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Killing-Yano symmetry in supergravity theories

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Ref.

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Key word

Killing-Yano symmetry with torsion

Hidden symmetry of vacuum BHs

- ***Vacuum BHs***

  (with S^n horizon topology)

$$R_{ab} + \frac{1}{2} g_{ab} R = 0$$

Kerr BH (4D), Myers-Perry BHs (higher dimensions)

■ ***Integrability on these backgrounds***

□ *Separation of variables in the H-J, K-G and Dirac Eqs.*

Killing-Yano symmetry

(closed conformal Killing-Yano (CCKY) 2-form)

■ ***The only vacuum solution with a CCKY 2-form***

⇒ *Vacuum BHs are completely characterized by KY symmetry.*

Hidden symmetry of charged BHs

- ***Charged BHs in supergravities***

  (with S^n horizon topology)

$$R_{ab} + \frac{1}{2} g_{ab} R = \kappa T_{ab}$$

Ex. Chong-Cvetic-Lu-Pope BH (5D minimal SUGRA)

Kerr-Sen BH (abelian heterotic SUGRA)

■ ***Integrability on these backgrounds***

□ *Separation of variables in the H-J, K-G and "Dirac" Eqs.*

Killing-Yano symmetry with torsion

(closed conformal Killing-Yano (CCKY) 2-form with torsion)

⇒ *Charged BHs are characterized by KY symmetry with torsion.*

What is KY symmetry?

Isometry		Killing-Yano symmetry
Killing vectors (vectors) $\nabla_a k_b + \nabla_b k_a = 0$	→	**Killing-Yano forms** (antisymmetric tensors) $\nabla_a k_{bc_1 \cdots c_{p-1}} + \nabla_b k_{ac_1 \cdots c_{p-1}} = 0$

Yano (1954)

Similarly,

Conformal Killing vectors (vectors)	→	**Conformal Killing-Yano (CKY) forms** (antisymmetric tensors)

$$\nabla_X k = \frac{1}{p+1} \iota_X dk - \frac{1}{D-p+1} X^* \wedge \delta k$$

Tachibana (1969), Kashiwada (1968)

KY symmetry with torsion

<u>*CKY p-form equation*</u>

$$\nabla_X k = \frac{1}{p+1}\iota_X dk - \frac{1}{D-p+1} X^* \wedge \delta k \ , \quad \text{for } {}^\forall X$$

$$\nabla_X k \to \nabla^T_X k = \nabla_X k - \frac{1}{2}\sum_a (\iota_X \iota_{e_a} T) \wedge (\iota_{e_a} k)$$

$$dk \to d^T k = dk - \sum_a (\iota_{e_a} T) \wedge (\iota_{e_a} k)$$

$$\delta k \to \delta^T k = \delta k - \frac{1}{2}\sum_{a,b} (\iota_{e_a} \iota_{e_b} T) \wedge (\iota_{e_a} \iota_{e_b} k)$$

- ***CKY p-form with torsion equation***

Kubiznak, Kunduri and Yasui (2010)

$$\nabla^T_X k = \frac{1}{p+1}\iota_X d^T k - \frac{1}{D-p+1} X^* \wedge \delta^T k \ , \quad \text{for } {}^\forall X$$

What we want to do/ we did

- Construct physically and mathematically interesting solutions in various supergravity theories, by using Killing-Yano symmetry with torsion.
- To do so, we attempted to classify spacetimes admitting a “closed” conformal Killing-Yano 2-form with torsion.

Results

- We reduced the classification problem to one algebraic Eq and a set of (integrable, coupled, nonlinear) PDEs.

Type A

A-1. *In D=2n+ε dimensions*

$$ds^2 = \sum_{\mu=1}^{n} \frac{U_\mu}{X_\mu} dx_\mu^2 + \sum_{\mu=1}^{n} \frac{f_\mu^2 X_\mu}{U_\mu} \Big(\sum_{k=0}^{n-1} A_\mu^{(k)} d\psi_k - \mathcal{A}_{(1)} \Big)^2$$
$$+ \varepsilon \left(\frac{k_1}{\sigma} \Big(\sum_{k=0}^{n} A^{(k)} d\psi_k - \mathcal{A}_{(1)} \Big) + k_2\, \sigma \Big(d\psi_n - \mathcal{A}_{(1)} \Big) \right)^2$$
$$\mathcal{A}_{(1)} = \frac{1}{\Phi} \sum_{\mu=1}^{n} \frac{N_\mu}{U_\mu} \sum_{k=0}^{n-1} A_\mu^{(k)} d\psi_k \,, \quad \Phi = 1 + \sum_{\mu=1}^{n} \frac{N_\mu}{U_\mu} \,, \quad \sigma = \prod_{\rho=1}^{n} x_\rho$$

- Includes ***3n*** unknown functions $f_\mu(x_\mu)$, $N_\mu(x_\mu)$ and $X_\mu(x_\mu)$ which depend on one variable.
- Covers Kerr-Sen BH metrics in abelian heterotic supergtavity.

A-2. *In odd dimensions*

$$ds^2 = \sum_{\mu=1}^{n} \frac{U_\mu}{X_\mu} dx_\mu^2 + \sum_{\mu=1}^{n} \frac{X_\mu}{U_\mu} \Big(\sum_{k=0}^{n-1} A_\mu^{(k)} d\psi_k \Big)^2$$
$$+ k^2\, \sigma^2 \Big(d\psi_n - \mathcal{B}_{(1)} \Big)^2$$
$$\mathcal{B}_{(1)} = \sum_{\mu=1}^{n} \frac{N_\mu}{U_\mu} \sum_{k=0}^{n-1} A_\mu^{(k)} d\psi_k \,, \quad \sigma = \prod_{\rho=1}^{n} x_\rho$$

- Includes ***2n*** unknown functions $N_\mu(x_\mu)$ and $X_\mu(x_\mu)$ which depend on one variable.
- Covers charged BH metrics in 5D minimal supergravity.

Type B & C

B. *In D=2n+ε dimensions*

$$ds^2 = \sum_{\mu=1}^{n} U_\mu \left(\frac{dx_\mu^2}{X_\mu} + \frac{dy_\mu^2}{Y_\mu}\right) + \varepsilon \left(\prod_{\rho=1}^{n} x_\rho^2\right) dz^2$$

- Includes **2n** unknown functions $X_\mu(x_\mu,y_\mu)$ and $Y_\mu(x_\mu,y_\mu)$ which depend on two variables.

C. *In 4 dimensions*

$$ds^2 =(x^2 - y^2)\left(\frac{dx^2}{X(x,\psi_1)} + \frac{dy^2}{Y(y)} + \Psi_1(x)^2 d\psi_1^2 + \Psi_2(y)^2(-\Xi_1(x)d\psi_1 + d\psi_2)^2\right)$$

- Includes **5** unknown functions $X(x,\psi_1), Y(y), \Psi_1(x), \Psi_2(y)$ and $\Xi_1(x)$.

C. *In 5 dimensions*

$$ds^2 =(x^2 - y^2)\left(\frac{dx^2}{X(x,\psi_1)} + \frac{dy^2}{Y(y)} + \Psi_1(x)^2 d\psi_1^2 + \Psi_2(y)^2(-\Xi_1(x)d\psi_1 + d\psi_2)^2\right) + x^2y^2\Big(d\psi_0 - \Xi_2(y)(-\Xi_1(x)d\psi_1 + d\psi_2)\Big)^2$$

- Includes **6** unknown functions $X(x,\psi_1), Y(y), \Psi_1(x), \Psi_2(y), \Xi_1(x)$ and $\Xi_2(y)$.

Recent developments (1)

- We found that (toric) Sasaki-Einstein metrics admit Killing-Yano symmetry with torsion.

(CCKY 3-form with torsion)

with Takeuchi and Yasui

- **Einstein** (Ric=Λ g)

  Ex. $T^{1,1},\ Y^{p,q},\ L^{abc},\ (S^5)$

- **Kahler sandwich structure**

2n+1 d Sasaki

Kahler

$$g_{cone} = dr^2 + r^2 g_{2n+1}$$

$$g_{2n+1} = g_{2n} + (d\psi + A)^2$$

Kahler

Recent developments (2)

- Employing Killing-Yano symmetry with torsion, we constructed new solutions of 5D and 11D supergravities which are deformations from Sasaki metrics.

with Takeuchi and Yasui

- **5d minimal SUGRA and 11d SUGRA**
- **Kahler with torsion sandwich structure**

Our solutions

KT

$$g_{cone} = dr^2 + r^2 g_{2n+1}$$

$$g_{2n+1} = g_{2n} + (d\psi + A)^2$$

KT

Summary & Outlook

- Killing-Yano symmetry completely characterizes vacuum BHs.
- (Some) charged BHs in supergravity theories admit generalized Killing-Yano symmetry with torsion.
- We have classified spacetimes admitting KY symmetry with torsion (a CCKY 2-form with torsion) and obtained three types of the metrics.
- We have constructed new solutions of 5D and 11D SUGRA, which generalize (toric) SE metrics.
- Application of Sasaki with torsion to AdS/CFT?