# Entropy inside Evaporating Black Holes

RIKEN iTHEMS
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with H.Kawai (Kyoto)

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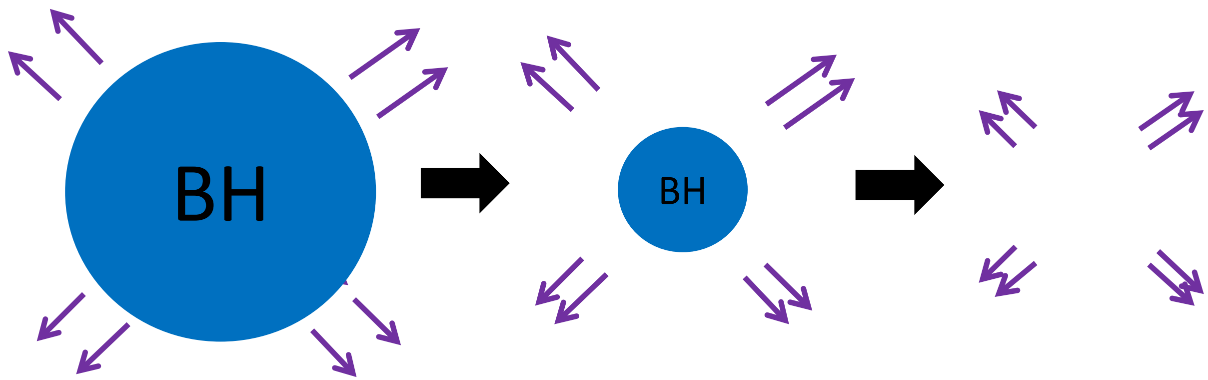
# BHs evaporate by nature.




Stefan-Boltzmann law of $T_H = \frac{\hbar}{8\pi GM}$:

$$\frac{dM}{dt} = -\frac{1}{\hbar^3} \times (N\text{: d.o.f. of fields}) \times (O(1)\text{: graybody factor}) \times \text{Area} \times T_H^4 \propto -\frac{\hbar N}{M^2}$$

⇒We parametrize this as

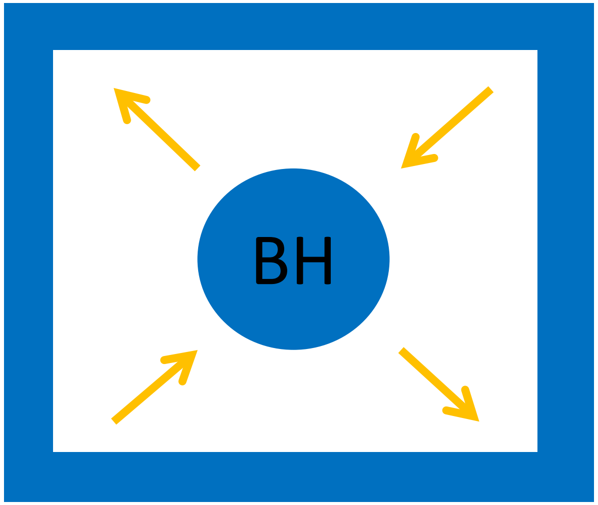
$\boxed{a = 2GM}$

$$\boxed{\frac{da(t)}{dt} = -\frac{2\sigma}{a(t)^2}}$$

Intensity of Hawking radiation
$\sigma = O(1) \propto N\hbar G = N\, l_p^2$

⇒BHs evaporate in $\Delta t_{life} \sim \frac{a^3}{\sigma}$

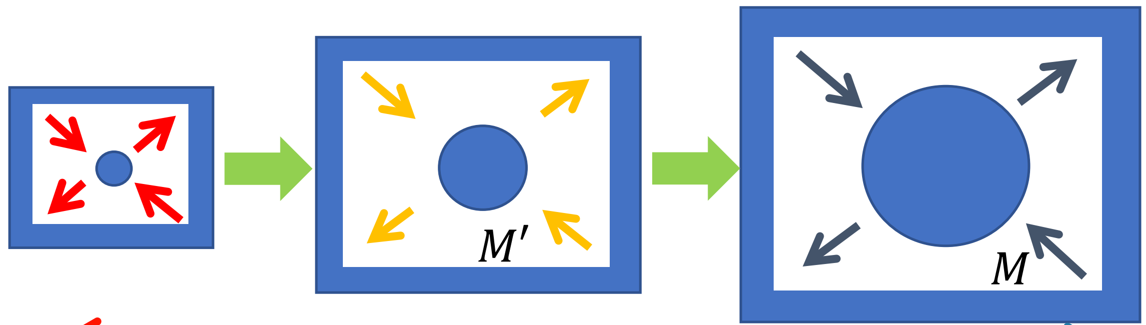
# BH can be in equilibrium with a heat bath.




$$T_H = \frac{\hbar}{4\pi a}$$

# BH has entropy.

- Suppose that we form a BH adiabatically in the heat bath by controlling the temperature and size properly.




- The entropy of radiation absorbed during this process is

$$S_{BH} = S_{rad} = \int \frac{dQ}{T} = \int_0^M \frac{dM'}{T_H(M')} = \frac{8\pi G}{\hbar}\frac{M^2}{2}$$

$$\boxed{T_H(M) = \frac{\hbar}{8\pi GM}}$$

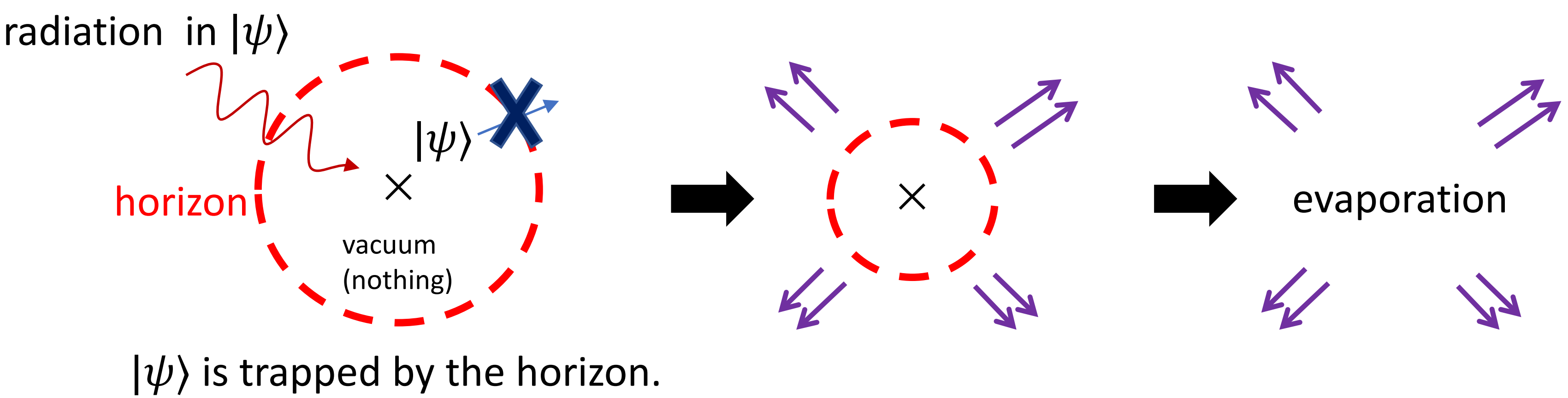
⇒BH has entropy:

$$\boxed{S_{BH} = \frac{A}{4l_p^2}}$$

$$a \equiv 2GM$$
$$A \equiv 4\pi a^2$$
$$l_p \equiv \sqrt{\hbar G}$$

# Where is the entropy?

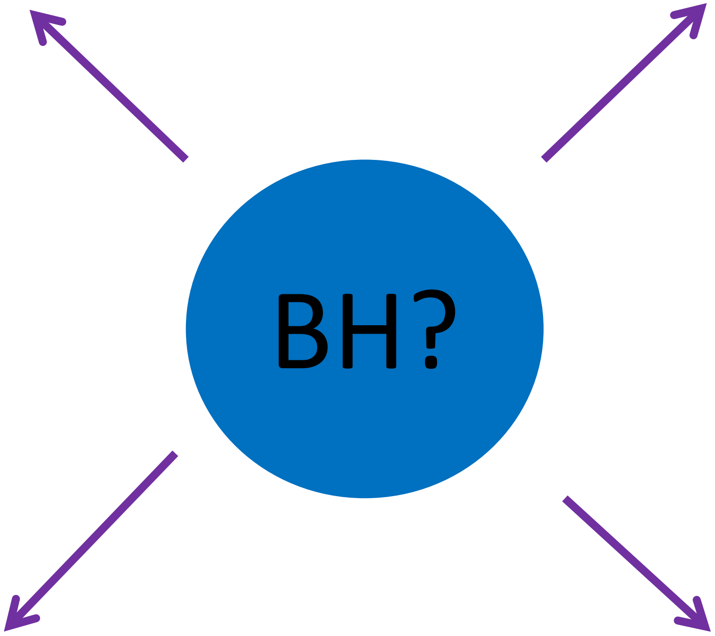
- Entropy should be inside the BH.
- In classical mechanics, BH is a vacuum region with the horizon, but the BH evaporates in a finite time.




⇒Information seems to be lost after evaporation.

⇒The existence of horizon is not consistent with QM?

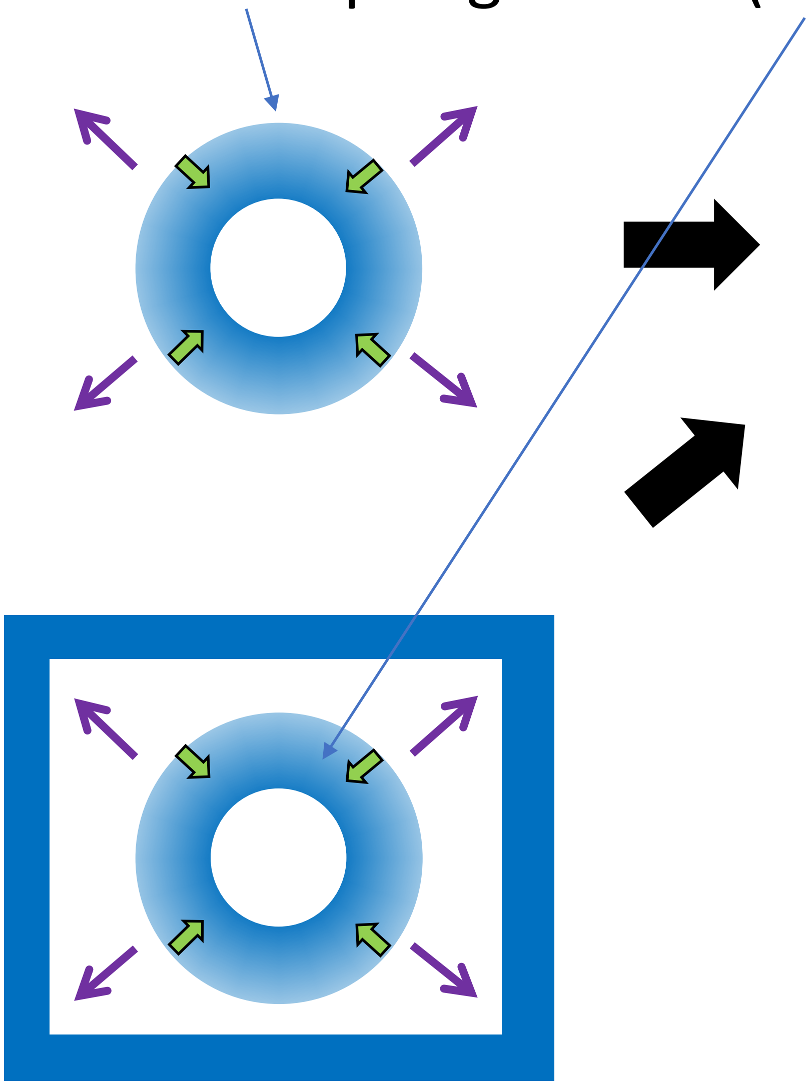
# What is BH in QM?




Kawai, Matsuo and Y. Y., Int. J. Mod. Phys. A 28, 1350050 (2013),
Kawai and Y. Y., Int. J. Mod. Phys. A 30, 1550091 (2015),
Kawai and Y. Y., Phys.Rev.D.93.044011 (2016),
Kawai and Y. Y., Universe 3, 51, (2017)

# Our approach

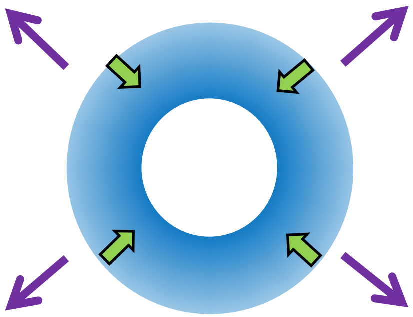
- Reconsider the time evolution of a 4D spherically-symmetric collapsing matter (radiation from the bath).



What happens?
Horizon is formed or not?

# Our approach

- Reconsider the time evolution of a 4D spherically-symmetric collapsing matter (radiation from the bath).



What happens?
Horizon is formed or not?

Generically, particle creation occurs in a time-dependent metric (even without horizon).
⇒ We need to include the back reaction of evaporation in the formation process.

Some people might wonder that such effect is negligible because

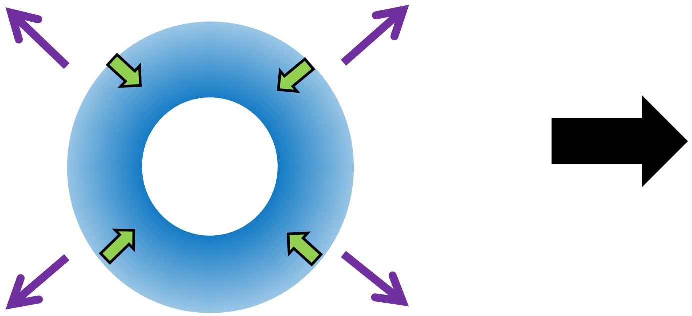
$$\text{time scale of evaporation} \sim \frac{a^3}{l_p^2} \gg a \sim \text{time scale of collapse.}$$

⇒No! because $\frac{a^3}{l_p^2}$ is measured at infinity and $a$ by a comoving observer.

⇒Consider dynamics of collapse and evaporation in a common time coordinate!

# Our approach

- Reconsider the time evolution of a 4D spherically-symmetric collapsing matter (radiation from the bath).



What happens?
Horizon is formed or not?

Generically, particle creation occurs in a time-dependent metric (even without horizon).
⇒ We need to include the back reaction of evaporation in the formation process.

⇒ We will solve the self-consistent eq:

$\hat{\phi}$= quantum matter fields

$$\Box\hat{\phi} = 0$$

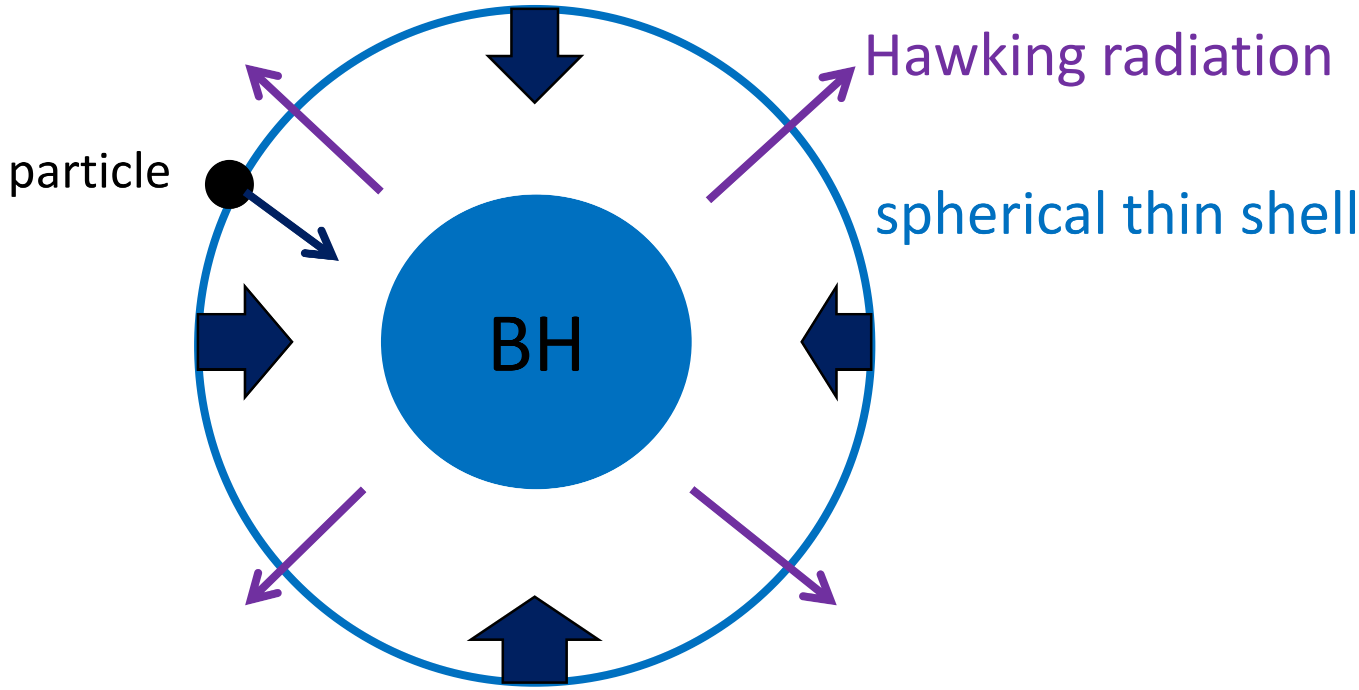
$$\boxed{G_{\mu\nu} = 8\pi G\langle\hat{T}_{\mu\nu}\rangle}$$

=collapsing matter
+ Hawking radiation

$g_{\mu\nu}$= classical metric

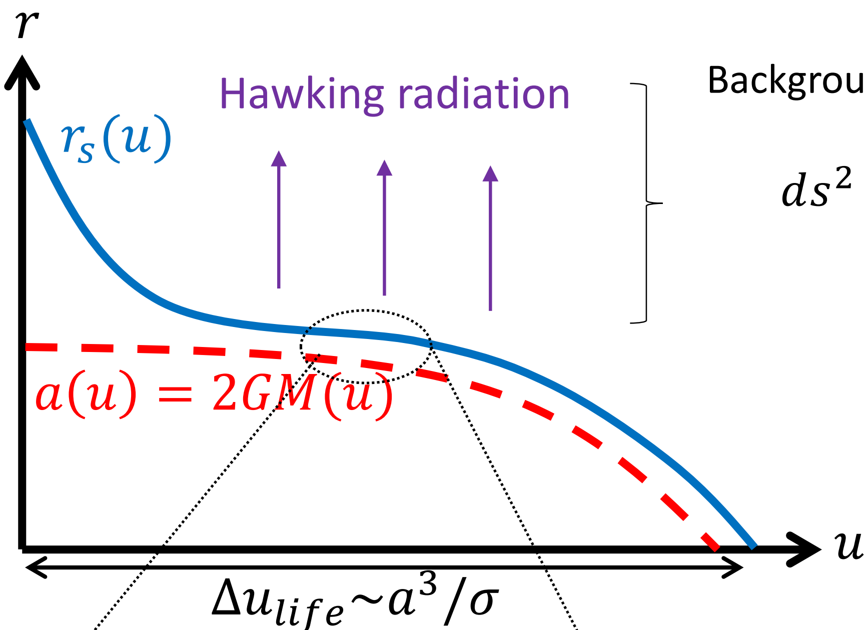
# Basic idea: step1

Imagine that a spherically-symmetric BH is evaporating in the vacuum.
Add a spherical thin shell (or a particle) to it.




What happens if dynamics of the matter and spacetime is considered?
⇒The shell will never reach the Schwarzschild radius.

# The shell never crosses "the horizon".



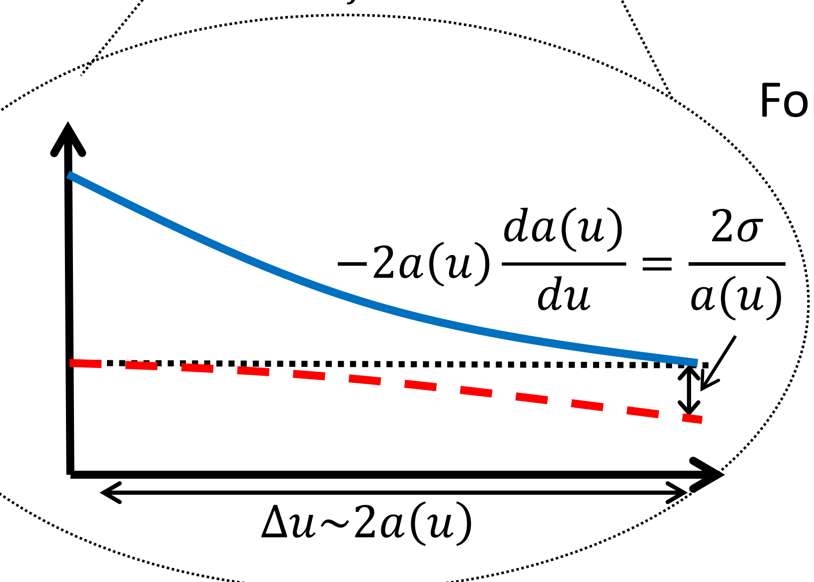

no gray-body factor

Background ≈ outgoing Vaidya metric

$$ds^2 = -\left(1 - \frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2$$

$$\frac{d}{du}a(u) = -\frac{\sigma}{a(u)^2}$$ ←Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$

$a \equiv 2GM$
intensity: $\sigma = O(1) \sim \hbar GN$
($N$=d.o.f. of fields)




For $r_s \sim a$, a particle with any $(l, m)$ behaves lightlike:

$$\frac{dr_s(u)}{du} = -\frac{r_s(u) - a(u)}{2r_s(u)}.$$

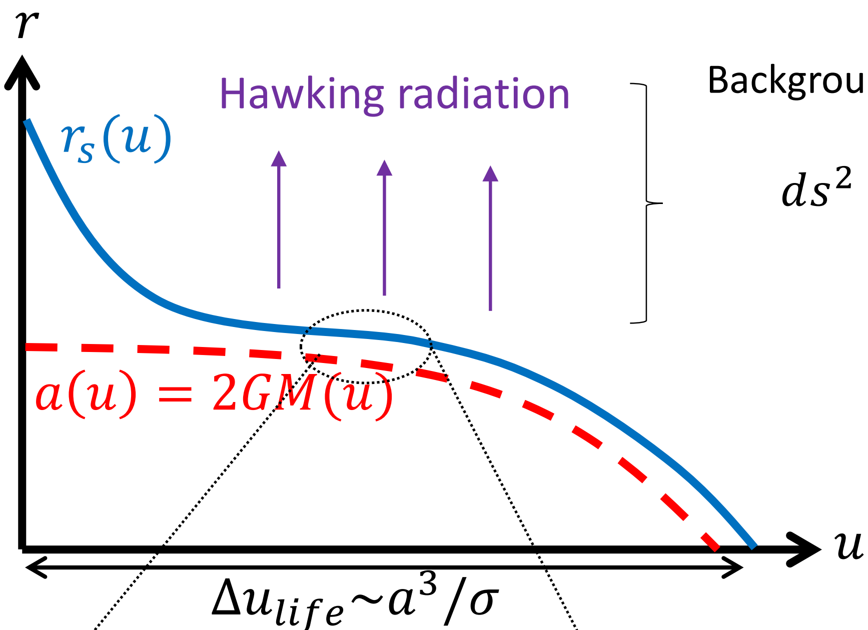
$$\Rightarrow r_s(u) \simeq a(u) - 2a(u)\frac{da(u)}{du} + Ca(u)e^{-\frac{u}{2a(u)}}$$

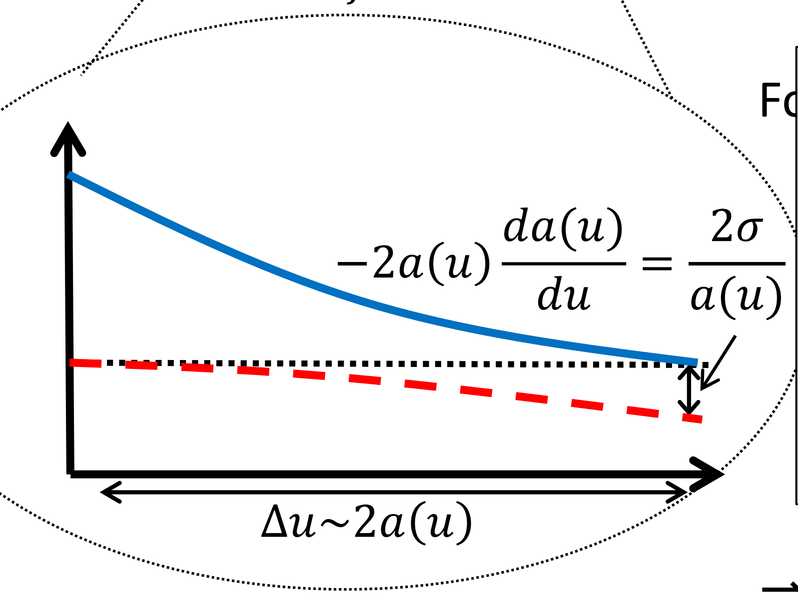
←Effect of back reaction

$$\to a(u) + \frac{2\sigma}{a(u)}$$

⇒Universally, particles will approach $a(u) + \frac{2\sigma}{a(u)}$.

# The shell never crosses "the horizon".




no gray-body factor

Background $\approx$ outgoing Vaidya metric

$$ds^2 = -\left(1 - \frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2$$

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←Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$

$a \equiv 2GM$
intensity: $\sigma = O(1) \sim \hbar GN$
($N$=d.o.f. of fields)

・The proper length of $\Delta r = \frac{2\sigma}{a}$ is given by

$$\sqrt{\sigma} \sim \sqrt{N} l_p \gg l_p$$

⇒If there are many d.o.f. of fields,

$$N \gg 1,$$

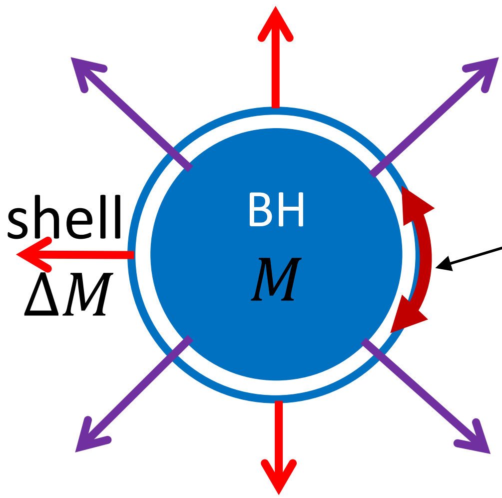
the shell is physically outside the "BH".

(This coordinate is complete, as we will see later.)

⇒Universally, particles will approach $a(u) + \frac{2\sigma}{a(u)}$.

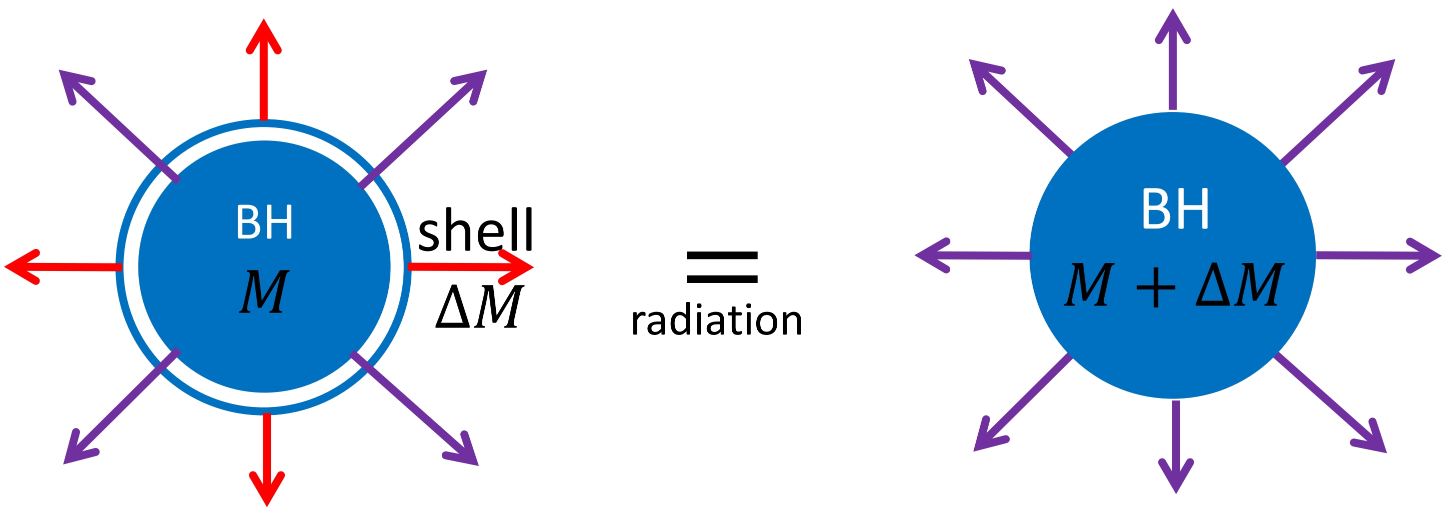
# Basic idea: step2

Radiation is emitted from the metric around the shell.




On the shell, a large tangential pressure occurs, which decelerates the shell.
⇒Dominant energy condition is broken.

# Basic idea: step2



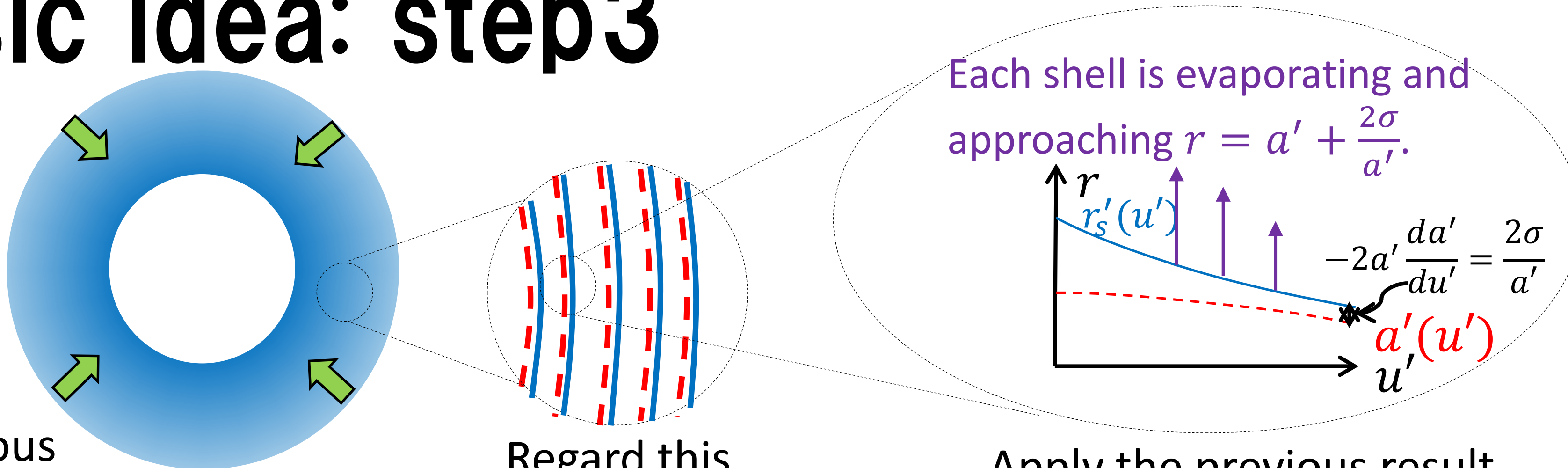

(radiation from the shell with $\Delta M$)

+(redshift factor) (radiation from the core BH with $M$)

= (radiation from a BH with $M + \Delta M$)

⇒This composite system (=shell+BH) behaves like a larger BH.

# Basic idea: step3

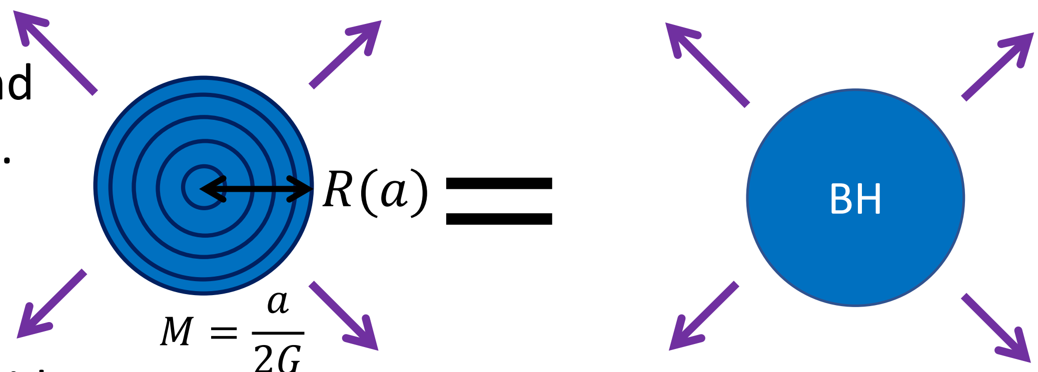



continuous
collapsing matter (not BH)

Regard this
as many shells.

Apply the previous result
to each shell recursively.

The shells pile up and
form a dense object.




The matter is filled inside

$$r = a + \frac{2\sigma}{a} \equiv R(a),$$

which is the boundary of the object (*surface*).

⇒ No horizon exists.

But this looks like an ordinary BH
from the outside.

⇒Any object we recognize as a
BH should be such an object.

# Talk Plan: How to obtain the solution?
# ⇒A self-consistent argument

We assume spherical symmetry. (**mean-field approximation** in field theory)

**<u>Step2</u>**: Assume conformal maters and evaluate $\langle T_{\mu\nu}\rangle$ on $g_{\mu\nu}$ by using $\nabla_\mu\langle T^\mu_\nu\rangle = 0$ and 4D Weyl anomaly formula.

$$\boxed{G_{\mu\nu} = 8\pi G\langle\hat{T}_{\mu\nu}\rangle}$$

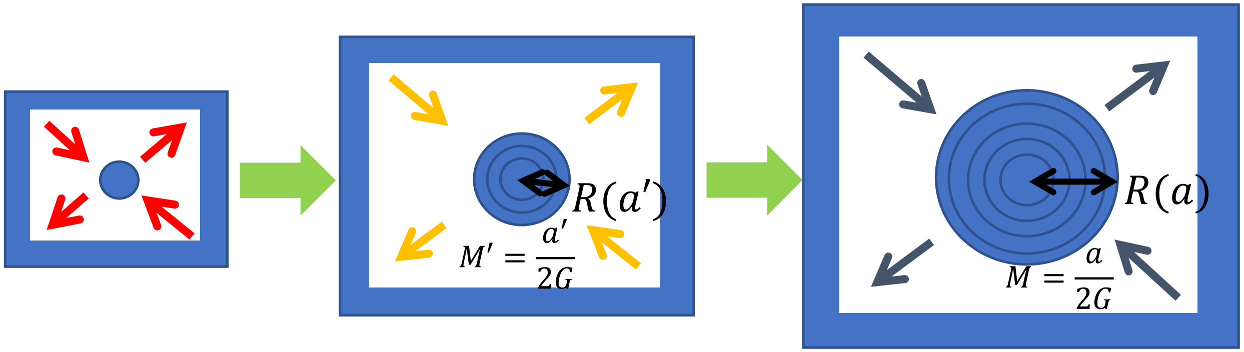
**<u>Step1</u>**: Construct a candidate metric $g_{\mu\nu}$ by a phenomenological discussion in the adiabatic formation.

**<u>Step3</u>**: Put this and determine the self-consistent $g_{\mu\nu}$.

**<u>Step4: Consistency check</u>**

Reproduce Stephan-Boltzmann law, Planck-like distribution, and entropy area law.

# Step1: Construct a candidate $g_{\mu\nu}$

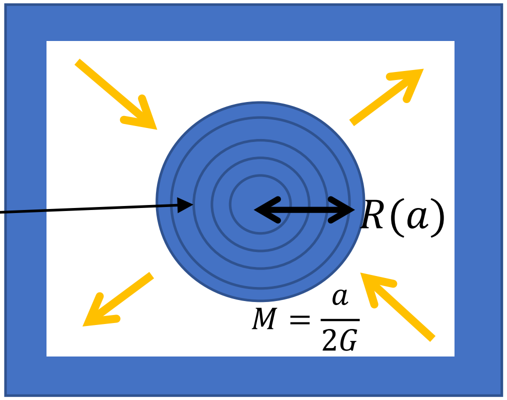



Kawai and Y. Y. IJMPA (2015)

# No $a$-dependence of $A(r)$, $B(r)$

Equilibrium and spherical symmetry at each step
⇒We can write the interior metric as

$$\boxed{ds^2 = -\frac{1}{B(r)} e^{A(r)} dt^2 + B(r) dr^2 + r^2 d\Omega^2.}$$




- In general, $A(r), B(r)$ depend on the size of the BH, $a$.
- But we can show that

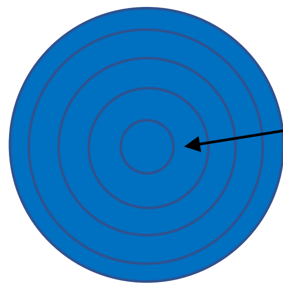
*their a-dependence is small:*

$$A(r;\cancel{a}), B(r;\cancel{a}) \qquad \text{for } r < R(a)$$

# Why? $A(r;\cancel{a}), B(r;\cancel{a})$ (1/2)

- Postulate 1: The redshift inside the BH is exponentially large.

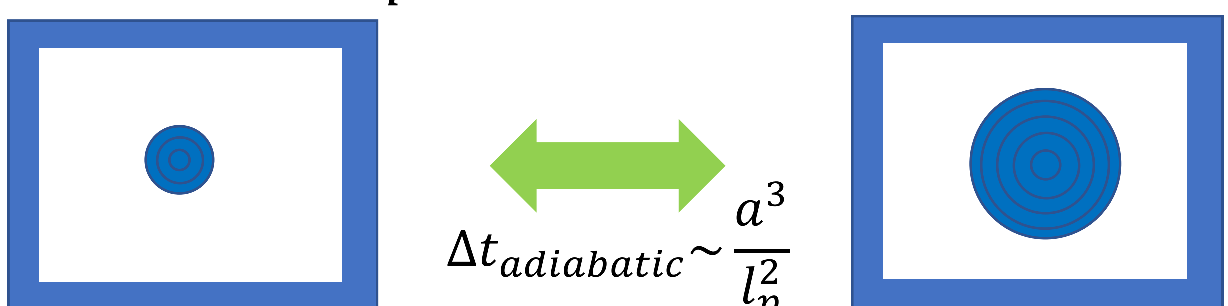
⇒We will check it later.



The time inside is almost frozen.

- Postulate 2 : The time scale of the adiabatic change of the BH is of order $O(a^3/l_p^2)$.

⇒We will check it later.



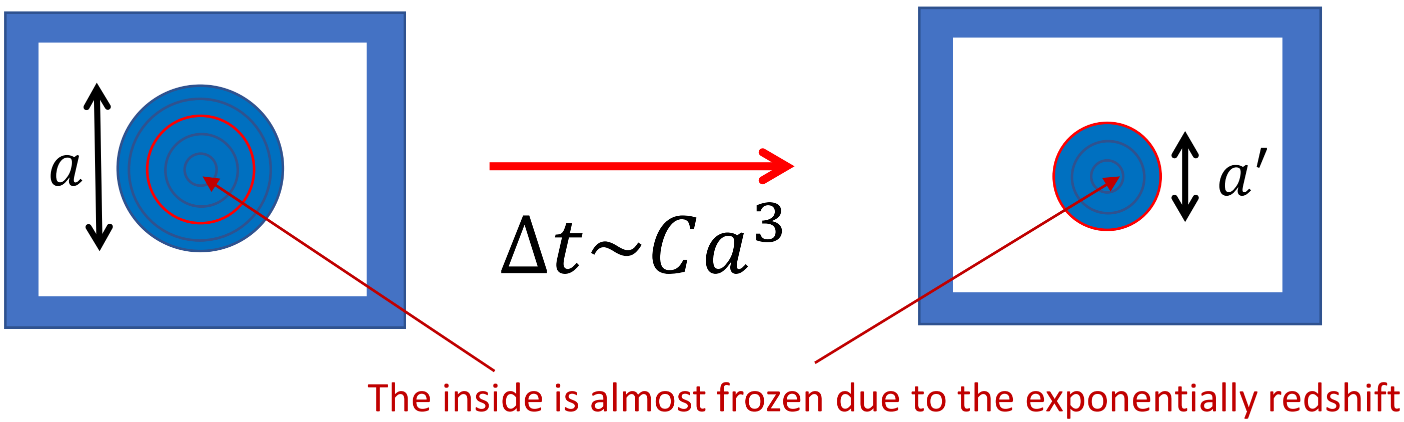
$$\Delta t_{adiabatic} \sim \frac{a^3}{l_p^2}$$

(Note: In the vacuum, a BH with $a$ evaporates in $\Delta t_{eva} \sim \frac{a^3}{l_p^2}$.

⇒The relaxation time scale should be at most order of $O(a^3/l_p^2)$.)

# Why? $A(r;\cancel{a}), B(r;\cancel{a})$ (2/2)

- Suppose a process in which a BH with radius $a$ is adiabatically shrunk to radius $a'$:



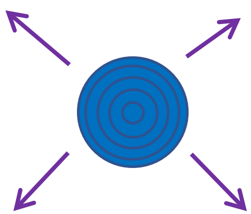

⇒The metric at $r < a'$ hardly changes:

$g_{\mu\nu}(r < a')$ of BH with $a = g_{\mu\nu}(r < a')$ of BH with $a'$

⇒As a first trial, we assume that $A(r)$ and $B(r)$ are universal functions independent of the size of the BH.

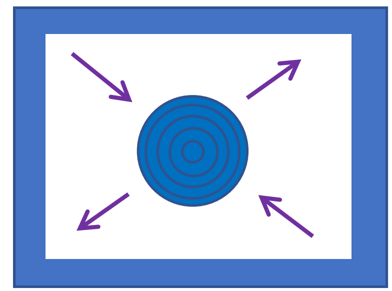
# Interior metric of evaporating BHs

The inside of the evaporating BH which has adiabatically grown has the same metric as that of the stationary BH in the heat bath.



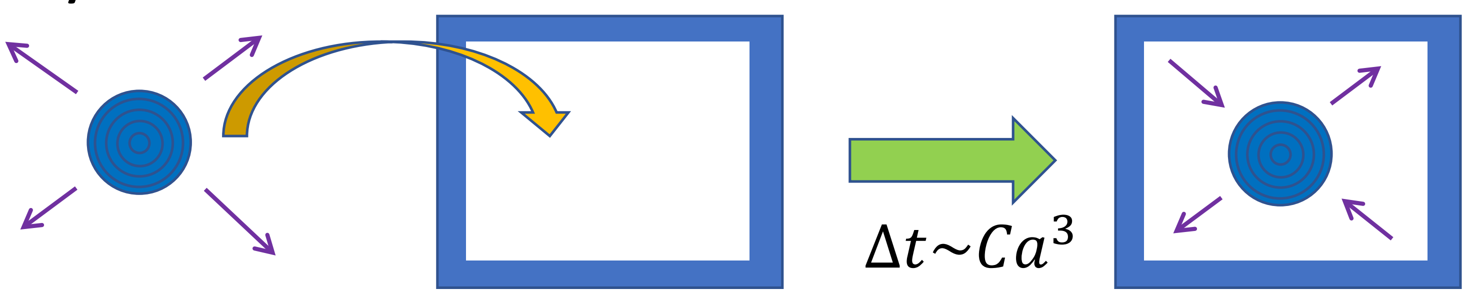
in vacuum

$$g_{\mu\nu}^{eva}(inside) = g_{\mu\nu}^{stat}(inside)$$



in heat bath

- Why?




When the size of the BH becomes $a$, we put it into the bath of $T = \frac{\hbar}{4\pi a}$.

It becomes stationary in $\Delta t \sim Ca^3$, in which the interior does not change due to the time freezing.

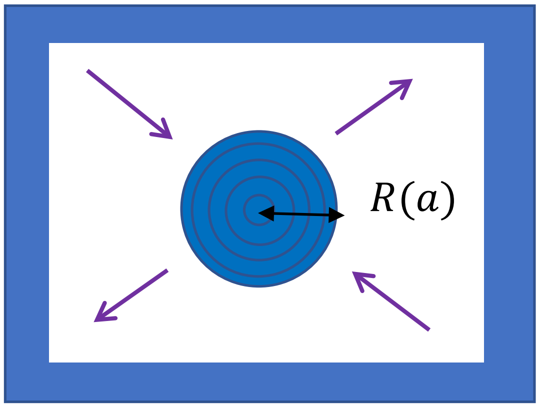
# Our ansatz

Thus, we can use universal function $A(r), B(r)$ and set

Stationary BH in the heat bath

$$ds^2 = \begin{cases} -\left(1-\frac{a}{r}\right)dt^2 + \frac{dr^2}{1-\frac{a}{r}} + r^2 d\Omega^2, & r > R(a) \\ -\frac{1}{B(r)}e^{A(r)}dt^2 + B(r)dr^2 + r^2 d\Omega^2, & r < R(a) \end{cases}$$

$$R(a) = a + \frac{2\sigma}{a}$$




$\Rightarrow$Let's determine $A(r), B(r)$ from now.

(Note: This is just a first approximation. In future, we want to consider the back reaction from the dilute radiation around the BH and the surface fluctuation with $\Delta r \sim \frac{l_p^2}{a}$ .)

# Determination of $B(r)$

$$ds^2 = \begin{cases} -\left(1-\frac{a}{r}\right)dt^2 + \frac{1}{1-\frac{a}{r}}dr^2 + r^2 d\Omega^2 \\ -\frac{1}{B(r)}e^{A(r)}dt^2 + B(r)dr^2 + r^2 d\Omega^2 \end{cases}$$

On the surface of BH, we have

$$g_{rr}\Big|_{r=R(a)} = \frac{R(a)}{R(a)-a} = \frac{aR(a)}{2\sigma} \approx \frac{R(a)^2}{2\sigma},$$

$$B(R(a)) =$$

Radial coordinate $r$ is uniquely fixed because of spherical symmetry.

$$\boxed{R(a) = a + \frac{2\sigma}{a}}$$

$$B(R(a)) = \frac{R(a)^2}{2\sigma}$$

Because this holds for any $a$ and $B(r)$ is independent of $a$, we have

$$\boxed{B(r) = \frac{r^2}{2\sigma}.}$$

# Determination of $A(r)$ (1/3)

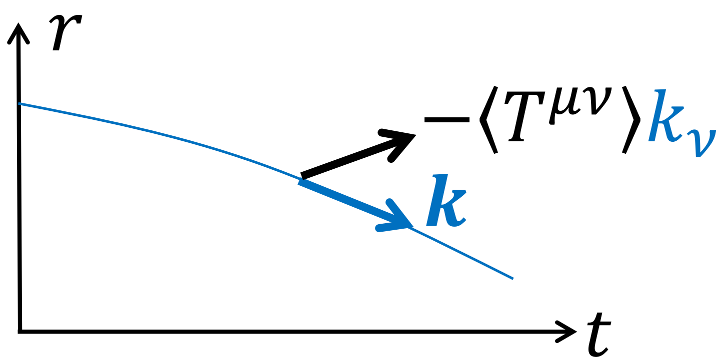
The system is in equilibrium.

⇒The energy-momentum flow inside the BH is time reversal invariant, which can be expressed covariantly as

$$-\langle T^{\mu\nu}\rangle k_\nu = \eta(l^\mu + f k^\mu)$$
$$-\langle T^{\mu\nu}\rangle l_\nu = \eta(k^\mu + f l^\mu)$$

Exchange in $t \to -t$

$$\begin{cases} \boldsymbol{k} = \frac{1}{\sqrt{B}} e^{\frac{A}{2}} dt - \sqrt{B} dr\text{: radially ingoing null vector } (\to -\boldsymbol{l}) \\ \boldsymbol{l} = \frac{1}{\sqrt{B}} e^{\frac{A}{2}} dt + \sqrt{B} dr\text{: radially outgoing null vector } (\to -\boldsymbol{k}) \end{cases}$$




Energy-momentum flow through a falling null spherical shell

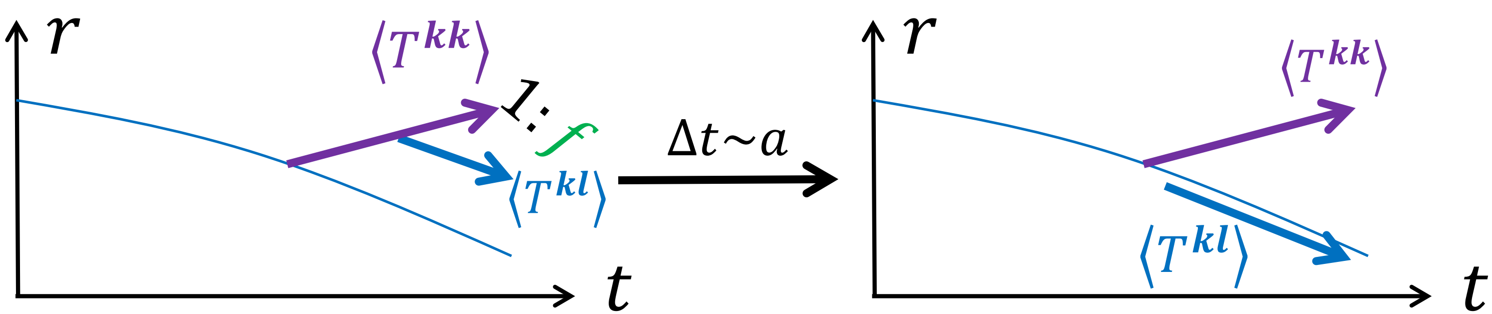
# Determination of $A(r)$ (2/3)

- $f(r)$ gives the ratio of outgoing and ingoing flow:

$$\langle T^{kk} \rangle : \langle T^{kl} \rangle = 1 : f \qquad (1 > f(r) \geq 0)$$

⇒Where does $f \neq 0$ come from?

⇒massive particles and reflection of the emitted particles




$f$ also appears in the ratio of energy density $-\langle T^t_t \rangle$ and radial pressure $\langle T^r_r \rangle$ :

$$\frac{\langle T^r_r \rangle}{-\langle T^t_t \rangle} = \frac{1 - f}{1 + f}$$

# Determination of $A(r)$ (3/3)

$$ds^2 = -\frac{1}{B(r)}e^{A(r)}dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

- If $f(r)$ is given, we can determine $A(r)$.

From Einstein eq., we have

$$G^{\boldsymbol{kk}} : G^{\boldsymbol{kl}} = \langle T^{\boldsymbol{kk}} \rangle : \langle T^{\boldsymbol{kl}} \rangle = 1 : f.$$

$$\Rightarrow \frac{dA}{dr} = \frac{2B}{(1+f)r}$$

$$B(r) = \frac{r^2}{2\sigma}$$

Here, for simplicity, we assume

$$f(r) = const.$$

In general,
$f(r)$ should vary slowly:

$$l_p \frac{df(r)}{dr} \ll f(r)$$

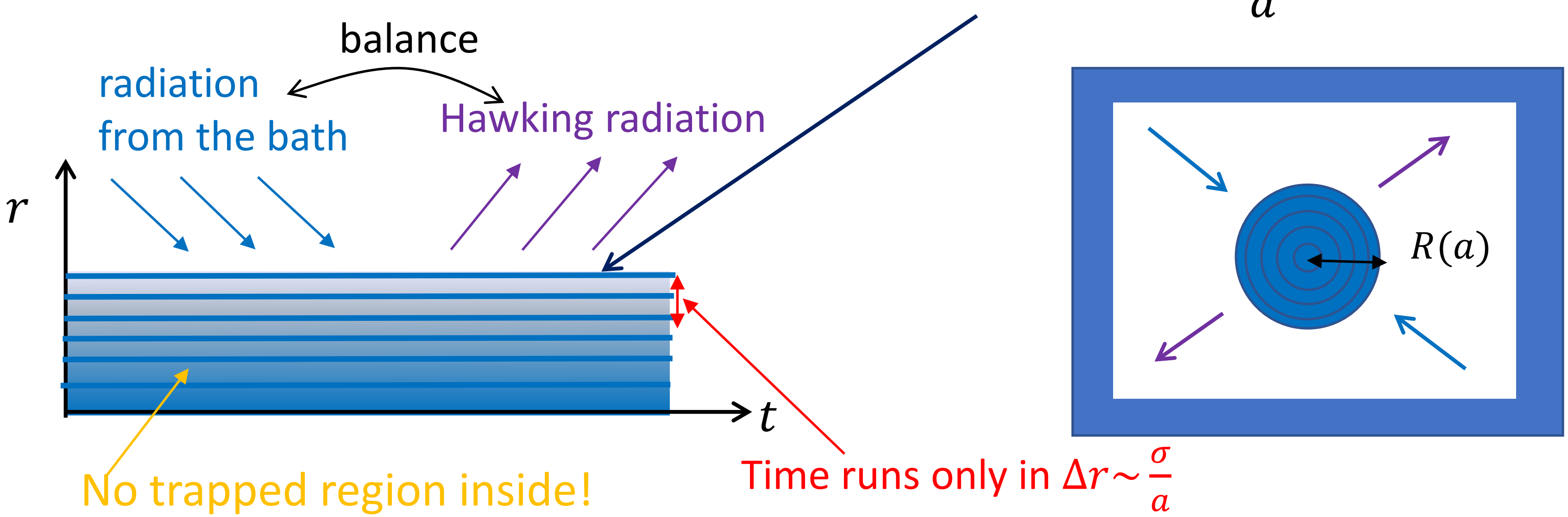
$$\Rightarrow \boxed{A(r) = \frac{r^2}{2(1+f)\sigma}}$$

# A candidate $g_{\mu\nu}$ of the stationary BH

$$ds^2 = \begin{cases} -\left(1-\frac{a}{r}\right)dt^2 + \frac{dr^2}{1-\frac{a}{r}} + r^2 d\Omega^2, & r > R(a) \\ -\frac{2\sigma}{r^2} \underline{e^{-\frac{R(a)^2-r^2}{2\sigma(1+f)}} dt^2} + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2, & r < R(a) \end{cases}$$

Exponentially large redshift
→The interior is frozen.

These are connected at $r = R(a) = a + \frac{2\sigma}{a}$ =const.



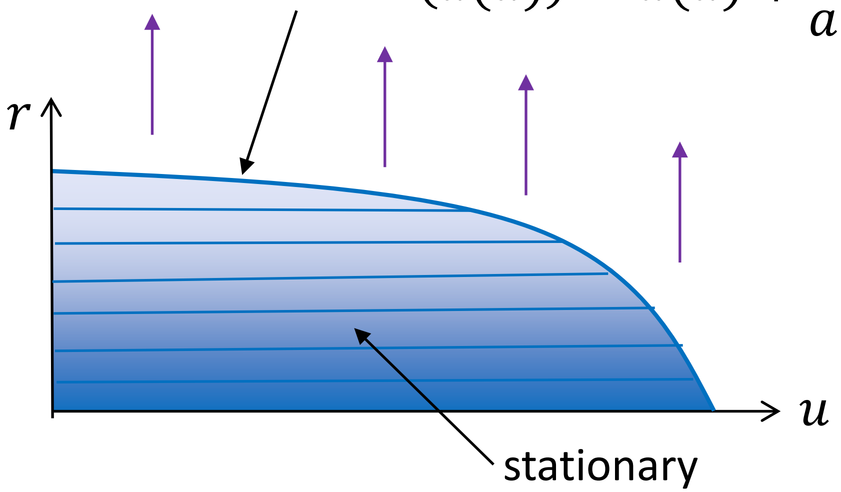
# A candidate $g_{\mu\nu}$ of the evaporating BH

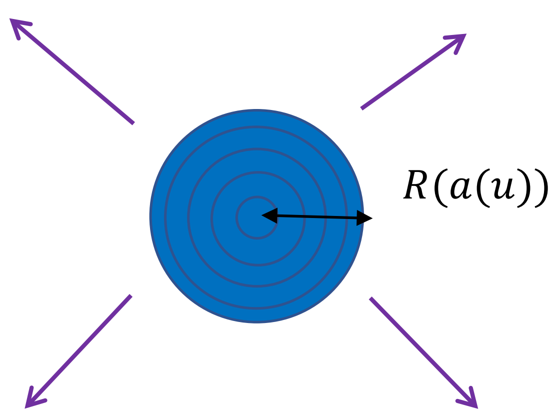
$$ds^2 = \begin{cases} -\dfrac{r-a(u)}{r}du^2 - 2dudr + r^2 d\Omega^2, & r > R(a(u)) \\ -\dfrac{2\sigma}{r^2} e^{-\frac{R(a(u))^2 - r^2}{2\sigma(1+f)}} du^2 - 2e^{-\frac{R(a(u))^2 - r^2}{4\sigma(1+f)}} dudr + r^2 d\Omega^2 & r < R(a(u)) \end{cases}$$

These are connected at

$$r = R(a(u)) = a(u) + \frac{2\sigma}{a(u)}$$

$$\frac{d}{du}a(u) = -\frac{\sigma}{a(u)^2}$$ ⇒We will show this later.

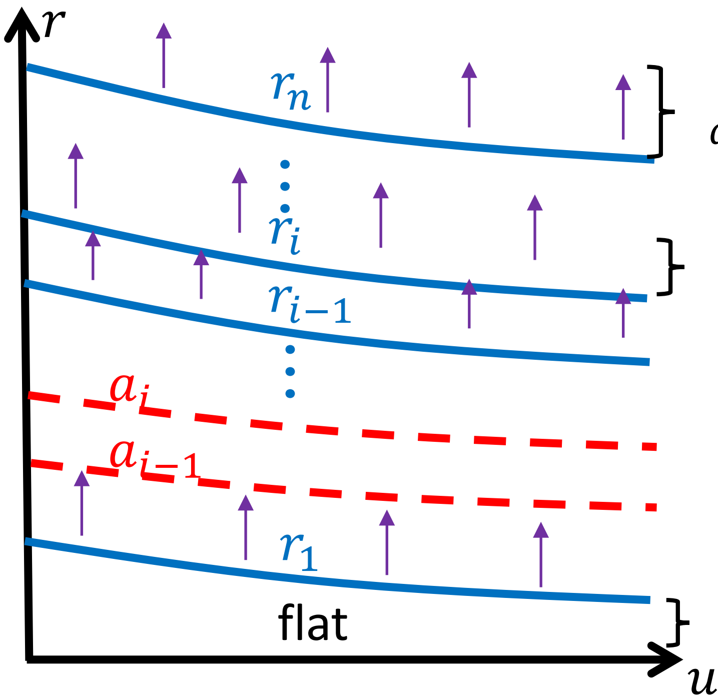


# Note: A multi-shell model

Kawai, Matsuo and Y. Y. IJMPA (2013)
Kawai and Y. Y. Universe (2017)

- We can use a multi-shell model and obtain the same interior metric with $f = 0$.




$$ds_{out}^2 = -\frac{r-a}{r}du^2 - 2dudr + r^2 d\Omega^2$$

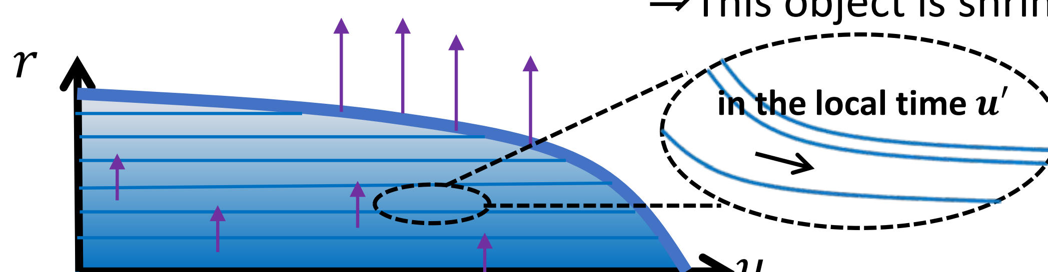
$$ds_i^2 = -\frac{r-a_i}{r}du_i^2 - 2du_i dr + r^2 d\Omega^2$$

$$\frac{da_i}{du_i} = -\frac{\sigma}{a_i^2}$$

This approaches the same stationary metric.

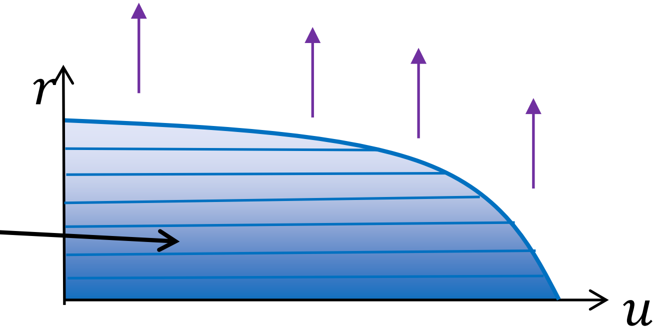
Without assumption about the stationarity, we reach the stationary interior metric!

Each shell keeps falling.
⇒This object is shrinking.



# Step2: Evaluation of $\langle T_{\mu\nu} \rangle$

# Setup




Consider the interior region.

The background metric is static:

$$ds^2 = -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma(1+f)}} dU^2 - 2e^{\frac{r^2}{4\sigma(1+f)}} dUdr + r^2 d\Omega^2$$
$$= -e^{\varphi(r(U,V))} dUdV + r(U,V)^2 d\Omega^2,$$

$\Rightarrow \langle T_{\mu\nu} \rangle$ also should be static:

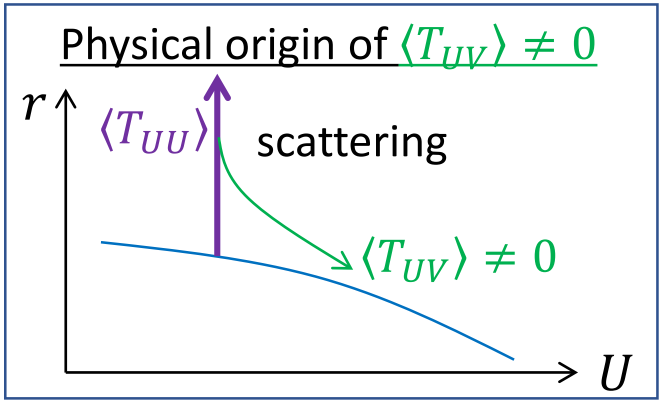
$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu}(r) \rangle, \qquad \langle T_{UU} \rangle = \langle T_{VV} \rangle$$

$\Rightarrow$We have the relation

$$\langle T_{UV} \rangle = f \langle T_{UU} \rangle,$$

we have to determine only

$$\langle T_{UU} \rangle, \qquad \langle T^{\theta}_{\theta} \rangle.$$



# The relations of $\langle T_{\mu\nu} \rangle$

・1st eq.

$\langle T^{\mu}_{\mu} \rangle = 2g^{UV} \underline{\langle T_{UV} \rangle} + 2\langle T^{\theta}_{\theta} \rangle$ leads to

$= f\langle T_{UU} \rangle$

$$\langle T^{\theta}_{\theta} \rangle = \frac{1}{2}\langle T^{\mu}_{\mu} \rangle + 2\mathrm{e}^{-\varphi} f \langle T_{UU} \rangle$$

・2nd eq.

Integrating $\nabla^{\mu}\langle T_{\mu U} \rangle = 0$ from $r = 0$ to $r$, we have

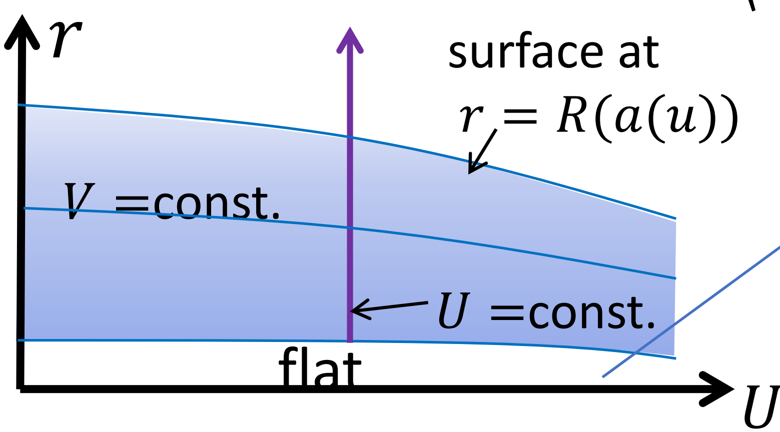
$$\langle T_{UU} \rangle = \frac{1+f}{2r^2} \sigma e^{\varphi} \langle T^{\mu}_{\mu} \rangle,$$

All components are expressed in terms of $\langle T^{\mu}_{\mu} \rangle$.

where we have used the boundary condition

$$\langle T_{\mu\nu}(r = 0) \rangle = 0$$

(⇔initial condition that the system started from the collapsing matter.)



# 4D Weyl anomaly

For simplicity, consider conformal matters.

$\Rightarrow \langle T^\mu_\mu \rangle$ is determined by the 4D Weyl anomaly:

$$\langle T^\mu_\mu \rangle = \hbar c_w \mathcal{F} - \hbar a_w \mathcal{G}$$ ←state-independent

where

$$\mathcal{F} \equiv C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}, \qquad \mathcal{G} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

⇒The candidate metric determines

$$\langle T^\mu_\mu \rangle = \frac{\hbar c_w}{3(1+f)^4 \sigma^2}$$

⇒We will discuss the case of non-conformal matters later.

step3: Determine the self-consistent $g_{\mu\nu}$

# The $g_{\mu\nu}$ is the self-consistent solution.

Thus , we have obtained

$$\langle T_\theta^\theta \rangle = \frac{\hbar c_W}{6(1+f)^4\sigma^2}, \qquad \langle T_{UU} \rangle = \langle T_{VV} \rangle = \frac{\hbar c_W}{3(1+f)^3 r^4} e^{\frac{r^2}{2(1+f)\sigma}},$$

$$\langle T_{UV} \rangle = f \langle T_{UU} \rangle$$

On the other hand, the metric gives

$$G_\theta^\theta = \frac{1}{2(1+f)^2\sigma}, \qquad G_{UU} = G_{VV} = \frac{\sigma}{(1+f)r^4} e^{\frac{r^2}{2(1+f)\sigma}},$$

$$G_{UV} = f G_{UU}$$

$\Rightarrow G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$ is satisfied if we identify

$$\sigma = \frac{8\pi l_p^2 c_W}{3(1+f)^2}.$$

(We can do the same program for the exterior and check the self-consistency.)

# Self-consistent solution

$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 \\ -\frac{2\sigma}{r^2} e^{-\frac{1}{2\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} du^2 - 2e^{-\frac{1}{4\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} dudr + r^2 d\Omega^2 \end{cases}$$

←Non-perturbative solution w.r.t. $\hbar$

$$\boxed{\sigma \equiv \frac{8\pi G\hbar c_W}{3(1+f)^2}}$$

The surface exists at

$$R(a(u)) \equiv a(u) + \frac{2\sigma}{a(u)}$$

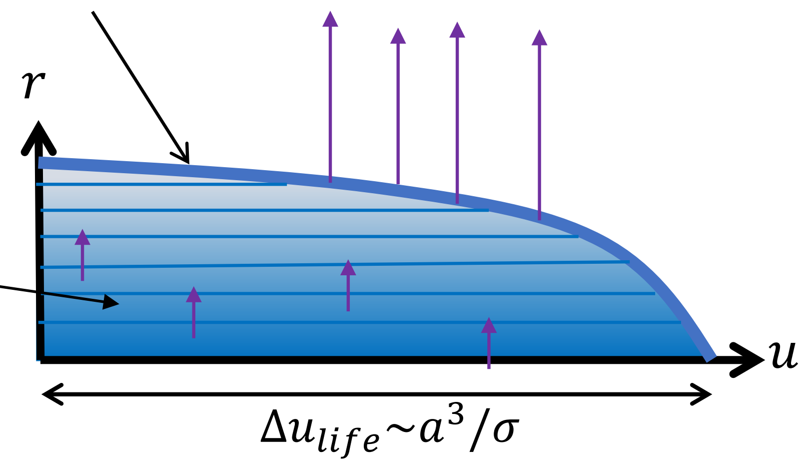
⇒Stefan-Boltzmann law is reproduced self-consistently:

Hawking radiation ∝ $c_W$

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}$$

$$\langle T^\mu_\mu \rangle = \hbar c_W C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - \hbar a_W \mathcal{G}$$

No trapped region




Note: This configuration is typical in a thermodynamical sense.

# Consistency check (1/3)
# Validity of the classical gravity

In the macroscopic region ($r > l_P$),

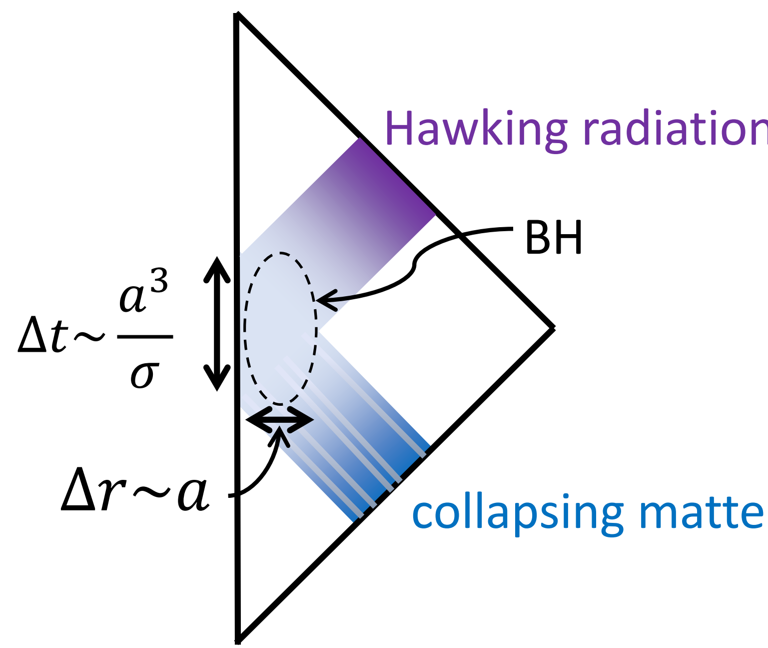
$$R, \sqrt{R_{\alpha\beta}R^{\alpha\beta}}, \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{1}{c_w l_p^2} \ll \frac{1}{l_p^2}$$

if $c_w \gg 1$

($c_w$ plays a role of $N$ in the introduction.)

⇒No singularity:

$G_{\mu\nu} = 8\pi G\langle \hat{T}_{\mu\nu}\rangle$ is valid as long as $a > l_p$.




Note: This is consistent with the singularity theorem because there is no trapped region and the dominant energy-condition is broken.

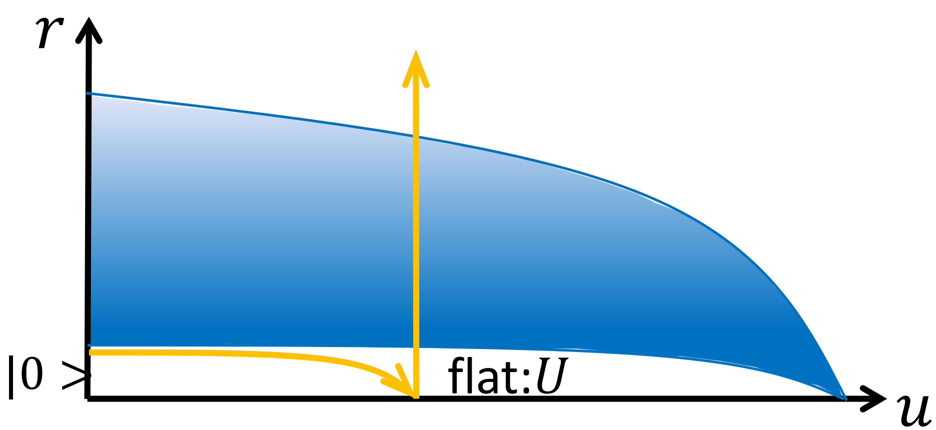
# Consistency check (2/3): Hawking radiation

- Hawking radiation appears self-consistently:

By a similar manner to Hawking's derivation,

we can show

$$\langle 0|\hat{N}_\omega|0\rangle = \frac{1}{e^{\hbar\omega/T}-1}, \qquad T = \frac{\hbar}{4\pi a(u)}$$ (for s-wave)




$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}$$

# Consistency check (3/3)
# Energy condition of $\langle T_{\mu\nu}\rangle$

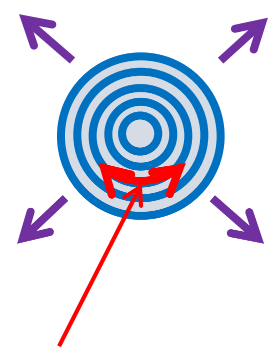
$$\boxed{4\pi \int_{\sim\sqrt{c_W}\, l_p}^{R(a)} dr\, r^2\left(-\langle T^t_t\rangle\right) \approx \frac{a}{2G} = M}$$

$$0 < -\langle T^t_t\rangle = \frac{1}{8\pi G}\frac{1}{r^2},\ \langle T^r_r\rangle = \frac{1}{8\pi G}\frac{1-f}{1+f}\frac{1}{r^2}$$

The dominant energy condition ($\rho \geq p_i > 0$) is broken.

anisotropic ⇒ Not a fluid

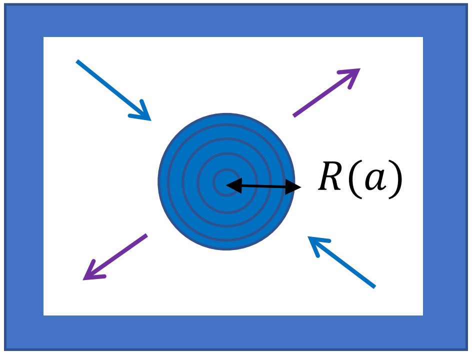
$$\langle T^\theta_\theta\rangle = \frac{1}{8\pi G}\frac{3}{16\pi c_W l_p^2}$$



This large angular pressure, which is consistent with 4D Weyl anomaly, supports the object.

# Why is $\langle T^{\theta}_{\theta} \rangle$ so large?

- To study it, we want to evaluate $\langle \psi | T_{\mu\nu} | \psi \rangle$, where $|\psi\rangle = |\text{equilibrium at each stage}\rangle$.




- As a first trial, we are now calculating $\langle 0 | T_{\mu\nu} | 0 \rangle$ direcly.

⇒<u>Result</u>: For $N$ massless scalar fields $\phi_i$,

$$\langle 0 | T^{\theta}_{\theta} | 0 \rangle \sim \langle 0 | T^{\mu}_{\mu} | 0 \rangle = O\left(\frac{1}{N l_p^2}\right), \qquad \Rightarrow \text{consistent!}$$

which appears after integrating angular modes $\sum_{l=0}^{\infty}$ .

[Kawai and Y.Y., work in progress]

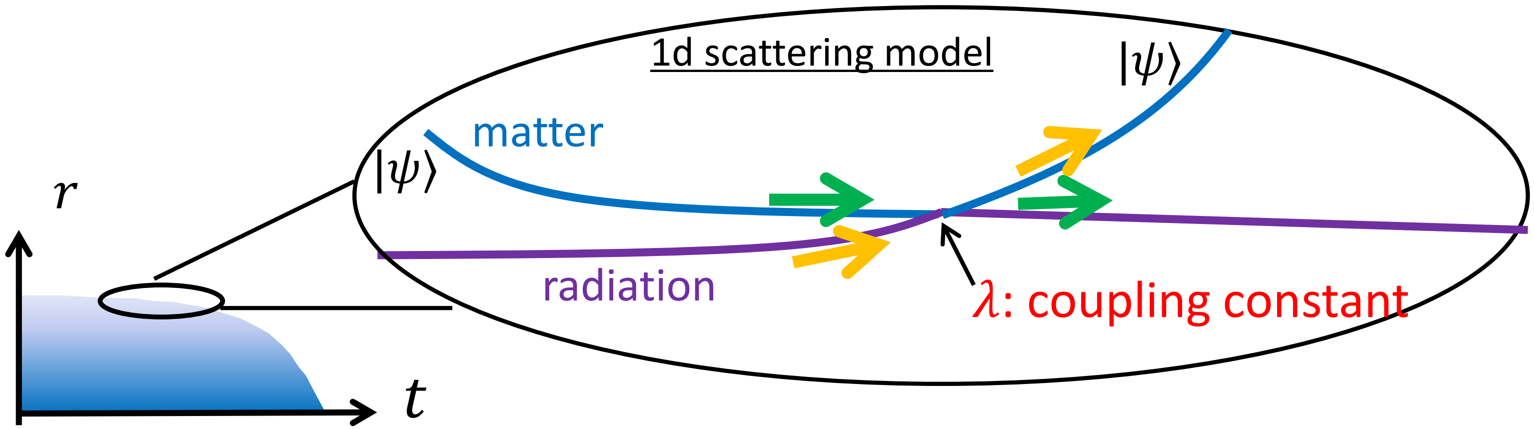
# Step4:
# Information problem and BH entropy

[Kawai and Y.Y. PRD 2016]

# Information recovery by interaction

- Hawking radiation is created inside the collapsing matter.

⇒The collapsing matter and Hawking radiation interact.

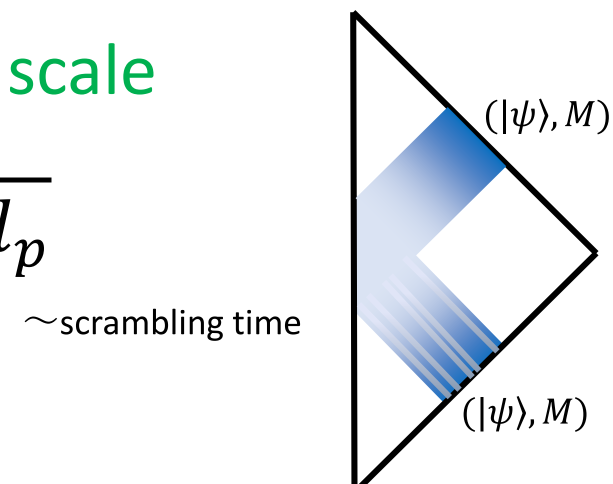



⇒We can estimate the scattering time scale

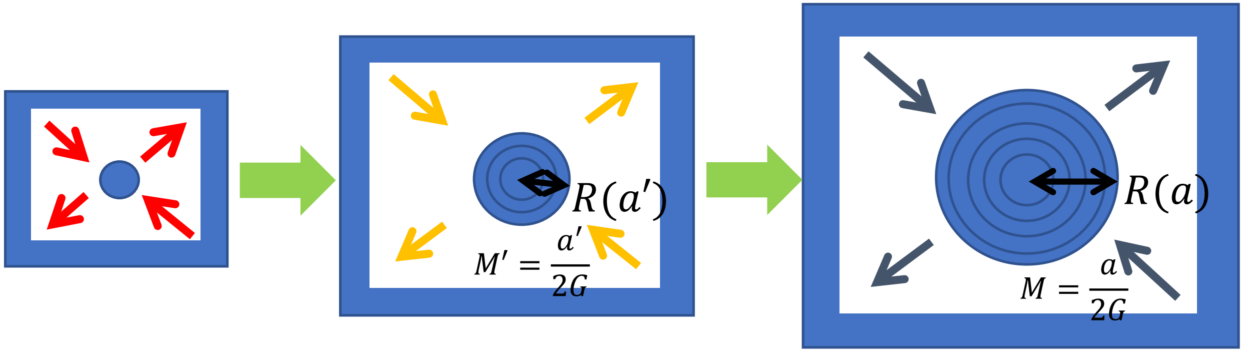
$$\Delta t_{scat} \sim a \log \frac{a}{\lambda N l_p}$$

⇒information flow = energy flow

⇒information recovery?



# Entropy from the interior




$$\sigma = \frac{8\pi l_p^2 c_W}{3(1+f)^2}$$

- The interior metric of the stationary BH is given by

$$ds^2 = -\frac{2\sigma}{r^2} e^{-\frac{1}{2(1+f)\sigma}(R(a)^2 - r^2)} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2.$$

- We can evaluate the entropy density as

$$s = \frac{1}{4\pi r^2} \frac{2\pi\sqrt{2\sigma}}{l_p^2}. \qquad \Rightarrow 4\pi r^2 s \sim \sqrt{N}/l_p$$

- Summing up it over the interior volume, we obtain

$$S = \int dV\, s = \int_0^{R(a)} dr \sqrt{g_{rr}}\, 4\pi r^2 s \approx \int_0^a dr \frac{r}{\sqrt{2\sigma}} \frac{2\pi\sqrt{2\sigma}}{l_p^2} = \frac{\pi a^2}{l_p^2} = \frac{A}{4 l_p^2}$$

⇒The information should be stored inside the BH.

# Conclusions

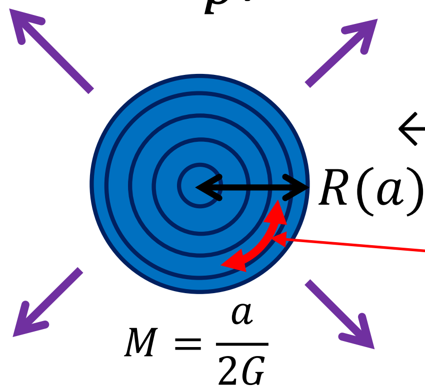
# Conclusion: The new picture of BH

BH evaporates by nature.

⇒Quantum BH is described by field theory $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ with $c_W \gg 1$ (as long as $a \gg l_p$).

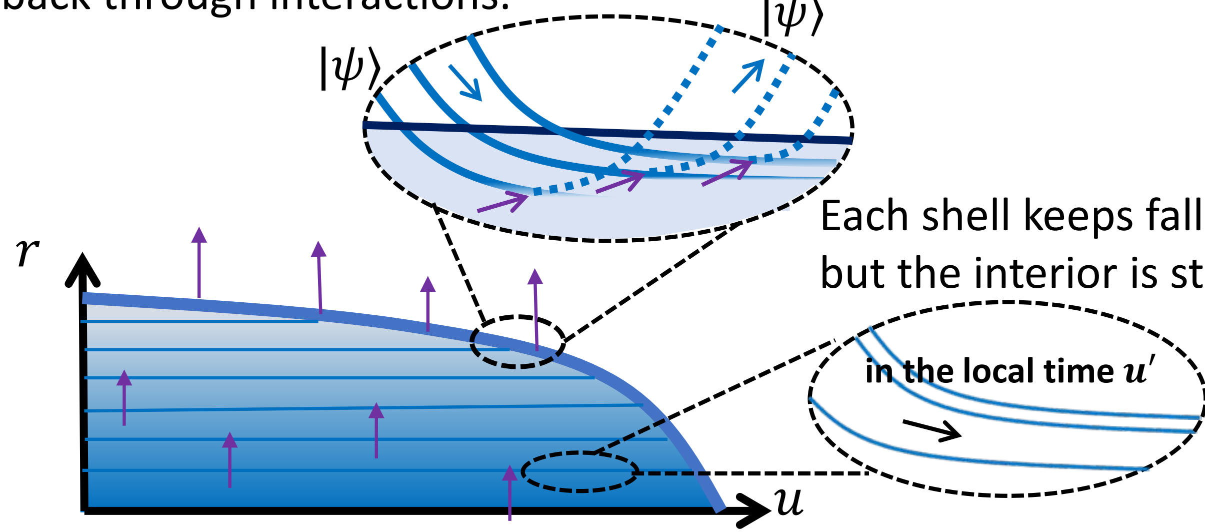
The surface exists at

$$R(a(u)) \equiv a(u) + \frac{2\sigma}{a(u)}$$

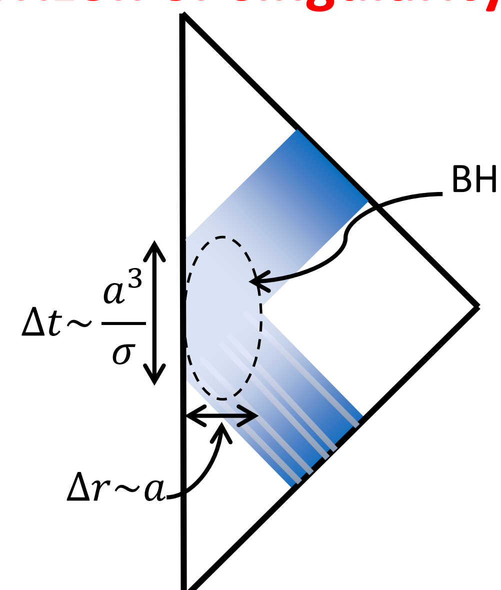



**No horizon or singularity**

Information may come back through interactions.




Each shell keeps falling, but the interior is stationary.



***No horizon but surface.***
***This is the BH.***

**Thank you very much!**