Weyl invariance of string theories in the generalized supergravity backgrounds

Junichi Sakamoto (Kyoto Univ.)

arXiv : 1703.09213, 1811.10600

These works are a collaboration with
Jose J. Fernandez-Melgarejoa, Yuho Sakatani, and Kentaroh Yoshida

One-day workshop for QFT and string theory @Osaka City University 2018/12/14

Introduction

- Well-known fact

String theory → Supergravity

Low energy limit

- An important progress

Recently, a new class of low energy effective theories has been discovered:

Generalized Supergravity

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795] [Tseytlin-Wulff, 1605.04884]

Introduction

There is a deep connection between the generalized SUGRA and the κ-symmetry of the GS superstring.

■ Old result

[Grisaru-Howe-Mezincescu-Nilsson-Townsend, 1985]
[Bergshoeff-Sezgin-Townsend, 1986]

A target spacetime solves the EOM of SUGRA

$\Downarrow$

The κ-symmetry of the GS superstring

The inverse of the statement had been conjectured.

Introduction

- New Result [Tseytlin-Wulff, 1605.04884]

The κ-symmetry of the GS action

$\Downarrow$

The generalized supergravity equations

$\Rightarrow$ Consistency at the classical level

A long-standing problem had been resolved!

Introduction

Question

Consistency at the quantum level?

$$\langle T^a{}_a \rangle = 0\,?$$

Our result

We showed that bosonic string theories defined on generalized SUGRA backgrounds are Weyl invariant.

Talk plan

0. Introduction

1. What is the generalized supergravity? 4 slides

2. Weyl invariance of bosonic string theory 5 slides

3. Weyl invariance for generalized supergravity backgrounds 4 slides

4. summary and discussions

1. What is the generalized SUGRA?

The generalized supergravity equations

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795] [Tseytlin-Wulff, 1605.04884]

The generalized supergravity equations (GSE)

$$R_{mn} - \frac{1}{4} H_{mpq} H_n{}^{pq} + 2 D_{(m} Z_{n)} = 0$$

$$-\frac{1}{2} D^k H_{kmn} + Z^k H_{kmn} + 2 D_{[m} I_{n]} = 0$$

$$R - \frac{1}{12} H_{mnp} H^{mnp} + 4 D_m Z^m - 4 (I^m I_m + Z^m Z_m) = 0$$

Here, we have ignored R-R fields, dilatino, and gravitino.

The modifications are characterized by *I* and *Z*.

The generalized supergravity equations

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795] [Tseytlin-Wulff, 1605.04884]

The generalized supergravity equations (GSE)

$$R_{mn} - \frac{1}{4} H_{mpq} H_n{}^{pq} + 2 \boxed{D_{(m} Z_{n)}} = 0$$

$$-\frac{1}{2} D^k H_{kmn} + \boxed{Z^k} H_{kmn} + 2 \boxed{D_{[m} I_{n]}} = 0$$

$$R - \frac{1}{12} H_{mnp} H^{mnp} + \boxed{4 D_m Z^m - 4 (I^m I_m + Z^m Z_m)} = 0$$

Here, we have ignored R-R fields, dilatino, and gravitino.

The modifications are characterized by *I* and *Z*.

The relation between *I* and *Z*

$$D_m I_n + D_n I_m = 0$$ (Killing equations)

$$I^p H_{pmn} + 2\,\partial_{[m} Z_{n]} = 0 \qquad Z_m I^m = 0$$

By fixing the gauge of *B* as $\mathcal{L}_I B = 0$,

$$\Longrightarrow \quad Z_m = \partial_m \Phi + I^n B_{nm}$$

The GSE can be characterized by the Killing vector *I*

Weyl or Scale invariance?

Originally, the GSE was derived from the conditions for the scale invariance condition of the string sigma model.

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

At that time, they could not find a local counterterm.

⇒ Inconsistency at the quantum level?

For this issue, it is useful to embed the GSE into Double Field Theory (DFT).

GSE from Double field theory

The GSE can be embedded into DFT by adding the dilaton to a linear dual coordinate dependence. [J.S.-Sakatani-Yoshida, 1703.09213]

$$\boxed{\Phi_* = \Phi + I^i\,\tilde{Y}_i}$$

In particular, the linear dual coordinate dependence is consistent with the section condition in DFT.

This modification enables us to construct an appropriate counterterm.

[J.S.-Sakatani-Yoshida, 1703.09213]

2. Weyl invariance of bosonic string theory

Weyl invariance of bosonic string theory

The action of bosonic string theory (D=26)

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-\gamma}\left(g_{mn}\,\gamma^{ab} - B_{mn}\,\varepsilon^{ab}\right)\partial_a X^m\,\partial_b X^n\,,$$

γ^{ab} : The world-sheet metric ε^{ab} : The anti-symmetric tensor

Here, (g_{mn}, B_{mn}) are the metric and the B-field of an arbitrary background.

■ Classical Weyl invariance of the action

$$T^a_{\ a} \equiv \frac{4\pi}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S}{\delta\gamma^{ab}} = 0$$

Weyl invariance of bosonic string theory

The Weyl invariance of the model is broken at the quantum level.

- The trace anomaly

$$2\alpha' \langle T^a{}_a \rangle = \left(\beta^G_{mn}\gamma^{ab} - \beta^B_{mn}\varepsilon^{ab}\right) \partial_a X^m \partial_b X^n$$

- The β-functions at the one-loop level [Callan-Friedan-Martinec-Perry, 1985]

$$\beta^G_{mn} = \alpha' \left(R_{mn} - \frac{1}{4} H_{mpq} H_n{}^{pq} \right) \quad \beta^B_{mn} = \alpha' \left(-\frac{1}{2} D^k H_{kmn} \right)$$

Weyl invariance of bosonic string theory

Quantum scale invariance condition [Hull-Townsend, 1986]

$$\beta^G_{mn} = -2\alpha' D_{(m}Z_{n)}, \quad \beta^B_{mn} = -2\alpha'(Z^k H_{kmn} + 2D_{[m}I_{n]}),$$

where I and Z are arbitrary vector fields.

- In fact, the trace anomaly on the *on-shell* becomes

$$\langle T^a{}_a \rangle = -\mathcal{D}_a \left[(Z_n \gamma^{ab} - I_n \varepsilon^{ab}) \partial_b X^n \right]$$

➡ No trace anomaly

- The quantum scale invariance condition has the same form as in the NS-NS sector of the GSE.

Weyl invariance of bosonic string theory

$$\boxed{\text{How do we improve the EM tensor to obtain quantum Weyl invariant string sigma models?}}$$

Let us consider a special case

$$Z_m = \partial_m \Phi \qquad I_m = 0$$

The Weyl anomaly reduces to

$$\boxed{\langle T^a{}_a \rangle = -\mathcal{D}^a \partial_a \Phi}$$

We can find the following local counterterm.

Weyl invariance of bosonic string theory

The Fradkin-Tseytlin term [Fradkin-Tseytlin, 1985]

$$S_{\mathrm{FT}} = \frac{1}{4\pi} \int \mathrm{d}^2\sigma \sqrt{-\gamma}\, R^{(2)} \Phi$$

Indeed,

$$\langle T^a{}_a \rangle_{\mathrm{FT}} = \frac{4\pi}{\sqrt{-\gamma}} \gamma^{ab} \frac{\delta S_{\mathrm{FT}}}{\delta \gamma^{ab}} = \mathcal{D}^a \partial_a \Phi \,. \quad \Rightarrow \quad \text{Cancel !}$$

1. The term is not classical Weyl invariant
2. $S_{\mathrm{FT}} = \mathcal{O}(\alpha'^0)$ $\Leftarrow$ a quantum correction

2. Weyl invariance for generalized supergravity backgrounds

A generalization of the Fradkin-Tseytlin term

Our proposal [J.S.-Sakatani-Yoshida, 1703.09213]

$$S_{\mathrm{FT}}^{(*)} = \frac{1}{4\pi}\int \mathrm{d}^2\sigma\,\sqrt{-\gamma}\,R^{(2)}\Phi_* \qquad \Phi_* = \Phi + I^i\,\tilde{Y}_i$$

We can obtain

$$\langle T^a{}_a\rangle_{\mathrm{FT}}^{(*)} = \frac{4\pi}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S_{\mathrm{FT}}^{(*)}}{\delta\gamma^{ab}} = +\mathcal{D}_a\left[\left(Z_n\gamma^{ab} - I_n\varepsilon^{ab}\right)\partial_b X^n\right]$$ ➡ Cancel !

Here, we have used the eom of the DSM: [Hull, 0406102]

$$\partial_a\tilde{Y}_i = g_{in}\,\varepsilon^b{}_a\,\partial_b X^n + B_{in}\,\partial_a X^n$$

$\tilde{Y}_i$ is non-local ➡ Is the c.t. non-local as well?

A generalization of the Fradkin-Tseytlin term

In 2d, the EH action is a total derivative,

$$\sqrt{-\gamma}R^{(2)} = \partial_a \alpha^a$$ vector density

By using the fact, the counterterm can be rewritten as

$$\boxed{S^{(*)}_{\mathrm{FT}} = \frac{1}{4\pi}\int \mathrm{d}^2\sigma \left(\sqrt{-\gamma}\, R^{(2)}\,\Phi + \varepsilon_{ab}\,\alpha^a\,J^b\right)}$$

where J^a is the Noether current associated with I^m.

$$J^a = I^m(g_{mn}\gamma^{ab} - B_{mn}\epsilon^{ab})\partial_b X^n$$

When $I^m = 0$, this reduces to the FT term.

A concrete construction of α

$\boxed{\text{Is there a local expression of } \alpha^a \text{ ?}}$

- A problem [Deser-Jackiw, 9510145] [Yale-Padmanabhan, 2010]

  If we construct α^a only from the metric,
  it is not covariant.

- A resolution [Yale-Padmanabhan, 2010]

$$\boxed{\sqrt{-\gamma}R^{(2)} = 2\,\sigma\,\partial_a\left[\sqrt{-\gamma}(n^b\mathcal{D}_b n^a - n^a\mathcal{D}_b n^b)\right]}$$

n^a : normalized vector field $\left(\ \gamma_{ab}n^a n^b = \sigma \quad \sigma = \pm 1\ \right)$

An important point

n^a itself does not contribute to the EH action at all.

A concrete construction of α

Assumption

α^a depends on γ^{ab} and X^m.

A possible candidate for *n* is the Noether current J^a :

$$n^a = \frac{1}{\sqrt{\sigma \gamma_{cd} J^c J^d}} J^a$$

By using the normalized vector, we can define α^a as

$$\alpha^a := 2\,\sigma \sqrt{-\gamma}(n^b \mathcal{D}_b n^a - n^a \mathcal{D}_b n^b)$$

This is manifestly *covariant* and a *local function* of the fundamental fields.

We obtain a local counterterm

Summary

- Weyl invariance of bosonic string theories in generalized supergravity backgrounds

- Construction of an appropriate counterterm by using the doubled formalism

- The counterterm is *local*

  In particular, the final expression can be described without introducing the doubled formalism.

> String theories may consistently be defined on generalized supergravity backgrounds.

Discussions

- Studying quantum structures of higher order in α'

  It may be useful to construct a simple solvable model.

- What is n^a ?

  At this moment, there is no good reason that
  n^a is proportional to a Noether current J^a .

  However, a Killing vector I^m always exists on
  generalized supergravity backgrounds.

  Therefore, it might be natural to expect that
  n^a is a world-sheet counterpart of the Killing vector I^m .

Thank you

A comment on the divergence of alpha

$$\alpha^a := 2\,\sigma\,\sqrt{-\gamma}(n^b\mathcal{D}_b n^a - n^a\mathcal{D}_b n^b)$$

Taking the divergence of it, we obtain

$$\partial_a\alpha^a = \sqrt{-\gamma}\,R^{(2)} + 2\sigma\,\sqrt{-\gamma}\,(\mathcal{D}_a n^b\mathcal{D}_b n^a - \mathcal{D}_a n^a\mathcal{D}_b n^b)$$

In two dimensions, the following identity holds:

$$\mathcal{D}_a n^b\mathcal{D}_b n^a = \mathcal{D}_a n^a\mathcal{D}_b n^b$$

$\partial_a\alpha^a$ is a local function of the world-sheet metric only.