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Conference “Progress in QFT and String Theory”
@Osaka City univ.

Holographic Nuclei

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arXiv/1003.4988 N. Iizuka, P.Yi (KIAS), KH

M-theory for nuclear physics?

Problem

Nuclear properties from QCD?

Cause

QCD is strongly coupled

Our solution

QCD + AdS/CFT = holographic QCD

→ Matrix model describing multi nucleons

→ Nuclear properties

Big Buddha at Todaiji temple

Plan

1. Nucleus is matrix. 3 pages
2. Few-body nucleons. 2 pages
3. Nucleus. 5 pages

M-theory for Nuclear Physics?

[Iizuka,Yi, KH 1003.4988]

$$S = c\int dt\ \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 - \frac{g^2}{4}[X^I, X^J]^2\right] + \cdots$$

X^M $(M = 1, 2, 3, 4)$: *A* x *A* Hermitian matrix

$X^{M=1,2,3}$ eigenvalues = *A* baryons' location

Only 2 parameters : $M_{\mathrm{KK}},\ \lambda \equiv N_c (g_{\mathrm{QCD}})^2$

Nuclear physics

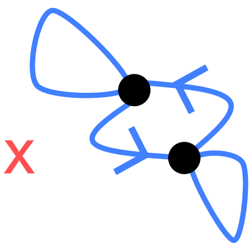
$$S \sim \int dt\left[\sum_{i=1}^{A}\frac{1}{2}m_N|\partial_t \vec{x}^{(i)}|^2 + \sum_{i\neq j} V(x^{(i)} - x^{(j)}, \cdots) + \cdots\right]$$

Nucleus is Matrix

Baryon is heavy at large N_c

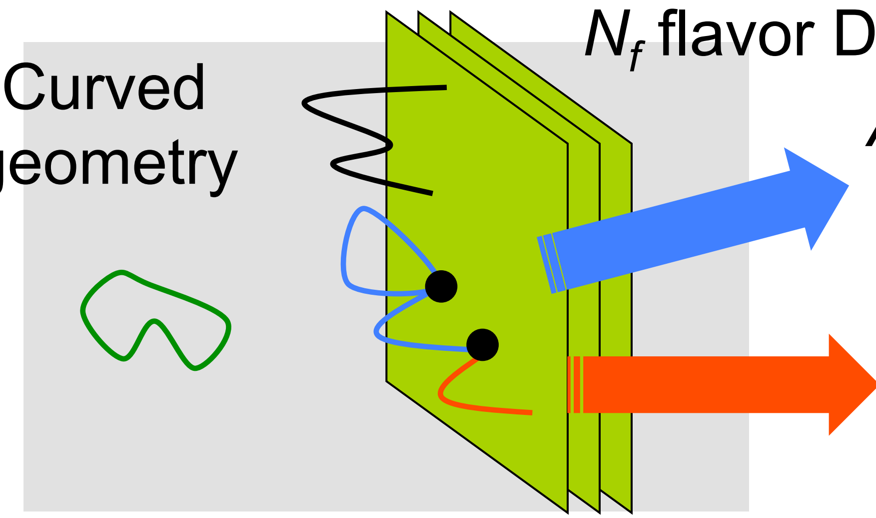
⇨ Baryon = D-brane

Multiple D-branes described by matrix



Nucleus is a matrix

Gravity side of AdS/CFT



N_f flavor D-branes

A x A matrix “X(t)”
Eigenvalues
= Baryon location

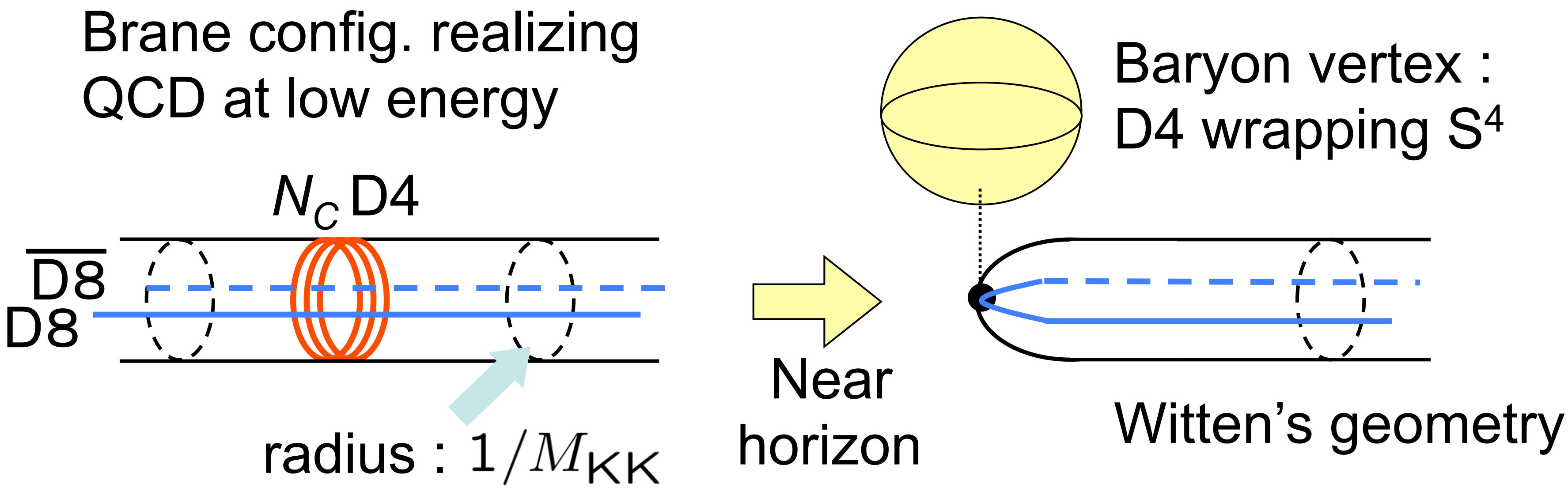
A x N_f matrix “w(t)”
Spin / Isospin

Stringy derivation

Effective action of *A* D4-branes in Witten's geometry
(with Sakai-Sugimoto flavor D8-branes)

[Iizuka,Yi, KH 1003.4988]

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt \, {\rm tr} \, \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot\alpha} D_0 w_{\dot\alpha i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot\alpha} w_{\dot\alpha i} - \frac{3^6 \pi^2}{4 \lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i \vec{D} \cdot \vec{\tau}^{\,\dot\alpha}{}_{\dot\beta} \bar{X}^{\dot\beta\alpha} X_{\alpha\dot\alpha} + i \vec{D} \cdot \vec{\tau}^{\,\dot\alpha}{}_{\dot\beta} \bar{w}_i^{\dot\beta} w_{\dot\alpha i} \Big] + 4 N_c \int dt \, {\rm tr} \, A_0$$



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Nucleon is fermion

[Iizuka, KH 1006.3612]

Rotation in 3dim. $w_{\dot\alpha i} \to U_{\dot\alpha}^{\ \dot\beta} w_{\dot\beta i}\,, \quad U = \exp\left[i\frac{\theta}{2}\tau^3\right]$ $\quad 0 \le \theta \le 2\pi$

= "electron" in a magnetic field $\quad D_0 w_{\dot\alpha i} \equiv \partial_0 w_{\dot\alpha i} - i w_{\dot\alpha i} A_0$

⇨ Phase of nucleon wave function = AB phase = $N_c\pi$

A=1 : Baryon spectrum

[Iizuka,Yi, KH 1003.4988]
Cf: [Hata,Sakai,Sugimoto,Yamato]

Spectrum from the Hamiltonian :

$$M = M_0 + \frac{1}{\sqrt{6}}\left[\sqrt{(I/2+1)^2 + N_c^2} + 2n_\rho + 2n_{X^4} + 2\right]$$

⇨ $I = J = 1/2 \quad 940^+, 1359^+, 1359^-, 1778^+, 1778^-, \ldots$

$M_N, M_\Delta \quad$ Experiments : $940^+, 1440^+, 1535^-, 1710^+, 1655^-, \ldots$

A=2 : Repulsive core

[Iizuka,Yi, KH 1003.4988]
[Iizuka, S.Aoki, KH, 1203.5386]

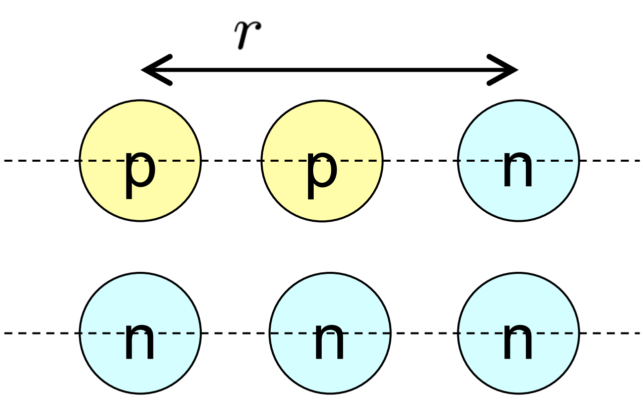
$$V_{\mathrm{T}}(\vec{r}) = 2\pi I_1^i I_2^i \frac{N_c}{\lambda M_{\mathrm{KK}}} \frac{1}{r^2}$$

$$V_{\mathrm{C}}(\vec{r}) = \pi \left(\frac{3^3}{2} - 8 I_1^i I_2^i J_1^j J_2^j \right) \frac{N_c}{\lambda M_{\mathrm{KK}}} \frac{1}{r^2}$$

Repulsive core, $1/r^2$ dependence, universal for multi flavors

A=3 : three-body force

[Iizuka, KH,1005.4412]



spins averaged

He-3 nucleus

$$\langle V_{3-\mathrm{body}} \rangle = \frac{2^{5/2} 3^{9/2} 5 \pi^2 N_c}{\lambda^2 M_{\mathrm{KK}}^3 |r|^4}$$

Neutron stars

$$\langle V_{3-\mathrm{body}} \rangle = \frac{2^{-1/2} 3^{15/2} \pi^2 N_c}{\lambda^2 M_{\mathrm{KK}}^3 |r|^4}$$

Positive !

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arXiv/1103.5688 T.Morita, KH

Holographic nucleus : Heavier, simpler

Large *A* → *w(t)* drops off
→ Dimensionally reduced Yang-Mills

$$S = c\int dt\ \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 - \frac{g^2}{4}[X^I, X^J]^2\right]$$

Question :

Solve this quantum mechanics at large *A*.

Formation of holographic nucleus

$$S = c \int dt \ \mathrm{tr} \left[\frac{1}{2} (D_t X^I)^2 - \frac{g^2}{4} [X^I, X^J]^2 \right]$$

Classical : diagonal $X^I = \begin{pmatrix} x^I_{(1)} & & \\ & x^I_{(2)} & \\ & & \ldots \end{pmatrix}$

Quantum mechanical : not diagonal, because…

For large diagonal value *x* and take $X = \begin{pmatrix} x & \delta x \\ \delta x & -x \end{pmatrix}$

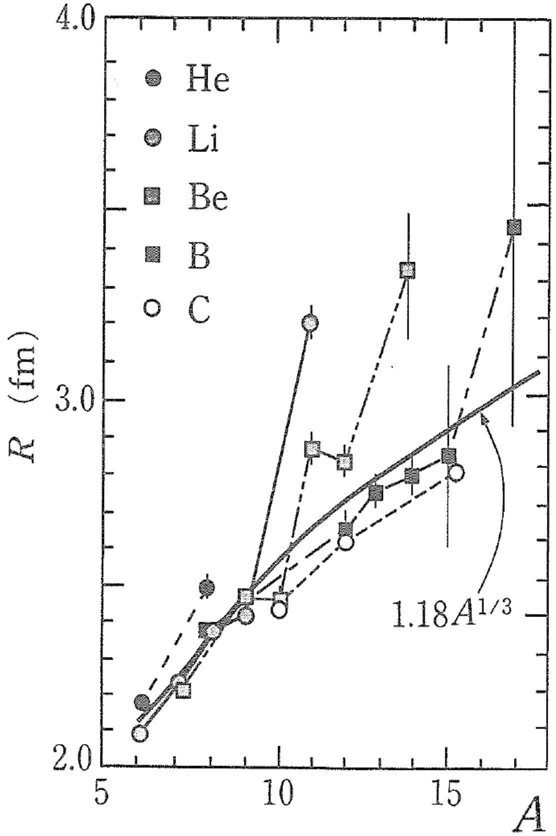
the potential is too narrow : $V \sim g^2 x^2 (\delta x)^2$

[Luscher, 1983]

Quantum Vacuum : Eigenvalues accumulate

= Nucleus inevitably forms

Holographic nuclear radius $\propto \mathbf{A}^{1/3}$



[『大学院原子核物理』(講談社)]

$$\sqrt{r^2_{\rm mean}} \simeq 1.0 A^{1/3} \ [\rm fm]$$

$$S = c \int dt \ {\rm tr} \left[\frac{1}{2}(D_t X^I)^2 - \frac{g^2}{4}[X^I, X^J]^2 \right]$$

'tHooft expansion: fixed $\lambda_A \equiv A g^2$

→ Dimensional analysis

$$\frac{1}{A^2} {\rm tr}\langle X^I X^I \rangle = c \lambda_A^{-1/3} + \cdots$$

→ Nuclear radius, saturation beahvior

$$\sqrt{r^2_{\rm mean}} \equiv \sqrt{\frac{1}{A} {\rm tr}\langle X^I X^I \rangle} \propto A^{1/3}$$

Analytic formula for the radius

Large *D*, Large *A* expansion （ $g^2 A D$ fixed）

(known technology) [9811220 Hotta,Nishimura,Tsuchiya]
[0910.4526 Mandal,Mahato,Morita]

Result :
$$\left.\sqrt{r^2_{\rm mean}}\right|_{T=0} = \frac{3^{5/2}\pi^{2/3}}{2^{5/6}5^{1/6}} \frac{A^{1/3}}{M_{\rm KK}(N_c\lambda^2)^{1/3}}$$

Input :
- Rho meson mass $M_{\rm KK} \sim 0.95[\rm GeV]$
- Yukawa $g_{\pi NN}$ $\lambda \sim 5.3$

⇨ $$\sqrt{r^2_{\rm mean}} \sim 0.7 A^{1/3}\ [\rm fm]$$

Experiments : $\sqrt{r^2_{\rm mean}} \simeq 1.0 A^{1/3}\ [\rm fm]$

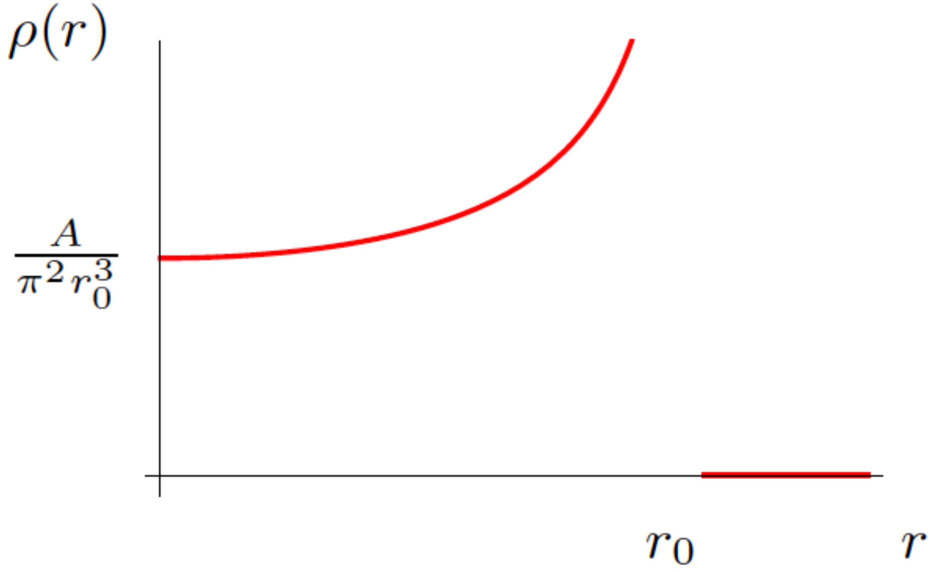
Nuclear density?

We may "try" D-brane distribution formula of M(atrix)-theory

[Taylor, vanRaamsdonk]

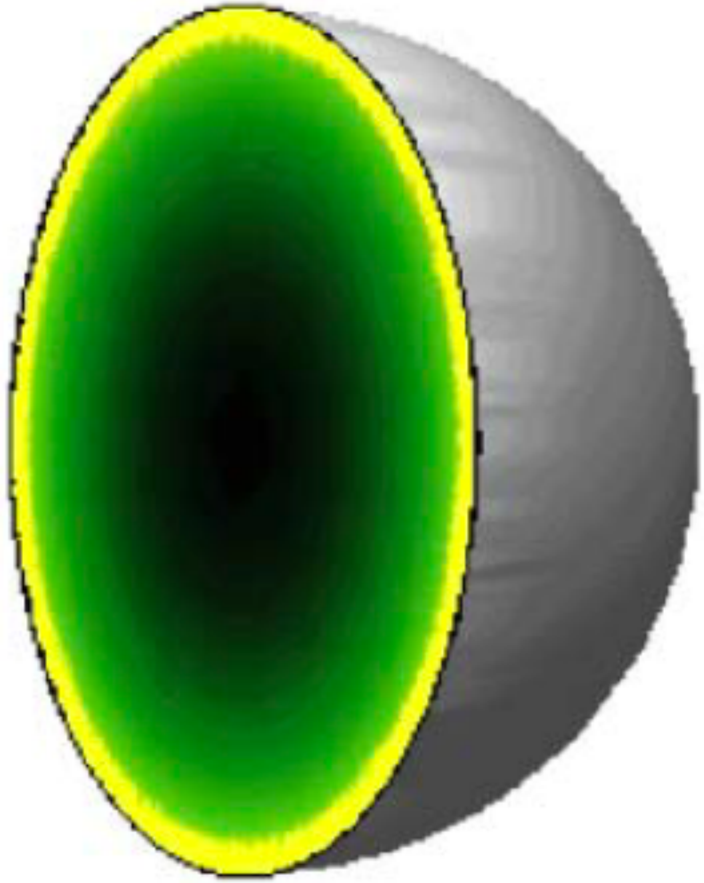
$$\rho(x) = \int d^3k \, e^{-ik\cdot x} \, \mathrm{tr} \, \exp[ik \cdot X]$$

$$= \begin{cases} \dfrac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} & (r < r_0) \\ 0 & (r_0 < r) \end{cases}$$

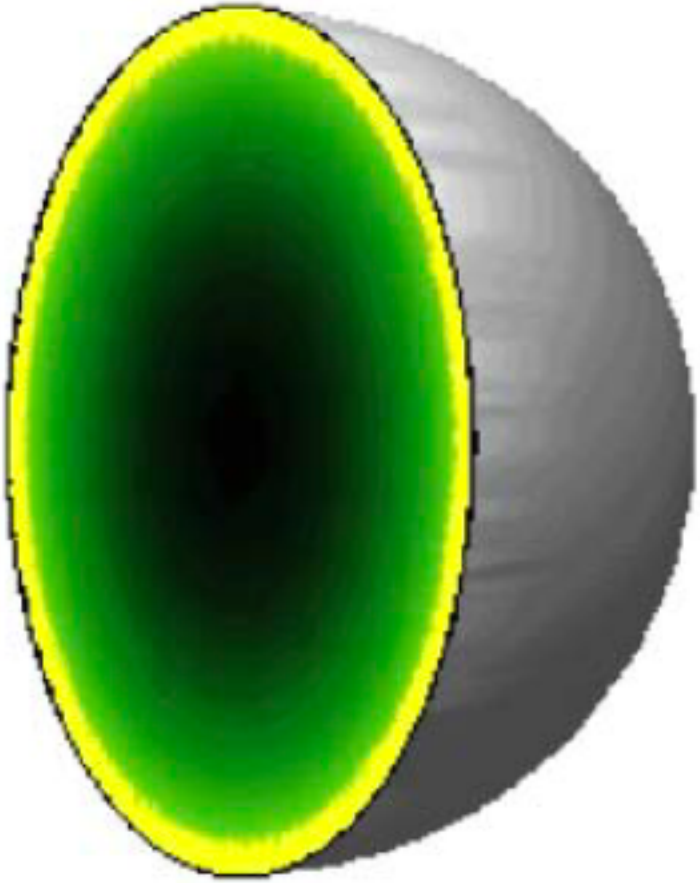


Finiteness of the nucleus.

$$r_0 \sim 0.8 \times A^{1/3} \; [\mathrm{fm}]$$



M-theory for Nuclear Physics?



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