Surprising trace anomaly from freakolography

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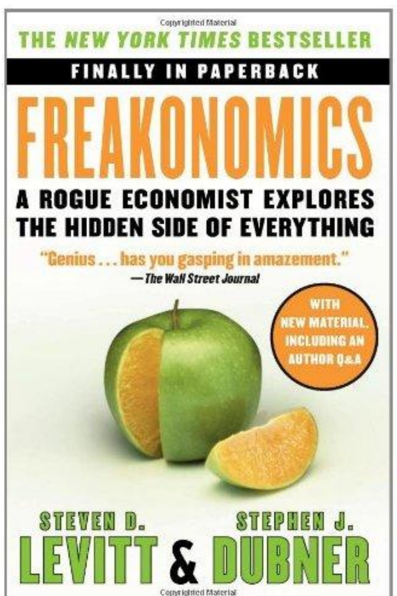
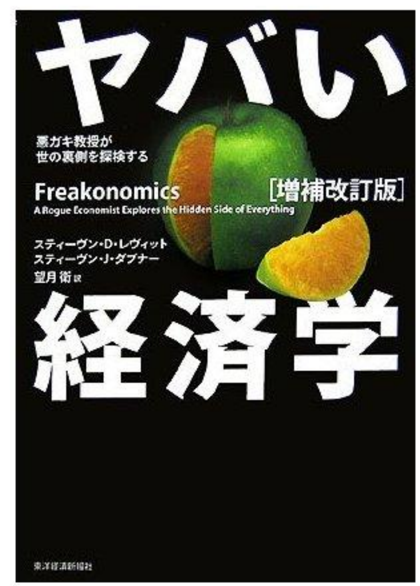
Freakonomics:

application of economics to what is traditionally beyond its scope, uncovering hidden side of everything

(Levitt, Dubner)





Freakolography:

application of holography to what is traditionally beyond its scope, uncovering
hidden side of everything
(Nakayama)

Two main questions:

- Scale inv = Conformal inv ?
- Surprising d=4 trace anomaly

Two main freakolographic methods:

- Space-time flipped Horava gravity
- Spontaneous Lorentz (AdS) symmetry breaking and violation of NEC

Scale = Conformal?

- It is not true. Counterexamples in higher dimension (e.g. 5d U(1) Maxwell theory)
- But in d=2, the equivalence was shown by Zamolodchikov-Polchinski under unitarity, causality etc.
- Not known in d=3,4. One of major obstructions for the proof of a-theorem

Conf vs Scale in EM tensor

- Scale invariance

$$x^\mu \to \lambda x^\mu$$

→Trace of energy-momentum (EM) tensor is a divergence of a so-called Virial current

$$T^\mu_\mu = \partial^\mu J_\mu \qquad D_\mu = x_\nu T_\mu{}^\nu - J_\mu$$

- Conformal invariance

$$x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + 2a^\mu x_\mu + a^2 x^2}$$

- EM tensor can be improved to be traceless

$$J_\mu = \partial^\nu L_{\mu\nu} \quad T^\mu_\mu \to \tilde{T}^\mu_\mu = 0 \qquad K_\mu = v_\nu \tilde{T}^\nu_\mu$$

Unexpected trace anomaly

- Dimensional analysis (4d):

$$T^{\mu}_{\ \mu} = a(\mathrm{Euler}) - c(\mathrm{Weyl}^2)$$
$$+ bR^2 + b'\Box R + e\epsilon^{\rho\sigma\alpha\beta} R_{\rho\sigma\mu\nu} R^{\mu\nu}{}_{\alpha\beta} + \text{non anomalous terms}$$

- $\mathrm{Euler} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$ is important in "a-theorem" ("a" decreases along RG flow)
- Hirzebruch-Pontryagin term is CP violating but can appear in CP-violating CFT (in principle)
- R^2 is inconsistent(?) for CFTs but it can appear in scale but non-CFT
- We'll see these unexpected terms from freakolography

Scenario one could imagine

- To get scale (but non-conf) inv, the beta function may not vanish

$$T^{\mu}_{\ \mu} = \beta^i O_i = \partial^\mu J_\mu$$

- If the virial current is chiral, then we expect gravitational chiral anomaly

$$T^{\mu}_{\ \mu} - bR^2 - a\mathrm{Euler} + c(\mathrm{Weyl})^2$$
$$= \beta^i O_i = D^\mu J_\mu - \epsilon^{\rho\sigma\alpha\beta} R_{\rho\sigma\mu\nu} R^{\mu\nu}_{\ \ \ \alpha\beta}$$

- R^2 and Hirzebruch-Pontryagin term can both appear (in principle)
- Is it easy if you try?
  - If you break unitarity, it is easy.
  - It is inconsistent with strongest a-theorem (if any)

$$\frac{da}{d\log\mu} = -g_{ij}\beta^i\beta^j$$

Holographic / freakolographic computation

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Holographay: Start from geometry

d+1 metric with d dim Poincare + scale invariance automatically selects AdS_{d+1} space

$$ds^2 = \frac{dz^2}{z^2} + f(z) dx_\mu dx^\mu$$

$$z \to \lambda z \ , \quad t \to \lambda t \ , \quad x \to \lambda x$$

$$ds^2 = \frac{dz^2 + dx_\mu^2}{z^2}$$

$$\delta x_\mu = 2(\epsilon^\nu x_\mu) x_\nu - (z^2 + x^\nu x_\nu)\epsilon_\mu \ , \delta z = 2(\epsilon^\nu x_\nu) z$$

Freakolography: space-time flipped Horava theory

Enhancement of "Isometry" requires d+1 diffeormorphism, so Horava theory which only preserves foliation preserving diffeomorphism does not work.

$$ds^2 = \frac{dz^2 + dx_\mu^2}{z^2}$$

$$\delta x_\mu = 2(\epsilon^\nu x_\mu)x_\nu - (z^2 + x^\nu x_\nu)\epsilon_\mu \ , \delta z = 2(\epsilon^\nu x_\nu)z$$

is not foliation preserving diff

$$\delta N = \partial_r (N f)$$

$$\delta N^\mu = \partial_r (N^\mu f) + \partial_r \xi^\mu + \mathcal{L}_\xi N^\mu$$

$$\delta g_{\mu\nu} = f \delta_r g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu} \ .$$

Alternatively Lorentz breaking

Non-trivial vector matter configuration may break AdS isometry spontaneously:
Identify its dual as Virial current.
(Horava gravity and Lorentz breaking are closely related)

Example: non-trivial vector field

$$A = A_M dx^M = \frac{a dz}{z}$$

Not invariant under special conformal

$$\delta x_\mu = 2(\epsilon^\nu x_\mu)x_\nu - (z^2 + x^\nu x_\nu)\epsilon_\mu \ , \delta z = 2(\epsilon^\nu x_\nu) z$$

Dual to Virial current $T^\mu_\mu = \partial^\mu J_\mu$

Interpretation: Holographic c-theorem

Holographic c-theorem gives: $ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2$

$$\frac{da}{dr} \sim \frac{A''}{(A')^d} \sim (T^t{}_t - T^r{}_r) = k^M k^N T_{MN}$$

Null energy condition (NEC) leads to strong c-theorem

$$k^M k^N T_{MN} \geq 0 \ , \quad k^2 = 0$$

Strict null energy condition leads to strongest c-theorem (= positivity of metric)

$$T^t{}_t - T^r{}_r \sim G^{IJ} \partial_r \Phi_I \partial_r \Phi_J \sim G^{IJ} \beta_I \beta_J \geq 0$$

Modification of the possibility $\beta^I O_I = \partial^\mu J_\mu$ → sigma model is gauged. $G^{IJ}(\partial_r - A_r)\Phi_I(\partial_r - A_r)\Phi_J$

$\Phi^I \sim z^{ia}$ corresponds to "cyclic" RG-flow

In unitary gauge, we conclude scale but non-conf ⬄ bulk vector condensation $A = A_M dx^M = \frac{a dz}{z}$

Such a non-trivial configuration violates (strict) Null Energy Condition

Null energy condition: $R_{MN}k^M k^N \geq 0 \ , \quad k^M k_M = 0$

(Ex)

$$L = -\frac{1}{4}F_{MN}F^{MN} + m^2 A_M A^M + \lambda (A_M A^M)^2$$

$$R_{zz} + R_{tt} = (m^2 + 2\lambda a^2)a^2 = 0$$

- More generically, we need strict NEC to completely exclude the possibility
- Equivalent for strongest holographic c-theorem
- It is true in supergravity compactification

Freakolography and trace anomaly

- Consider space-time flipped Horava gravity whose dual is scale but non-conformal

$$\mathrm{ds}^2 = N^2 dr^2 + G_{\mu\nu}(dx^\mu + N^\mu dr)(dx^\nu + N^\nu dr)$$

$$S = \int N dr \sqrt{-G} d^d x (K^{\mu\nu} K_{\mu\nu} - \lambda K^2 + R + \Lambda) \ .$$

$$K_{\mu\nu} = \frac{1}{2N}(\partial_r G_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu)$$

- Introduce the Graham-Fefferman ansatz

$$ds^2 = l^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{g_{\mu\nu}(\rho, x) dx^\mu dx^\nu}{\rho} \right)$$

$$g = g^{(0)} + \rho g^{(2)} + \cdots + \rho^{d/2} g^{(d)} + \rho^{d/2} \log \rho h^{(d)} + \mathcal{O}(\rho^{d/2+1})$$

- Solve EOM and study the log counterterm

$$S = \frac{l^3}{4} \log \epsilon \int d^4 x \sqrt{-g_0} \left(R^{(0)}_{\mu\nu} R^{\mu\nu(0)} - \frac{\lambda}{4\lambda - 1} R^{(0)2} \right) \ .$$

Space-time flipped Horava gravity

- Holographic (freakolographic) trace anomaly is

$$\langle T^{\mu}_{\ \mu} \rangle = -2c \left(R_{\mu\nu}R^{\mu\nu} - \frac{\lambda}{4\lambda - 1} R^2 \right)$$
$$= c \left((\mathrm{Euler} - \mathrm{Weyl}^2) - \frac{2}{3}\frac{\lambda - 1}{4\lambda - 1} R^2 \right) \ .$$

- We found R^2 term! Cannot be conformal!
- One may further add $\int N dr \sqrt{-G} d^4x K \epsilon^{\rho\sigma\alpha\beta} R_{\rho\sigma\mu\nu} R^{\mu\nu}{}_{\alpha\beta}$ to generate CP violating trace anomaly

$$T^{\mu}_{\ \mu} = e\epsilon^{\rho\sigma\alpha\beta} R_{\rho\sigma\mu\nu} R^{\mu\nu}{}_{\alpha\beta}$$

- Or, CS-gravity coupling with vector condensation directly gives CP-odd as expected from anomaly in virial current

$$\int d^5x \sqrt{g} \epsilon_{MNLPQ} A^M R^{NL}{}_{AB} R^{PQAB} = a \log \epsilon \int d^4x \sqrt{g} \epsilon^{\rho\sigma\alpha\beta} R_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta}$$

Can/should (strict) NEC kill freakolography?

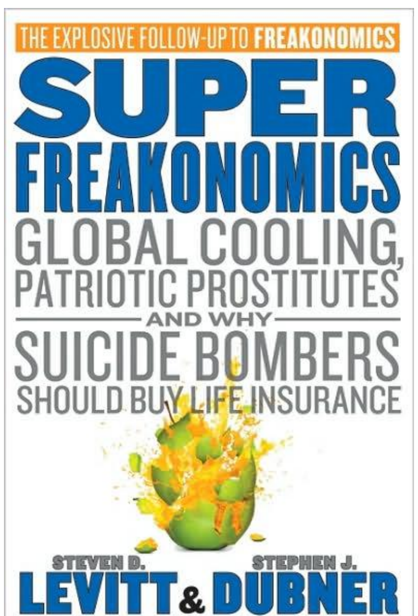
- In d=2, boundary (d=3 bulk), we can show strongest c-theorem, and scale = inv, so effectively, strict NEC must be true
- (strict) NEC is related to unitarity?
- NEC gives area non-decreasing theorem for black hole holography
- No information in zero-energy states (= strict NEC)
- Counterexample in higher dimension? d = 4- ϵ result by Grinstein et al?

What we leaned from holography

- Full space-time diff is tightly related to the emergence of conformal invariance
- It is possible to construct scale but non-conf geometry at the sacrifice of full space-time diff (spontaneous Lorentz symmetry breaking, Horava-like gravity…)
- Are they good? (violation of NEC, unitarity?…)
- No holography? Or Freakoholography?

After the success, Levitt and Dubner wrote the second book ``super freakonomics”.

Naturally, we expect we’ll hear about ``super freakolography” next time.



Stay tuned!!