NITEP Workshop 18/12/14

Symmetry Breaking in Quantum Curves & Super Chern-Simons Matrix Models

Sanefumi Moriyama (Osaka City Univ)

Main References:

S.M., T.Nosaka, JHEP, 2015;

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T.Furukawa, S.M., to appear (see TF's poster).

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Geometry

Symmetry Breaking in Quantum Curves & Super Chern-Simons Matrix Models

M2-brane Physics

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0. Overview

String Theory

Type IIA

Hetero $E_8 \times E_8$

Qunatum Gravity

Exciting for

$E_3 \times u(1)$

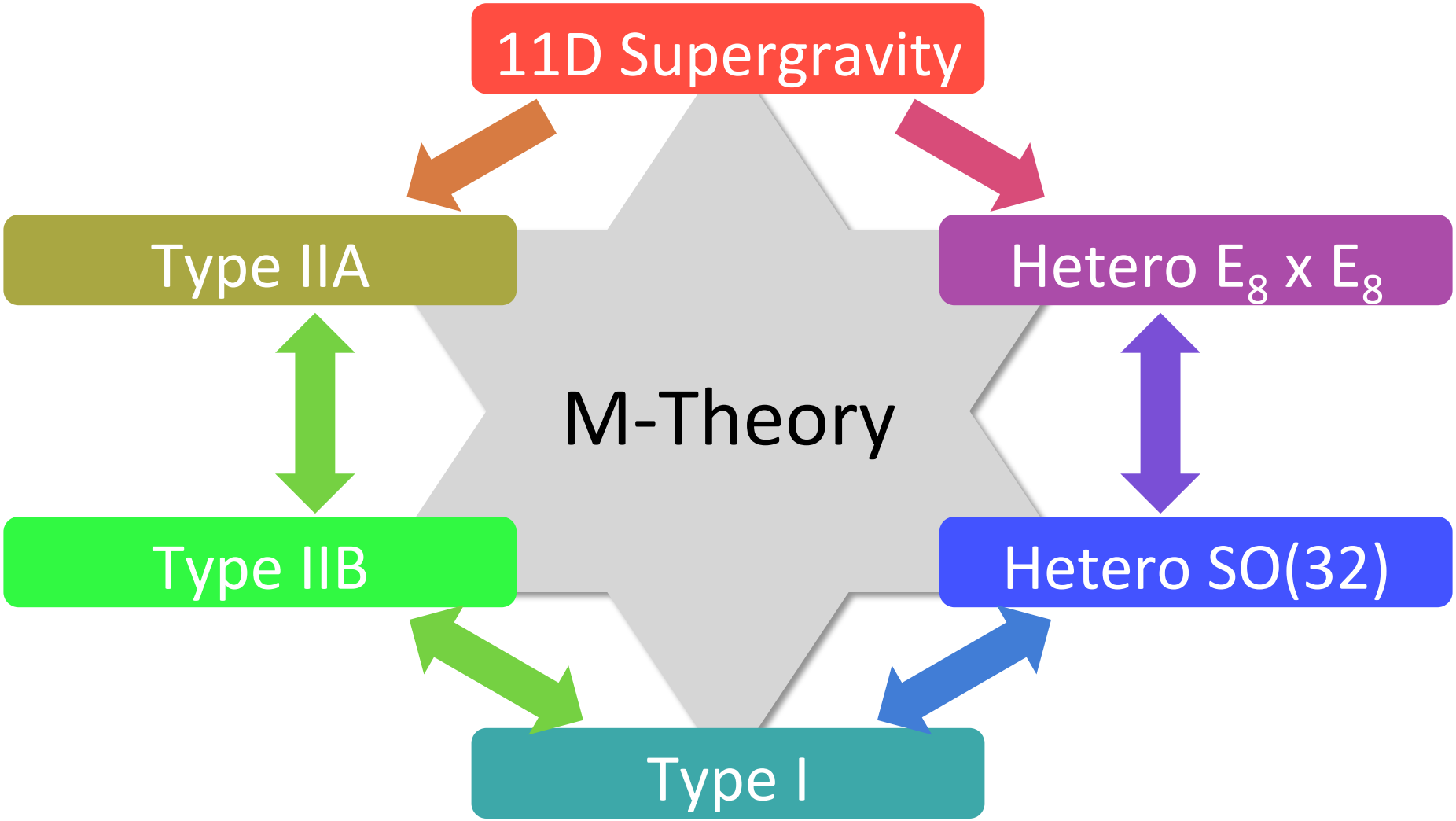
$= [su(3) \times su(2)]_{u(1)}$

Type IIB

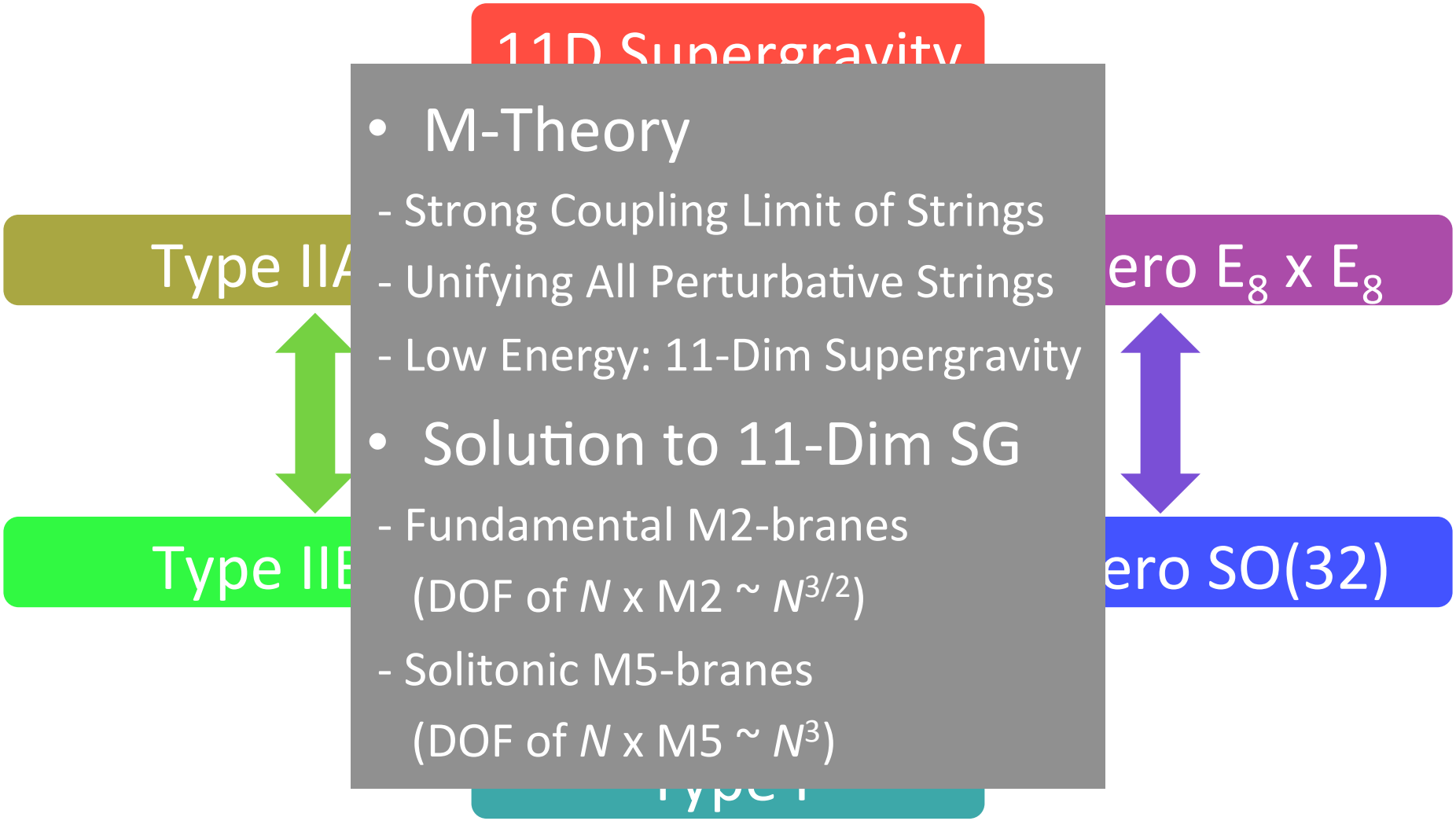
Hetero SO(32)

Type I

String Duality & M-Theory



String Duality & M-Theory



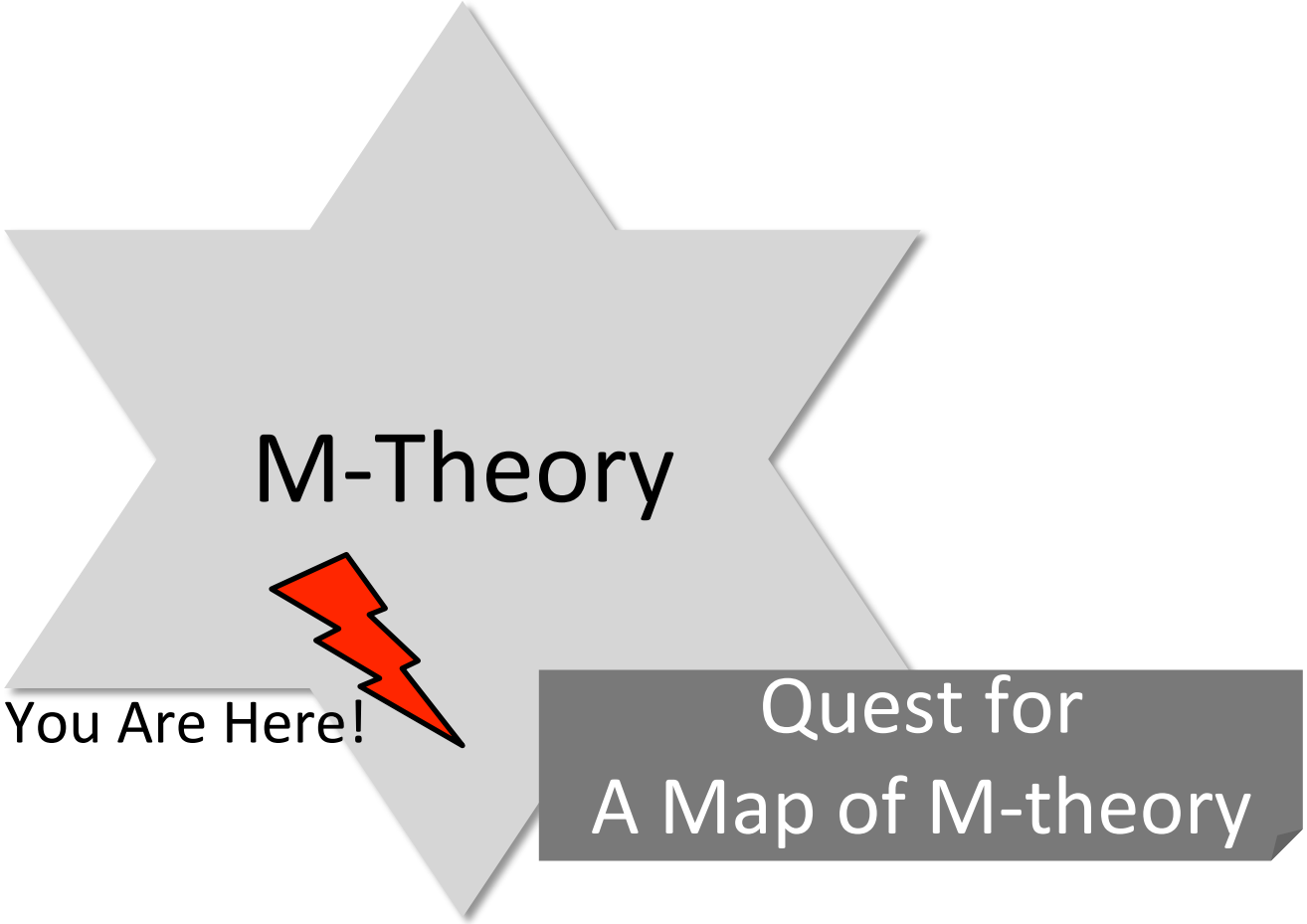
- M-Theory
  - Strong Coupling Limit of Strings
  - Unifying All Perturbative Strings
  - Low Energy: 11-Dim Supergravity
- Solution to 11-Dim SG
  - Fundamental M2-branes
    (DOF of N x M2 ~ $N^{3/2}$)
  - Solitonic M5-branes
    (DOF of N x M5 ~ N^3)

M is for Mother / Membrane / Mystery

- Mother theory for Strings ?
- Theory of Membranes ?
- Mysterious Theory ?

  - Worldvolume Theories for M2 / M5 ?
  - Degrees of Freedom $N^{3/2}$ / N^3 ?
  - Whole Moduli Space ?

You Are Here! (Hopefully)



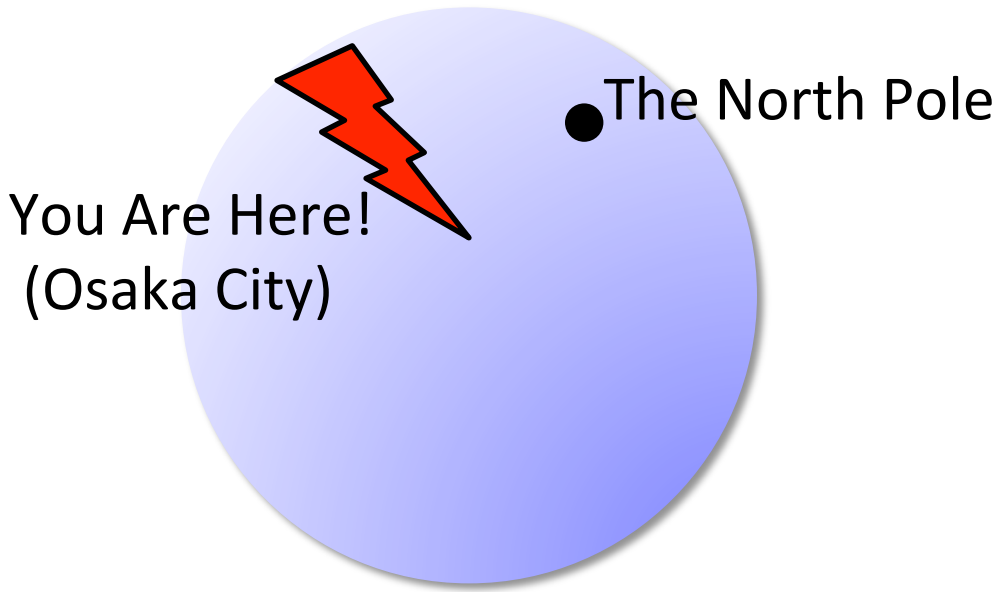
A Map of The Earth

- Investigate Special Points

  e.g. Osaka City, Equator, Polar Points, ..., The Center of Earth

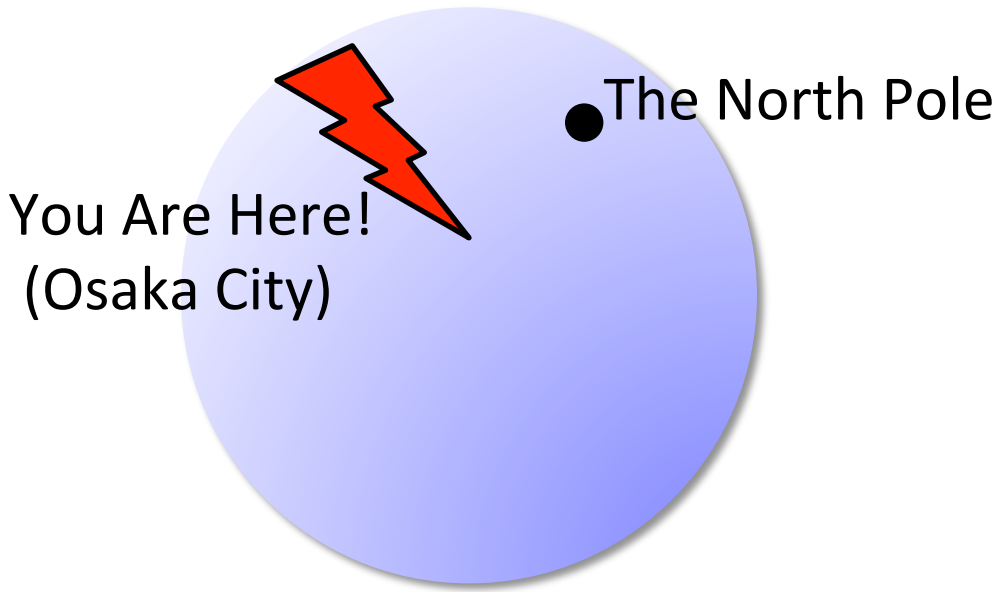
- Understand the Symmetry of the Earth

  "The Earth is approximately A Sphere." "so(3) Symmetry"



Our History

- Osaka (Phenomenological)
- Sphere (Theoretical)
- The Center (Ultra-Theoretical)



History of String Theory

- Osaka (Phenomenological)
- Sphere (Theoretical)
- The Center (Ultra-Theoretical)

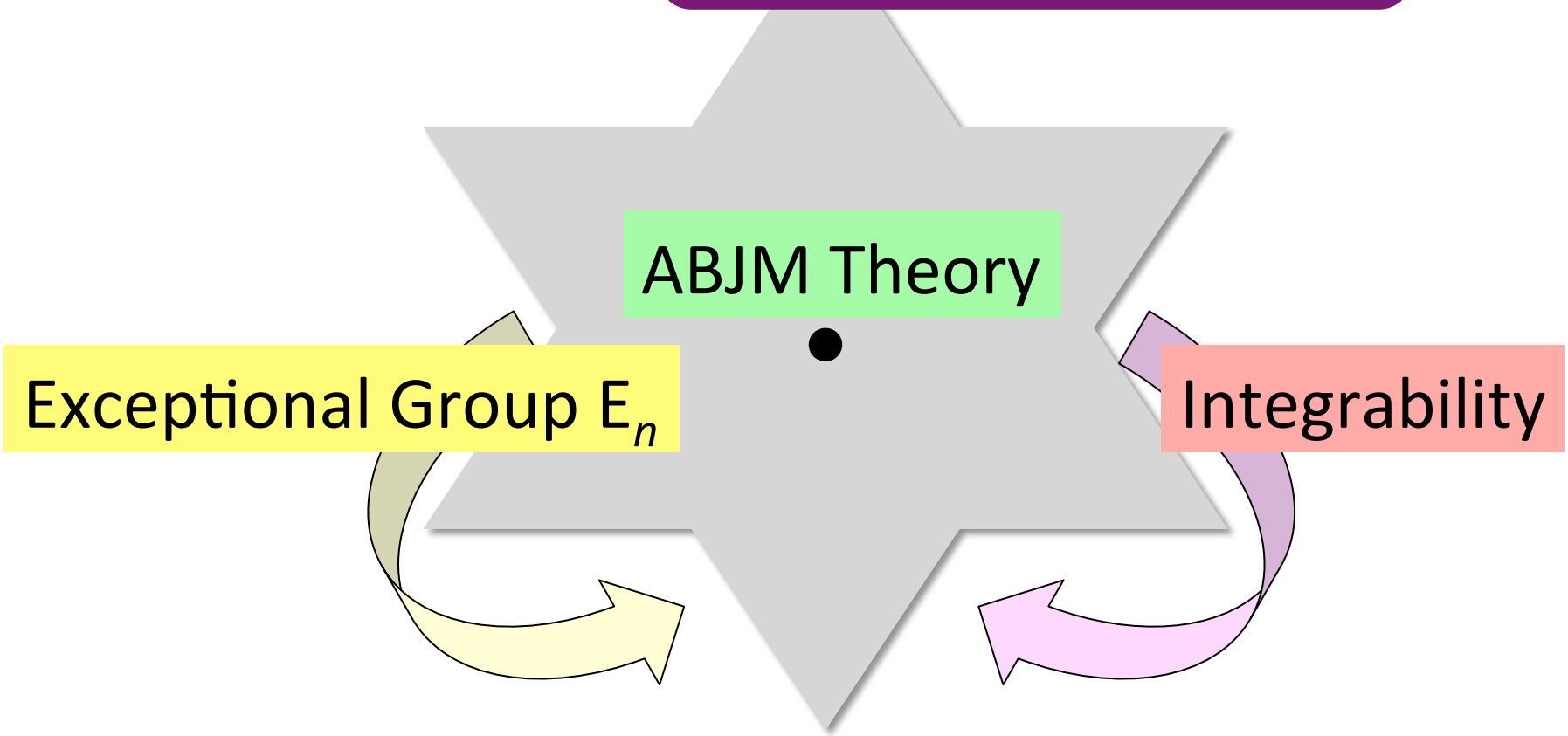
Our History

History of String Theory
or M-Theory

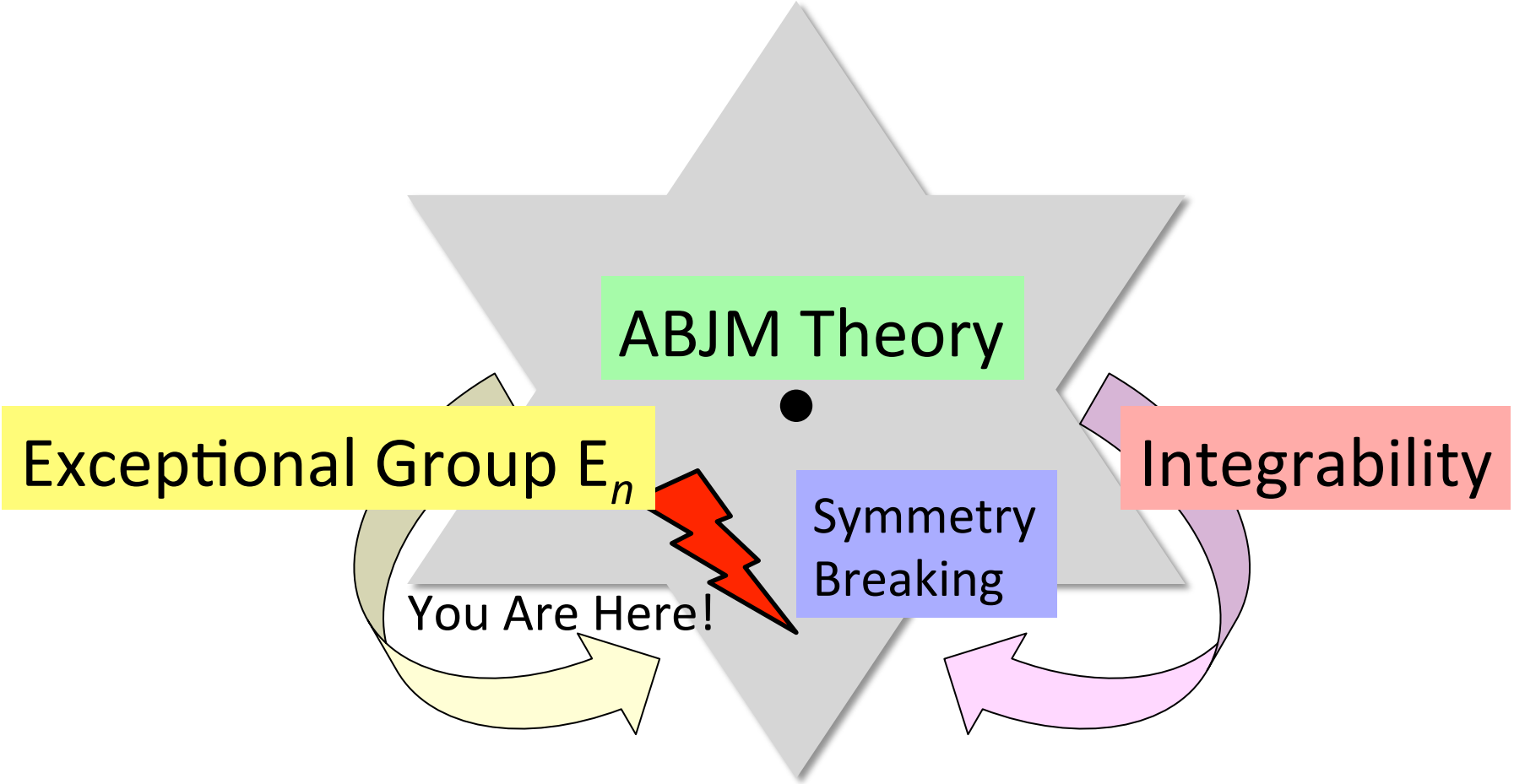
Consistency in Quantum Theory, Constraints from Symmetry, ...

A Map of M-Theory

Correctly Speaking, A Map of Supersymmetric Backgrounds for M2-branes



A Map of M-Theory



Contents

1. ABJM Theory

The Worldvolume Theory Describing M2 on Background with Largest SUSY

"The Center of Earth"

Worldvolume Theory of M2-branes

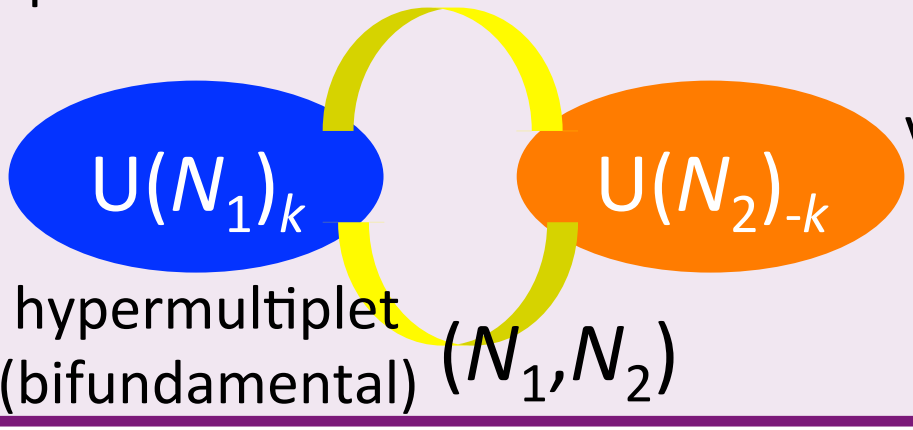
- Single M2-brane (Breaking Transl. Sym. & SUSY)
  - * 8 bosons (spin 0) + 8 fermions (spin 1/2)
- Multiple M2-branes ???
  - * gauge field (spin 1)
- Supersymmetrize Chern-Simons field to $\mathcal{N}$=8!

  [J.H.Schwarz 2004]
  - * No DOF for CS in 3D
- Only $\mathcal{N}$=3 for Arbitrary Gauge/Representation

ABJM Theory

$\mathcal{N}$=6 Chern-Simons Theory

(Expected to Enhance to $\mathcal{N}$=8 for k=1)



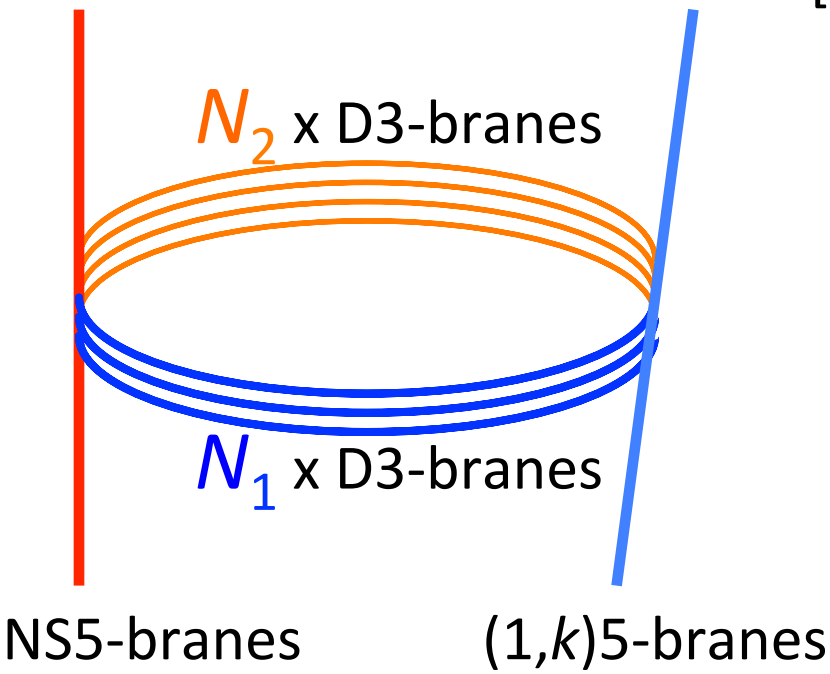
vector multiplet
(adjoint)

[Aharony-Bergman-Jafferis-Maldacena N_1=N_2,
Hosomichi-Lee-Lee-Lee-Park $N_1 \neq N_2$,
Aharony-Bergman-Jafferis $N_1 \neq N_2$, 2008]

Brane Configuration in IIB

From Large Supersymmetries

[Kitao-Ohta-Ohta 1998, ...]



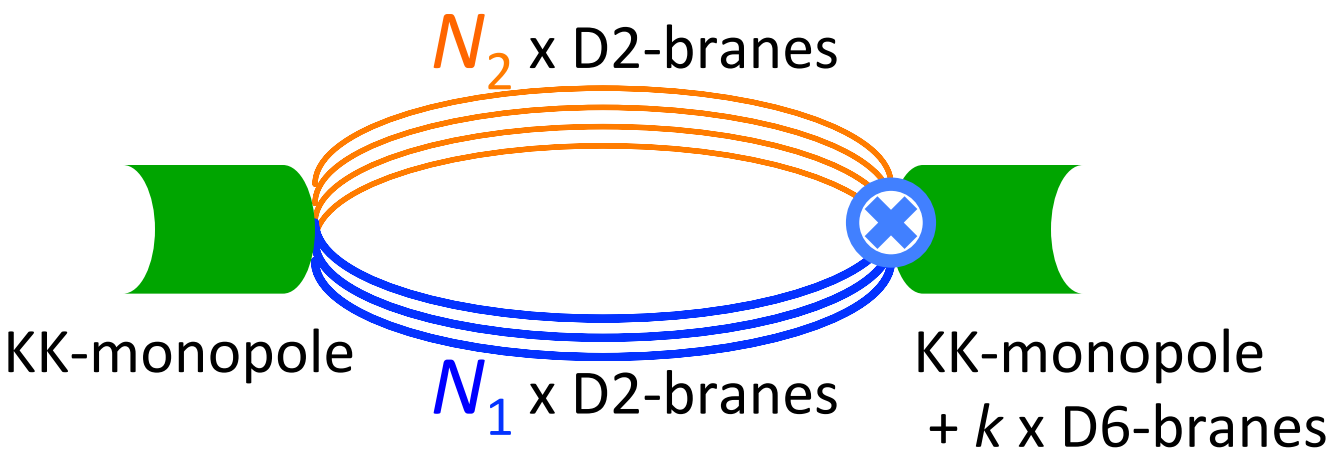
→ T-duality to IIA

→ Lift to M-Theory

(IIB String Theory)

Brane Configuration in IIA

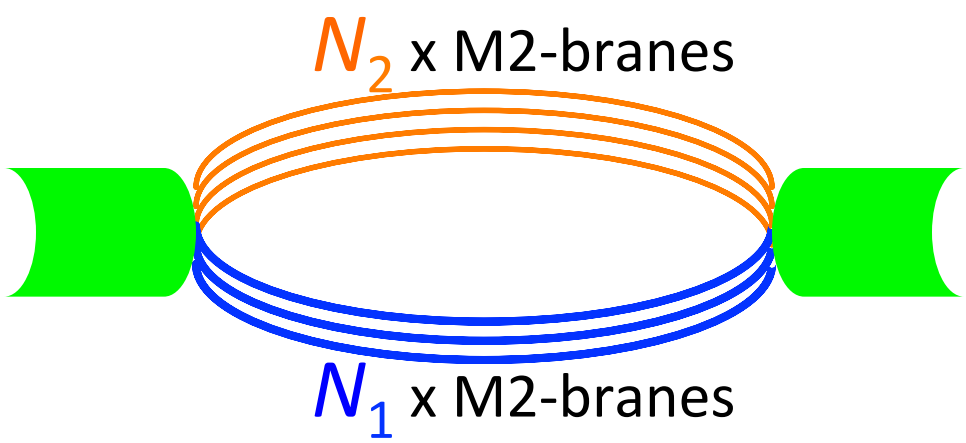
→ T-duality to IIA



(IIA String Theory)

Brane Configuration in M

→ Lift to M-Theory

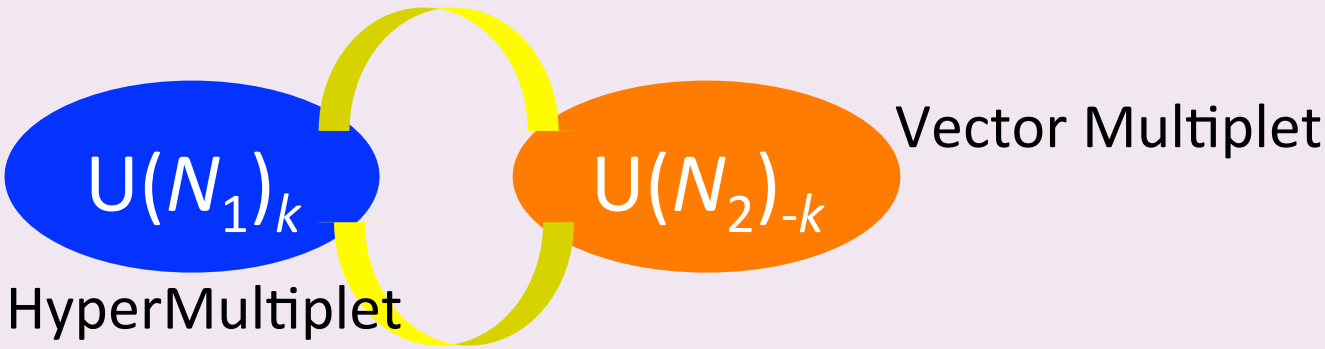


(1,0) KK-monopole + (1,k) KK-monopole ⇒ C^4/Z_k

(M-Theory)

ABJM Theory

$\mathcal{N}$=6 Chern-Simons Theory



Min(N_1,N_2) x M2 & |N_2-N_1| x fractional M2 on C^4 / Z_k

Partition Function OR VEV

$\langle s_Y \rangle_k (N_1 | N_2)$

Partition Function OR
VEV of Half-BPS Wilson Loop in Rep Y
on Min(N_1,N_2) x M2 & $|N_2 - N_1|$ x Fractional M2

$\langle s_Y \rangle_k (N_1 | N_2)$

$= \int \ldots DA \ldots \exp(-S_{\mathrm{ABJM}}[A,\ldots]) \operatorname{tr} P \exp \int A_\mu dx^\mu + \ldots$

$= \ldots$

After Localization Technique, ...

[Kapustin-Willett-Yaakov 2009]

ABJM Matrix Model

$$\left(\begin{array}{l} N_1, N_2 \in \mathbb{Z}_{\geq 0} \\ k \in \mathbb{R} \quad \text{Physically, } k \in \mathbb{Z}_{>0} \end{array}\right)$$

Characters labeled by Young Diagram

$$\langle s_\lambda \rangle_k (N_1 \mid N_2) = \frac{i^{-\frac{1}{2}\left(N_1^2 - N_2^2\right)}}{N_1! N_2!} \int \prod_{m=1}^{N_1} \frac{d\mu_m}{2\pi} \prod_{n=1}^{N_2} \frac{d\nu_n}{2\pi} \, s_\lambda\left(e^\mu \mid e^\nu\right)$$

Vector Multiplet

$$\frac{\prod_{m<m'}^{N_1}\left(2\sinh\frac{\mu_m - \mu_{m'}}{2}\right)^2 \prod_{n<n'}^{N_2}\left(2\sinh\frac{\nu_n - \nu_{n'}}{2}\right)^2}{\prod_{m=1}^{N_1}\prod_{n=1}^{N_2}\left(2\cosh\frac{\mu_m - \nu_n}{2}\right)^2}$$

HyperMultiplet

1-loop

$$\exp\frac{ik}{4\pi}\left(\sum_{m=1}^{N_1}\mu_m^2 - \sum_{n=1}^{N_2}\nu_n^2\right)$$

Classical

Grand Canonical Ensemble

- Canonical Partition Function

$$Z_k(N) \;=\; \langle s_Y{=}1 \rangle_k (N|N)$$

(Partition Function without Fractional Branes (Rank Difference) or Wilson loops (s_Y))

- Grand Canonical Partition Function

$$\Xi_k(z) \;=\; \Sigma_{N=0}^{\infty} z^N Z_k(N)$$

(Generating Function, N : Particle Number, z : Dual Fugacity)

From Matrix Models To Curves

[Marino-Putrov 2011] [..., Hatsuda-Marino-M-Okuyama 2013]

Grand Partition Function $\Xi_k(z)$

=

Spectral Det
Det ($1 + z\, H^{-1}$)

=

Free Energy of Top Strings
exp [$\Sigma\, N^d_{jL,jR}\, F^d_{jL,jR}\,(T)$]

$H=(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$
(Curve Eq of **Local $\mathbf{P^1 \times P^1}$**)
$Q = e^q$, $P = e^p$, $[q,p] = i\, 2\pi k$

$N^d_{jL,jR}$: BPS index of degree d, spins (j_L,j_R) on **Local $\mathbf{P^1 \times P^1}$**

$T=T(z)$: Kahler Parameters

From Matrix Models To Curves

[Grassi-Hatsuda-Marino 2014]

(Correspondence Without Referring To M2)

Spectral Det

$\mathrm{Det}\,(\,1 + z\,H^{-1}\,)$

=

Free Energy of Top Strings

$\exp\,[\,\Sigma\, N^{d}_{jL,jR}\, F^{d}_{jL,jR}\,(T)\,]$

$H = f(Q,P)$ (Curve Eq)

$Q = e^{q},\ P = e^{p},\ [q,p] = i\,2\pi k$

$N^{d}_{jL,jR}$: BPS index on the Curve

From Matrix Models To Curves

 for Matrix Models Because

- Interpretation in M2 is Clearer
- Generalization to Other Orbifolds, Orientifolds, Rank Deformations, ... is Easier

 for Curves Because

- Correspondence is Clearer
- Geometrical Viewpoint Helps

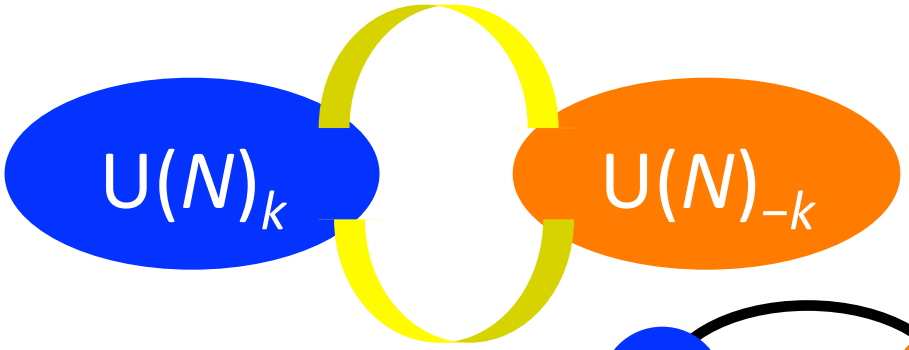
2. Super Chern-Simons Theories

Generalizing ABJM From
Matrix Model Viewpoint

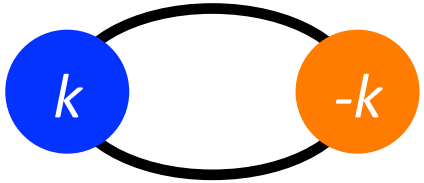
"The Earth is a sphere"

Motivation

- ABJM Theory = The Center of Earth
- Generalization is inevitable for the Map
- Regarding ABJM Quiver Diagram

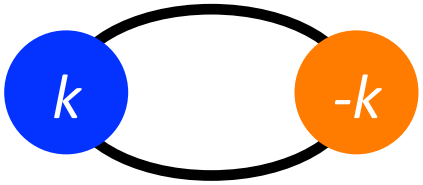


as Affine Dynkin Diagram A_1,

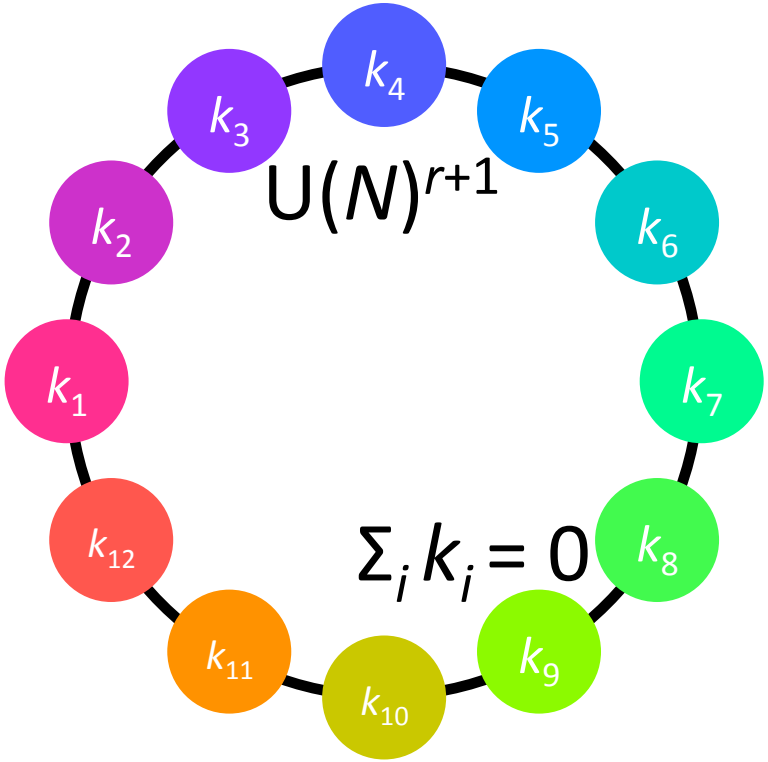


ADE Generalization with $\mathcal{N}$=3

ADE Generalization

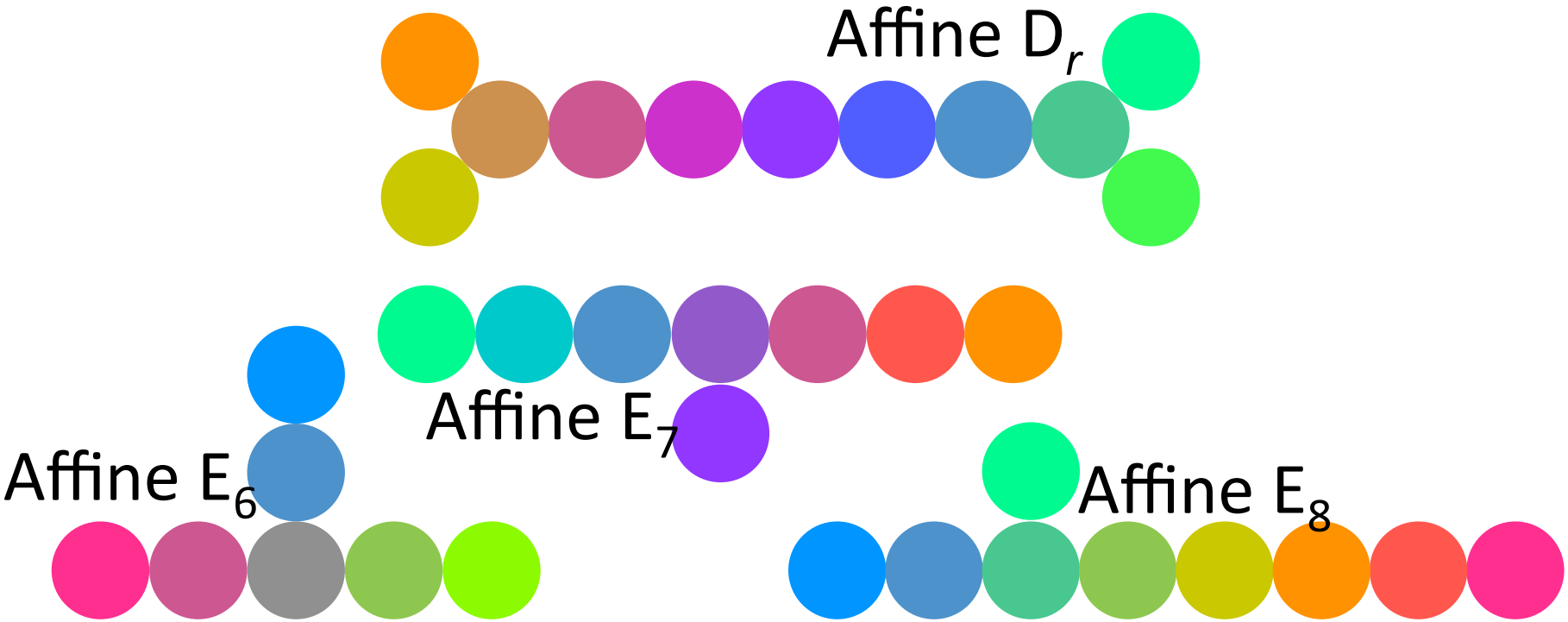


Affine A_1
(=ABJM)



Affine A_r

ADE Generalization



Spectral Pfaffian (instead of det) For Affine D

[M-Nosaka, Assel-Drukker-Felix 2015]

Special Class of A_r Quiver

[Imamura-Yokoyama 2008]

- For $k_a=(k/2)(s_a-s_{a-1})$ with $s_a=\pm 1$

SUSY Is Enhanced to $\mathcal{N}=4$

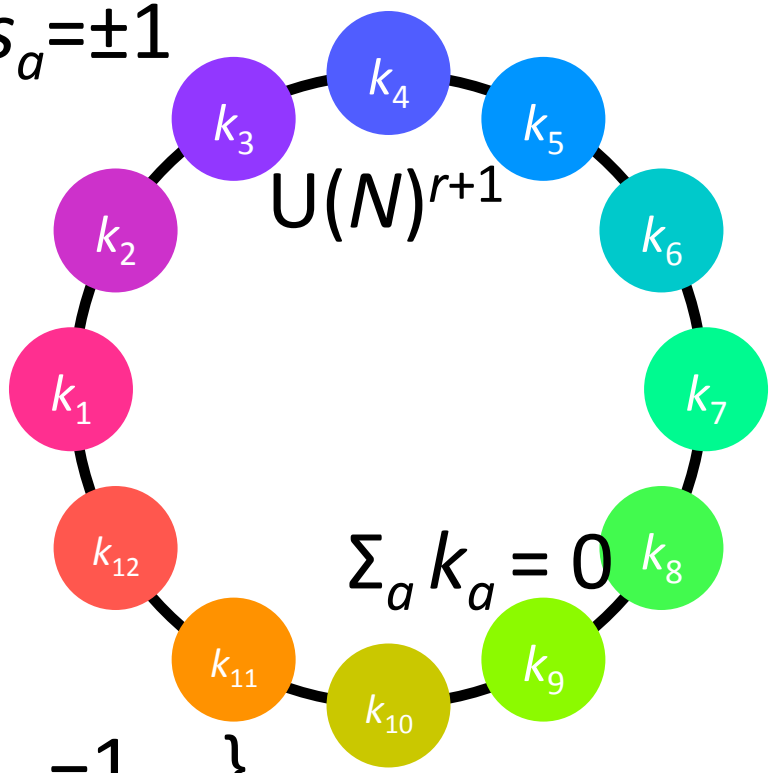
with

$s_a=+1$: NS5-branes

$s_a=-1$: $(1,k)$5-branes

- $\mathcal{N}=4$ Theories

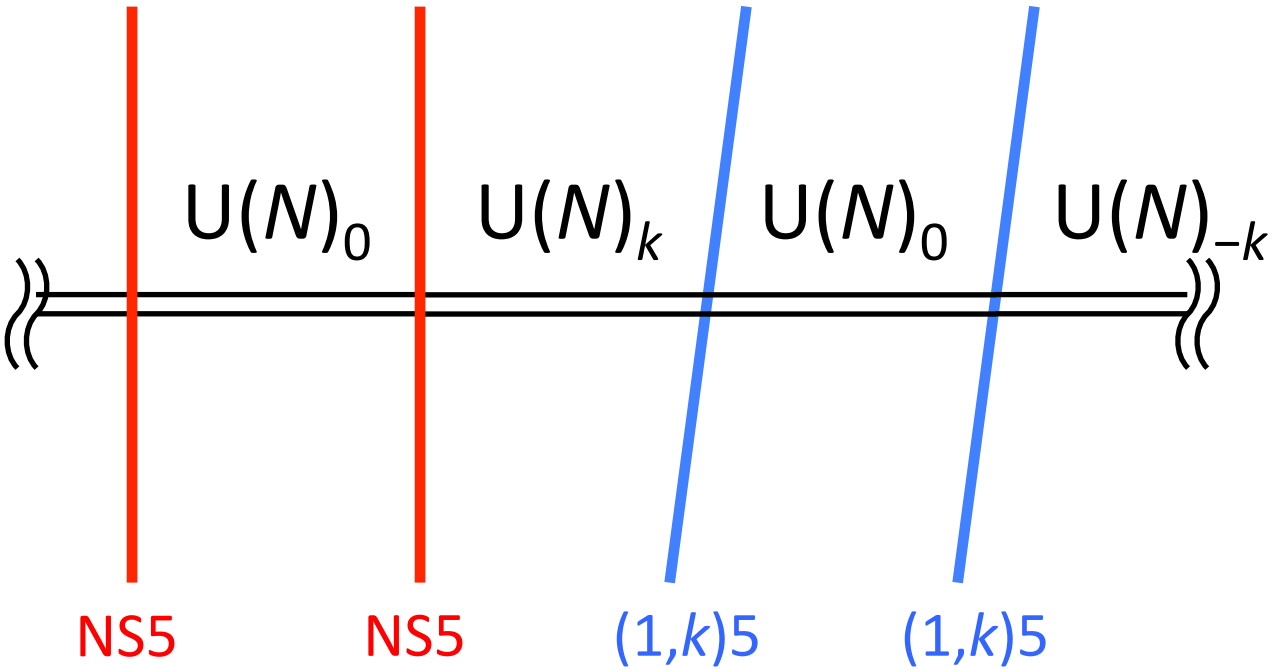
$\{s_a\} = \{+1,+1,\ldots,-1,-1,\ldots,+1,\ldots,-1,\ldots\}$

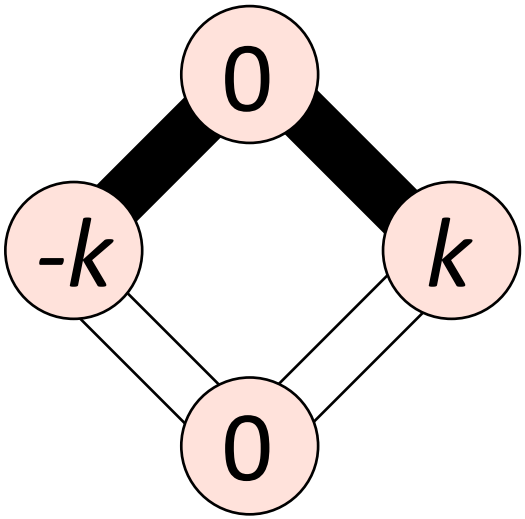


As a simple case

(2,2) Model (={+1,+1,−1,−1}) was studied

[M-Nosaka 2014]





Results in D5 Del Pezzo

(2,2) Model Is Described By Topological Strings

Free Energy of Top Strings
$\exp [\, \Sigma\, N^{d}_{jL,jR}\, F^{d}_{jL,jR}(T)\,]$

$N^{d}_{jL,jR}$: BPS index of degree d, spins (j_L, j_R) on **Local D5 Del Pezzo**

Natural Because

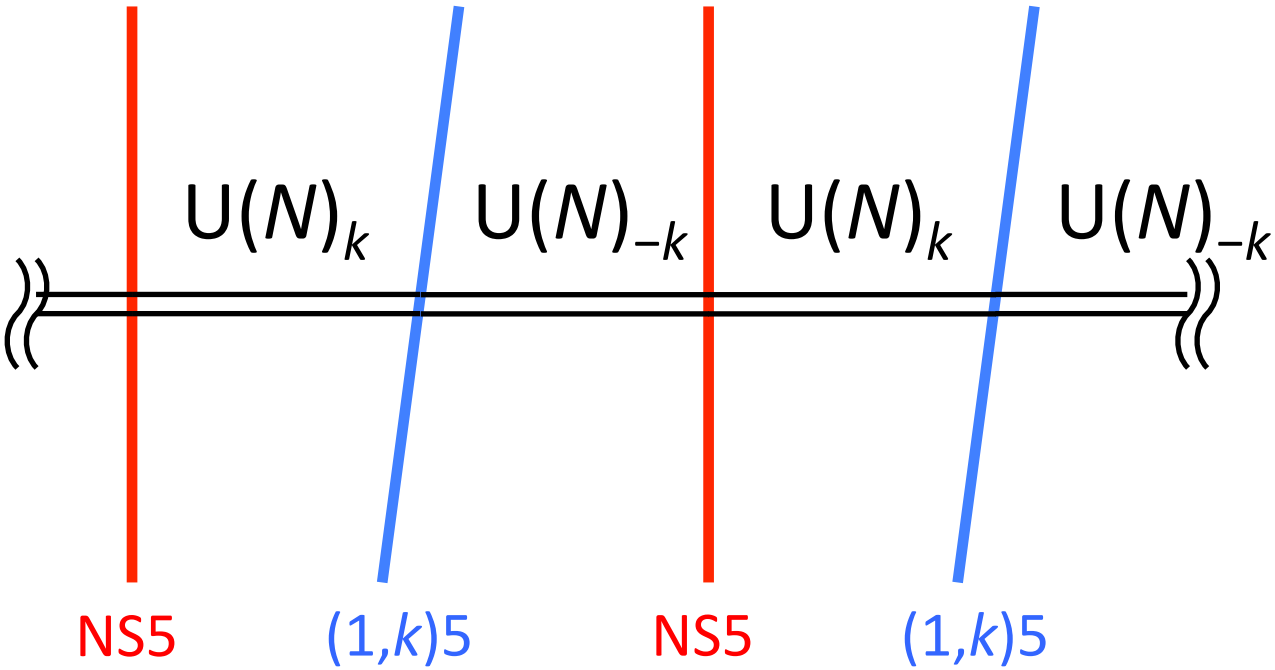
$H = (Q^{1/2}+Q^{-1/2})^{2}\,(P^{1/2}+P^{-1/2})^{2}$

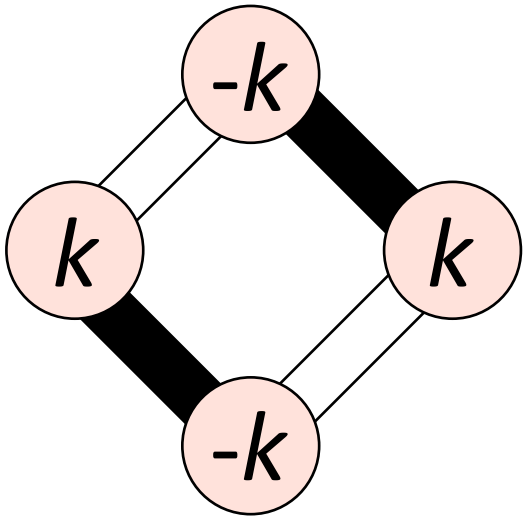
(Curve Eq of D5(=so(10)) Del Pezzo)

As further generalizations

(2,2)Model({+1,+1,−1,−1}),(1,1,1,1)Model({+1,−1,+1,−1})
& Rank Deformations (N_1,N_2,N_3,N_4)

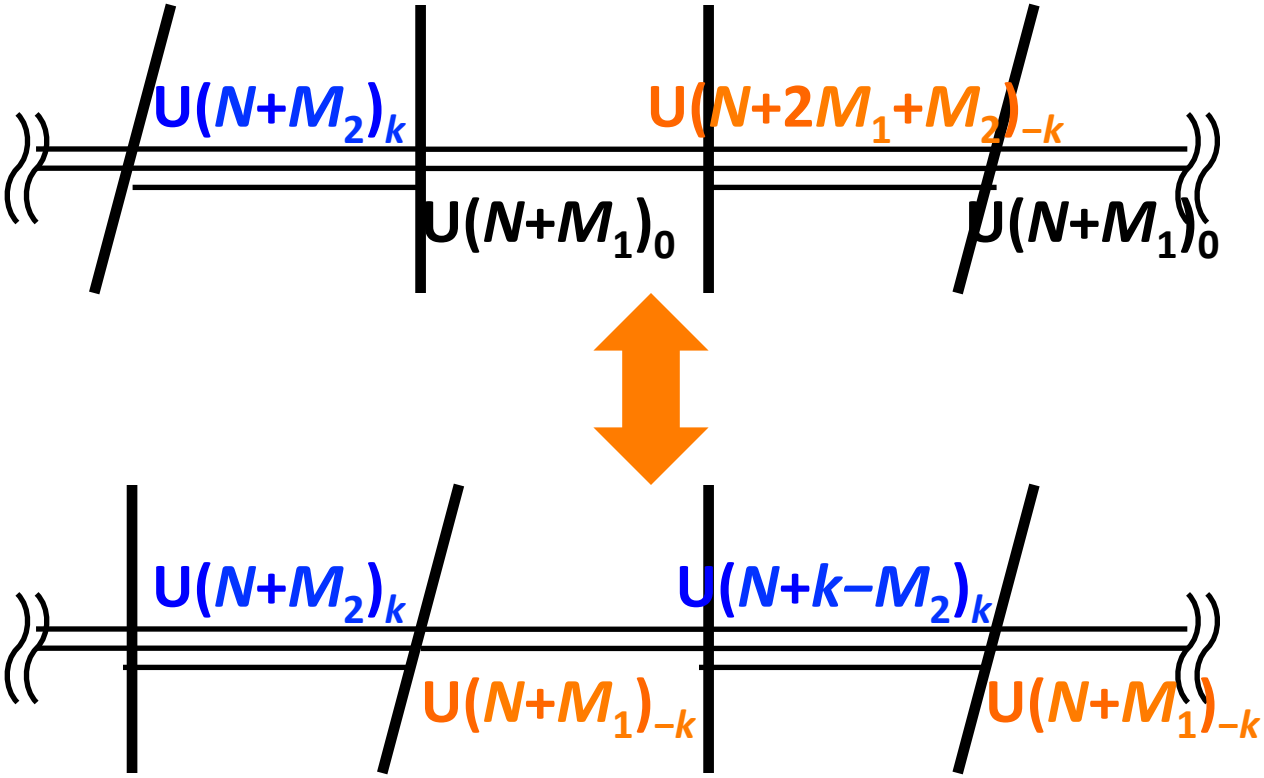
[M-Nakayama-Nosaka 2017]





Moduli Space is Connected

(2,2) Model & (1,1,1,1) Model Are Connected Via Hanany-Witten Transitions



Moduli Space is Connected

Both (2,2) & (1,1,1,1) Models Are Described By Topological Strings In A Single Function

Free Energy of Top Strings
$\exp [\Sigma N^{d}_{jL,jR} F^{d}_{jL,jR} (T)]$

- Six Kahler Parameters

$$T_i^{\pm} (\mu, M_1, M_2) = \ldots, \quad (i = 1,2,3)$$

- Total BPS index is Distributed by Various Combinations of Kahler Parameters.

Explicit Decomposition of BPS index

- 6 Degrees for 6 Kahler Parameters

$$\Sigma\, N^{\boldsymbol{d}}_{(j_L,j_R)}(d_1^+,d_2^+,d_3^+;d_1^-,d_2^-,d_3^-)\cdot(T_1^+,T_2^+,T_3^+;T_1^-,T_2^-,T_3^-)$$

- BPS Index

- d=1, (j_L,j_R)=(0,0)

$$16 \rightarrow 2(1,0,0;0,0,0)+4(0,1,0;0,0,0)+2(0,0,1;0,0,0)$$
$$+2(0,0,0;1,0,0)+4(0,0,0;0,1,0)+2(0,0,0;0,0,1)$$

Numerical Table by Experts
[Huang-Klemm-Poretschkin 2013]

How About Higher Degrees?

Explicit Decomposition of BPS index

$\lvert\boldsymbol{d}\rvert$	$\{\boldsymbol{d} = (d_1^+, d_2^+, d_3^+; d_1^-, d_2^-, d_3^-)\}$		$\pm N^{\boldsymbol{d}}_{j_L,j_R}(j_L, j_R)$
1	$(1,0,0;0,0,0)$	$(0,0,0;1,0,0)$	$2(0,0)$
	$(0,1,0;0,0,0)$	$(0,0,0;0,1,0)$	$4(0,0)$
	$(0,0,1;0,0,0)$	$(0,0,0;0,0,1)$	$2(0,0)$
2	$(0,2,0;0,0,0), (1,0,1;0,0,0)$	$(0,0,0;0,2,0), (0,0,0;1,0,1)$	$(0,\frac{1}{2})$
	$(1,0,0;0,1,0), (0,1,0;0,0,1)$	$(0,1,0;1,0,0), (0,0,1;0,1,0)$	$2(0,\frac{1}{2})$
	$(1,0,0;1,0,0), (0,1,0;0,1,0), (0,0,1;0,0,1)$		$4(0,\frac{1}{2})$
3	$(2,0,0;1,0,0), (0,2,0;0,0,1$ $(1,1,0;0,1,0), (1,0,1;0,0,1$	$(1,0,0;2,0,0), (0,0,1;0,2,0),$ $0;1,1,0), (0,0,1;1,0,1)$	$2(0,1)$
	$(0,2,0;0,1,0), (1,0$ $(1,1,0;1,0,0), (0,1$		$4(0,1)$
	$(0,0,2;0,0,1), (0,2$ $(0,1,1;0,1,0), (1,0$		$2(0,1)$

Decompositions Not Unique
Due to Relations among T's
$2T_2^{\pm} = T_1^{\pm} + T_3^{\pm}$,
$T_1^{+}+T_1^{-} = T_2^{+}+T_2^{-} = T_3^{+}+T_3^{-}$, ...
A Trouble ... ...

Organizing BPS Index Differently

[M-Nosaka-Yano 2018]

In $(M_1,M_2)=(M,0)$ Deformation,

d	$(j_{\rm L}, j_{\rm R})$	BPS	$(-1)^{d-1}\sum_{d_{\rm I}}\left(\sum_{d_{\rm II}} N^{(d,d_{\rm I},d_{\rm II})}_{j_{\rm L},j_{\rm R}}\right)_{d_{\rm I}}$
1	$(0,0)$	16	$8_{+1}+8_{-1}$
2	$(0,\frac{1}{2})$	10	$1_{+2}+8_0+1_{-2}$
3	$(0,1)$	16	$8_{+1}+8_{-1}$
4	$(0,\frac{1}{2})$	1	1_0
	$(0,\frac{3}{2})$	45	$8_{+2}+29_0+8_{-2}$
	$(\frac{1}{2},2)$	1	1

Reminiscent of $45 \to 28_0 + 8_{+2} + 8_{-2} + 1_0$ in so(10) → so(8)

Organizing BPS Index Differently

In General (M_1,M_2) Deformation,

d	(j_L, j_R)	d_I	BPS	$(-1)^{d-1}\sum_{d_{II}}\left(N^{(d,d_I,d_{II})}_{j_L,j_R}\right)_{d_{II}}$
1	$(0,0)$	± 1	8	$2_{+1}+4_0+2_{-1}$
2	$(0,\frac{1}{2})$	0	8	$2_{+1}+4_0+2_{-1}$
		± 2	1	1_0
3	$(0,1)$	± 1	8	$2_{+1}+4_0+2_{-1}$
4	$(0,\frac{1}{2})$	0	1	1_0
	$(0,\frac{3}{2})$	0	29	$1_{+2}+8_{+1}+11_0+8_{-1}+1_{-2}$

Interpreted As Further Decomposition so(8)→[su(2)]^3

e.g. 28→(3,1,1,1)+(1,3,1,1)+(1,1,3,1)+(1,1,1,3)+(2,2,2,2)

in so(8) → [su(2)]^4

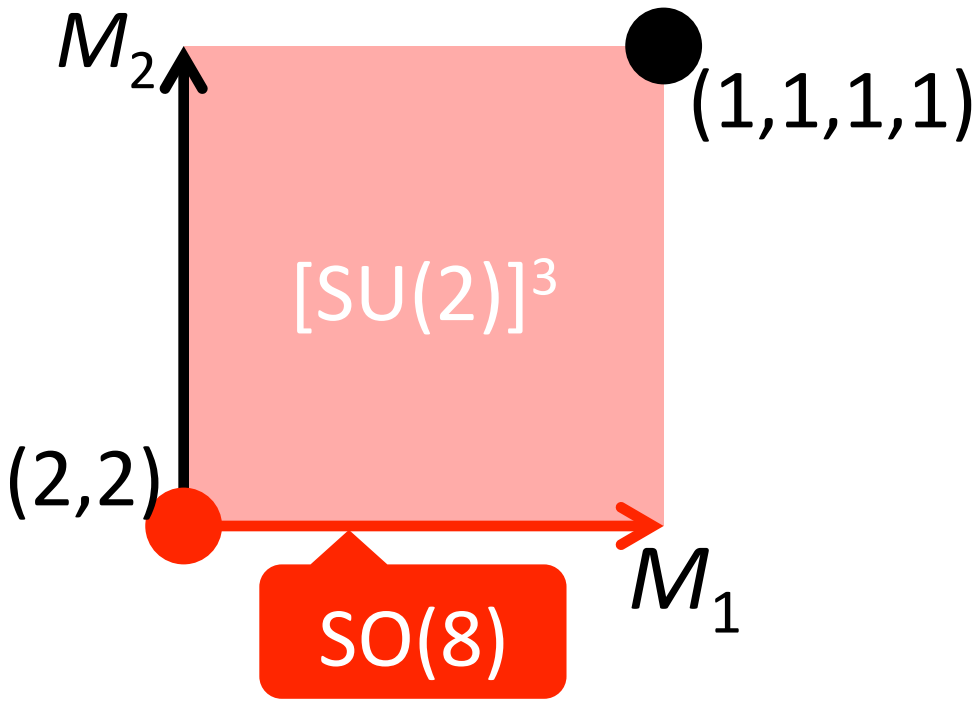
More Fun

d	(j_L, j_R)	BPS	representations
8	$(0, \frac{7}{2})$	4726	$\mathbf{1386} + \mathbf{1050} + 2 \times \mathbf{945} + \mathbf{210} + \mathbf{54} + 3 \times \mathbf{45} + \mathbf{1}$
	$(0, \frac{5}{2}), (\frac{1}{2}, 4)$	3431	$\mathbf{1050} + \mathbf{945} + \mathbf{770} + 2 \times \mathbf{210} + 2 \times \mathbf{54} + 3 \times \mathbf{45} + 3 \times \mathbf{1}$
	$(\frac{1}{2}, 3)$	1602	$\mathbf{945} + 2 \times \mathbf{210} + \mathbf{54} + 4 \times \mathbf{45} + 3 \times \mathbf{1}$
	$(0, \frac{3}{2}), (1, \frac{9}{2})$	1345	$\mathbf{945} + \mathbf{210} + \mathbf{54} + 3 \times \mathbf{45} + \mathbf{1}$
	$(\frac{1}{2}, 2), (1, \frac{7}{2})$	357	$\mathbf{210} + \mathbf{54} + 2 \times \mathbf{45} + 3 \times \mathbf{1}$
	$(0, \frac{1}{2}), (\frac{3}{2}, 5)$	311	$\mathbf{210} + \mathbf{54} + \mathbf{45} + 2 \times \mathbf{1}$
	$(0, \frac{9}{2})$	257	$\mathbf{210} + \mathbf{45} + 2 \times \mathbf{1}$
	$(\frac{1}{2}, 1), (\frac{1}{2}, 5), (1, \frac{5}{2}), (\frac{3}{2}, 4)$	46	$\mathbf{45} + \mathbf{1}$
	$(2, \frac{11}{2})$	45	$\mathbf{45}$
	$(1, \frac{3}{2}), (1, \frac{11}{2}), (\frac{3}{2}, 3), (2, \frac{9}{2}), (\frac{5}{2}, 6)$	1	$\mathbf{1}$

Numerical Table by Experts [Huang-Klemm-Poretschkin 2013]

Identifying as Representations using Matrix Models

Finally, "A Map of M-theory"



A Natural Question

Nice to Summarize Numerical Results by

$so(10) \to so(8)$ & $so(8) \to [su(2)]^3$

- But Why ?
- Any Explanations ?

[Also Raised by Y.Hikida & S.Sugimoto,
YITP Workshop "Strings & Fields 2017"]

- Now We Have Answer From Curve Viewpoint

3. Symmetry, Symmetry Breaking

Understanding Symmetry Breaking
in Super Chern-Simons Theories
From Curve Viewpoint

Question: How Symmetry is Breaking ?

Spectral Det
Det ($1 + z\, H^{-1}$)

$Q = e^{q}$, $P = e^{p}$, $[q,p] = i\, 2\pi k$

(2,2) Model so(10) → so(8)

$H = (Q^{1/2}+Q^{-1/2})^{2}\,(P^{1/2}+P^{-1/2})^{2}$

(1,1,1,1) Model so(10) → $[su(2)]^3$

$H = (Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$

In Either Case, $H = \Sigma_{(m,n)=\{-1,0,1\}\times\{-1,0,1\}}\, c_{mn}\, Q^{m}\, P^{n}$

(Since $Q^{\alpha}P^{\beta} = e^{i2\pi k\alpha\beta}\, P^{\beta}Q^{\alpha}$)

D5 Quantum Curve

As Classical Curves Defined by Zeros of Polynomial Rings

Definition: Spectral Problem of

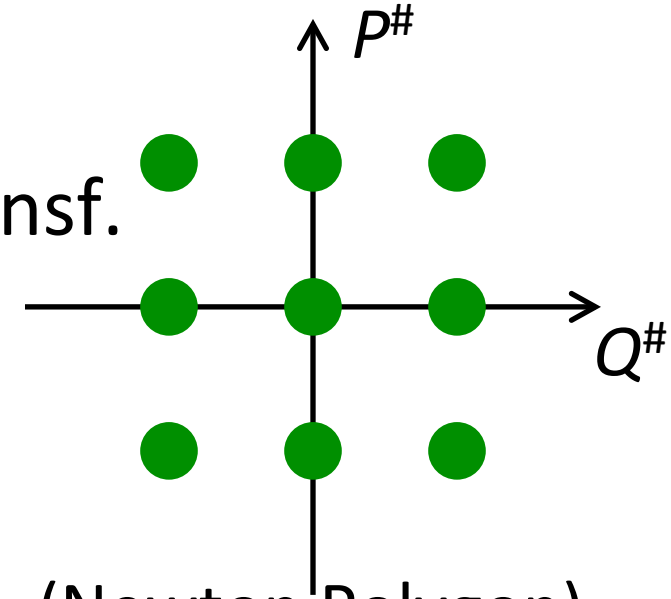
$$H = \Sigma_{(m,n)=\{-1,0,1\}\times\{-1,0,1\}}\, c_{mn}\, Q^m\, P^n$$

With

$$Q^\alpha\, P^\beta = e^{2\pi i k \alpha\beta}\, P^\beta\, Q^\alpha$$

Invariant under Similarity Transf.

$$H \sim G\, H\, G^{-1}$$



(Newton Polygon)

D5 Classical Curve

[Kajiwara-Noumi-Yamada 2015]

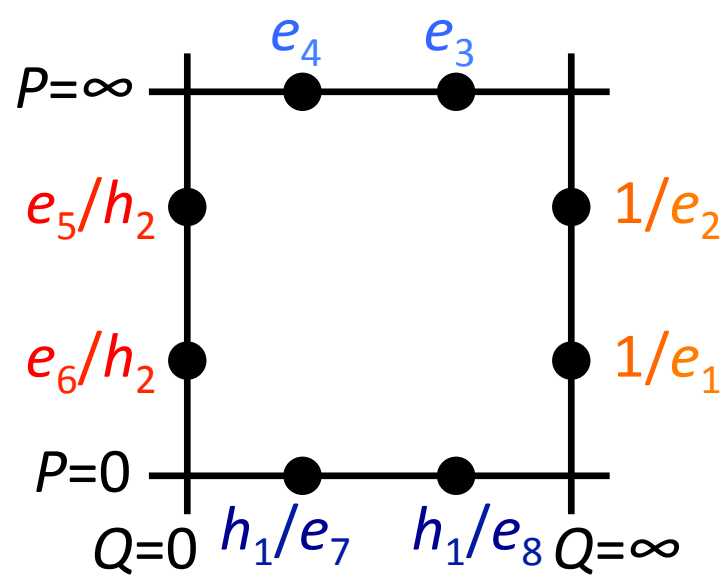
D5 Curve Parameterized by Asymptotic Values

$$0 = QP - (e_3+e_4)P + e_3e_4Q^{-1}P$$
$$-(e_1^{-1}+e_2^{-1})Q + E/\alpha - \ldots Q^{-1}$$
$$+(e_1e_2)^{-1}QP^{-1} - \ldots P^{-1} + \ldots Q^{-1}P^{-1}$$

Subject to Vieta's Formula
解と係数の関係 $(h_1h_2)^2 = e_1 \ldots e_8$

Enjoys D5 Weyl Group

Under Rational Maps



Similarly, D5 Quantum Curve

[Kubo-M-Nosaka 2018]

Rational Map in Classical Curve, Lifted to Similarity Transformation in Quantum Curve

- Same Parameterization

$$
\begin{aligned}
H/\alpha = \quad & QP && -(e_3+e_4)P && +e_3e_4Q^{-1}P \\
& -(e_1^{-1}+e_2^{-1})Q && +E/\alpha && -\ldots Q^{-1} \\
& +(e_1e_2)^{-1}QP^{-1} && -\ldots P^{-1} && +\ldots Q^{-1}P^{-1}
\end{aligned}
$$

But Normal Order by Q: Left & P: Right

D5 Weyl Transformation

Trivial Transformations

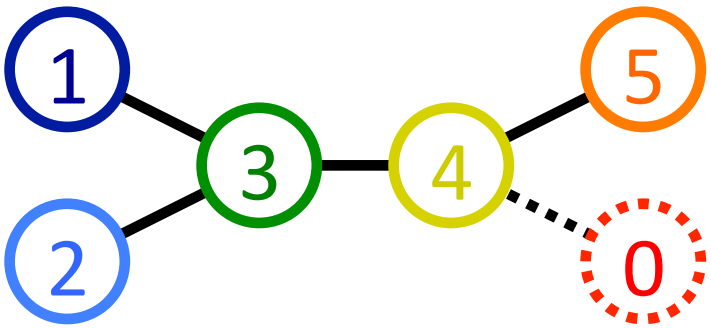
(Switching Asymptotic Values)

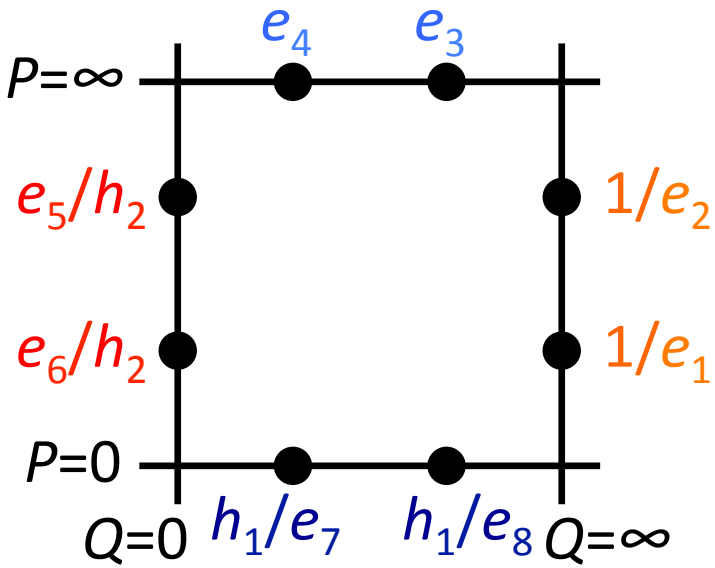
s_1: $h_1/e_7 \Leftrightarrow h_1/e_8$

s_2: $e_3 \Leftrightarrow e_4$

s_5: $1/e_1 \Leftrightarrow 1/e_2$

s_0: $e_5/h_2 \Leftrightarrow e_6/h_2$





D5 Weyl Transformation

For Nontrivial s_3, we choose $G = \exp[F_3(q) - F_7(q)]$

$\exp[F_3(q) - F_3(q - 2\pi i k)] = \exp[q] - e_3$, $\exp[F_7(q + 2\pi i k) - F_7(q)] = \exp[q] - h_1/e_7$

Then

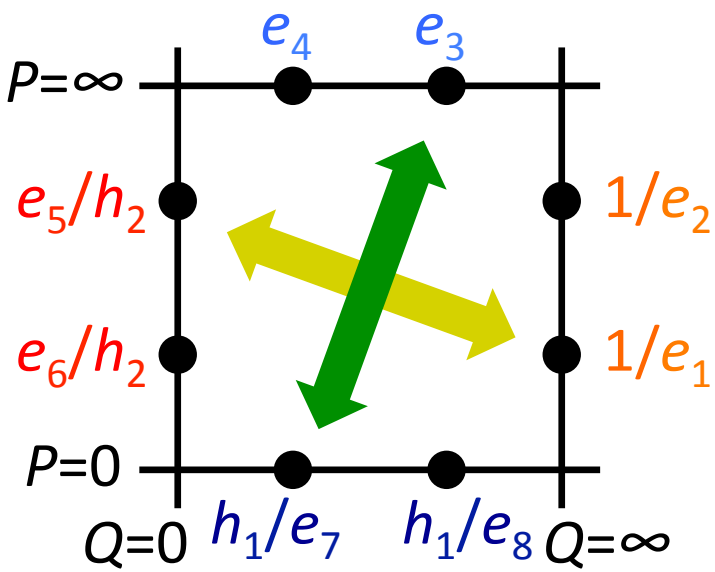
$$Q' = GQG^{-1} = Q$$

$$P' = GPG^{-1} = (Q - e_3)P(Q - h_1/e_7)^{-1}$$

$$s_3 : e_3 \Leftrightarrow h_1/e_7$$

Similarly, $s_4 : 1/e_1 \Leftrightarrow e_5/h_2$

Totally, D5=so(10) Weyl Transf.



Gauge Fixing

- $(h_1, h_2, e_1, \ldots, e_8)$ 10 Param. For 8 Asympt. Values
- Similarity Transformation

$$G = \exp[\mathrm{i}ap/(2\pi k)],\ G = \exp[-\mathrm{i}bq/(2\pi k)]$$

Then, Identification $(Q,P) \sim (e^a Q, P)$, $(Q,P) \sim (Q, e^b P)$

- Totally, 4 Gauge Fixing Conditions

$$e_2 = e_4 = e_6 = e_8 = 1$$

- 6 Param. Subject to 1 Vieta's Constraint

$$\rightarrow 5 \text{ DOF}$$

Matrix Models

- (2,2), $H = (Q^{1/2}+Q^{-1/2})^{\mathbf{2}}\,(P^{1/2}+P^{-1/2})^{\mathbf{2}}$

$(h_1,h_2,e_1,e_3,e_5,e_7)=(1,1,1,1,1,1)$

- (1,1,1,1), $H = (Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$

$(h_1,h_2,e_1,e_3,e_5,e_7)=(1,1,\mathrm{e}^{-\pi \mathrm{i} k},\mathrm{e}^{+\pi \mathrm{i} k},\mathrm{e}^{-\pi \mathrm{i} k},\mathrm{e}^{+\pi \mathrm{i} k})$

Subject to $(h_1h_2)^2 = e_1e_3e_5e_7$

Matrix Models

- (2,2), $H = (Q^{1/2}+Q^{-1/2})^2\,(P^{1/2}+P^{-1/2})^2$

$(h_1,h_2,e_1,e_3,e_5,e_7)=(1,1,1,1,1,1)$

Remaining Symmetry

$\rightarrow$ so(8)

- (1,1,1,1), $H = (Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})(Q^{1/2}+Q^{-1/2})(P^{1/2}+P^{-1/2})$

$(h_1,h_2,e_1,e_3,e_5,e_7)=(1,1,e^{-\pi ik},e^{+\pi ik},e^{-\pi ik},e^{+\pi ik})$

$\rightarrow$ su(3) x $[\mathrm{su}(2)]^2 \neq [\mathrm{su}(2)]^3$

Subject to $(h_1h_2)^2 = e_1e_3e_5e_7$

Symmetry Enhancement For (1,1,1,1)

- (1,1,1,1) Model Without Deformations

  $[su(2)]^3$ Observed In Matrix Models

  $su(3) \times [su(2)]^2$ Observed in Curves

- Reason

In Matrix Model Remaining Sym Was Studied As Further Deformations from (2,2) Model. In Fact,

- Intersection of so(8) & $su(3) \times [su(2)]^2$ Is $[su(2)]^3$ and,

- Symmetry For (1,1,1,1) Is Enhanced Accidentally.

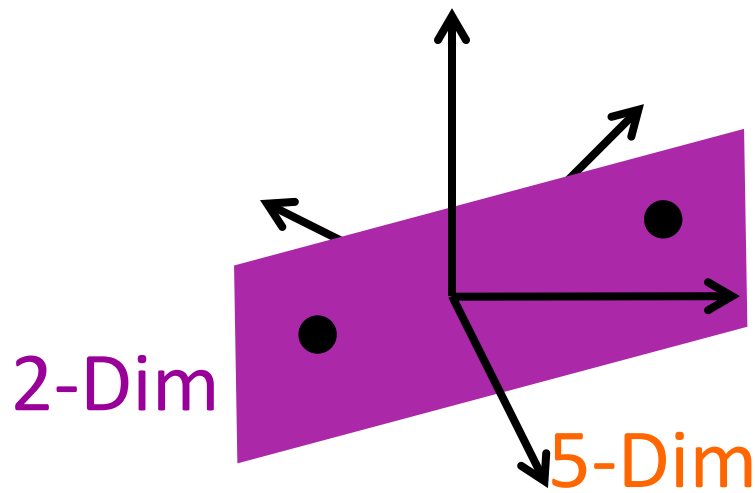
Matrix Model Subspace

Out of 5-Dim Parameter Space,

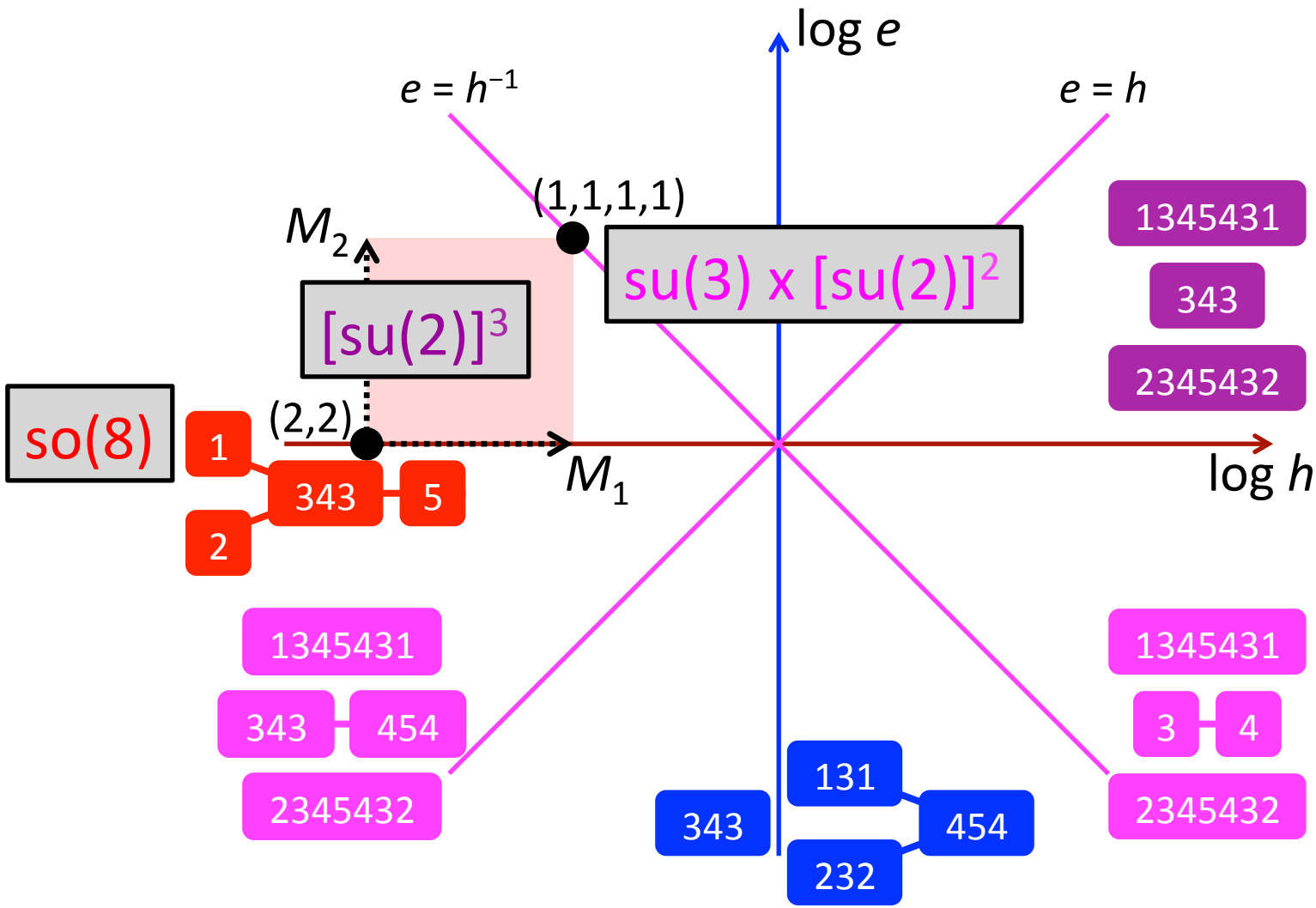
Identify (M_1,M_2) 2-Dim Subspace

Containing (2,2) & (1,1,1,1) Models

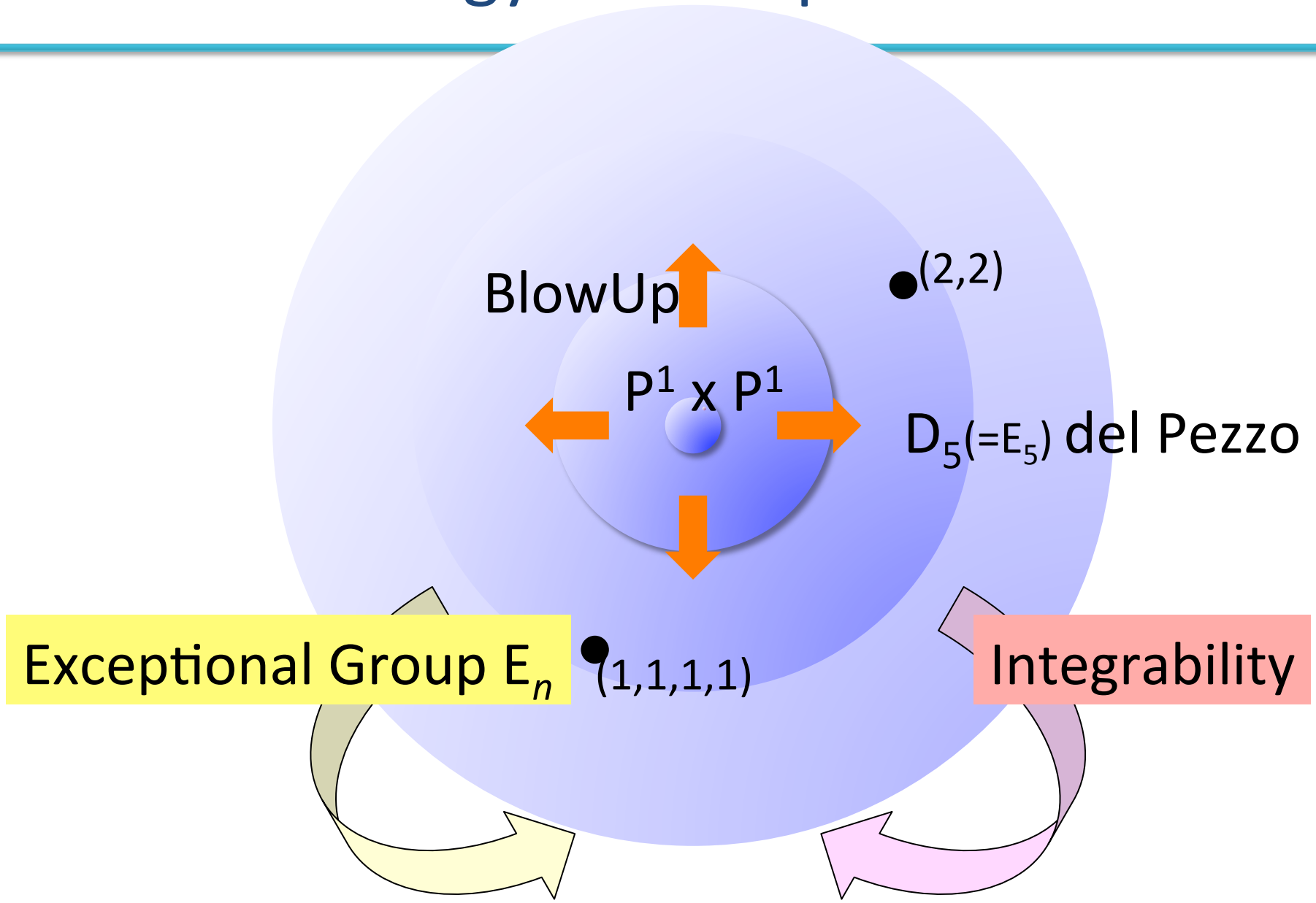
$(h_1,h_2,e_1,e_3,e_5,e_7)=(e^{-2\pi ik}h^{-1}e, e^{+2\pi ik}he^{-1}, e^{-1}, e, e^{-1}, e)$



"A Map of M-Theory ?"



Analogy in A Map of Earth



Summary

- M2-brane Backgrounds Are Parameterized By Curve Parameters, Forming "A Map".
- Exceptional Weyl Group Acts On The Map.

$$(2,2): \quad D_5 \rightarrow D_4$$
$$(1,1,1,1): \quad D_5 \rightarrow A_2 \times (A_1)^2$$
$$(2,1): \quad D_5 \rightarrow A_3$$
$$(2,1,2,1): \quad E_7 \rightarrow D_5 \times A_1$$

- All Symmetry Breakings from Matrix Models Are Reproduced

Discussions

- So Far Analytical Studies for CS Matrix Models. A New Avenue to Algebraic Studies.
- Reviews on Curves [Kajiwara-Noumi-Yamada 2015] are Motivated for Study of Painleve Equations. More Explicit Relations? Integrability?

  [Bonelli-Grassi-Tanzini 2017, Furukawa-M to appear]
- So Far Curves of Genus 1. Curves of Higher Genus?