KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

On S-duality in non-SUSY gauge theory

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- special thanks to Y. Ookouchi -

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① Introduction

In this talk, we consider the following duality

S-dual in type IIB

$$O3^{+}\text{-}\overline{D3} \longleftrightarrow \widetilde{O3}^{-}\text{-}\overline{D3}$$

[Uranga' 99]

$\downarrow$ Low energy $\qquad\qquad$ $\downarrow$ Low energy

Electric theory

		gauge	global
	ele	$USp(2n)$	$SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
scalar	Φ^I	$\square\square$	6

Magnetic theory

		gauge	global
	mag	$SO(2n-1)$	$SO(6)$
gauge	a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion	q^i	$\square\square$	4_+
scalar	ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

Electric theory

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\square$	1
Q^i	$\substack{\square\\\square}$	4_+
Φ^I	$\square\square$	6

Strongly coupled at IR

dual

↔

Magnetic theory

mag	gauge $SO(2n-1)$	global $SO(6)$
a_μ	$\substack{\square\\\square}$	1
q^i	$\square\square$	4_+
ϕ^I	$\substack{\square\\\square}$	6

Weakly coupled at IR

- Conjectured properties:

① Confinement

② Dynamical Sym Breaking

$$\langle Q^i Q^j \rangle \propto \delta^{ij}$$

$$SO(6) \simeq SU(4) \to SO(4)$$

- Decoupling limit is ambiguous.
- Physics at IR should be easier than electric theory

Q : Why are they dual?

Q : Can we understand ① , ② using the duality?

CAUTION

- I am not going to “prove” or “derive” confinement and dynamical symmetry breaking,

  but I will just “try to understand” what is going on under duality.

- Some of the arguments are based on speculative model.

Please be generous!

Plan of Talk

✓ ① Introduction

② Brief review of O3-planes

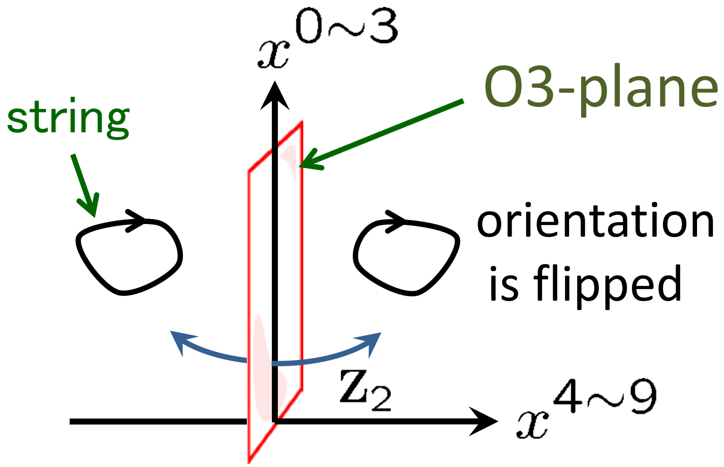
③ O3-$\overline{D3}$ system

④ Confinement and DSB

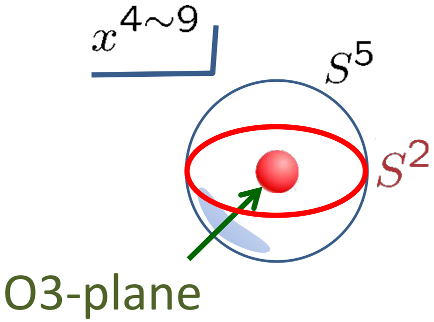
⑤ Summary

② Brief review of O3-planes

- O3-plane: fixed plane of $\mathbf{Z}_2$ $\begin{cases} x^{4\sim9} \to -x^{4\sim9} \\ \text{and flip orientation of strings} \end{cases}$



- Discrete torsion $H_2(S^5/\mathbf{Z}_2, \tilde{\mathbf{Z}}) \simeq \mathbf{Z}_2$ [Witten '98]



$$\tau_{\mathrm{NS}} = \exp\left(i\int_{S^2} B_2\right) = \pm 1 \qquad \tau_{\mathrm{RR}} = \exp\left(i\int_{S^2} C_2\right) = \pm 1$$

4 types:

	$\mathrm{O3}^-$	$\mathrm{O3}^+$	$\widetilde{\mathrm{O3}}^-$	$\widetilde{\mathrm{O3}}^+$
$(\tau_{\mathrm{NS}}, \tau_{\mathrm{RR}})$	$(+,+)$	$(-,+)$	$(+,-)$	$(-,-)$

- S-duality of O3-planes [Witten '98]

$$\tau_{NS} \xleftrightarrow{\text{S-dual}} \tau_{RR} \qquad \begin{tikzcd} O3^- \arrow[loop right, "\text{S-dual}"] \end{tikzcd} \qquad O3^+ \xleftrightarrow{\text{S-dual}} \widetilde{O3}^-$$

- O3 + *n* D3 system

$$\begin{tikzcd}
O3^- + n\,\mathrm{D3} \arrow[r, Rightarrow] & \mathcal{N}=4\ SO(2n)\ \mathrm{SYM} \arrow[loop right, "\text{S-dual}"] \\
O3^+ + n\,\mathrm{D3} \arrow[r, Rightarrow] & \mathcal{N}=4\ USp(2n)\ \mathrm{SYM} \arrow[d, bend left, leftrightarrow, "\text{S-dual}"] \\
\widetilde{O3}^- + n\,\mathrm{D3} \arrow[r, Rightarrow] & \mathcal{N}=4\ SO(2n+1)\ \mathrm{SYM}
\end{tikzcd}$$

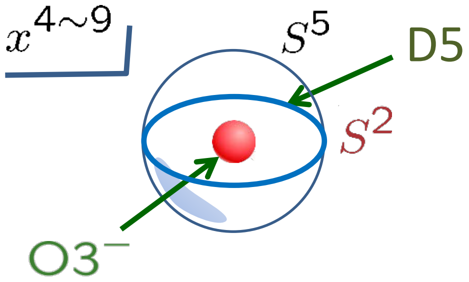
Consistent with the S-duality in N=4 SYM.

- $\widetilde{O3}^- \sim O3^- + \frac{1}{2}\mathrm{D3}$

However, this picture is misleading to understand non-perturbative properties, such as S-duality.

- Better picture: [Hyakutake-Imamura-S.S. ’00]

$$\boxed{\widetilde{\mathrm{O3}}^- \sim \mathrm{O3}^- + \text{spherical D5}}$$ (topologically)



magnetic flux on D5

$$\frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = \frac{1}{2} \quad \Rightarrow$$

RR charge of ½ D3-brane

Similarly, $$\boxed{\mathrm{O3}^+ \sim \mathrm{O3}^- + \text{spherical NS5}}$$

- Strings ending on O3-planes

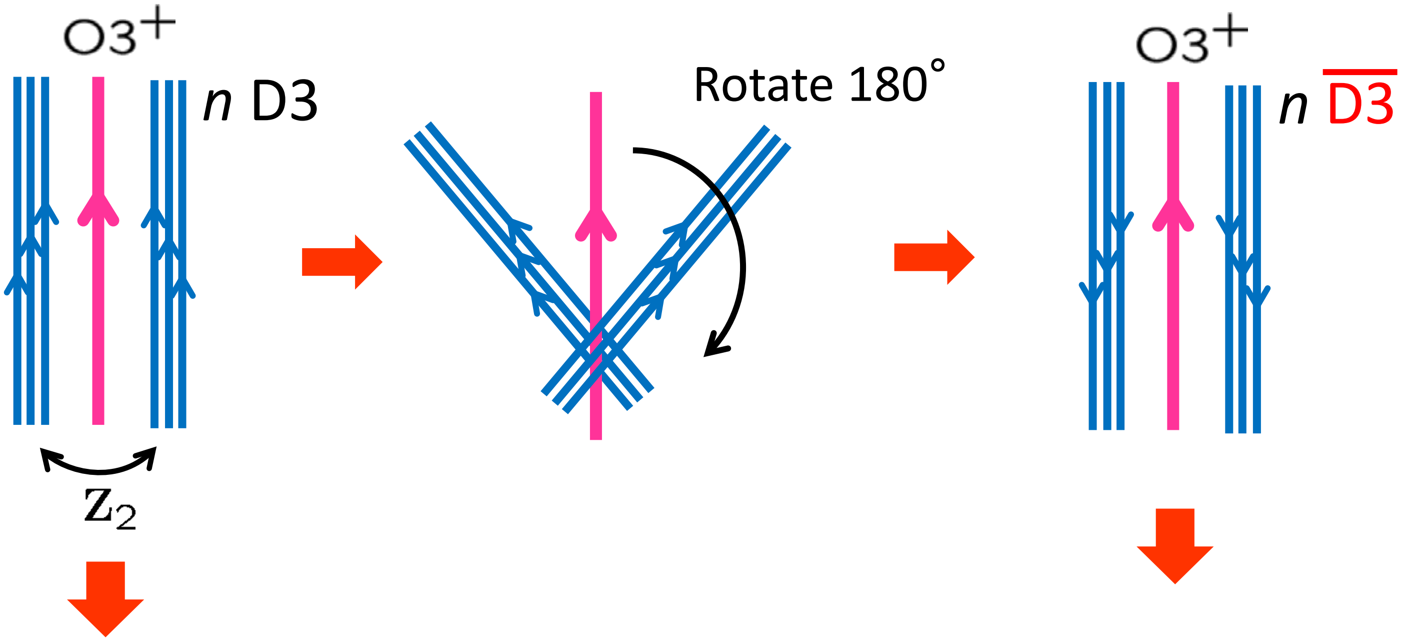
F1 can end on $\widetilde{\mathrm{O3}}^-$ $\overset{\text{S-dual}}{\longleftrightarrow}$ D1 can end on $\mathrm{O3}^+$

F1 cannot end on $\mathrm{O3}^+$ $\overset{\text{S-dual}}{\longleftrightarrow}$ D1 cannot end on $\widetilde{\mathrm{O3}}^-$

Cf) F1 can end on D5, but not on NS5
D1 can end on NS5, but not on D5

③ O3-$\overline{\text{D3}}$ system

[S.S. '99, Uranga '99]



		gauge	global
		$USp(2n)$	$SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\square\square$	4_+
scalar	Φ^I	$\square\square$	6

$\mathcal{N} = 4$ $USp(2n)$ SYM

	ele	gauge	global
		$USp(2n)$	$SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\begin{array}{c}\square\\\square\end{array}$	4_+
scalar	Φ^I	$\square\square$	6

non-SUSY

- $\mathrm{O3}^+ + n\,\overline{\mathrm{D3}}$ $\rightarrow$

	ele	gauge	global
		$USp(2n)$	$SO(6)$
gauge	A_μ	□□	1
fermion	Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
scalar	Φ^I	□□	6

$\Big\updownarrow$ S-dual

- $\underbrace{\widetilde{\mathrm{O3}}^-}$ $+ n\,\overline{\mathrm{D3}}$

$$\downarrow$$

$$\overbrace{\mathrm{O3}^- + \frac{1}{2}\mathrm{D3}} + n\,\overline{\mathrm{D3}} \xrightarrow[\langle t\rangle \neq 0]{\text{Tachyon condensation}} \mathrm{O3}^- + \frac{1}{2}(2n-1)\,\overline{\mathrm{D3}}$$

[Sen ’98] [Uranga ’99]

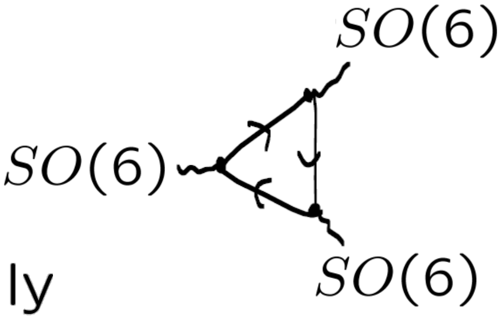
	mag A	gauge	global
		$SO(2n)$	$SO(6)$
gauge	a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion	q^i	□□	4_+
scalar	ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
tachyon	t	□	1
fermion	ψ_i	□	4_-

$\}$ D3-$\overline{\mathrm{D3}}$ string

	mag B	gauge	global
		$SO(2n-1)$	$SO(6)$
gauge	a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion	q^i	□□	4_+
scalar	ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

- ’t Hooft anomaly matching condition for SO(6)^3 is satisfied

ele	gauge	global
	$USp(2n)$	$SO(6)$
A_μ	$\square\square$	1
Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\square$	6



$SO(6)^3$ anomaly

$$\propto \frac{1}{2}\underset{\substack{\uparrow\\ Q^i}}{2n(2n-1)} = \underline{2n^2-n}$$

mag A	gauge	global
	$SO(2n)$	$SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
t	$\square$	1
ψ_i	$\square$	4_-

$$\propto \frac{1}{2}\underset{\substack{\uparrow\\ q^i}}{2n(2n+1)} - \underset{\substack{\uparrow\\ \psi_i}}{2n} = \underline{2n^2-n}$$

④ Confinement and DSB

- *n*=1 case

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\Box\!\Box$	1
Q^i	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	4_+
Φ^I	$\Box\!\Box$	6

↓ *n*=1

$USp(2) \simeq SU(2)$

Q^i : $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ is gauge singlet

Φ^I become massive
via quantum effect

$\Bigg\}$ ➡ SU(2) pure YM !

Magnetic description of pure YM theory

mag A

	gauge $SO(2n)$	global $SO(6)$
a_μ	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	1
q^i	$\square\!\square$	4_+
ϕ^I	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	6
tachyon t	$\square$	1
ψ_i	$\square$	4_-

$$\xrightarrow{\langle t\rangle \neq 0}$$

mag B

	gauge $SO(2n-1)$	global $SO(6)$
a_μ	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	1
q^i	$\square\!\square$	4_+
ϕ^I	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	6

mag A $\downarrow$ *n*=1

mag B $\downarrow$ *n*=1

$SO(2) \simeq U(1)$

t is magnetic monopole

$$\xrightarrow{\langle t\rangle \neq 0}$$

"monopole condensation"

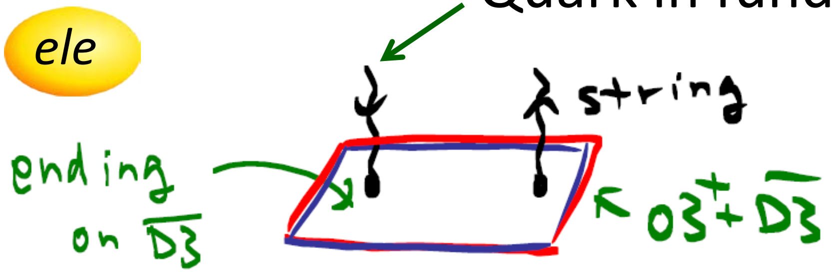
Gauge symmetry is completely Higgsed

Manifestation of "dual Meissner effect" !

[Nambu, 't Hooft, Mandelstam, Polyakov, …]

Q-$\overline{Q}$ potential

ele

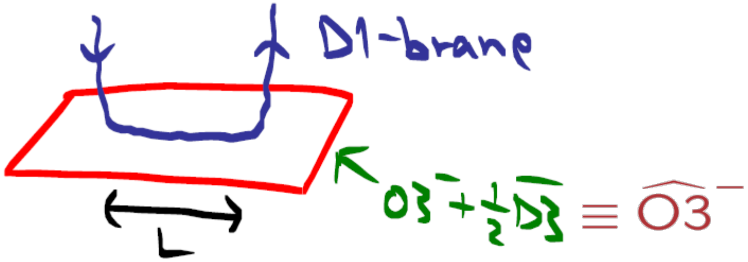


Quark in fund rep.

This picture is valid only when the coupling is small

mag B

$$O3^- + \frac{1}{2}\overline{D3} \sim \left(O3^- + \text{spherical D5 with } \frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = -\frac{1}{2}\right) \equiv \widehat{O3}^-$$



D1 cannot end on $\widehat{O3}^-$

- ➡ Should be connected
- ➡ linear potential $V(L) \propto L$
- ➡ Confinement !
  (in electric theory)

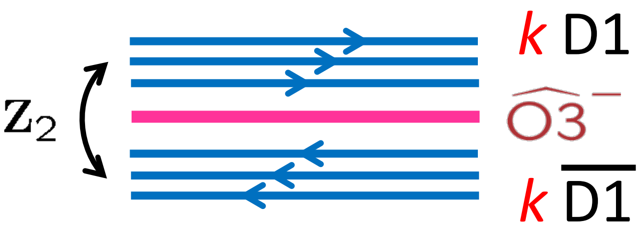
- 2-ality

ele All the fields are rank two tensors $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $\square\square$ of USp(2n)

Flux tube associated with fundamental rep. is stable, but rank 2 tensor can be screened.

➡ The fluxes are classified by $\mathbf{Z}_2$

mag B Consider k flux tubes,



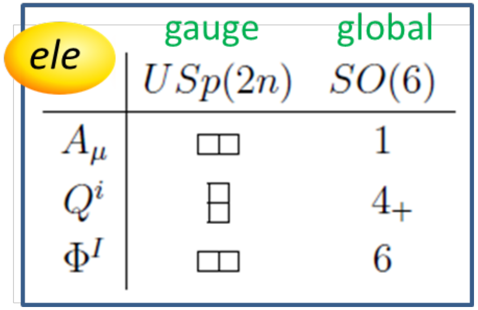
D1 world-volume gauge theory is $U(k)$ theory with tachyon in $\begin{smallmatrix}\square\\\square\end{smallmatrix}$

➡ k=1 is stable (no tachyon), while two $\overline{\text{D1}}$-D1 pairs can be annihilated via tachyon condensation.

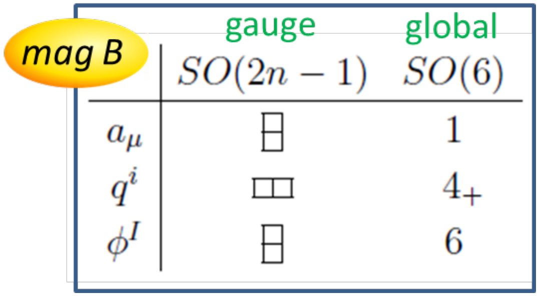
➡ Consistent with the above $\mathbf{Z}_2$ property !

Cf) non-BPS D7 in type I is a $\mathbf{Z}_2$ charged object. [Witten '98]

- $n > 1$ case



ele	gauge	global
	$USp(2n)$	$SO(6)$
A_μ	$\square\!\square$	1
Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\!\square$	6



mag B	gauge	global
	$SO(2n-1)$	$SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\!\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

Weakly coupled at UV

$m_\Phi^2 > 0$ (1-loop)

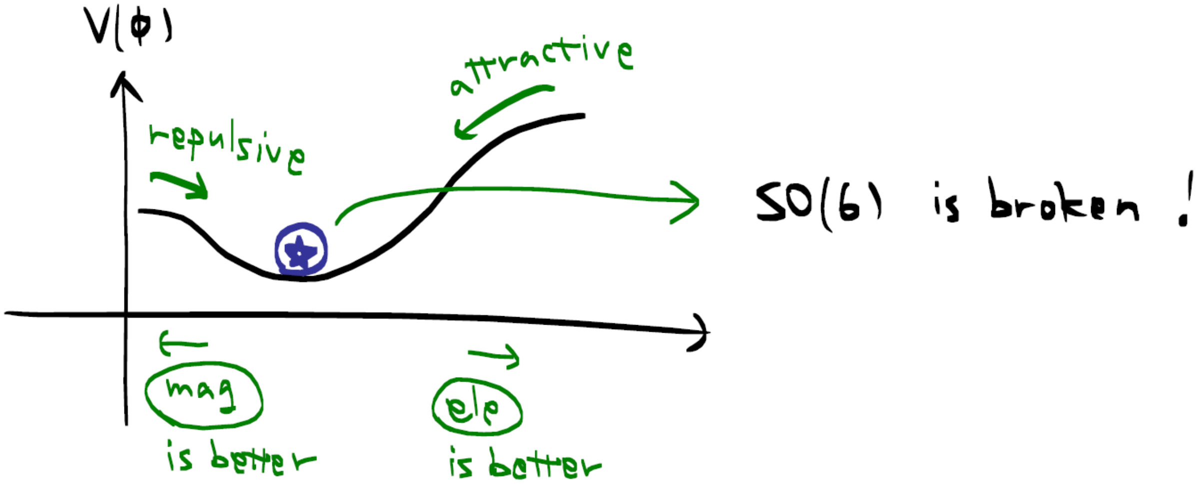
$O3^+$ and $\overline{D3}$ are attractive

Weakly coupled at IR

$m_\phi^2 < 0$ (1-loop)

$\widehat{O3}^-$ and $\overline{D3}$ are repulsive

➡ We expect:



- Unfortunately, we do not know the precise form of the potential.
- To proceed, we consider the following *speculative model* that seems to capture some of the qualitative features in the magnetic description.

$$V(\phi^I) = -\frac{\mu^2}{2}\,\mathrm{tr}(\phi^I\phi^I) - \frac{g}{4}\,\mathrm{tr}\left([\phi^I,\phi^J]^2\right) + \frac{\lambda}{2}\,\mathrm{tr}\left((\phi^I\phi^I)^2\right)$$

One-loop
tachyonic mass term

Tree level potential

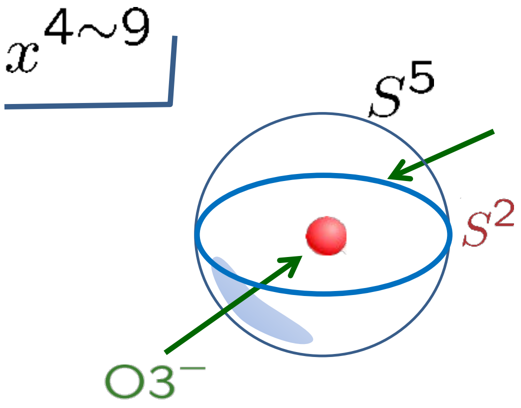
Added to stabilize
the potential

- This model has a fuzzy sphere solution:

$$\phi^{1\sim3} = aJ^{1\sim3} \qquad \phi^{4\sim6} = 0$$

$$\begin{cases} J^i & \text{: spin (n-1) representation of SU(2)} \\ a = \sqrt{\dfrac{\mu^2}{2g + 2\lambda n(n-1)}} \end{cases}$$

- This fuzzy sphere solution corresponds to



fuzzy S^2 ~ spherical D5 embedded in $x^{4\sim6}$ with finite radius

➡ $\underset{x^{4\sim9}}{SO(6)}$ is broken to $\underset{x^{4\sim6}}{SO(3)} \times \underset{x^{7\sim9}}{SO(3)} \simeq SU(2) \times SU(2)$

- This breaking pattern is consistent with the dynamical symmetry breaking expected in electric theory.

$$SO(6) \simeq SU(4) \underset{\langle Q^i Q^j \rangle \propto \delta^{ij}}{\longrightarrow} SO(4) \simeq SU(2) \times SU(2)$$

- Then, the gauge symmetry SO(2n-1) is completely broken.

➡ Confinement in electric theory

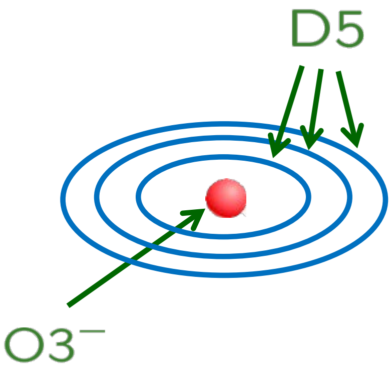
- Note:

For large enough λ, we can show the following:

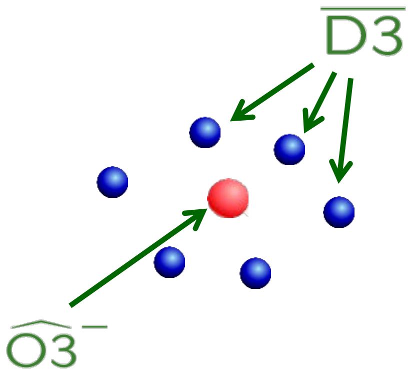
(i) This solution is stable with respect to small fluctuation.

(ii) This solution has lower energy than the combination of

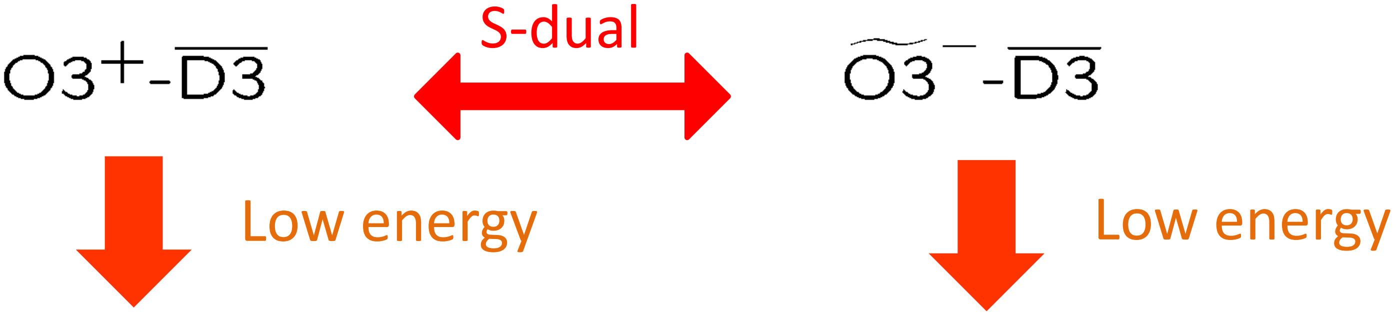
multiple spheres



isolated $\overline{D3}$'s



5 Summary



O3^{+}-$\overline{\mathrm{D3}}$

S-dual

$\widetilde{\mathrm{O3}}^{\,-}$-$\overline{\mathrm{D3}}$

Low energy

Low energy

Electric theory

ele	gauge	global
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A_μ	$\square\square$	1
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Magnetic theory

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Confinement
Dynamical Sym Breaking

↔

Tachyon condensation
Fuzzy sphere configuration