Quantum Gravity and the Naturalness Problem

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I will discuss the low energy effective action of quantum gravity and the possibility of solving the naturalness problem by using it.

Contents

1. Low energy effective theory of quantum gravity / string theory

Low energy effective theory of quantum gravity / string theory is obtained by integrating out the short distance physics.

Because of the symmetry, it should be

$$S_{\text{eff}} = \int d^D x \sqrt{g} \left(\kappa R + \Lambda + \text{gauge} + \text{matter} + \ldots \right)$$

Is that all?

Usually, action is additive.

$$S = S_0 + S_{\text{int}},$$

$$S_0 = \int d^4 x \left(\frac{-1}{4} F_{\mu\nu}{}^2 + \overline{\psi} \gamma^\mu \partial_\mu \psi \right),$$

$$S_{\text{int}} = \int d^4 x \left(e A_\mu \overline{\psi} \gamma^\mu \psi \right).$$

why not

$$S = S_0 \, S_{\text{int}}.$$ (Sugawara ~1982)

By replacing each factor to its expectation value, we have

$$S_{\text{eff}} \simeq \langle S_{\text{int}} \rangle S_0 + \langle S_0 \rangle S_{\text{int}} - \langle S_0 \rangle \langle S_{\text{int}} \rangle.$$

The coupling constant is determined by the history of the universe.

Actually, in quantum gravity or matrix model, there are some mechanisms that the low energy effective theory becomes

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} \frac{1}{2} c_{ij} S_i S_j + \sum_{ijk} \frac{1}{6} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

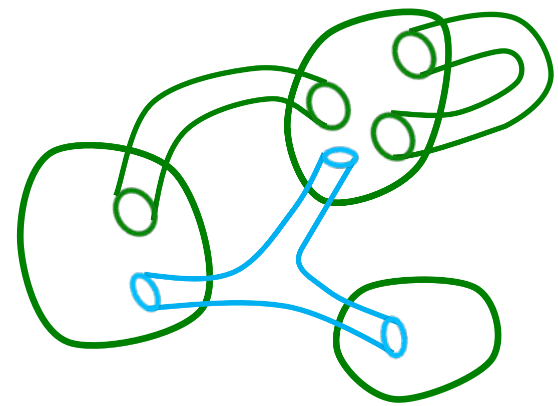
$O_i(x)$: local operators

Multilocal action from wormhole

Coleman

Path integral of quantum gravity

$$\sum_{\text{topology}} \int [dg] \exp(-S)$$



Effect of a wormhole can be expressed as

$$\int [dg] \sum_{i,j} \frac{1}{2} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x) O^j(y)\, \exp(-S) \ .$$

Summing up wormholes, we have

$$\int [dg] \exp\left(-S + \frac{1}{2} \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x) O^j(y) \right) .$$

bifurcated wormholes

$\Rightarrow$ cubic terms, quartic terms, …

Multilocal action from IIB matrix model (1)

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu, \Psi]\right)$$

Various possibilities for the emergence of space-time

(1) A_μ as the space-time coordinates

mutually commuting A_μ $\Rightarrow$ space-time

(2) A_μ as non-commutative space-time

non-commutative A_μ $\Rightarrow$ NC space-time

(3) A_μ as derivatives

Multilocal action from IIB matrix model (2)

A_μ as derivative

Hanada, HK, Kimura

A_μ can be regarded as a covariant derivative on any manifold with less than ten dimensions.

$$(A_a\varphi)_\alpha = C_{(a)\alpha}{}^{b,\beta}\nabla_b\varphi_\beta$$

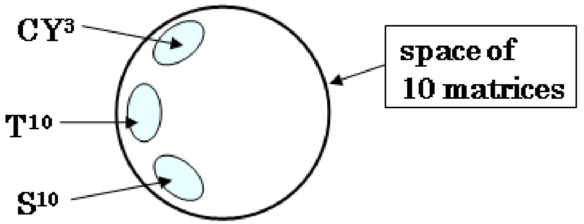
φ_α : regular representation field on manifold *M*

$C_{(a)\alpha}{}^{b,\beta}$, $(a=1,..,D)$: the Clebsh-Gordan coefficients

$V_{\text{vector}}\otimes V_r \simeq V_r\oplus\cdots\oplus V_r$ *r* : regular representation

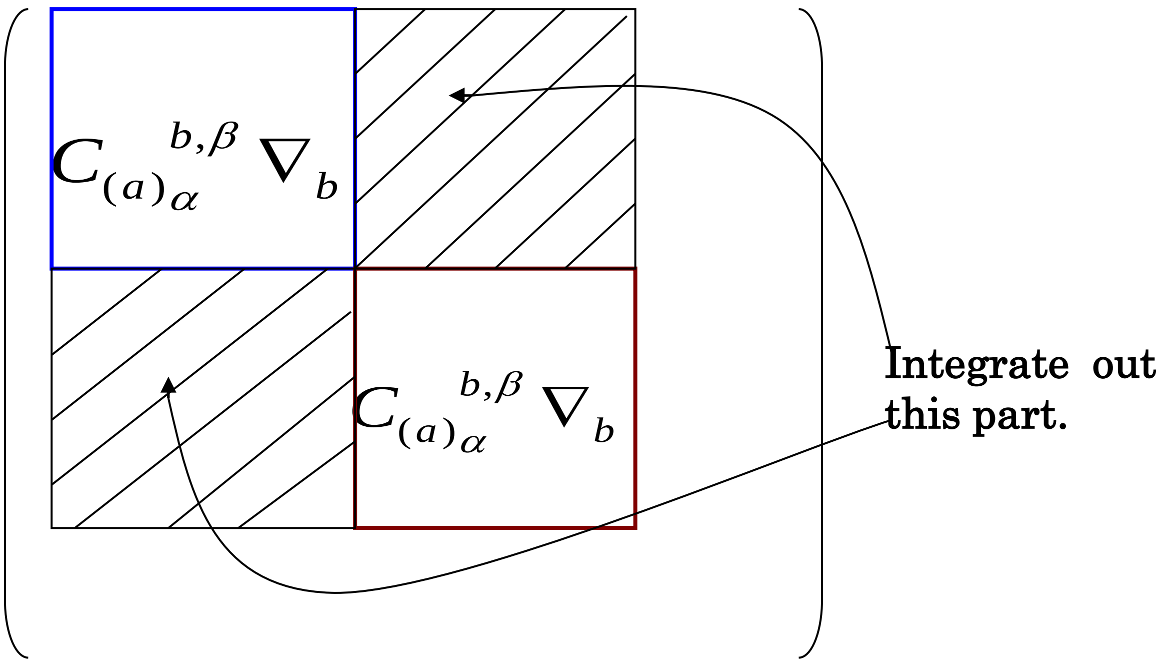
ex. derivative on flat space

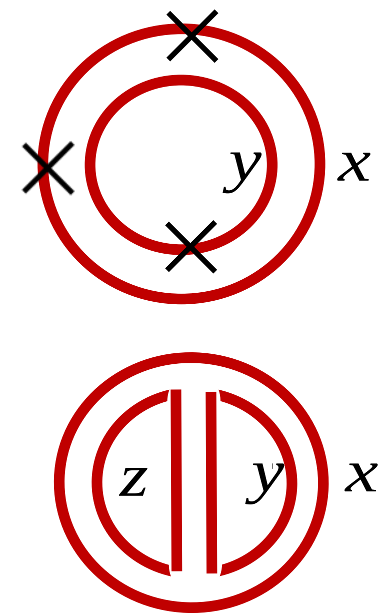
$$(A_a)_\alpha{}^\beta = C_{(a)\alpha}{}^{b,\beta}\partial_b$$



Multilocal action from IIB matrix model (3)

integration of (off) diagonal blocks





The path integral gives

$$S_{\text{eff}} = \sum_i c_i\, S_i + \sum_{i\,j} c_{i\,j}\, S_i S_j + \sum_{i\,jk} c_{i\,jk}\, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)}\, O_i(x).$$

Y. Asano and HK
(to appear)

We have seen

the effective action of quantum gravity/string is given by

$$S_{\text{eff}} = \sum_i c_i\, S_i + \sum_{i\,j} c_{i\,j}\, S_i S_j + \sum_{i\,jk} c_{i\,jk}\, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)}\, O_i(x).$$

the coupling constants are determined by the state

$$S_{\text{eff}} \simeq \sum_i c_i S_i + \sum_{i,j} 2c_{i\,j} \langle S_i \rangle S_j + \sum_{i,j,k} 3c_{i\,jk} \langle S_i \rangle \langle S_j \rangle S_k + \cdots.$$

More precisely, the path integral is given by

$$Z = \int [d\phi] \exp\left(i\, S_{\text{eff}}\right) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i\, S_i \right).$$

Coupling constants are not merely constant but to be integrated.

<u>the question</u>

Is there a natural or the most probable state?

Are some special values of the coupling constants favored?

In the following, as an ad hoc assumption, we postulate the probabilistic interpretation for the universe wave function.

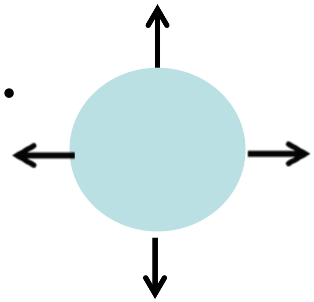
2. Finding the most probable state

What is the most probable (natural) state?

The case of the ordinary field theory

Ground state is the most probable or natural.

Our vacuum is close to the ground state.
This is because the universe is expanding,
and our vacuum is being cooled down.



minimum of the Hamiltonian H

$$\downarrow\uparrow$$

automatically chosen by the Euclidean path integral

$$Z=\int[d\phi]\exp\left(-S_E\right)\sim e^{-TE_0}$$

The case of quantum gravity / string theory

Total energy is always zero:

WDW eq. $$H_{\text{total}} |\Psi\rangle = 0$$

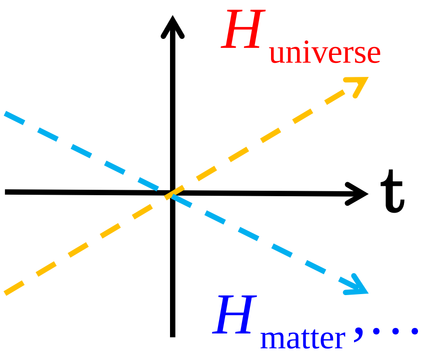
$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}} + \cdots$$

$$H_{\text{universe}} = -z\left(\frac{1}{2} p_z^{\ 2} + \cdots\right)$$ **←wrong sign**

⇓ $z = a^3$: volume of the universe

"Ground state" does not make sense.

Wick rotation is not well defined.



⇒ We reexamine the Lorentzian wave function.

cf. Coleman's original argument

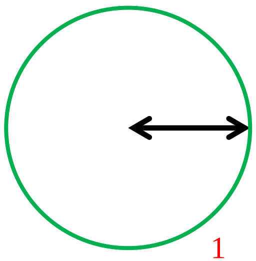
The Euclidean path integral is dominated by the 4-sphere.

⇒ strong peak at $\Lambda=0$.

single universe

$$Z_{\text{universe}} = \int d\Lambda\, w(\Lambda) \int [dg] \exp\left(-\int d^4x \sqrt{g}\,(R+\Lambda)\right)$$

$$\sim \int d\Lambda\, w(\Lambda) \exp\left(\frac{1}{\Lambda}\right). \qquad \leftarrow S_E\left(\mathrm{S}^4\right) = -\frac{1}{\Lambda}$$



$r \sim \Lambda^{-\frac{1}{2}}$

multiverse

$$Z_{\text{multi}} \sim \int d\Lambda\, w(\Lambda) \exp\left(\exp\left(\frac{1}{\Lambda}\right)\right).$$

However,

S^4 is a bounce that describes an emerging universe.

Tunneling seems enhanced! ← wrong Wick rotation.

Wave function of a universe

We assume that the universe emerges with a small size ε.

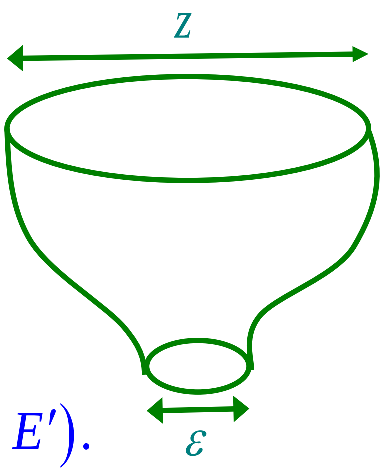
$$\psi(z,\lambda) = \mu_0 \int_{z(0)=\varepsilon,\, z(1)=z} [dz\,dp\,dN] \exp\left(i \int_0^1 dt \left(p\dot{z} - N H_\lambda \right) \right)$$

$$= \mu_0 \langle z | \delta(\hat{H}_\lambda) | \varepsilon \rangle \qquad \hat{H}_\lambda = -\frac{1}{2}\sqrt{z}\,p^2\sqrt{z} + \cdots$$

Forget about λ for a while.

$$= \mu_0 \phi^*_{E=0}(\varepsilon)\, \phi_{E=0}(z) \qquad \hat{H}|\phi_E\rangle = E|\phi_E\rangle,\ \langle \phi_E | \phi_{E'} \rangle = \delta(E-E').$$

$$= \mu\, \phi_{E=0}(z)$$



z : volume of the universe

The wave function of the universe is given by

$$|\psi\rangle = \mu |\phi_{E=0}\rangle,$$

μ : probability amplitude of a universe emerging.

Probabilistic interpretation (1)

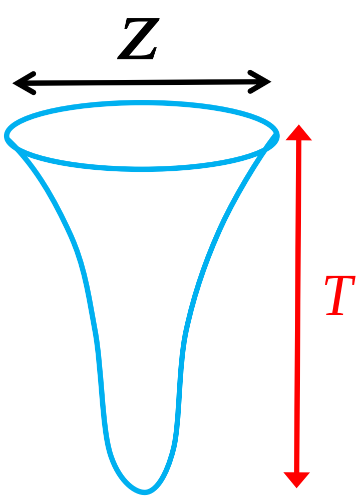
postulate $\psi(z) = \mu\,\phi_{E=0}(z)$

$$|\psi(z)|^2 dz \propto \text{probability of finding a universe of size } z$$

meaning of this measure

$$\int dz\,|\phi_{E=0}(z)|^2 \sim \int dz \frac{1}{z\,p(z)}$$

$$\phi_{E=0}(z) \sim \frac{1}{\sqrt{z\,p(z)}} \exp\left(i\int^z dz'\,p(z')\right)$$



$$= \int dz \frac{1}{\dot{z}} = \int dT$$

$$H = z\left(-\frac{1}{2}p^2 + \cdots\right) \to \dot{z} = \frac{\partial H}{\partial p} = -zp$$

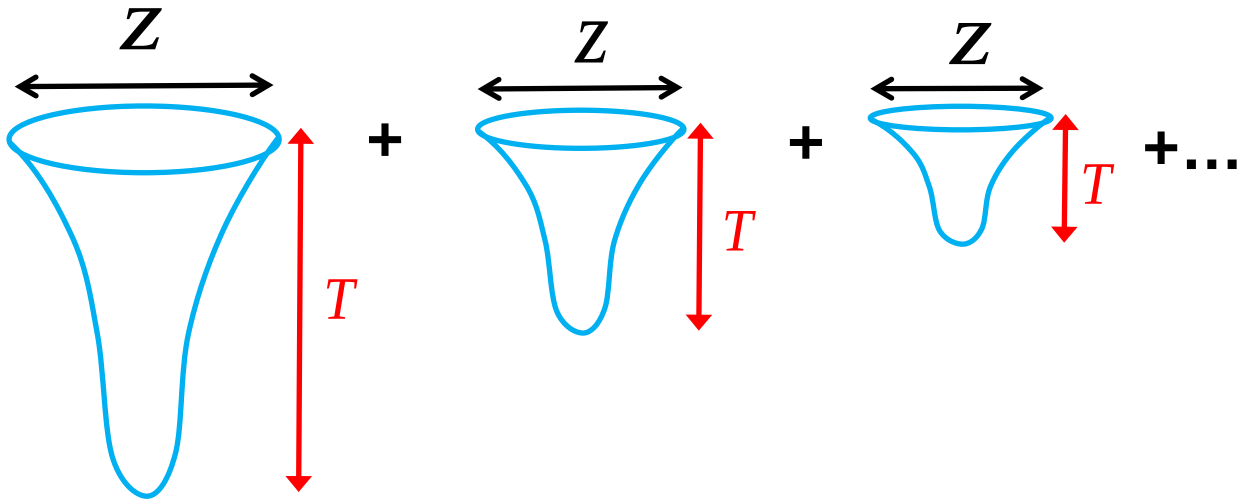
T : age of the universe
the time that has passed
after the universe is created

$$\Rightarrow |\psi(z)|^2 dz \sim |\mu|^2 dT$$

$|\mu|^2 =$ probability of a universe emerging in unit time

Probabilistic interpretation (2)

$|\psi\rangle$ **is a superposition of the universe with various age,**

$|\psi\rangle =$ 

$\left|\psi(z)\right|^2 dz \sim \left|\mu\right|^2 dT$ **gives the probability of finding a universe of age** $T \sim T + dT$ **.**

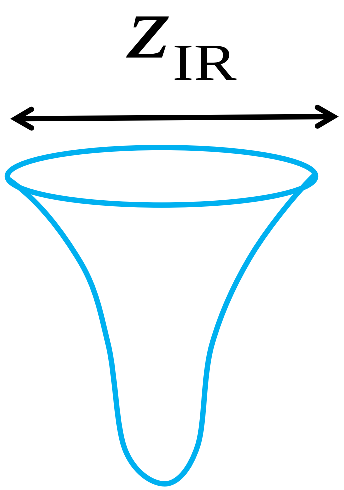
Lifetime of the universe

$$\int \left|\psi(z)\right|^2 dz \sim |\mu|^2 \int dT = |\mu|^2 \times (\text{life time of the universe})$$

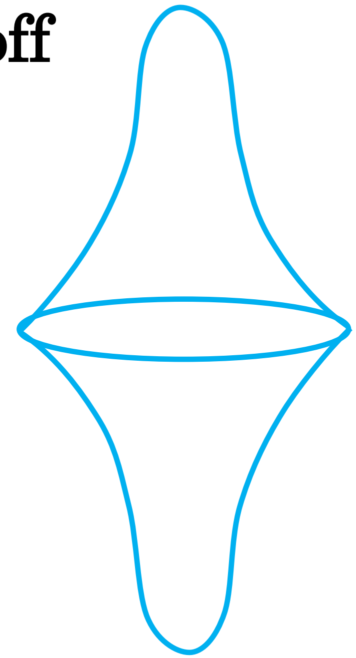
dimensionless

infrared cutoff

We introduce an infrared cutoff for the size of universes.



ceases to exist or

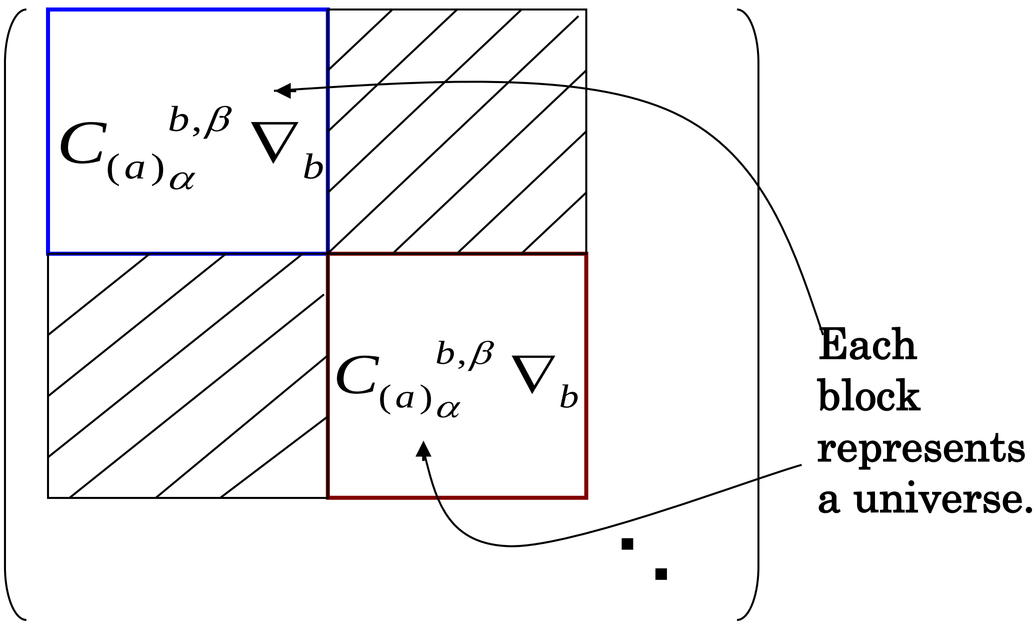


bounces back

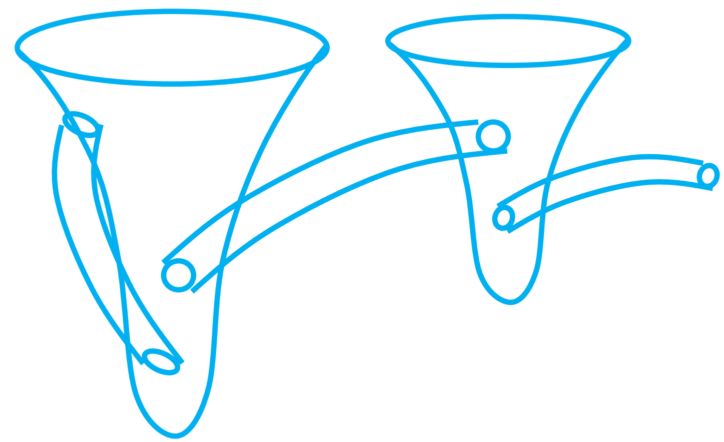
Wave Function of the multiverse (1)

Okada, HK

Multiverse appears naturally in quantum gravity / string theory.



matrix model



quantum gravity

Wave Function of the multiverse (2)

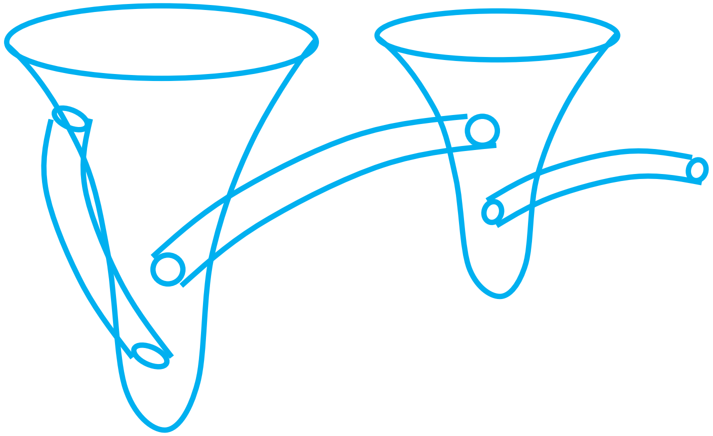
The multiverse sate is a superposition of N-verses.

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle \quad \leftarrow Z = \int d\lambda w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)$$

$$\Psi_N\left(z_1, \cdots, z_N, \lambda\right) = \psi\left(z_1, \lambda\right) \cdots \psi\left(z_N, \lambda\right)$$

$$\left|\psi, \lambda\right\rangle \otimes \left|\psi, \lambda\right\rangle \otimes \cdots$$

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_N$$



Wave Function of the multiverse (3)

Probabilistic interpretation

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle$$

$$\Psi_N(z_1, \cdots, z_N, \lambda) = \psi(z_1, \lambda) \cdots \psi(z_N, \lambda)$$

$\left|\Psi_N(z_1, \cdots, z_N, \lambda)\right|^2 dz_1 \cdots dz_N \left|w(\lambda)\right|^2 d\lambda$ **represents**

the probability of finding N universes with size

$$z_1 \sim z_1 + dz_1,\ \cdots,\ z_N \sim z_N + dz_N$$

and finding the coupling constants in

$$\lambda \sim \lambda + d\lambda .$$

Probability distribution of λ

$$P(\lambda) = \sum_{N=0}^{\infty} \int \frac{dz_1 \cdots dz_N}{N!} \left|\Psi_N(z_1,\cdots,z_N,\lambda)\right|^2 \left|w(\lambda)\right|^2$$

$$= \exp\left(\int dz \left|\psi(z,\lambda)\right|^2\right) \left|w(\lambda)\right|^2 \qquad \leftarrow \Psi_N(z_1,\cdots,z_N,\lambda) = \psi(z_1,\lambda)\cdots\psi(z_N,\lambda)$$

$$= \exp\left(\left|\mu\tau(\lambda)\right|^2\right) \left|w(\lambda)\right|^2 \qquad \leftarrow \left|\psi\right\rangle = \mu\left|\phi_{E=0}\right\rangle$$

$$\tau(\lambda) = \int dz \left|\phi_{E=0}(z,\lambda)\right|^2 \sim \text{(life time of the universe)}$$

$\tau(\lambda)$ can be very large.

⬇

λ is chosen in such a way that $\tau(\lambda)$ is maximized, irrespectively of $w(\lambda)$.

We have seen

the coupling constants are chosen in such a way that the lifetime of the universe becomes maximum.

the question

What values of the coupling constants make the lifetime maximum?

3. Naturalness and Big Fix

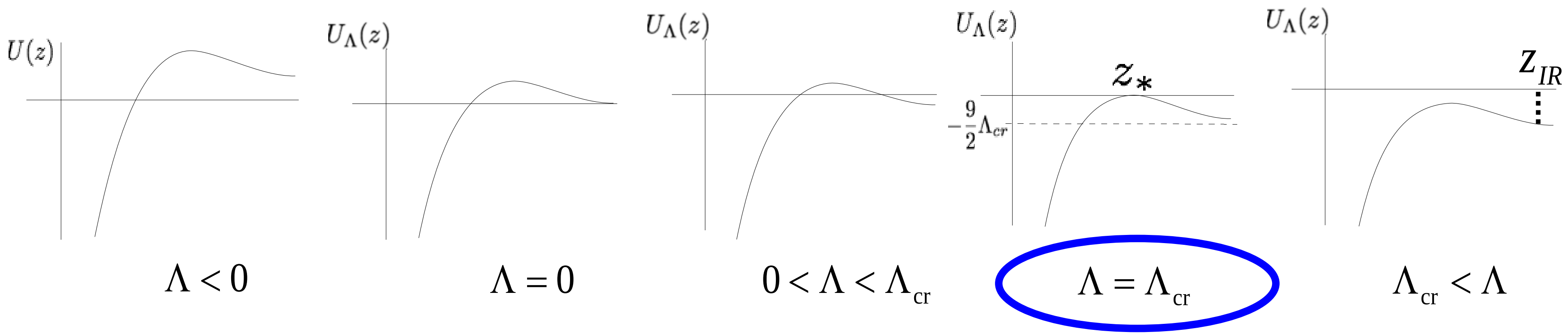
Cosmological constant

What value of Λ maximizes $\int dz \left|\mu \phi_{E=0}(z,\lambda)\right|^2$?

WKB sol $\phi_{E=0}(z,\lambda) \sim \frac{1}{\sqrt{z\, p(z,\lambda)}}$ with $p(z,\lambda) = \sqrt{-2U(z)}.$ S^3 topology

$$U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{matt}}{z} - \frac{C_{rad}}{z^{4/3}}$$

assuming all matters decay to radiation



The cosmological constant in the far future is predicted to be very small.

Λ∼curvature∼energy density

$\Lambda_{cr} \sim 1/C_{rad}$ (extremely small)

The other couplings (Big Fix) (1)

$$P(\lambda) = \exp\left(\left|\mu\tau(\lambda)\right|^2\right)\left|w(\lambda)\right|^2 \quad \leftarrow \tau(\lambda) = \int dz \left|\phi_{E=0}(z,\lambda)\right|^2$$

The exponent is divergent, and regulated by the IR cutoff :

$$\int dz \left|\phi_{E=0}(z,\lambda)\right|^2 \sim \int_0^{z_{IR}} \frac{1}{z\sqrt{\Lambda_{\mathrm{cr}}}} \sim \sqrt{C_{\mathrm{rad}}} \log z_{IR} . \quad \leftarrow \Lambda_{\mathrm{cr}} \sim 1/C_{\mathrm{rad}}$$

assuming all matters decay to radiation

BIG FIX 2

λ are determined in such a way that $C_{\mathrm{rad}}(\lambda)$ is maximized.

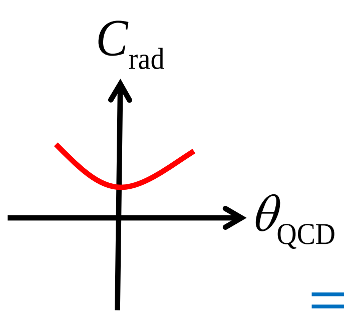
The other couplings (Big Fix) (2)

Quantities like $C_{\text{rad}}(\lambda)$ are almost determined by the long distance physics.

$\Rightarrow$ If the cosmological evolution is completely understood, we can calculate $C_{\text{rad}}(\lambda)$ theoretically, and all of the renormalized couplings are in principle determined.

However, some of the couplings can be determined without knowing the detail of the cosmological evolution.

case 1. Symmetry example θ_{QCD} Nielsen, Ninomiya

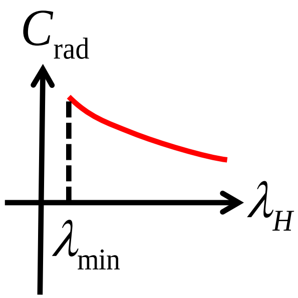


1. It becomes important only after the QCD phase transition.
2. The effective action at QCD scale is invariant under $\theta_{\text{QCD}} \to -\theta_{\text{QCD}}$.

$\Rightarrow C_{\text{rad}}$ is minimum or maximum at $\theta_{\text{QCD}} = 0$ at least locally.

The other couplings (Big Fix) (3)

case 2. End point **example** **Higgs coupling λ_H**



1. **Some (renormalized) couplings are bounded.**
2. $C_{\rm rad}$ **can be monotonic in them.**

$\Rightarrow$ $C_{\rm rad}$ **is maximized at the end point.**

A scenario for λ_H.

Fix v_h to the observed value and vary λ_H.

assuming the leptogenesis

$\lambda_H \searrow$ $\Rightarrow$ sphaleron process $\nearrow$

$\Rightarrow$ baryon number $\nearrow$

$\Rightarrow$ radiation from baryon decay $\nearrow$

$\Rightarrow$ **Higgs mass is at its lower bound.** $m_{\rm Higgs} \sim 125 \pm 5\,{\rm GeV}$

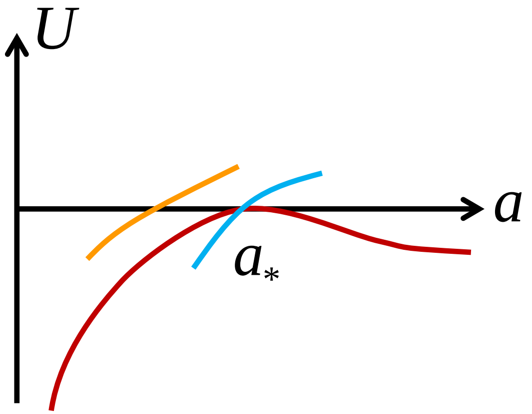
without SUSY

The other couplings (Big Fix) (4)

<u>**non-trivial example**</u> **QCD coupling or proton mass** m

We assume that dark matters decay faster than protons, and do not consider matter dominant era by leptons after the protons decay.

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\text{matt}}}{a^3} - \frac{C_{\text{rad}}}{a^4}$$



If the curvature term balances with matter before the proton decay, the universe bounce back when the protons decay.

The earlier the protons decay, the less C_{rad} remains.

C_{rad} **is maximized if the curvature term balances with the energy density when the protons decay.**

The other couplings (Big Fix) (5) (cont'd)

The curvature term balances with the energy density when the protons decay.

$$\frac{1}{{a_*}^2} = \frac{GM}{{a_*}^3}, \quad M = N_B\, m.$$

$$a_* = \left(GM\tau^2\right)^{\frac{1}{3}}$$

N_B : total baryon number

m : proton mass

τ : proton life time

$$\Rightarrow \quad \tau = GM$$

$$\tau = \frac{{m_{GUT}}^4}{g^4 m^5}, \quad GM = \frac{N_B m}{{m_P}^2}$$

$$\Rightarrow \quad m^6 = \frac{{m_P}^2 {m_{GUT}}^4}{g^4 N_B}$$

$$N_B = \frac{{m_P}^2 {m_{GUT}}^4}{g^4 m^6} \sim 10^{105}$$

Cf. 10^{78} protons in $\left(10^{10}\ \mathrm{ly}\right)^3$ **in our universe**

$$\Rightarrow \quad a_{\text{present}} = 10^9 \times 10^{10}\ \mathrm{ly}$$

Reasonable?

4. Summary

Summary

In the quantum gravity or string theory, the low energy effective action is not a simple local one but the multi-local one.

The multiverse naturally appears, and it becomes a superposition of states with various values of the coupling constants.

The coupling constants are fixed is such a way that the lifetime of the universe is maximized.

For example the cosmological constant in the far future is predicted to be very small: $\Lambda\left(z \to \infty\right) \sim 0$.

The Higgs mass is predicted at its lower bound.

Future problems

- Some scenario other than the probabilistic interpretation.
- Comparison of different dimensional space-time?
- Generalization to the landscape?

Appendix　IIB matrix model

IIB Matrix Model

Ishibashi, HK, Kitazawa, Tsuchiya

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu [A^\mu, \Psi]\right)$$

A candidate of the constructive definition of string theory.

Evidences

(1) World sheet regularization

Green-Schwartz action in the Schild Gauge

$$S = \int d^2\xi \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu \{X^\mu, \Psi\}\right)$$

Regularization by matrix

$$\{\ ,\ \} \to [\ ,\ ]$$

$$\int \to Tr$$

$$\downarrow$$

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu [A^\mu, \Psi]\right)$$

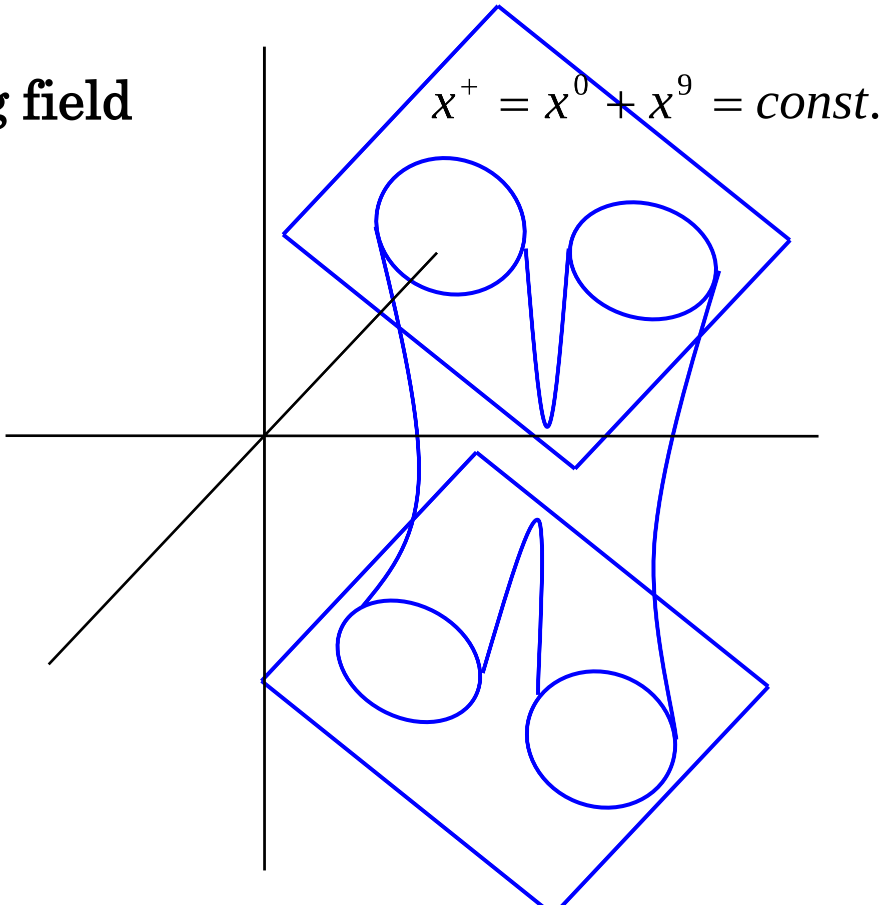
(2) Loop equation and string field

Wilson loop = string field

$$w(k_\mu(\cdot\cdot)) = Tr(P\exp(i\oint d\sigma\, k_\mu(\sigma) A^\mu + fermion))$$

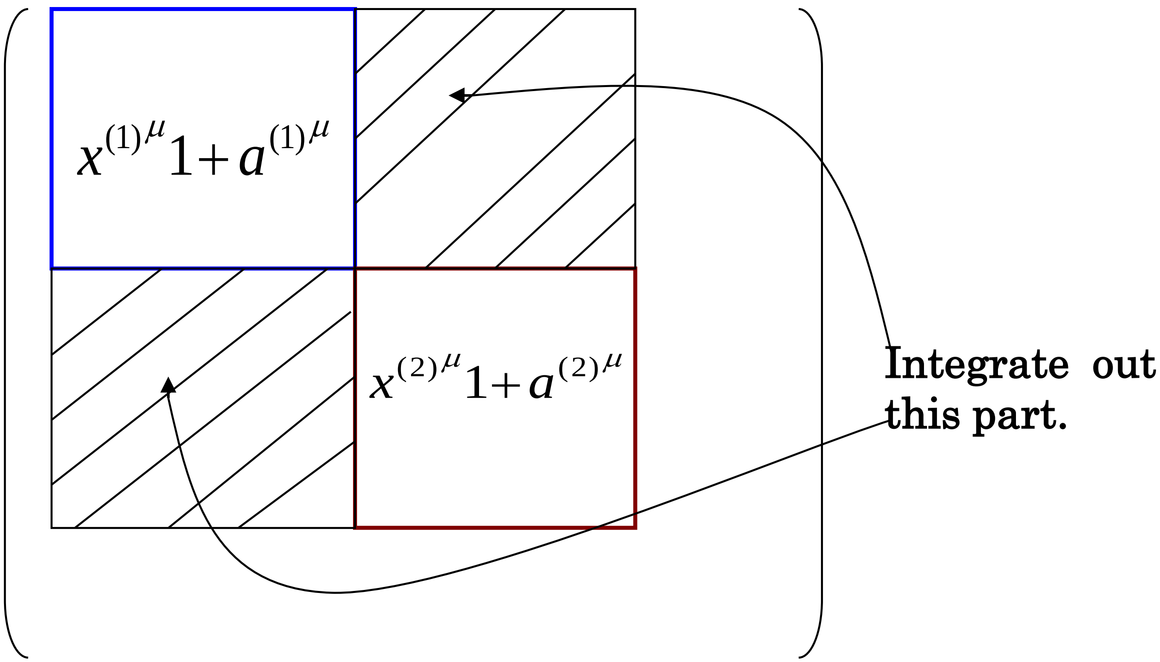
$\Leftrightarrow$ **creation annihilation operator of** $|k_\mu(\cdot\cdot)>$

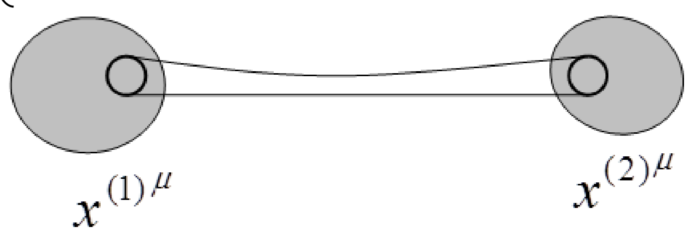
loop equation $\rightarrow$ **light-cone string field**



This can be shown with some assumptions .

(3) effective Lagrangian and gravity





The loop integral gives the exchange of graviton and dilaton.

$$S_{eff} = -\frac{1}{(x^{(1)} - x^{(2)})^8}\{const \cdot tr(f^{(1)}{}_{\mu\lambda} f^{(1)}{}_{\nu\lambda})\, tr(f^{(2)}{}_{\mu\lambda} f^{(2)}{}_{\nu\lambda})$$

$$-const \cdot tr(f^{(1)}{}_{\mu\nu} f^{(1)}{}_{\mu\nu})\, tr(f^{(2)}{}_{\lambda\rho} f^{(2)}{}_{\lambda\rho}) + \cdots\}$$