(3+1)-dimensional expanding universe from a Lorentzian matrix model for superstring theory in (9+1)-dimensions

"Progress in Quantum Field Theory and String Theory",
Osaka City University, 2012.4.3-7

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Ref.) Kim-J.N.-Tsuchiya: PRL 108 (2012) 011601
arXiv:1108.1540

1. Introduction

fundamental questions concerning our universe

1. Why (3+1)-dimensions ?

2. Why expanding ?

Answers from a nonperturbative formulation of superstring theory in (9+1)-dimensions

type IIB matrix model

$$S = \frac{1}{g^2} \mathrm{tr} \left\{ -\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\psi_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta] \right\}$$

Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

nonperturbative formulation of type IIB superstring theory in10d

SSB of SO(10) in the Euclidean model

J.N.-Okubo-Sugino, JHEP1110(2011)135, arXiv:1108.1293

- free energy of SO(d) symmetric vacua (d=2,3,4,5,6,7)
  minimum at d=3
- extent of space-time finite in all directions

→ motivated us to consider the Lorentzian model

Seiberg's rapporteur talk (2005)
at the 23rd Solvay Conference in Physics

"Emergent Spacetime"

Understanding how time emerges will undoubtedly shed new light on some of the most important questions in theoretical physics including the origin of the Universe.

Indeed, the emergent spacetime seems to be naturally realized in the Lorentzian matrix model !

Plan of the talk

1. Introduction
2. Previous works on type IIB matrix model
3. Defining the Lorenzian matrix model
4. Monte Carlo results for the Lorentzian matrix model
5. Summary and discussions

2. Previous works on type IIB matrix model

type IIB matrix model   Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

$$S = \frac{1}{g^2}\,\mathrm{tr}\left\{-\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\psi_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right\}$$

proposed as a nonperturbative definition of type IIB superstring theory in 10 dim.

c.f.) Matrix Theory   Banks-Fischler-Shenker-Susskind ('96)

- matrix regularization of the Green-Schwarz worldsheet action in the Schild gauge

- interactions between D-branes

- string field theory from SD eqs. for Wilson loops

Fukuma-Kawai-Kitazawa-Tsuchiya ('98)

The action of type IIB matrix model

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

$N \times N$ Hermitian matrices SO(9,1) symmetry

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

raised and lowered by the metric

$$\eta = \mathrm{diag}(-1, 1, \cdots, 1)$$

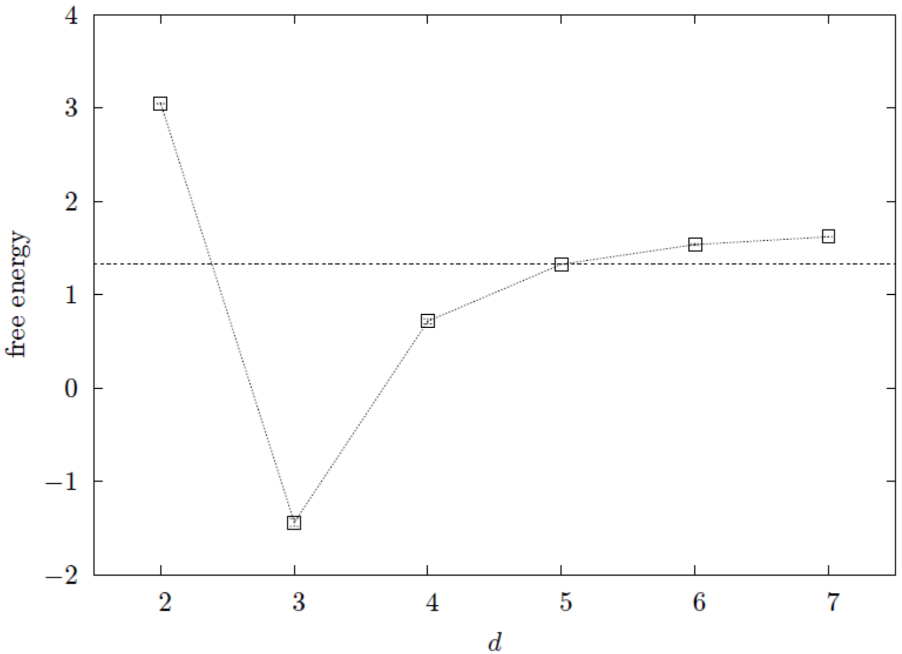
Previous works

Wick rotation $(A_0 = iA_{10} \;, \quad \Gamma^0 = -i\Gamma_{10})$

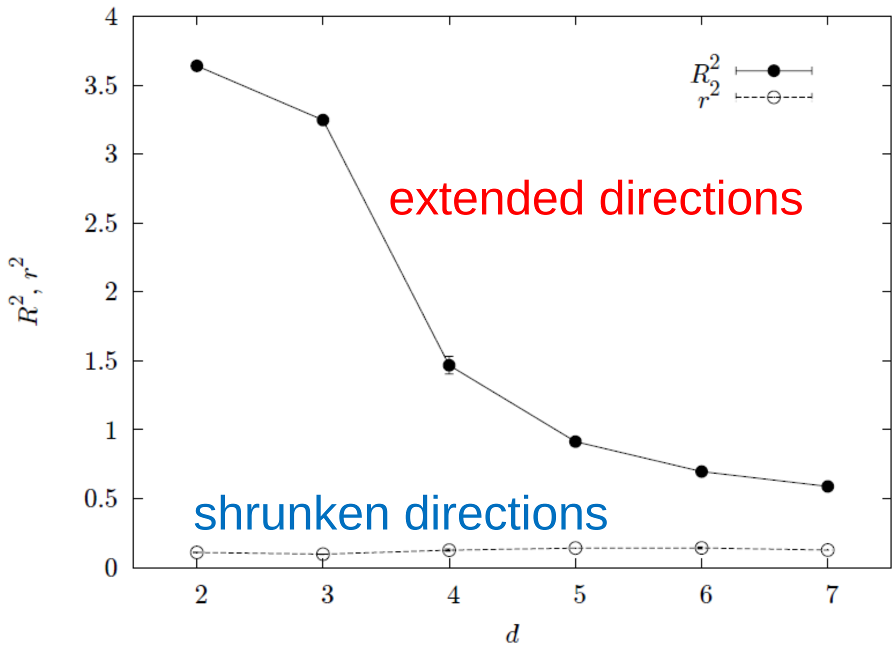
⇒ Euclidean model with SO(10) symmetry

Recent results by the Gaussian expansion method

J.N.-Okubo-Sugino, JHEP1110(2011)135,   arXiv:1108.1293



Minimum of the free energy occurs at d=3



Extent of space-time finite in all directions

SSB of SO(10) down to SO(3) :
interesting dynamical property of the Euclidean model,
but the connection to the real space-time is unclear…

3. Defining the Lorentzian matrix model

Difference between Euclidean and Lorentzian (I) bosonic action

- Euclidean model

$$S_b \propto \mathrm{tr}\,(F_{\mu\nu})^2 \qquad F_{\mu\nu} = -i[A_\mu, A_\nu]$$

positive definite

Classical flat direction is lifted up by quantum effects.

⇒ The model is well defined without any cutoff.

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

- Lorentzian model

$$S_b \propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) = -2\,\underbrace{\mathrm{tr}\,(F_{0i})^2 + \mathrm{tr}\,(F_{ij})^2}_{\text{opposite sign !}}$$

Looks extremely unstable ! ⇒ Hence, no one ever dared to study this model seriously!

Difference between Euclidean and Lorentzian (II) Pfaffian (obtained by integrating out fermions)

$$\int d\Psi \, e^{iS_{\mathsf{f}}} = \mathsf{Pf}\mathcal{M}(A)$$

- Euclidean model $\quad \mathsf{Pf}\mathcal{M}(A) \in \mathbf{C}$

The phase plays a crucial role in SSB of SO(10),
But it makes Monte Carlo studies extremely difficult.

J.N.-Vernizzi ('00), Anagnostopoulos-J.N.('02)

- Lorentzian model $\quad \mathsf{Pf}\mathcal{M}(A) \in \mathbf{R}$

Good news for Monte Carlo studies, but
we lose a source of SSB.

The definition of the partition function

- Euclidean model

$$Z = \int dA\, d\Psi\, e^{-S} = \int dA\, e^{-S_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

- Lorentzian model

$$Z = \int dA\, d\Psi\, e^{iS} = \int dA\, e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

connection to the worldsheet theory

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$\xi_0 \equiv -i\xi_2$ (We need to Wick rotate the worldsheet coordinate, too.)

How to deal with the phase factor $e^{iS_{\mathrm{b}}}$ in

$$Z = \int dA \, d\Psi \, e^{iS} = \int dA \, e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

Under the scale transformation $A_\mu \mapsto \rho A_\mu$

$$\begin{aligned} S_{\mathrm{b}} &\mapsto \rho^4 S_{\mathrm{b}} \\ dA &\mapsto \rho^{10(N^2-1)} dA \\ \mathrm{Pf}\mathcal{M}(A) &\mapsto \rho^{8(N^2-1)} \mathrm{Pf}\mathcal{M}(A) \end{aligned}$$

Integrating over the scale factor first, we get $\delta(S_{\mathrm{b}})$

Regularizing the Lorentzian model

(1) IR cutoff in the temporal direction

$$\frac{1}{N}\mathrm{tr}\,(A_0)^2 \le \kappa\frac{1}{N}\mathrm{tr}\,(A_i)^2 \qquad \text{(invariant under } A_\mu \to \rho A_\mu\text{)}$$

(2) IR cutoff in the spatial direction

$$\frac{1}{N}\mathrm{tr}\,(A_i)^2 \le L^2$$

(SO(9) symmetry is still manifest.)

Thus we arrive at

$$\begin{aligned} Z &= \int dA\,\delta\left(\frac{1}{N}\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu})\right)\mathrm{Pf}\mathcal{M}(A) \\ &\quad\times\delta\left(\frac{1}{N}\mathrm{tr}\,(A_i)^2-1\right)\theta\left(\kappa-\frac{1}{N}\mathrm{tr}\,(A_0)^2\right) \end{aligned}$$

Monte Carlo simulation : Rational Hybrid Monte Carlo algorithm

no sign problem unlike in the Euclidean model

4. Monte Carlo results for the Lorentzian matrix model

The emergent time

Eigenvalue distribution of A_0

extends smoothly to infinity as $\kappa \to \infty$

thanks to SUSY !

c.f.) bosonic case

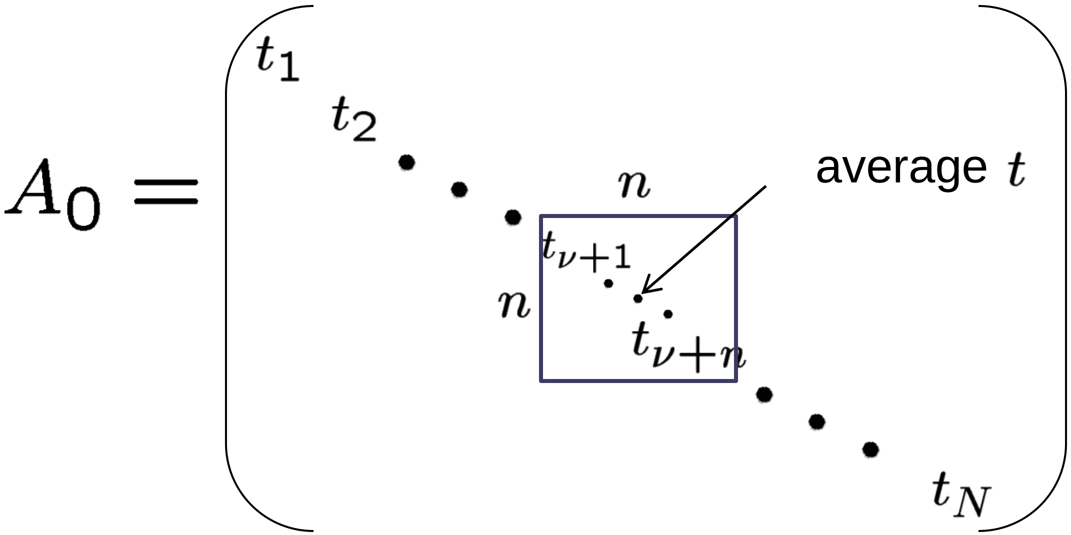
Attractive potential between the eigenvalues
induced by the one-loop effects

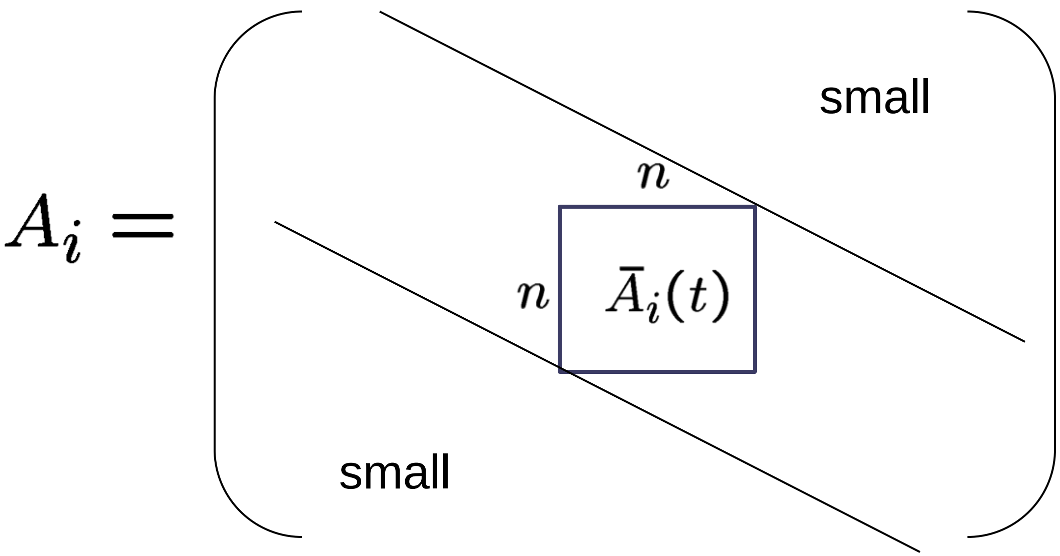
(The eigenvalue distribution of A_0
has finite extent even in the $\kappa \to \infty$ limit.)

SUSY plays a crucial role in the emergent time !

This does not guarantee that the notion of "time evolution" emerges, too…

Extracting the “time evolution”





We observe band-diagonal structure.

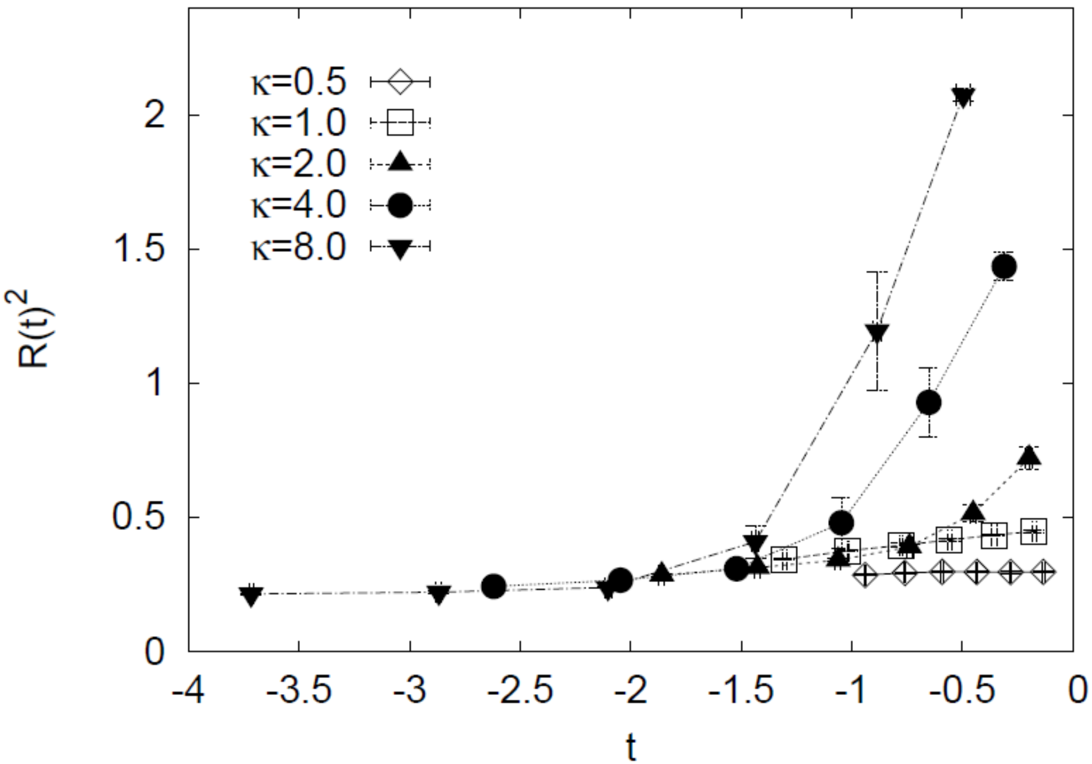
(space-time NC is small.)

$\bar{A}_i(t)$ represents space structure at fixed time t

The size of the space v.s. time

$$R(t)^2 \equiv \frac{1}{n} \mathrm{tr}\, \bar{A}_i(t)^2$$

peak at $t = 0$ starts to grow for $\kappa > \kappa_{\mathrm{cr}}$



$N = 16\ , \quad n = 4$

Symmetric under $t \mapsto -t$

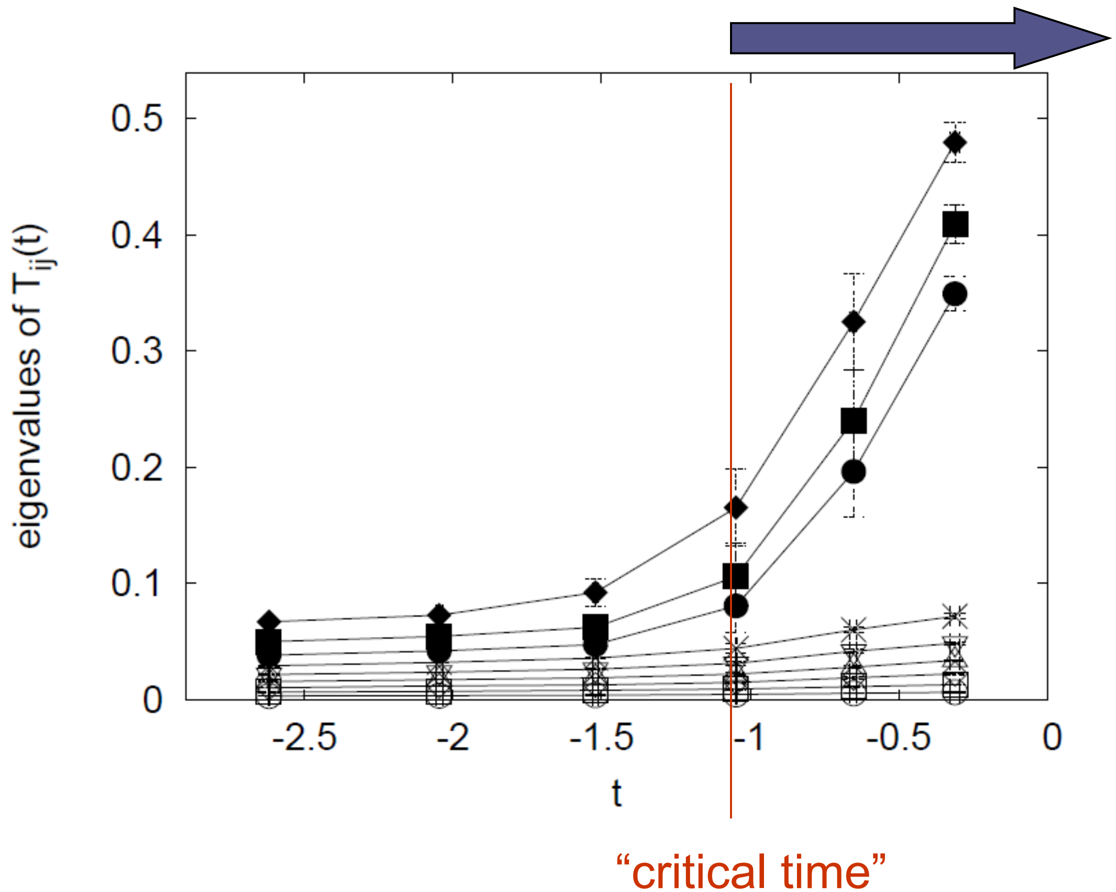
We only show $t < 0$

Spontaneous breaking of SO(9)

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{\bar{A}_i(t)\bar{A}_j(t)\right\}$$

$$\mathrm{SO}(9) \overset{\mathrm{SSB}}{\to} \mathrm{SO}(3)$$

$N = 16$, $\kappa = 4.0$



Removing the two cutoffs κ and L in the $N \to \infty$ limit

1) $\kappa \to \infty$ with $N \to \infty$ (continuum limit)

$$\kappa = \beta N^{1/4}$$

2) $L \to \infty$ with $\beta \to \infty$ (infinite volume limit)

fix the scale by $R(t_c)$

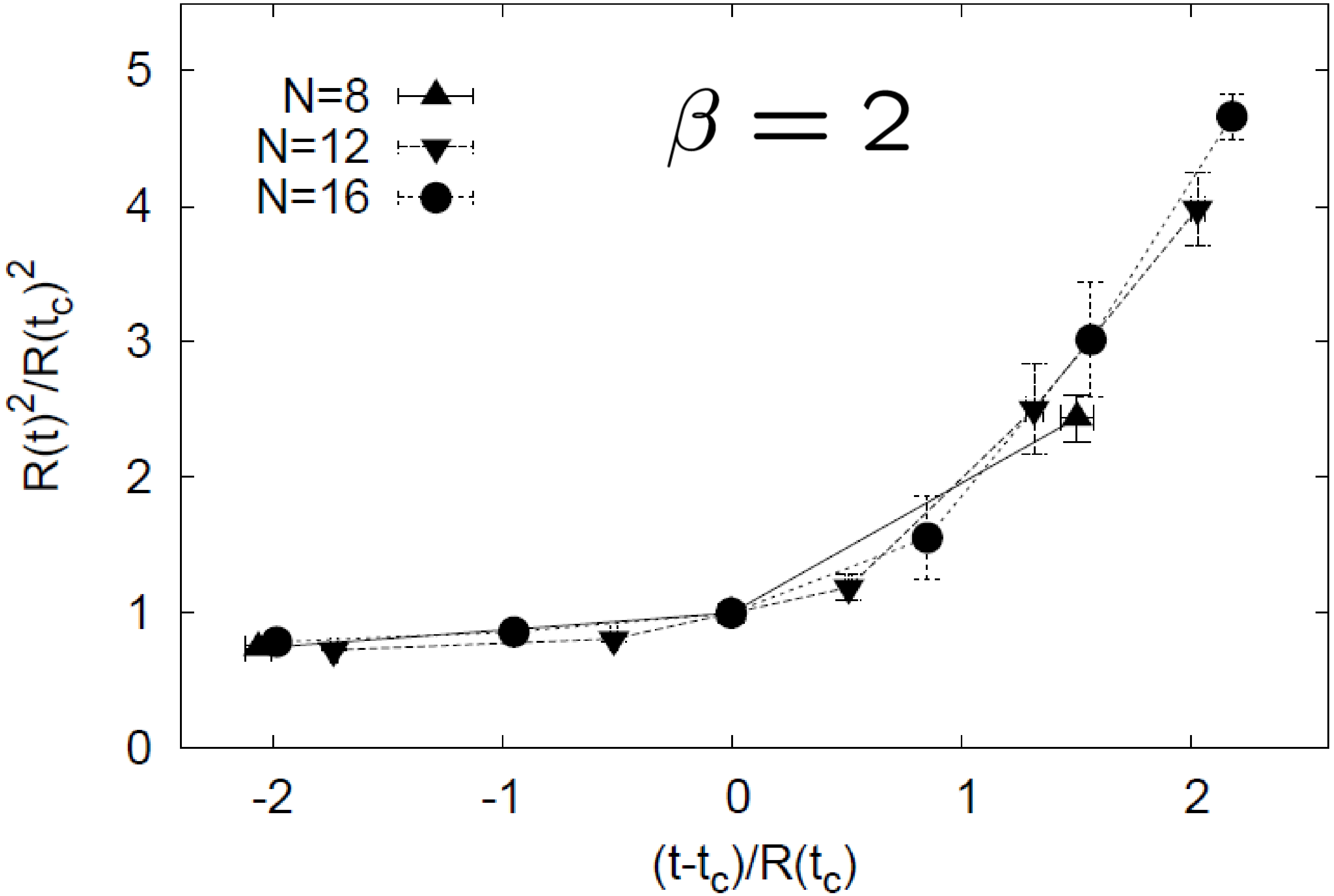
The theory thus obtained has
no parameters other than one scale paramter !

A property that nonperturbative superstring theory is expected to have !

Large-N scaling

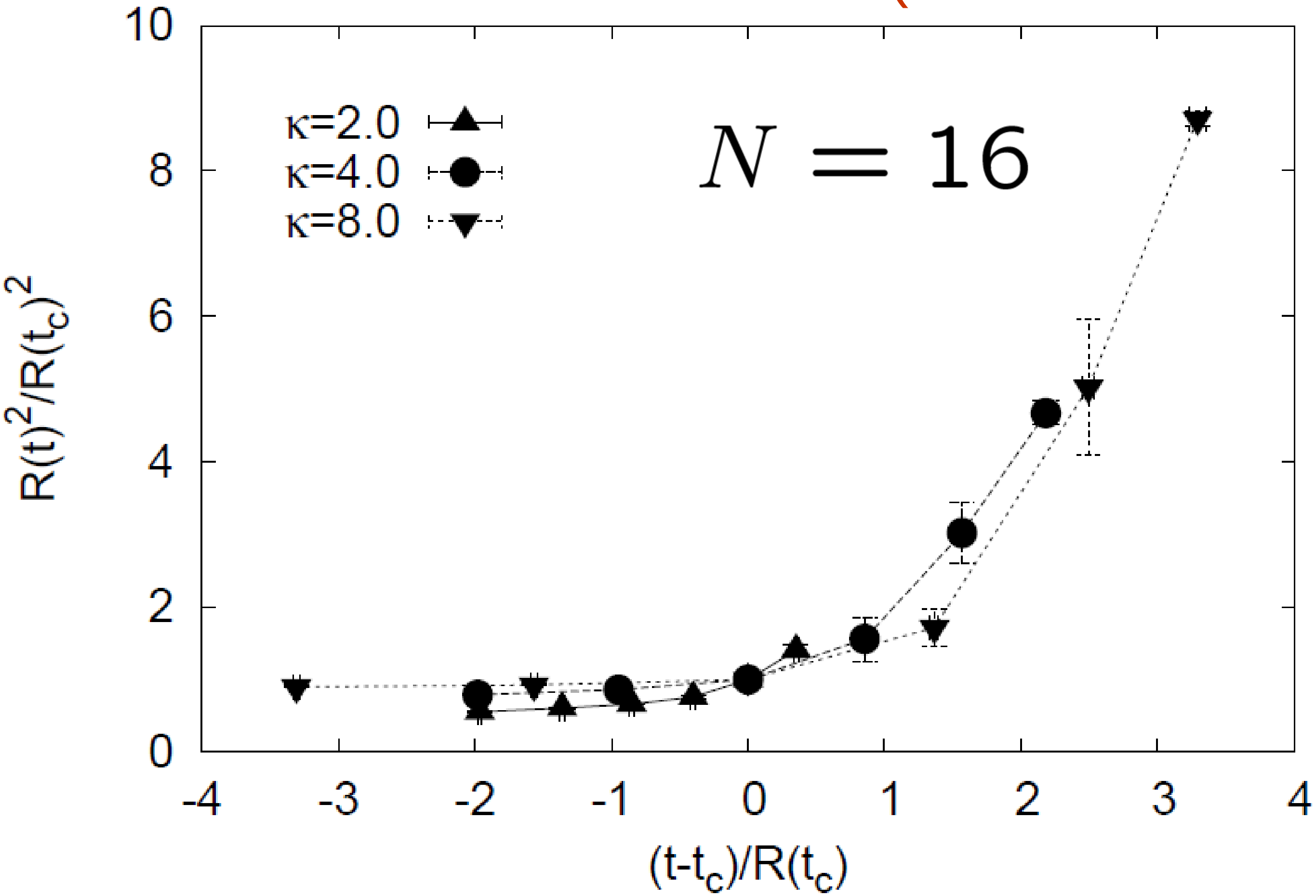
Clear large-N scaling behavior observed with $\kappa = \beta\, N^{1/4}$

(continuum limit)



Infinite volume limit

The extent of time increases and
the size of the universe becomes very large at later time.
(infinite volume limit)



7. Summary and discussions

type IIB matrix model :

nonperturbative formulation of
type IIB superstring theory in (9+1)-dim.

Instead of making Wick rotation, we defined the Lorentzian matrix model introducing IR cutoffs .

Monte Carlo studies revealed: Kim-J.N.-Tsuchiya ('11)

- IR cutoffs can be removed in the large-N limit.
- The theory thus obtained has no dim.less parameter.
  supporting the validity as a nonperturbative formulation of superstring theory
- "Time" emerges dynamically thanks to SUSY.
- "Time evolution" of 9d space emerges.
- 3 out of 9 spatial directions start to expand at some "critical time"

$$\mathrm{SO}(9) \rightarrow \mathrm{SO}(3)$$

Future directions

- Does a local field theory on a commutative space-time appear at later time ?

- How do 4 fundamental interactions and the matter fields appear at later time ?

Monte Carlo simulation AND
Studies of classical solutions (+ quantum corrections)

See S.-W.Kim's poster

We hope the Lorentzian matrix model
provides a new perspective on
- particle physics beyond the standard model
- microscopic descriptions of inflation, dark energy, etc..