On S-duality in non-SUSY gauge theory

Shigeki Sugimoto (Kavli IPMU)

- special thanks to Y. Ookouchi -
Introduction

In this talk, we consider the following duality

\[ \mathbb{O}3^+ - \mathbb{D}3 \xrightarrow{\text{S-dual in type IIB}} \mathbb{O}3^- - \mathbb{D}3 \]

Low energy

Electric theory

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Magnetic theory

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[Uranga’ 99]
Electric theory

- Conjectured properties:
  ① Confinement
  ② Dynamical Sym Breaking

\[ \langle Q^i Q^j \rangle \propto \delta^{ij} \]
\[ SO(6) \simeq SU(4) \rightarrow SO(4) \]

Q : Why are they dual?
Q : Can we understand ①, ② using the duality?

Magnetic theory

- Decoupling limit is ambiguous.
- Physics at IR should be easier than electric theory

Strongly coupled at IR
Weakly coupled at IR

dual

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CAUTION

I am not going to “prove” or “derive” confinement and dynamical symmetry breaking, but I will just “try to understand” what is going on under duality.

Some of the arguments are based on speculative model.

Please be generous!
Plan of Talk

1. Introduction
2. Brief review of $O_3$-planes
3. $O_3$-$D_3$ system
4. Confinement and DSB
5. Summary
Brief review of O3-planes

- **O3-plane**: fixed plane of $\mathbb{Z}_2$ and flip orientation of strings

- **Discrete torsion**
  \[ H_2(S^5/\mathbb{Z}_2, \tilde{\mathbb{Z}}) \simeq \mathbb{Z}_2 \]  
  
  \[ \tau_{\text{NS}} = \exp \left( i \int_{S^2} B_2 \right) = \pm 1 \]  
  \[ \tau_{\text{RR}} = \exp \left( i \int_{S^2} C_2 \right) = \pm 1 \]

4 types:

<table>
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<tr>
<th>$(\tau_{\text{NS}}, \tau_{\text{RR}})$</th>
<th>O3$^{-}$</th>
<th>O3$^{+}$</th>
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[Witten ’98]
**S-duality of O3-planes**

\[
\tau_{\text{NS}} \leftrightarrow \tau_{\text{RR}} \quad \text{S-dual} \quad \text{O3}^{-} \leftrightarrow \text{S-dual} \quad \text{O3}^{+} \leftrightarrow \text{O3}^{-}
\]

**O3 + n D3 system**

- \(\text{O3}^{-} + n \text{D3} \rightarrow \mathcal{N} = 4 \ SO(2n) \ \text{SYM} \)  
- \(\text{O3}^{+} + n \text{D3} \rightarrow \mathcal{N} = 4 \ USp(2n) \ \text{SYM} \)  
- \(\text{O3}^{-} + n \text{D3} \rightarrow \mathcal{N} = 4 \ SO(2n + 1) \ \text{SYM} \)

Consistent with the S-duality in \(\mathcal{N}=4\) SYM.

\[\text{O3}^{-} \sim \text{O3}^{-} + \frac{1}{2} \text{D3}\]

However, this picture is misleading to understand non-perturbative properties, such as S-duality.

[Witten ‘98]
Better picture:

\[ \tilde{O}^3^- \sim O3^- + \text{spherical D5} \] (topologically)

\[ \frac{1}{2\pi} \int_{S^2/\mathbb{Z}_2} F = \frac{1}{2} \quad \text{RR charge of } \frac{1}{2} \text{ D3-brane} \]

Similarly,

\[ O3^+ \sim O3^- + \text{spherical NS5} \]

Strings ending on O3-planes

F1 can end on \( \tilde{O}^3^- \) \( \leftrightarrow \) S-dual \( \text{D1 can end on } O3^+ \)

F1 cannot end on \( O3^+ \) \( \leftrightarrow \) S-dual \( \text{D1 cannot end on } \tilde{O}^3^- \)

Cf) F1 can end on D5, but not on NS5
D1 can end on NS5, but not on D5

[Hyakutake-Imamura-S.S. '00]
$\mathcal{O}_3$–$D_3$ system

$\mathcal{N} = 4 \ USp(2n) \ SYM$

$[S.S. \ '99, Uranga \ '99]$
\[ \mathcal{O}_3^+ + n \overline{D3} \]

S-dual

\[ \mathcal{O}_3^- + n \overline{D3} \]

Tachyon condensation

\[ \mathcal{O}_3^- + \frac{1}{2}D3 + n \overline{D3} \]

\[ \langle t \rangle \neq 0 \]

\[ \mathcal{O}_3^- + \frac{1}{2}(2n - 1) \overline{D3} \]

**ele**

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**mag A**

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**mag B**

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D3-\( \overline{D3} \) string
‘t Hooft anomaly matching condition for $SO(6)^3$ is satisfied

\[
SO(6)^3 \text{ anomaly} \quad \propto \frac{1}{2} 2n(2n - 1) = 2n^2 - n
\]

\[
\frac{1}{2} 2n(2n + 1) - 2n = 2n^2 - n
\]
4 Confinement and DSB

- $n=1$ case

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$U Sp(2n) \simeq SU(2)$

$Q^i$ : is gauge singlet

$\Phi^I$ become massive via quantum effect

$SU(2)$ pure YM!
Magnetic description of pure YM theory

\[ SO(2) \simeq U(1) \]

\[ t \] is magnetic monopole

Gauge symmetry is completely Higgsed

Manifestation of “dual Meissner effect”!

[Nambu, ’t Hooft, Mandelstam, Polyakov, …]
Q-\bar{Q} potential

This picture is valid only when the coupling is small.

D1 cannot end on \( \tilde{O}_3^- \)

- Should be connected
- Linear potential \( V(L) \propto L \)
- Confinement!

(in electric theory)
2-ality

All the fields are rank two tensors $\mathbb{Z}_2$ of $\text{USp}(2n)$.

Flux tube associated with fundamental rep. is stable, but rank 2 tensor can be screened.

$\Rightarrow$ The fluxes are classified by $\mathbb{Z}_2$.

Consider $k$ flux tubes, $\mathbb{Z}_2$:

D1 world-volume gauge theory is $U(k)$ theory with tachyon in $\mathcal{D}$. $\Rightarrow$ $k=1$ is stable (no tachyon), while two $\overline{\text{D1}}$-D1 pairs can be annihilated via tachyon condensation.

$\Rightarrow$ Consistent with the above $\mathbb{Z}_2$ property!

Cf) non-BPS D7 in type I is a $\mathbb{Z}_2$ charged object. [Witten '98]
\( n > 1 \) case

Weakly coupled at UV
\[ m^2_\phi > 0 \quad (1\text{-loop}) \]
\( O3^+ \) and \( D3 \) are attractive

Weakly coupled at IR
\[ m^2_\phi < 0 \quad (1\text{-loop}) \]
\( \bar{O3}^- \) and \( \bar{D3} \) are repulsive

We expect:

\( SO(6) \) is broken!
Unfortunately, we do not know the precise form of the potential.

To proceed, we consider the following *speculative model* that seems to capture some of the qualitative features in the magnetic description.

\[ V(\phi^I) = -\frac{\mu^2}{2} \text{tr}(\phi^I \phi^I) - \frac{g}{4} \text{tr} ([\phi^I, \phi^J]^2) + \frac{\lambda}{2} \text{tr} ((\phi^I \phi^I)^2) \]

- One-loop tachyonic mass term
- Tree level potential
- Added to stabilize the potential

This model has a fuzzy sphere solution:

\[ \phi^{1 \sim 3} = a J^{1 \sim 3} \]
\[ \phi^{4 \sim 6} = 0 \]

\[ a = \sqrt{\frac{\mu^2}{2g + 2\lambda n(n - 1)}} \]

\[ J^i : \text{spin (n-1) representation of SU(2)} \]
This fuzzy sphere solution corresponds to

\[ x^{4 \sim 9} \quad S^5 \quad \text{fuzzy } S^2 \quad \sim \text{spherical D5} \quad \text{embedded in } x^{4 \sim 6} \]

with finite radius

\[ SO(6) \quad \text{is broken to} \quad SO(3) \times SO(3) \simeq SU(2) \times SU(2) \]

This breaking pattern is consistent with the dynamical symmetry breaking expected in electric theory.

\[ SO(6) \simeq SU(4) \rightarrow SO(4) \simeq SU(2) \times SU(2) \]

\[ \langle Q^i Q^j \rangle \propto \delta^{ij} \]

Then, the gauge symmetry SO(2n-1) is completely broken.

Confinement in electric theory
Note:

For large enough $\lambda$, we can show the following:

(i) This solution is stable with respect to small fluctuation.

(ii) This solution has lower energy than the combination of multiple spheres with isolated D3’s.
Summary

O3⁺-D3 \( \overset{\text{S-dual}}{\longrightarrow} \) O3⁻-D3

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Low energy

Confinement
Dynamical Sym Breaking

Tachyon condensation
Fuzzy sphere configuration