International Symposium in Honour of Professor Nambu for the 10th Anniversary of his Nobel Prize in Physics

From spontaneous symmetry breaking to quantum turbulence

W F Vinen

School of Physics and Astronomy, University of Birmingham

Introductory Comments

A great honour to be asked to contribute to this symposium.

Professor Nambu's view that condensed matter physics.is less attractive than nuclear and particle physics.

Professor Nambu's Nobel Prize was for his contributions to an understanding of ***spontaneous symmetry breaking in sub-atomic physics***, but his interest in symmetry breaking was probably generated by an interest in symmetry breaking in condensed matter physics, and in particular in ***superconductivity***,

My aim: an ***elementary talk*** that connects a ***simple but quite subtle form of symmetry breaking*** - the ***transition to superfluidity in liquid ^{4}He*** - with an area of research that is very active here in OCU, namely ***quantum turbulence.***

Transitions to turbulence also involve symmetry breaking, as I shall mention briefly.. I shall not refer to the complicated case of superfluidity in liquid ^{3}He, in spite of its having been much studied at OCU. But I shall refer briefly to ***superconductivity*** and ***cold superfluid gases.***

Superfluidity and Bose condensation

Superfluidity is always associated with some form of ***Bose condensation***.

Superfluidity arises from what used to be called an "**ordering process**". Bose condensation was originally seen as an ordering in momentum space rather than an ordering in real space.

*An **inadequate view of Bose condensation***. Putting a macroscopic number of identical Bosons into a single quantum state leads to the formation of a ***coherent matter wave*** (cf a coherent electromagnetic wave). This coherent matter wave leads to superfluidity.

In practice Bose condensation occurs in systems of interacting particles. What is the effect of this interaction? It leads to some virtual excitation out of the condensate, but the condensate remains.

A formal description is in terms of the form of the ***single-particle density matrix*** (Penrose and Onsager)

$$\langle \psi^{+}(r_1)\psi(r_2)\rangle \rightarrow \Psi^{*}(r_1)\Psi(r_2) \quad \text{as} \quad r_1 - r_2 \rightarrow \infty$$

where $\Psi(r) \equiv \langle \psi(r) \rangle$ is a "***classical" phase-coherent matter field***.

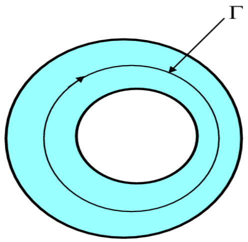
The superfluid order parameter, superfluid velocity, and supercurrents

$\Psi(r)$ is the ***complex order parameter*** for the superfluid transition, called the "***condensate wave function***". It is not a quantum-mechanical wave function in the ordinary sense, rather a classical field. The ordering implied by the existence of this field is often called **O**ff-**D**iagonal **L**ong **R**ange **O**rder.

In general $\Psi(r) = |\Psi(r)| exp\big(iS(r)\big)$ where $v_s = (\hbar/m)\nabla S(t)$ is the local velocity with which the condensate is moving. v_s is the **superfluid velocity.**.

This superfluid velocity must be ***irrotational*** (curl $v_s = 0$) and the ***superfluid circulation must be quantized***

$\Gamma = \oint v_s . dl = n\kappa$, where $\kappa = h/m$ is the quantum of circulation.



But does Bose condensation necessarily lead to superfluidity?.

Spontaneous symmetry breaking

Superfluidity depends on the existence of coherent matter wave in the fluid, with its well-defined phase. Changing that phase alters the state of the superfluid.. In other words it is the loss of global U(1) phase rotation symmetry that characterizes the superfluid transition. (Gauge symmetry)

Observation of the phase: coupled superfluids.

Number-phase uncertainty.

The stability of superflow, the excitation spectrum, the normal fluid and the condensate fractions

Is ^{4}He a superfluid? Is frictionless superflow stable, or at least metastable?

The ideal Bose-condensed gas is not a superfluid because its flow is unstable against single particle excitation, with spectrum $\varepsilon = p^2/2m$.

Interactions change the spectrum, leading to only phonons at the lowest energies: $\varepsilon = cp$, where *c* is the velocity of sound. These low-lying phonon states are the Nambu-Goldstone modes associated with the phase transition.

At $T = 0$ this phonon spectrum ensures that $\rho_s = \rho$. $(J_s = \rho_s v_s)$ (Landau)

At a finite *T* thermal excitation of phonons (+ rotons) leads to a "normal fluid". density ρ_n.. As $T \to T_c$ $\rho_n \to \rho$ $(\rho_s \to 0)$.

The superfluid density must not be confused with the condensate fraction.

These ideas apply to any simple Bose superfluid: liquid ^{4}He, and Bose condensed cold atomic gases

Superconductivity

Another superfluid system is the conduction electron gas in a superconductor. Electrons are Fermions, so Bose condensation of electrons cannot occur. But a form of Bose condensation of Cooper pairs can occur, leading to a two-particle density matrix having a form similar to that of the single particle density matrix in helium

$$\left\langle \psi_{\uparrow}^{+}(r_1)\psi_{\downarrow}^{+}(r_1)\psi_{\downarrow}(r_2)\psi_{\uparrow}(r_2)\right\rangle \to \Psi^{*}(r_1)\Psi(r_2) \quad \text{as} \quad r_1 - r_2 \to \infty$$

where $\Psi(r)$ has long-range phase coherence. Again superconductivity arises from broken gauge symmetry.

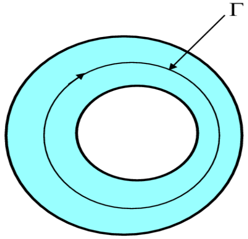
But there are important differences:

- Electrons are charged: $v_s = (\hbar/m)\nabla S(t) - (e/m)A(r)$.
- The Bose condensing particles are not present in the non-interacting system, which is degenerate Fermi gas.
- The Cooper pairs are very large compared with the electron spacing.
- There is an energy gap in the spectrum of excitations.

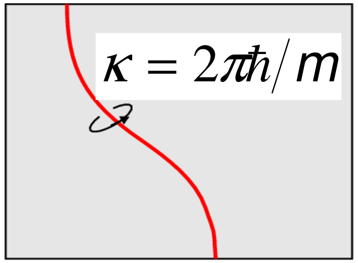
Observation of transition from BCS condensation to ordinary Bose condensation of particle pairs as interaction is increased.

Quantized vortices

A non-zero Γ can exist only in a multiply-connected volume.

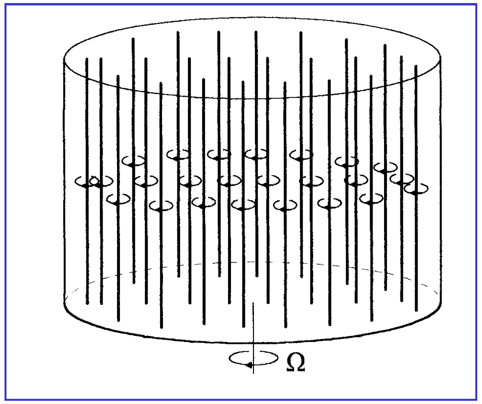


A possible topological defect: a ***free vortex line*** (Onsager, Feynman): $|\Psi| = 0$ along a line in the fluid, a phase change of 2π round the line, and hence a circulation of κ.



Core size determined by a coherence length.

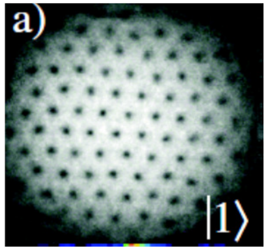
In an ***uncharged superfluid*** velocity ~ $1/r$ for all r *(Biot-Savart)*. In a ***charged superfluid*** velocity is cut off at distances greater than the penetration depth.



A quantized vortex is quite different from a vortex in a classical fluid, which can decay by viscous dissipation..

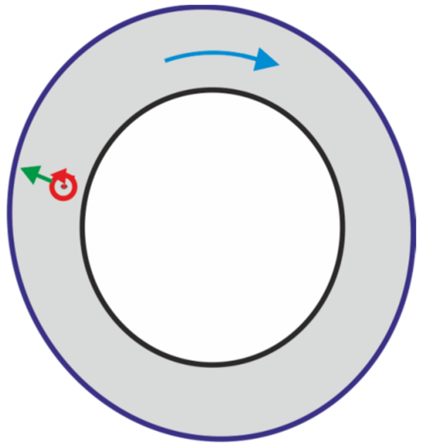
Quantized vortices play an important role in superfluids::

- They allow bulk rotation (or penetration of a magnetic field);
- They allow dissipation of a superflow.

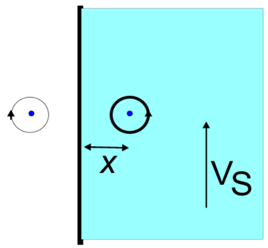


In an uncharged superfluid this dissipation is often associated with a irregular array of vortices: ***Quantum Turbulence.***

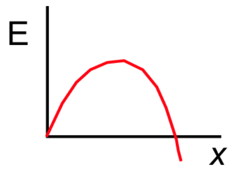
The stability of superflow; vortex nucleation; superfluid turbulence



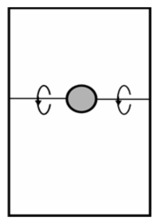
Can vortex motion lead to a decay of superfluid flow? Not necessarily, because this motion is opposed by potential barrier (interaction with image)



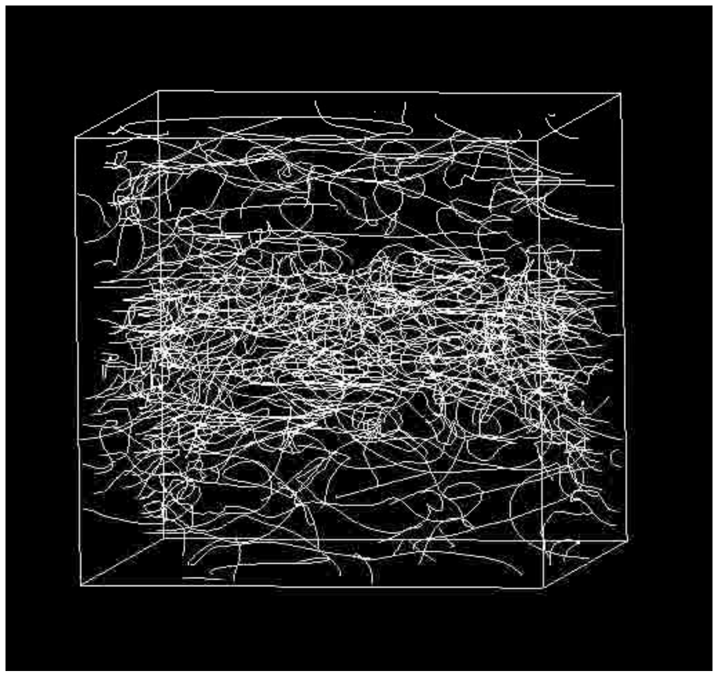
Intrinsic nucleation of vortices very difficult.



Therefore vortex nucleation is usually extrinsic and depends on expansion of existing (remanent) vortices.



Such expansion can often lead to a moving tangle of vortices: i.e. a turbulent superfluid.



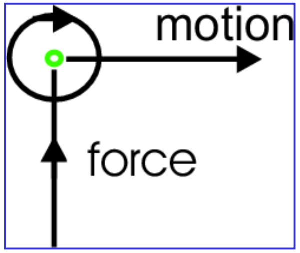
At the same time the normal fluid may become turbulent (in ^{4}He it has a very small viscosity), so we can have two ***interpenetrating turbulent fluids.***

Vortex dynamics

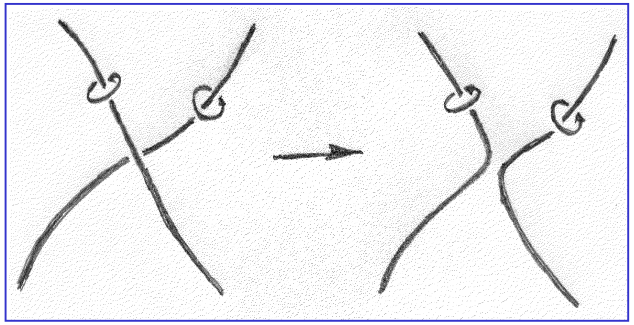
The weakly interacting Bose-condensed gas is described by a non-linear Schrodinger equation (the Gross-Pitaevskii eqn), from which we can deduce that a ***vortex will behave classically*** except on scales comparable with the coherence length. Assume that same applies to superfluid ^{4}He.

At $T = 0$ (no normal fluid) a vortex moves with the local fluid velocity, due often to other vortices (Biot-Savart)...

At $T \neq 0$ there are excitations forming the normal fluid. These are scattered by a vortex $\longrightarrow$ force of "**mutual friction**" between the two fluids. The vortex responds according to the Magnus effect.



If two vortices come very close they can reconnect (a non-classical effect).
Reconnections are very important.

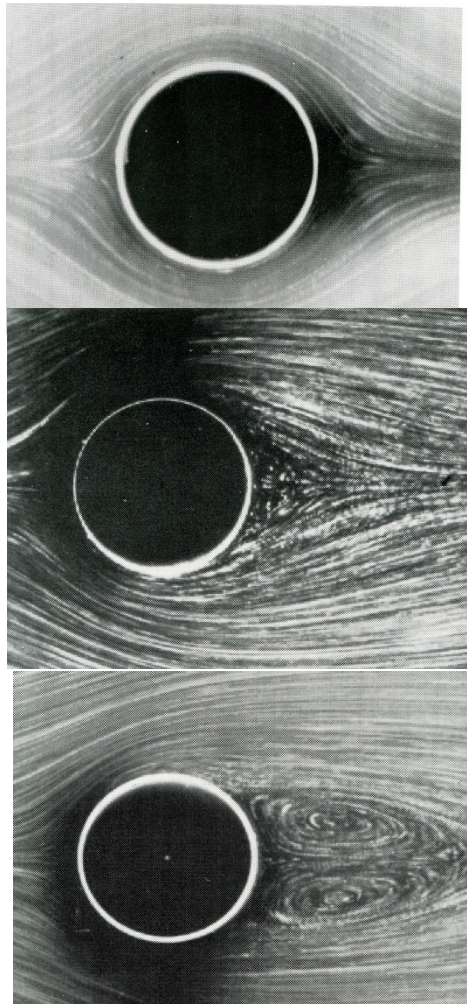


Classical turbulence

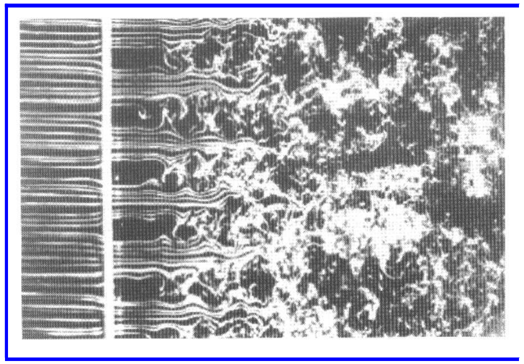
We can think of classical turbulence in terms of two problems.

- Understanding the initial instability of the (usually) laminar flow and how the corresponding flow structures develop. They tend not to be chaotic and may or may not be time dependent.. They will always involve symmetry breaking...

- Understanding the maintenance and properties of fully turbulent flows, which tend to be chaotic, with some recovery of (statistical) symmetry..



Homogeneous turbulence produced by flow through a grid →



Kármán vortex street →

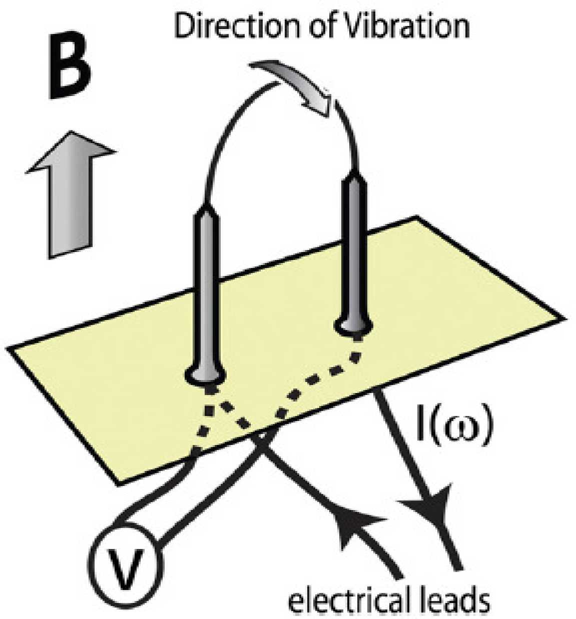
Superflow past an obstacle

Little experimental study of steady motion of an obstacle in a superfluid..

Much study of the drag on objects oscillating in a superfluid: typically an oscillating cylinder in the form of a vibrating wire..

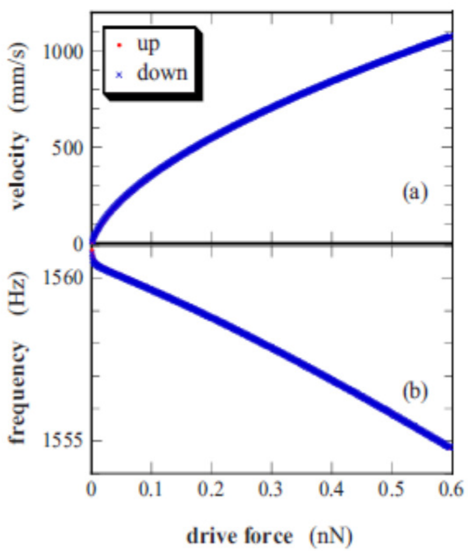
Results often difficult to interpret:

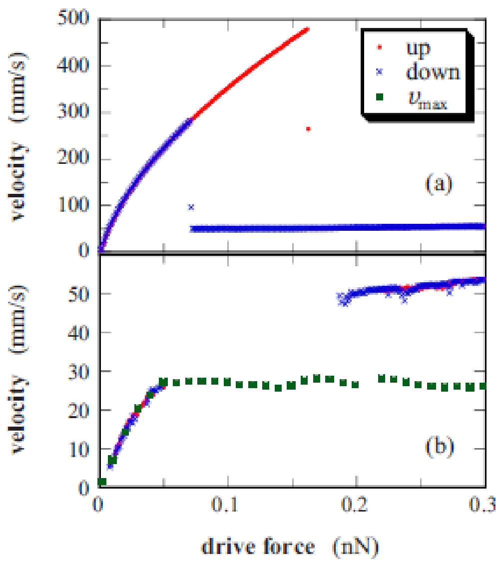
- initial instability due to remanent vortices with unknown geometry
- ***flow patterns depend crucially on the boundary conditions.***
- flow is complicated and cannot yet be visualized.



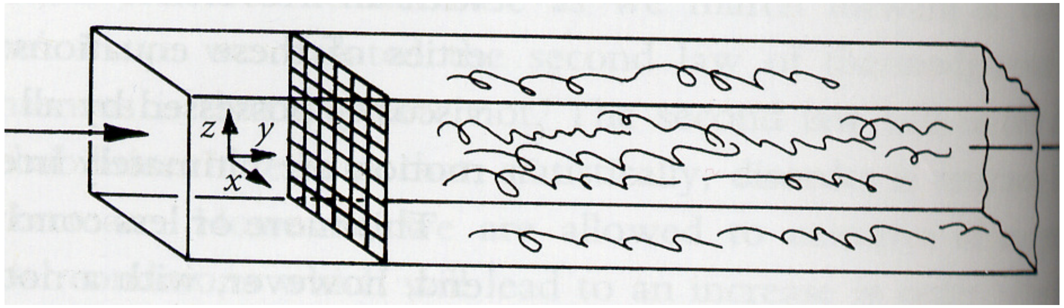
Best to avoid boundaries

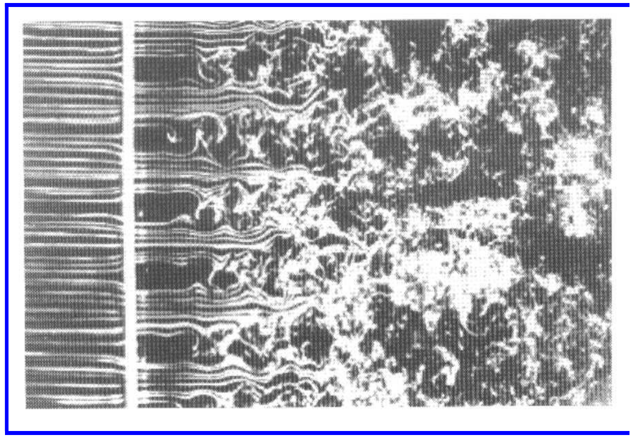
But some nice experiments have been done, especially by Hideo Yano here at OCU.





Homogeneous turbulence in a classical fluid

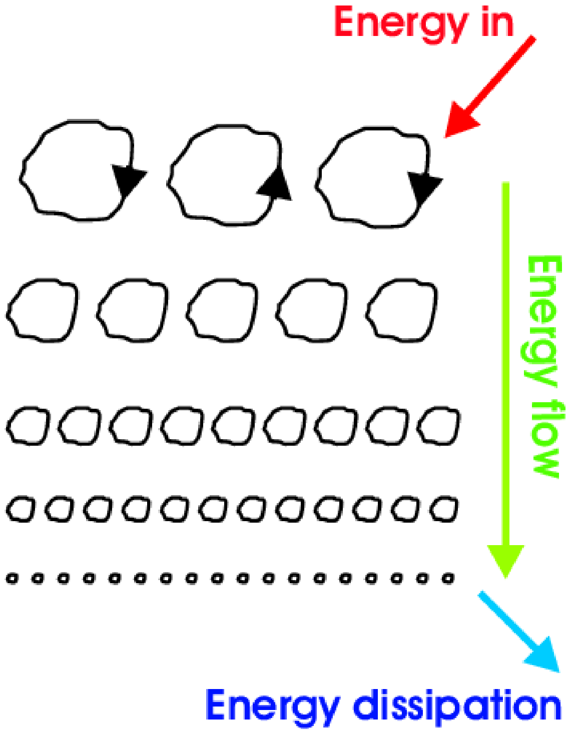


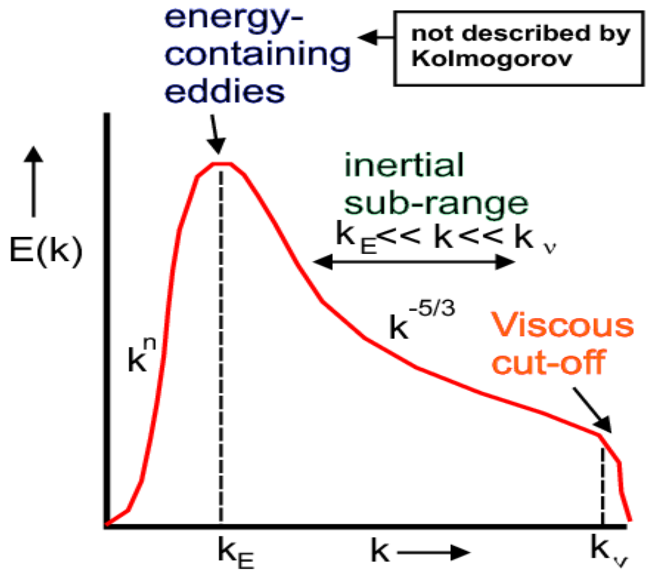


$$\frac{\partial v}{\partial t}+(v\cdot\nabla)v=-\frac{1}{\rho}\nabla p+\nu\nabla^2 v$$

Richardson cascade

Energy is injected into turbulent eddies at large Reynolds number. It then decays through non-linear interactions in a cascade of smaller and smaller eddies, until the eddies are so small that they have a Reynolds number ~ 1 and there is viscous dissipation.





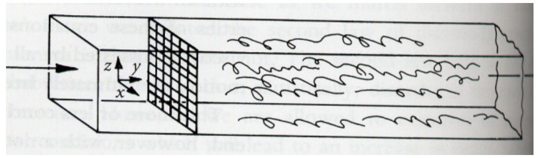
Kolmogorov energy spectrum

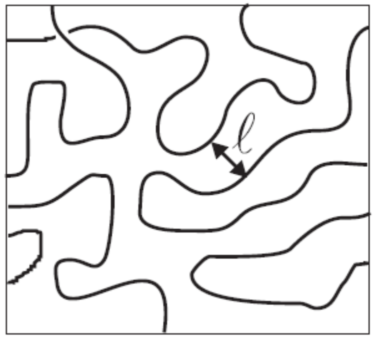
(Local interactions)

Homogeneous turbulence in superfluid ^{4}He at $T = 0$

Compared with the **classical** case

- No viscous dissipation.
- Polarization of vortices $\longrightarrow$ large eddies
- Quantized circulation $\longrightarrow$ minimum eddy size characterized by a length scale ℓ.





On **large scales** ($>>\ell$): eddies containing many quanta. Therefore classical behaviour? Energy cascade with Kolmogorov spectrum? Experimental evidence (Maurer & Tabeling – pressure fluctuations); evidence from computer simulations (Tsubota)

On **small scales** ($\leq \ell$): quantum dominated ((quantized vorticity; discrete vortex lines; vortex reconnections)), but apparently no dissipation! Rules out Kolmogorov? Can ℓ get smaller and smaller? No, because a decreasing ℓ implies an increasing velocity!

What happens to the **Reynolds number** Re= UL/ν? Re$\rightarrow \infty$?
We might define a "**quantum**" **Reynolds number** $\mathrm{Re_q} = UL/\kappa$. .
$\mathrm{Re_q} = 1$ corresponds to motion on scale ℓ. (dominated by quantum effects).
But **dissipation**??

Dissipation in homogeneous quantum turbulence

A moving (oscillating) vortex emits sound.

Vortex reconnections emit sound

In a cold Bose-condensed gas phonon emission due to reconnections is sufficient to terminate the Kolmogorov cascade.

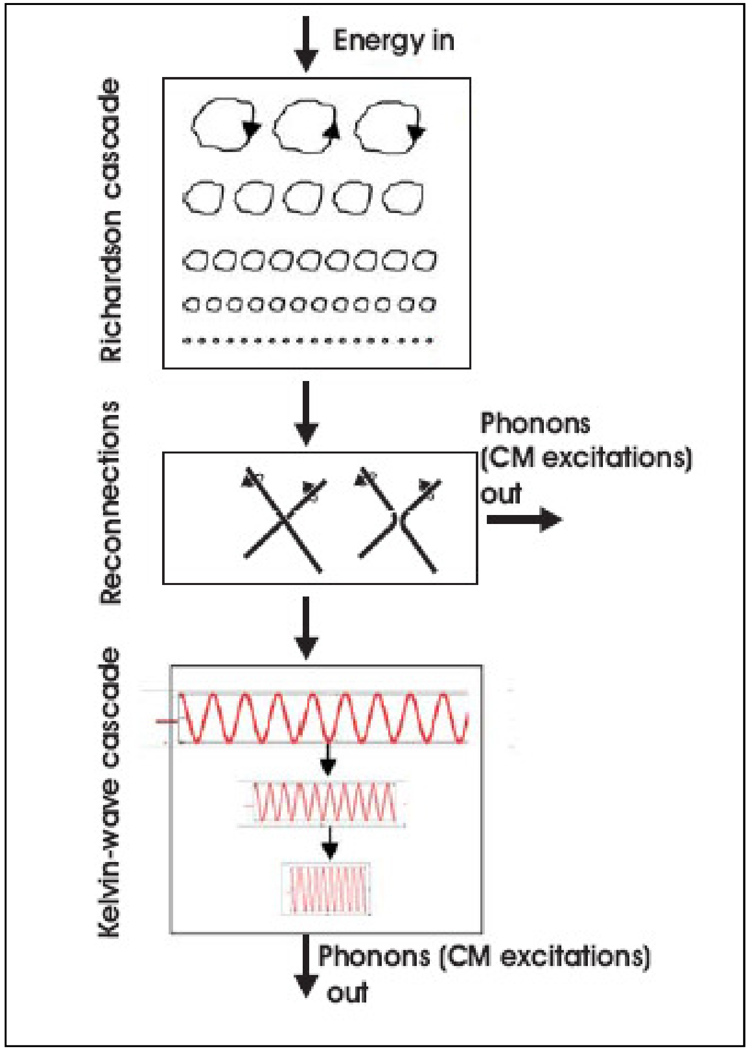
But not in superfluid ^{4}He

But reconnections also produce helical waves on vortices (Kelvin waves).

These Kelvin waves can build up in amplitude, start interacting, generating waves of higher and higher frequency. A process we call **wave turbulence**.

At a high enough frequency phonon emission becomes sufficient to terminate the wave turbulent spectrum.

All this is speculation. Exptal confirmation a challenge

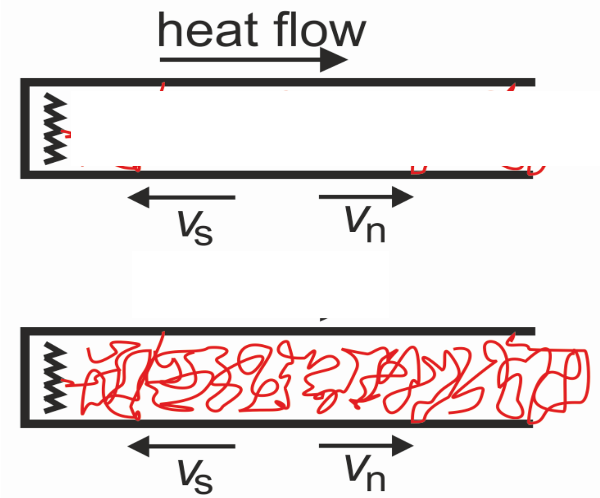


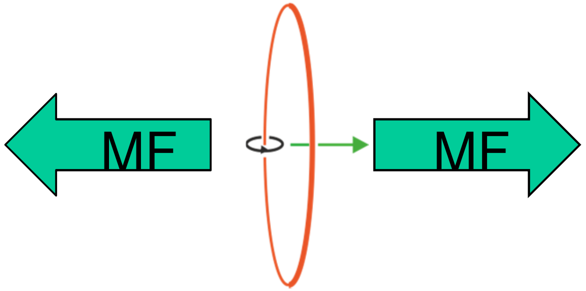
What happens at a finite *T*?

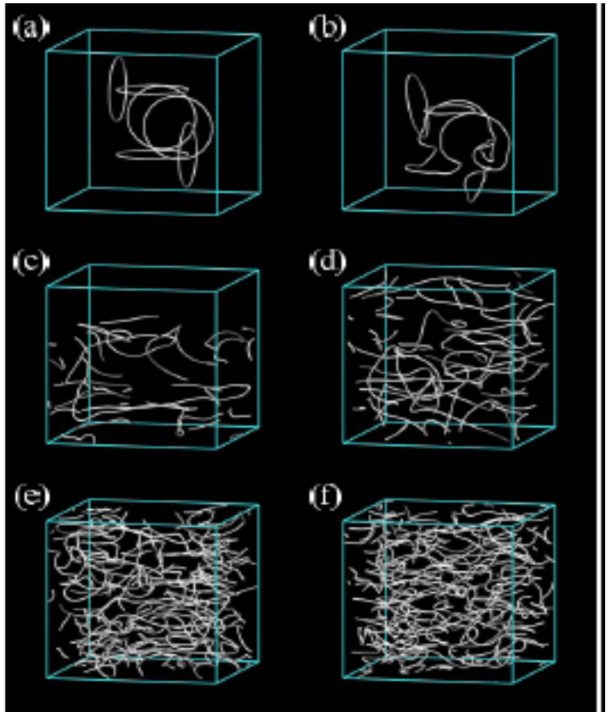
Forcing the two fluids to move at different velocities

Leads to the generation of ***self sustaining small-scale turbulence*** in superfluid component, with laminar flow of normal fluid.

The pioneering simulations of **Schwarz,** and the important developments in these simulations by **Tsubota**.







Crucial role of reconnections.

More recent developments: large-scale, partially coupled turbulence in both fluids**. A major challenge, for both experiment and theory**. (Coupled dynamics in numerical modelling especially challenging- Tsubota et al)

Why study quantum turbulence? The future.

New types of turbulence. Can we understand them? Can such understanding enrich our understanding of turbulence in general?

Can the discrete structure of quantum turbulence make it easier to study and understand than is the case with classical turbulence? For example can we see and understand better the non-linear processes that lead to turbulent energy cascades? Can we understand intermittency better? Can we look for and understand coherent structures?

This improved understanding will depend on our ability to ***see what is really going on in quantum turbulence***. This requires ***visualization of the flow***. For many years such visualization eluded us. But recent developments are changing this situation (Lathrop, Van Sciver, Wei Guo Skrbek): finding tracer particles that track the motion of one or other of the two fluids (as in the discovery of large scale turbulence in counterflow).

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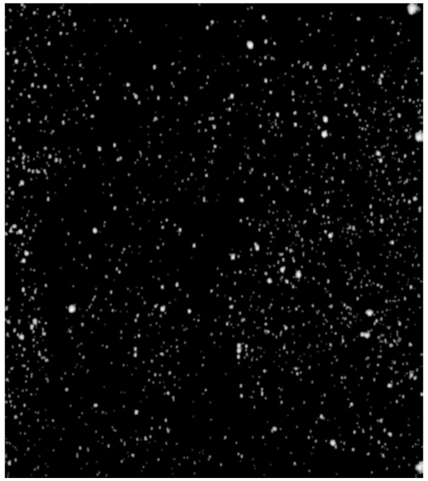

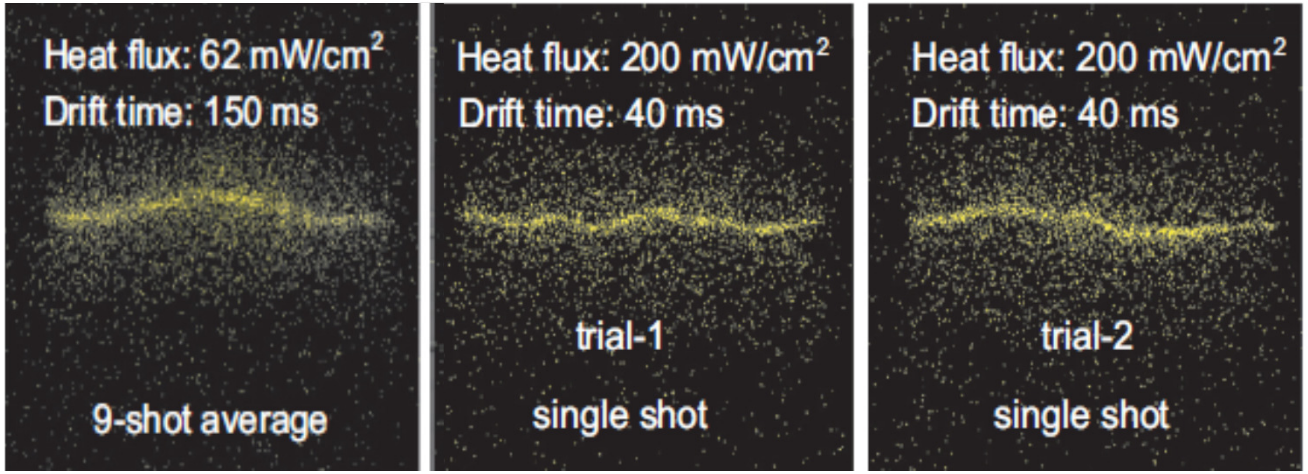

Exploiting these new techniques for visualizing flow patterns in a superfluid promise to transform the subject. After 80 years superfluidity is still interesting.

Thank you

Notes

Intermittency – inertial range decay into smaller and smaller eddies may not fill space.

Very high Rn.

Cold gases. 3He. Mention at end.