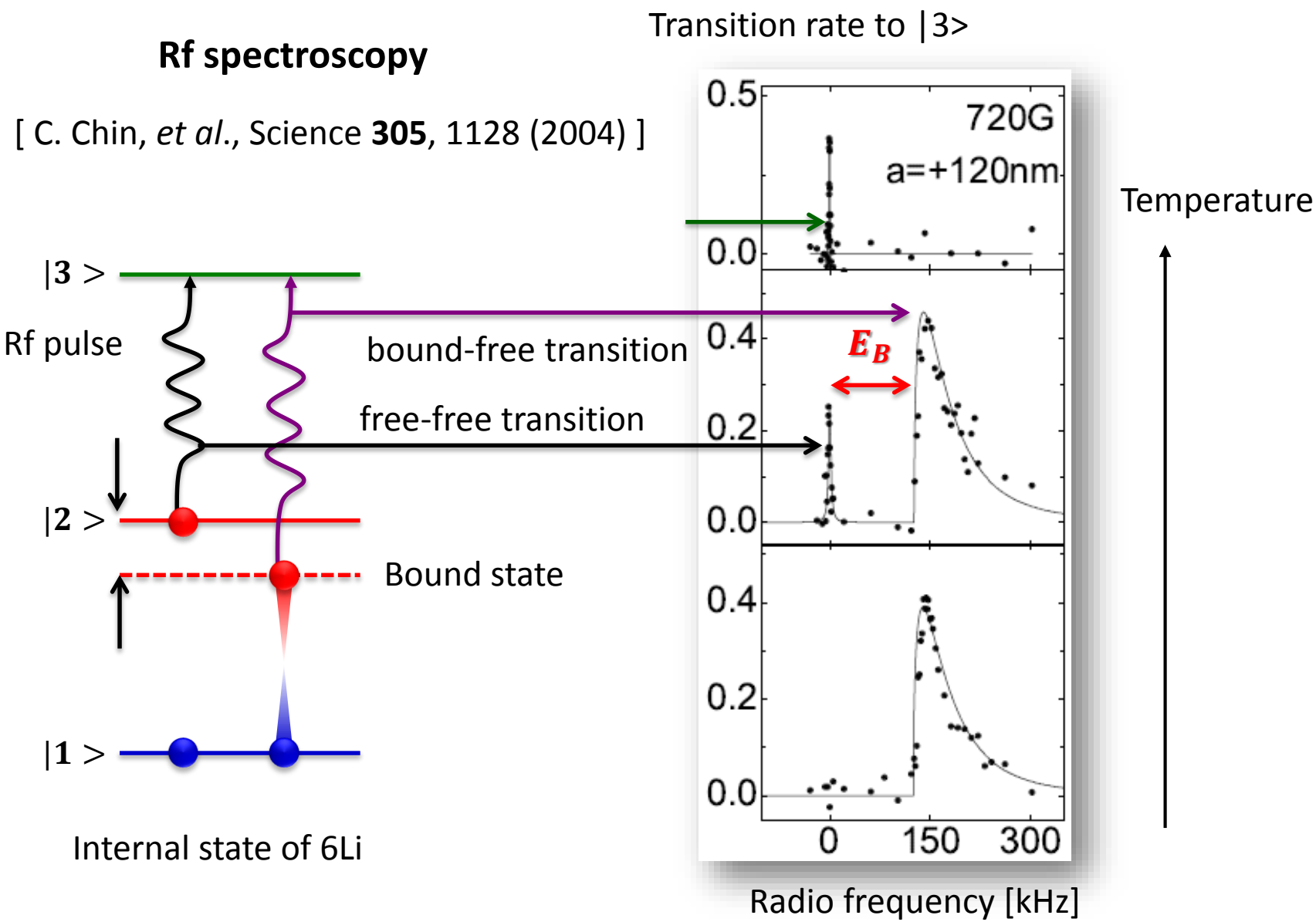
Binding energy: $E_b(a) = -\dfrac{\hbar^2}{2m_r a^2}$

Reduced mass: $m_r = \dfrac{m_1 m_2}{m_1 + m_2}$

Measurment of binding energy

Rf spectroscopy

[C. Chin, *et al.*, Science **305**, 1128 (2004)]



重ね合わせ状態の波動関数の測定

フェッシュバッハ分子(^{6}Li原子)：

ほとんどこの状態→普遍的

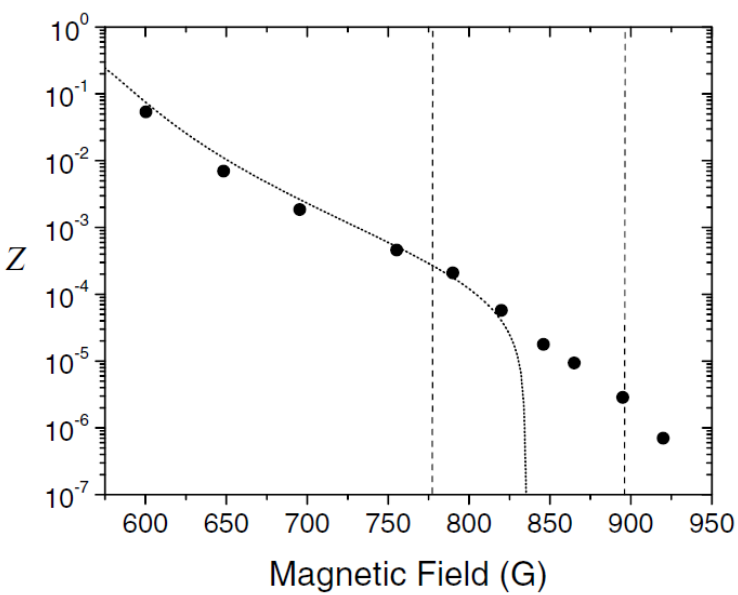
$$\psi(r) = \sqrt{Z}\cdot\psi_m(r)|\psi_{v=38}(S)> + \sqrt{1-Z}\cdot\frac{1}{\sqrt{2\pi a}}\frac{\exp\left(-\frac{r}{a}\right)}{r}|\phi_a(T)>$$

粒子の詳細に依存(非普遍性)

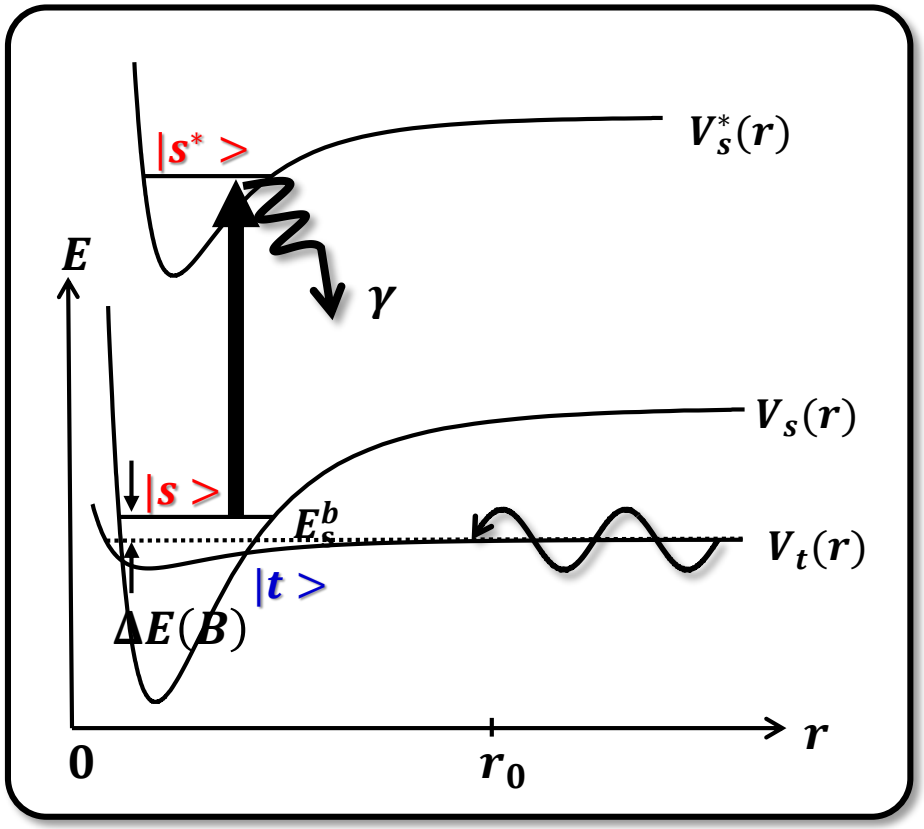
分子度：$Z \simeq \left[1+\frac{g_{am}^2 m^{3/2}}{4\pi\hbar^3\sqrt{|E_b|}}\right]^{-1}$

g_{am} : atom-molecule coupling constant

[R.A. Duine, Physics Reports 396, 115 (2004)]



[G. B. Partridge, Phys. Rev. Lett. 95, 020404 (2005)]

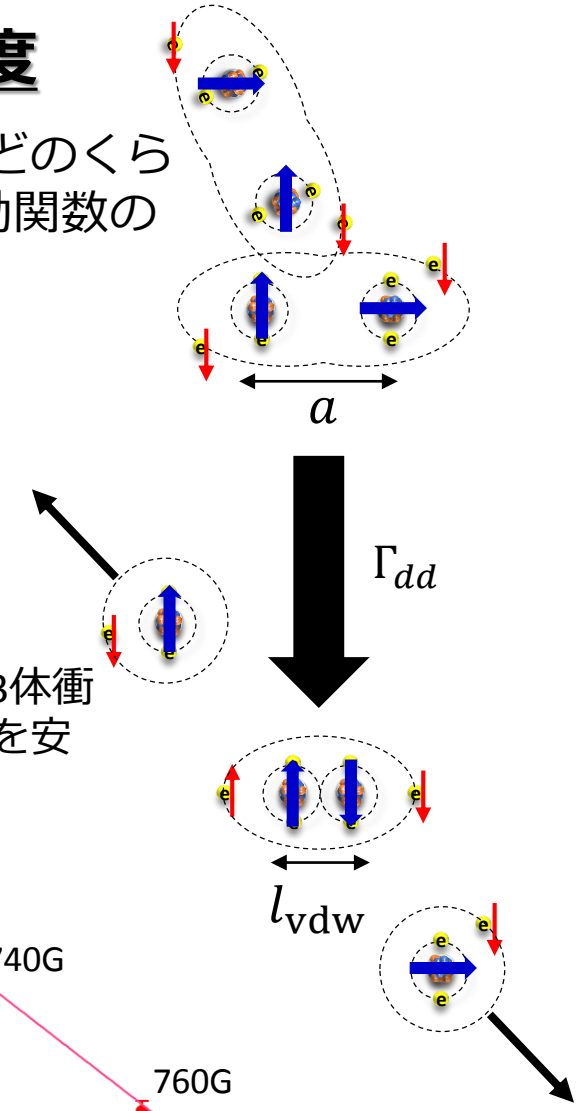


フェッシュバッハ分子(クラスター)の安定度

分子のロス(非弾性散乱)レート：
[D. S. Petrov, PRL **93**, 090404 (2004)]

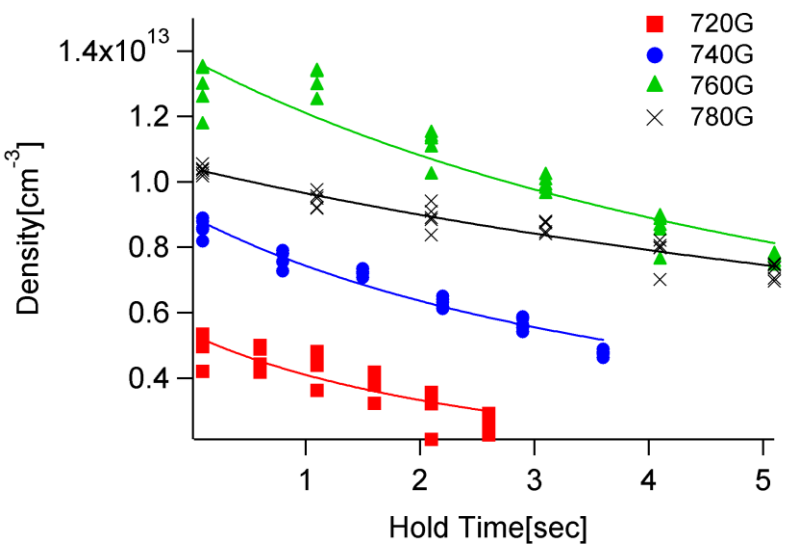
- **分子－分子：** $\frac{dn_d}{dt} = -\Gamma_{dd} n_d^2, \quad \Gamma_{dd} \propto a^{-2.55}$
- **分子－フェルミ粒子：** $\frac{dn_d}{dt} = -\Gamma_{fd} n_d n_f, \quad \Gamma_{fd} \propto a^{-3.33}$

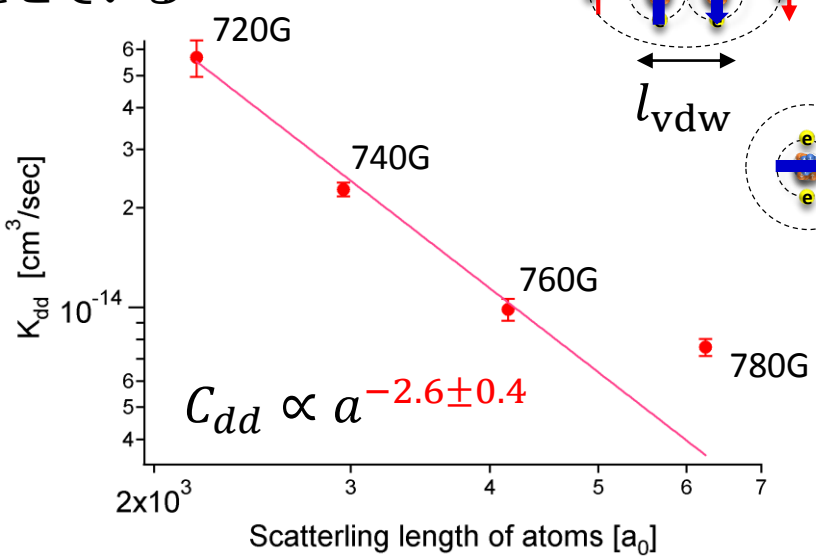
l_{vdw}^3の領域にどのくらい3粒子の波動関数の振幅があるか



パウリ効果が強く働き、3体衝突を抑制し、クラスターを安定化させている

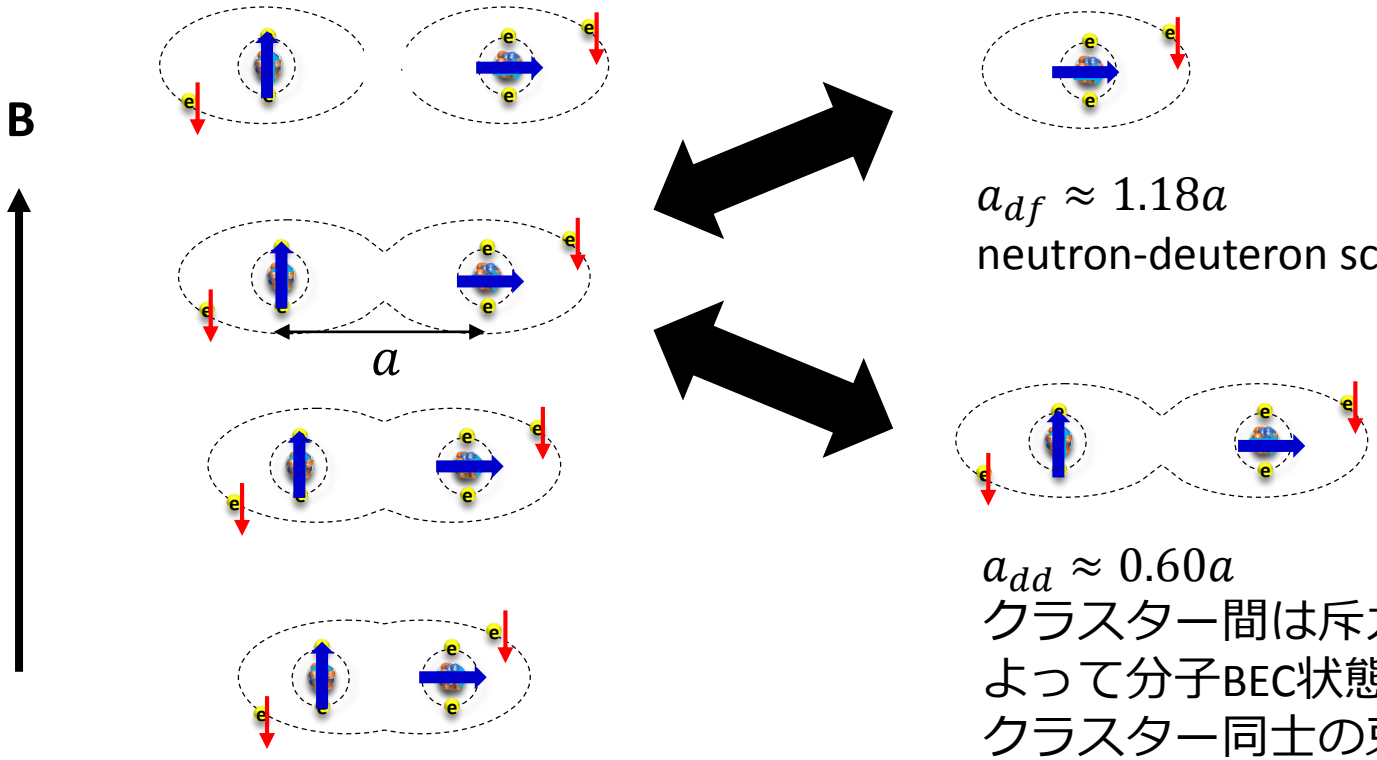
2007年12月の我々のデータ





クラスター間相互作用(弾性散乱)

パウリ効果が強く働く



$a_{df} \approx 1.18a$

neutron-deuteron scatteringと類似

$a_{dd} \approx 0.60a$

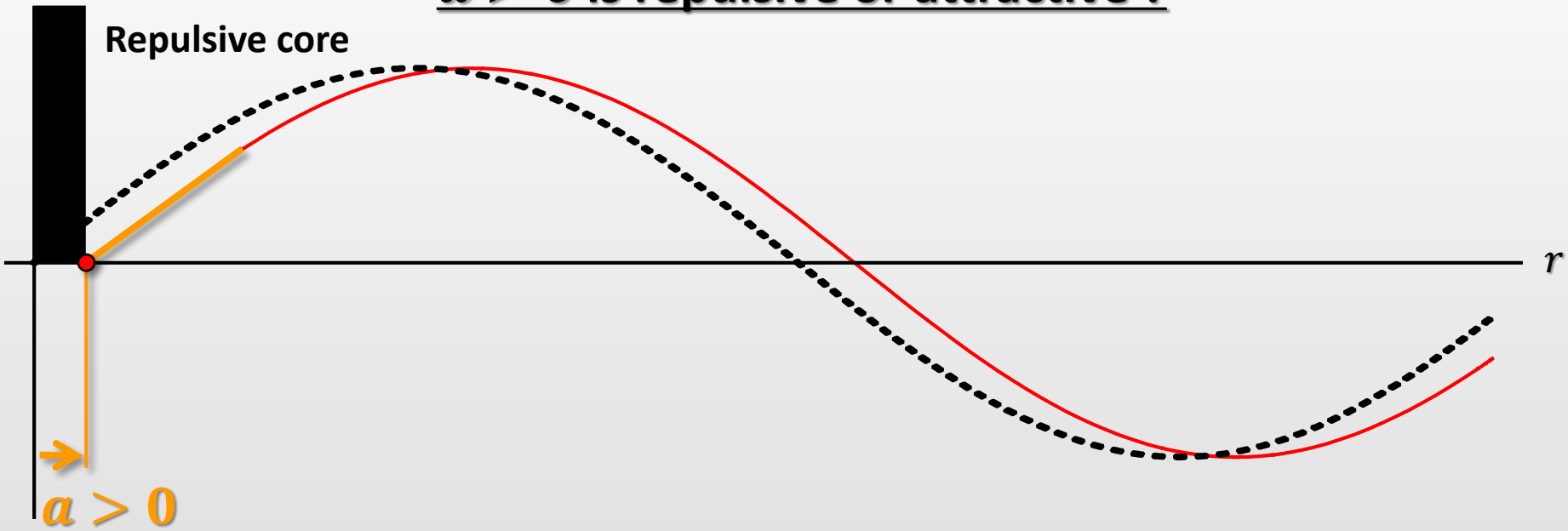
クラスター間は斥力相互作用
よって分子BEC状態は安定
クラスター同士の束縛状態はない

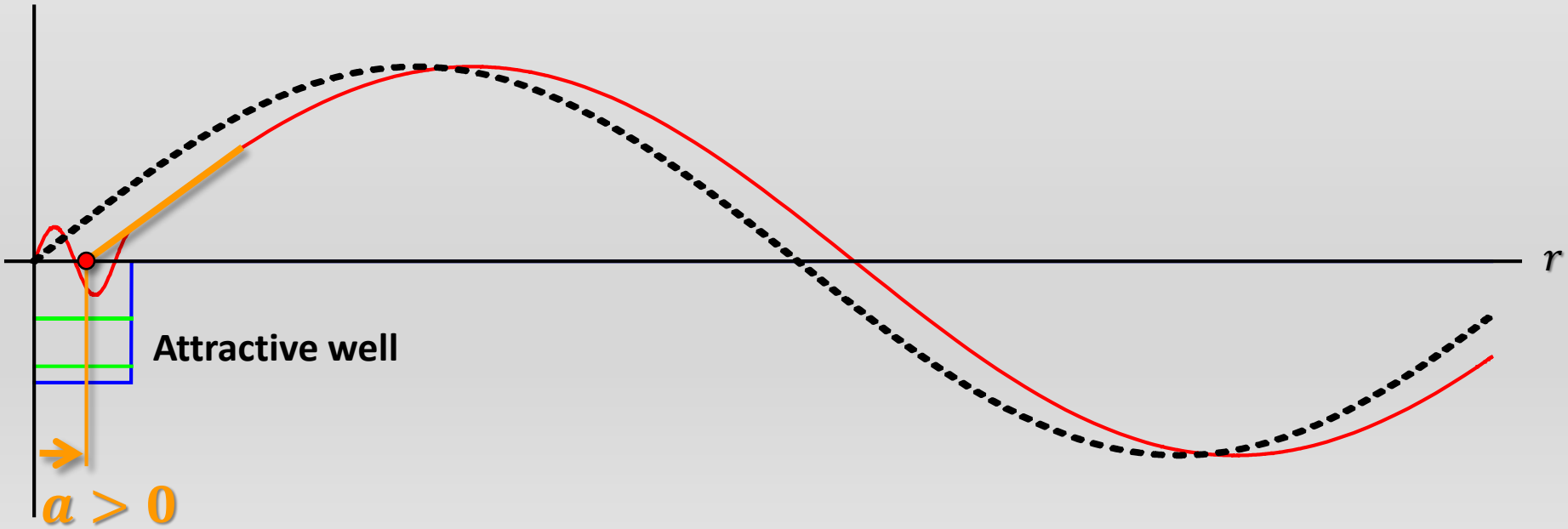
4つのフェルミオンから成るα粒子は
クラスターを構成できる。
フェッシュバッハ分子との違いは何か？

[D. S. Petrov, PRL **93**, 090404 (2004)]

[J. Levinsen, PRA **73**, 053607 (2006)]

$a > 0$ is repulsive or attractive ?





相互作用ポテンシャルと散乱長

- **Grand canonical Hamiltonian :**

$$\hat{\mathcal{H}} - \mu\hat{\mathcal{N}} = \sum_{\sigma}\int\left(\frac{\hbar^2}{2m}\nabla\hat{\Psi}_{\sigma}^{\dagger}(\boldsymbol{r})\nabla\hat{\Psi}_{\sigma}(\boldsymbol{r}) - \mu\right)d\boldsymbol{r}$$

$$+\iint\hat{\Psi}_{\uparrow}^{\dagger}(\boldsymbol{r}_{\uparrow})\hat{\Psi}_{\downarrow}^{\dagger}(\boldsymbol{r}_{\downarrow})\underline{\underline{U_{\mathrm{int}}(\boldsymbol{r}_{\uparrow}-\boldsymbol{r}_{\downarrow})}}\hat{\Psi}_{\downarrow}(\boldsymbol{r}_{\downarrow})\hat{\Psi}_{\uparrow}(\boldsymbol{r}_{\uparrow})d\boldsymbol{r}_{\uparrow}d\boldsymbol{r}_{\downarrow}$$

Detail of the particle

Non-universal

Universal

デルタ型擬ポテンシャル

$$U_{\mathrm{int}}(\boldsymbol{r}_{\uparrow}-\boldsymbol{r}_{\downarrow}) = -\frac{\hbar^2}{m}g(a)\delta(\boldsymbol{r}_{\uparrow}-\boldsymbol{r}_{\downarrow})$$

$$\frac{1}{g(a)} = -\frac{1}{4\pi a} + \frac{1}{V}\sum_{k}\frac{1}{k^2} > 0$$

$$\hat{\mathcal{H}} - \mu\hat{\mathcal{N}} = \sum_{\sigma}\int\left(\frac{\hbar^2}{2m}\nabla\hat{\Psi}_{\sigma}^{\dagger}(\boldsymbol{r})\nabla\hat{\Psi}_{\sigma}(\boldsymbol{r}) - \mu\right)d\boldsymbol{r}$$

$$\underline{\underline{-\frac{\hbar^2}{m}g(a)}}\int\hat{\Psi}_{\uparrow}^{\dagger}(\boldsymbol{r})\hat{\Psi}_{\downarrow}^{\dagger}(\boldsymbol{r})\hat{\Psi}_{\downarrow}(\boldsymbol{r})\hat{\Psi}_{\uparrow}(\boldsymbol{r})d\boldsymbol{r}$$

Renormalized coupling constant

Many-body Hamiltonian and thermodynamics

- **Grand canonical Hamiltonian :**

Interaction

$$\hat{\mathcal{H}} - \mu\hat{\mathcal{N}} = \sum_{\sigma} \int \left(\frac{\hbar^2}{2m} \nabla \hat{\Psi}_{\sigma}^{\dagger}(\boldsymbol{r}) \nabla \hat{\Psi}_{\sigma}(\boldsymbol{r}) - \mu \right) d\boldsymbol{r} - \frac{\hbar^2}{m} g(a) \int \hat{\Psi}_{\uparrow}^{\dagger}(\boldsymbol{r}) \hat{\Psi}_{\downarrow}^{\dagger}(\boldsymbol{r}) \hat{\Psi}_{\downarrow}(\boldsymbol{r}) \hat{\Psi}_{\uparrow}(\boldsymbol{r}) d\boldsymbol{r}$$

$$\left(\begin{array}{c} g(a) \geq 0 \\ \dfrac{dg(a)}{da^{-1}} = \dfrac{g^2(a)}{4\pi} \end{array} \right)$$

- **Eigenstate and eigenvalue of state *i* :** $K_i = \langle \Psi_i | \hat{\mathcal{H}} - \mu\hat{\mathcal{N}} | \Psi_i \rangle$

- **Grand partition function :** $Z_G = \sum_{i=0}^{\infty} \exp\left(-\frac{K_i}{k_B T} \right)$ *Excitation*

- **Thermodynamic potential :** $\Omega(V, T, \mu, a^{-1}) = -k_B T \ln Z_G$

- **Thermodynamic relation :**

Hellmann–Feynman theorem

$$\left(\frac{\partial \Omega}{\partial a^{-1}} \right)_{V,T,\mu} = \frac{\sum_i \boxed{\frac{\partial K_i(a^{-1})}{\partial a^{-1}}} \exp\left(-\frac{K_i}{k_B T} \right)}{\sum_i \exp\left(-\frac{K_i}{k_B T} \right)} = \frac{\sum_i \boxed{\left\langle \Psi_i \middle| \frac{\partial(\hat{\mathcal{H}} - \mu\hat{\mathcal{N}})}{\partial a^{-1}} \middle| \Psi_i \right\rangle} \exp\left(-\frac{K_i}{k_B T} \right)}{\sum_n \exp\left(-\frac{K_i}{k_B T} \right)}$$

$$\equiv -\frac{\hbar^2}{4\pi m} \int \boxed{C(\boldsymbol{r})} d\boldsymbol{r}$$

Contact density

Few-body physics and Virial coefficients

- **Grand partition function :** $Z_G(V,T,\mu,a^{-1}) = \sum_{N=0}^{\infty} Z_N(V,T,N,a^{-1}) e^{N\frac{\mu}{k_B T}}$

$$= 1 + Z_1 e^{\frac{\mu}{k_B T}} + Z_2 e^{2\frac{\mu}{k_B T}} + Z_3 e^{3\frac{\mu}{k_B T}} + \cdots$$

Partition function

$$Z_N = \sum_{i=0}^{\infty} \exp\left(-\frac{E_{N_i}}{k_B T}\right)$$

Excitation

High temperature region : $z \equiv e^{\frac{\mu}{k_B T}} \ll 1$

- **Thermodynamic potential :** $\Omega(V,T,\mu,a^{-1}) \to -k_B T Z_1 (z + B_2 z^2 + B_3 z^3 + \cdots B_n z^n)$

- **Virial coefficients :** $B_2 = (Z_2 - Z_1^2/2)/Z_1$

$$B_3 = (Z_3 - Z_1 Z_2 - Z_1^3/3)/Z_1$$

$$\vdots$$

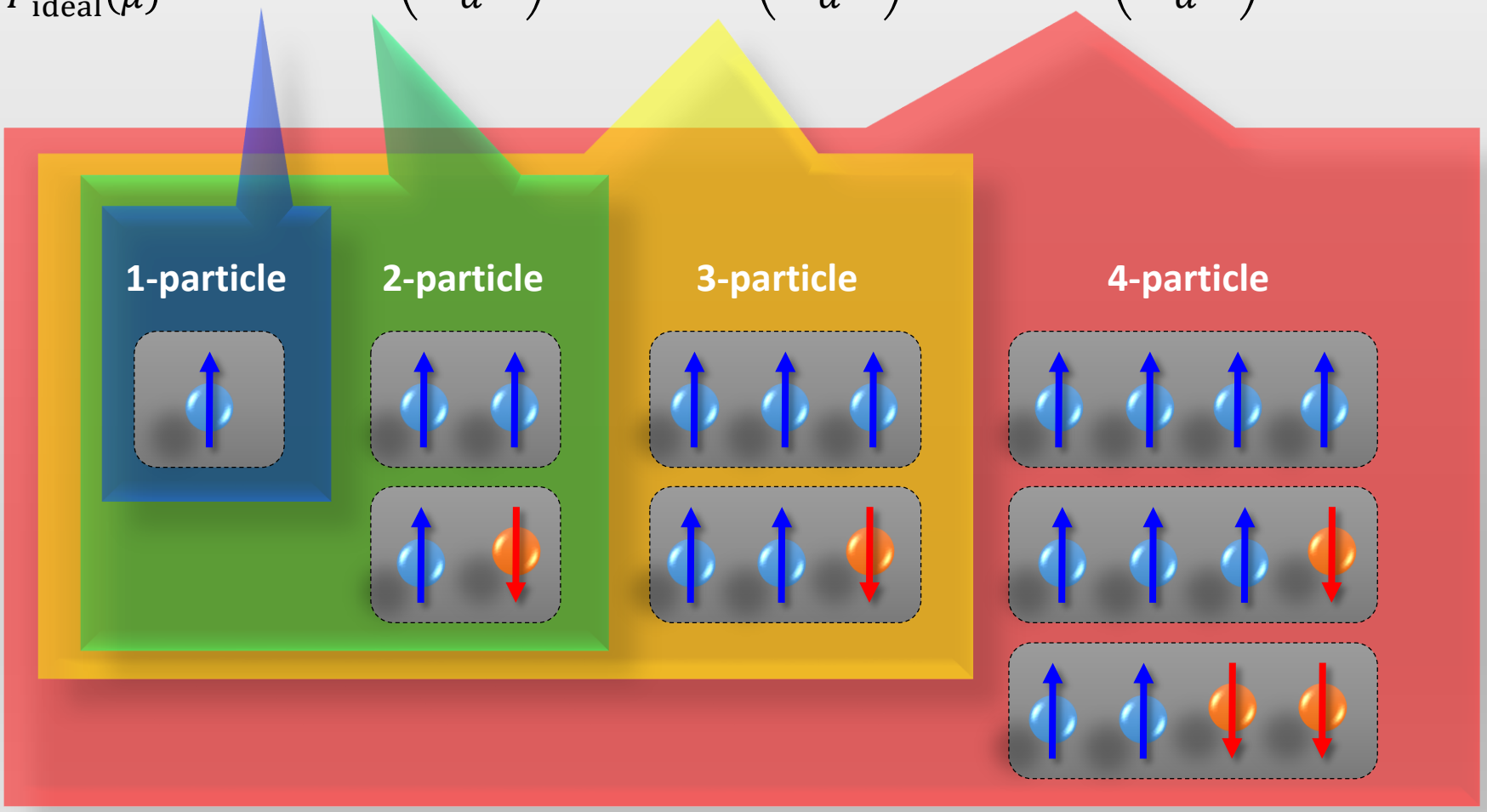
- **Universal equation of state :** $\frac{P}{P_{\text{ideal}}(\mu)} = f_P\left(\frac{\mu}{k_B T}, \frac{\Lambda_{\mathrm{T}}(\mathrm{T})}{a}\right)$

$$\to e^{\frac{\mu}{k_B T}} + B_2\left(\frac{\Lambda_{\mathrm{T}}(\mathrm{T})}{a}\right) e^{2\frac{\mu}{k_B T}} + B_3\left(\frac{\Lambda_{\mathrm{T}}(\mathrm{T})}{a}\right) e^{3\frac{\mu}{k_B T}} + \cdots$$

Few-body physics and higher Virial coefficients

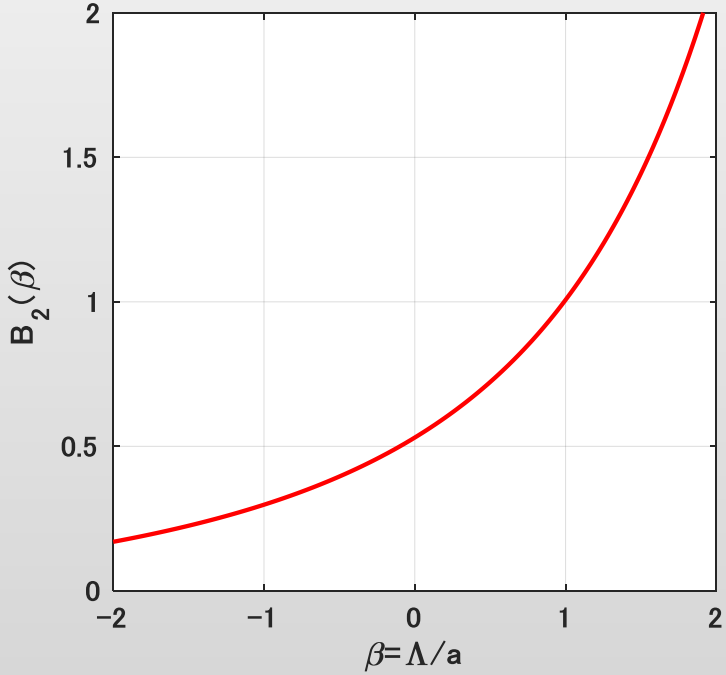
- **Universal equation of state at high temperature :**

$$\frac{P}{P_{\text{ideal}}(\mu)} = e^{\frac{\mu}{k_B T}} + B_2\left(\frac{\Lambda_{\mathrm{T}}(T)}{a}\right)e^{2\frac{\mu}{k_B T}} + B_3\left(\frac{\Lambda_{\mathrm{T}}(T)}{a}\right)e^{3\frac{\mu}{k_B T}} + B_4\left(\frac{\Lambda_{\mathrm{T}}(T)}{a}\right)e^{4\frac{\mu}{k_B T}} + \cdots$$



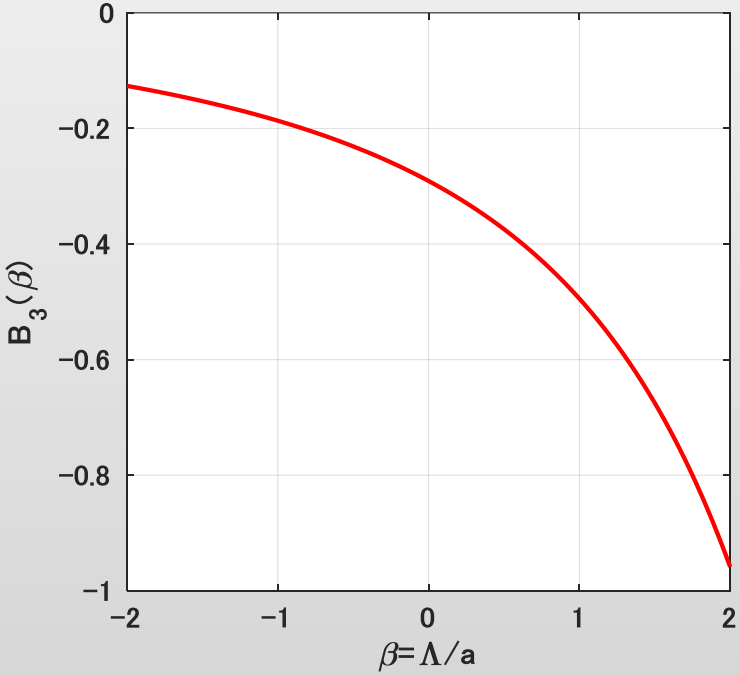
2nd and 3rd Virial coefficients

2nd Virial coefficients



N. Sakumichi,
Phys. Rev. A 89, 033622 (2014)

3rd Virial coefficients



X. Leyronas,
Phys. Rev. A 84, 053633 (2011)
「Virial expansion with Feynman diagrams」

Many-body Hamiltonian and thermodynamics

- **Thermodynamic relation :** $\left(\frac{\partial \Omega}{\partial a^{-1}}\right)_{V,T,\mu} = -\frac{\hbar^2}{4\pi m}\int C(\boldsymbol{r})d\boldsymbol{r}$

- **Contact density :** $C(\boldsymbol{r}) = \left\langle \left(\boldsymbol{g}(\boldsymbol{a})\widehat{\Psi}_{\uparrow}^{\dagger}(\boldsymbol{r})\widehat{\Psi}_{\downarrow}^{\dagger}(\boldsymbol{r})\right)\left(\boldsymbol{g}(\boldsymbol{a})\widehat{\Psi}_{\downarrow}(\boldsymbol{r})\widehat{\Psi}_{\uparrow}(\boldsymbol{r})\right)\right\rangle$

For homogeneous system $\left(\int C(\boldsymbol{r})d\boldsymbol{r} \to CV\right)$

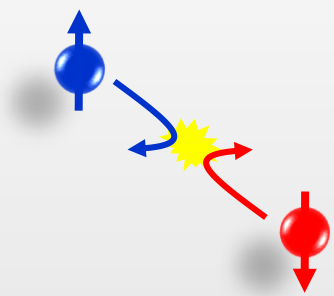
- **Gibbs-Duhem equation :** $dP = sdT + nd\mu + \left(\frac{\hbar^2}{4\pi m}C\right)da^{-1}$

$\mathcal{F} = \mu n - P$ $\updownarrow$ $\therefore \boldsymbol{P = P(T, \mu, a^{-1})}$

- **Total differential of free energy density :** $d\mathcal{F} = -sdT + nd\mu - \left(\frac{\hbar^2}{4\pi m}C\right)da^{-1}$

$$\therefore \boldsymbol{\mathcal{F} = \mathcal{F}(T, n, a^{-1})}$$

Phase diagram for Spin-1/2 fermions



***s*-wave contact interaction**

$$\hat{\mathcal{H}}_{\text{int}}\big(U_{\text{int}}(\boldsymbol{r}_\uparrow - \boldsymbol{r}_\downarrow)\big) \rightarrow \hat{\mathcal{H}}_{\text{int}}(\underline{\underline{a}})$$

Tunable by Feshbach resonance