β -deformed matrix models and Nekrasov Partition Functions

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1. Introduction

(Random) matrix models 0-dim. field theory

$$Z = \int [dM] \exp\left(\frac{1}{g_s}\mathrm{Tr}W(M)\right).$$

Various types of matrix models:

- choice of "ensemble" (type of matrix)
- choice of the "action" W

→various usage in 2d gravity (non-critical string), IKKT model, Dijkgraaf-Vafa (DV) theory,……

- (Example) Hermitian one-matrix model
  - matrix M: an NxN Hermitian matrix
  - Large N limit→ matrix model curve

$$y^2(z) = \frac{1}{4}W'(z)^2 + f(z).$$

- DV theory('02): gauge theory/matrix model correspondence identify this curve with the Seiberg-Witten curves of the N=2 gauge theories
- Alday-Gaiotto-Tachikawa (AGT) conjecture ([0906.3219]) →revisit this gauge theory/matrix model correspondence

AGT conjecture

from the view point of gauge theory/matrix model correspondence

→ renewed interests in β -deformed matrix models

Itoyama-T.O.-Maruyoshi [0911.4244], Eguchi-Maruyoshi [0911.4797], Sulkowski [0912.5476],
Mironov-Morozov-Shakirov [1001.0563], Itoyama-T.O. [1003.2929], Mironov-Morozov-Morozov [1003.5752],
Awata-Yamada [1004.5122], Eguchi-Maruyoshi [1006.0828], Itoyama-T.O.-Yonezawa [1008.1861], Maruyoshi-Yagi [1009.5553],
Mironov-Morozov-Shakirov [1010.1734], Marshakov-Mironov-Morozov [1011.4491],
Mironov-Morozov-Shakirov [1011.5629; 1012.3137], Mironov-Morozov-Popolitov-Shakirov [1103.5470],
Itoyama-Yonezawa [1104.2738], Bonelli-Maruyoshi-Tanzini [1104.4016], Itoyama-T.O. [1106.1539], Nishinaka-Rim [1112.3545],
Morozov [1201.4595], Mironov-Morozov-Zakirova [1202.6029], …..

Many variants of β -deformed matrix models

Here we concentrate on a type which has close connection with the Selberg integral.

4d/2d connection through 0d matrices (DV,[0909.2453])

4d N=2 SU(2) with N_f=4

instanton part of
Nekrasov part. fn.

AGT

2d CFT

four-point
conformal block

DF multiple integral

0d β -deformed matrix model
of Selberg type

Mironov-Morozov-Shakirov [1001.0573], Itoyama-T.O [1003.2929]

- Matrix model technology
  - ✓ Saddle point evaluation in the large N limit
  - ✓ Orthogonal polynomials
  - ✓ Recursion relations
  - ✓ ……

- message:

(a generalization of) the Selberg integral also plays important role in the β -deformed matrix models, which have connection with N=2 gauge theories

→Kadell integral (1997)
(Macdonald's ex-conjecture (1987))

- Euler beta function

$$B(\alpha_1, \alpha_2) = \int_0^1 t^{\alpha_1 - 1}(1-t)^{\alpha_2 - 1} dt = \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)}{\Gamma(\alpha_1 + \alpha_2)}.$$

- Selberg integral (1944)
  (multiple generalization of the beta fn.)

$$S_N(\alpha_1, \alpha_2, \beta) = \left(\prod_{I=1}^{N} \int_0^1 dx_I \right) \prod_{I=1}^{N} x_I^{\alpha_1 - 1}(1 - x_I)^{\alpha_2 - 1} \prod_{1 \le I < J \le N} |x_I - x_J|^{2\beta}$$

$$= \prod_{j=1}^{N} \frac{\Gamma(1 + j\beta)\Gamma(\alpha_1 + (j-1)\beta)\Gamma(\alpha_2 + (j-1)\beta)}{\Gamma(1+\beta)\Gamma(\alpha_1 + \alpha_2 + (N + j - 2)\beta)}.$$

Atle Selberg, "Bemerkinger om et multipelt integral (Remarks on a multiple integral)", Norsk Matematisk Tidsskrift 26 (1944), 71-78 (in Norwegian)

- Macdonald-Kadell formula (Kadell integral)

$$\prod_{I=1}^{N}\left(\int_0^1 dx_I\right) P_\lambda^{(1/\beta)}(x)\prod_{I=1}^{N} x_I^{\alpha_1-1}(1-x_I)^{\alpha_2-1}\prod_{I<J}|x_I-x_J|^{2\beta}$$

$$=\frac{[\alpha_1+(N-1)\beta]_\lambda^{(\beta)}}{[\alpha_1+\alpha_2+2(N-1)\beta]_\lambda^{(\gamma)}}P_\lambda^{(1/\beta)}(1^N)S_N(\alpha_1,\alpha_2,\beta).$$

$\lambda=(\lambda_1,\lambda_2,\cdots)$:a partition $\lambda_1\geq\lambda_2\geq\cdots\geq 0.$

$[\alpha]_\lambda^{(\beta)}=\prod_{i=1}^{\infty}\frac{\Gamma(\alpha+\lambda_i+(1-i)\beta)}{\Gamma(\alpha+(1-i)\beta)}.$ $P_\lambda^{(1/\beta)}(x)$: the Jack symmetric polynomial

Plan of this talk

1. Introduction
2. β -deformed matrix model
3. AGT(4d/2d connection) through 0d matrices
4. On a basis of q-expansion
5. Summary

2. β -deformed matrix model

- Hermitian One Matrix Model:

M: $N \times N$ Hermitian matrix,
λ_I: eigenvalue of M
$W(x)$: a potential

$$Z = \int [\mathrm{d}M] \exp\left(\frac{1}{g_s}\mathrm{Tr}\, W(M)\right)$$
$$= \int_{-\infty}^{\infty} \mathrm{d}^N\lambda \prod_{I=1}^{N} \exp\left(\frac{1}{g_s} W(\lambda_I)\right) \prod_{1\le I<J\le N} |\lambda_I - \lambda_J|^2.$$

- β-deformed MM(β-ensemble)

$$Z = \int_{-\infty}^{\infty} \mathrm{d}^N\lambda \prod_{I=1}^{N} \exp\left(\frac{\sqrt{\beta}}{g_s} W(\lambda_I)\right) \prod_{1\le I<J\le N} |\lambda_I - \lambda_J|^{2\beta}.$$

$$\beta = \begin{cases} 1/2 & \text{Orthogonal matrix} \\ 1 & \text{Hermitian matrix} \\ 2 & \text{Symplectic matrix} \end{cases}$$

- (generalized) β -deformed MM

$$Z = \int_C d^N x \prod_{I=1}^{N} \exp\left(\sqrt{\beta} W(x_I)\right) \prod_{1\le I<J\le N} |x_I - x_J|^{2\beta},$$

(for some contour C)

- β -deformed MM of Selberg Type

Take $C = [0,1]$

$$\exp\left(\sqrt{\beta} W(x)\right) = x^{\alpha_1 - 1}(1-x)^{\alpha_2 - 1} \exp\left(\sqrt{\beta}\tilde{W}(x)\right).$$

If $\tilde{W}(x) = 0$ ⇒ Selberg integral

- MM of Selberg Type:

$$Z(\alpha_1, \alpha_2, \beta; \{g_i\})$$

$$= \int_0^1 d^N x_I \prod_{I=1}^{N} x_I^{\alpha_1 - 1}(1 - x_I)^{\alpha_2 - 1} \prod_{I<J}(x_I - x_J)^{2\beta} \ \exp\left(\sum_{I=1}^{N} W(x_I; g)\right)$$

$$= S_N(\alpha_1, \alpha_2, \beta) \left\langle\!\!\left\langle \exp\left(\sum_{I=1}^{N} W(x_I; g)\right) \right\rangle\!\!\right\rangle_N$$

↑ average w.r.t. the Selberg integral

$$W(x; g) = \sum_{i=0}^{\infty} g_i x^i.$$

- Exactly calculable: Itoyama-T.O. [1003.2929]
  expansion in Jack polynomial basis and by using Kadell integral

- the expansion into the Jack polynomials

$$\exp\left(\sum_{I=1}^{N} W(x_I;\{g_i\})\right) = \sum_{\lambda} C_\lambda^{(\beta)}(g)\, P_\lambda^{(1/\beta)}(x).$$

Here $P_\lambda^{(1/\beta)}(x)$ is a polynomial of $x = (x_1, \cdots, x_N)$ and $\lambda = (\lambda_1, \lambda_2, \cdots)$ is a partition: $\lambda_1 \geq \lambda_2 \geq \cdots \geq 0$.

- Jack polynomials are the eigenstates of

$$\sum_{I=1}^{N}\left(x_I\frac{\partial}{\partial x_I}\right)^2 + \beta \sum_{1\leq I<J\leq N}\left(\frac{x_I + x_J}{x_I - x_J}\right)\left(x_I\frac{\partial}{\partial x_I} - x_J\frac{\partial}{\partial x_J}\right)$$

with homogeneous degree $|\lambda| = \lambda_1 + \lambda_2 + \cdots$

- normalization

$$P_\lambda^{(1/\beta)}(x) = m_\lambda(x) + \sum_{\mu<\lambda} a_{\lambda\mu} m_\mu(x).$$

Here $m_\lambda(x)$ is the monomial symmetric polynomial.
" $<$ " is dominance ordering.

- Explicit forms of the Jack polynomials for $|\lambda| \leq 2$

$$P_{(1)}^{(1/\beta)}(x) = m_{(1)}(x) = \sum_{I=1}^{N} x_I,$$

$$P_{(2)}^{(1/\beta)}(x) = m_{(2)}(x) + \frac{2\beta}{1+\beta} m_{(1^2)}(x) = \sum_{I=1}^{N} x_I^2 + \frac{2\beta}{1+\beta} \sum_{1\leq I<J\leq N} x_I x_J,$$

$$P_{(1^2)}^{(1/\beta)}(x) = m_{(1^2)}(x) = \sum_{1\leq I<J\leq N} x_I x_J.$$

- The Macdonald-Kadell formula:

$$\langle\langle P_\lambda^{(1/\beta)}(x)\rangle\rangle_N = \prod_{s\in\lambda} \frac{\{a(s)+\alpha_1+(N-1-l'(s))\beta\}}{\{a(s)+\alpha_1+\alpha_2+(2N-2-l'(s))\beta\}} \frac{\{a(s)+(N-l'(s))\beta\}}{\{a(s)+(l(s)+1)\beta\}}.$$

where λ' is the conjugate partition of λ,

$$a(s)=\lambda_i-j,\quad l(s)=\lambda'_j-i,\quad a'(s)=j-1,\quad l'(s)=i-1.$$

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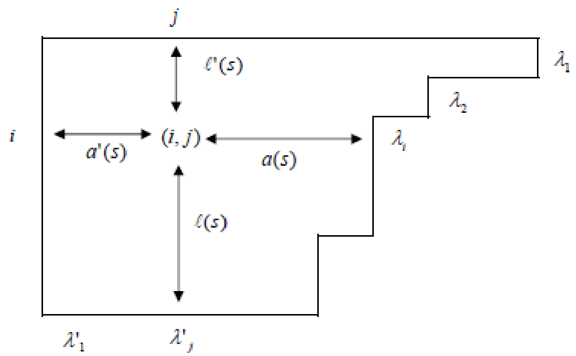


$$s=(i,j)\in\lambda$$

3. AGT(4d/2d connection) through 0d matrices

$$Z_{\text{inst}}(q) = (1-q)^{-(1/2)\alpha_2\alpha_3}\mathcal{B}(q) = \langle\langle \widehat{M}(x,y;q) \rangle\rangle.$$

4d: instanton part of Nekrasov part. fn.

2d: conformal block

0d: average in matrix model

$$\widehat{M}(x,y;q) = \prod_{I=1}^{N_L}(1-qx_I)^{\sqrt{\beta}\alpha_3}\prod_{J=1}^{N_R}(1-qy_J)^{\sqrt{\beta}\alpha_2}\prod_{I=1}^{N_L}\prod_{J=1}^{N_R}(1-qx_Iy_J)^{2\beta}.$$

- 4d: Instanton part of Nekrasov partition function for N=2 SU(2) gauge theory with N_f=4 (antifundamental matters)

$$Z_{\text{inst}}(q) = \sum_{\lambda_1,\lambda_2} q^{|\lambda_1|+|\lambda_2|} Z_{\lambda_1,\lambda_2}.$$

$$q = \exp(2\pi i \tau_{\text{UV}}), \qquad \tau_{\text{UV}} = \frac{4\pi i}{g_{\text{UV}}^2} + \frac{\theta_{\text{UV}}}{2\pi}.$$

$$Z_{(1),(0)} = -\frac{1}{\epsilon_1\epsilon_2}\frac{\prod_{r=1}^4(a+m_r)}{2a(2a+\epsilon)},$$

$$Z_{(0),(1)} = -\frac{1}{\epsilon_1\epsilon_2}\frac{\prod_{r=1}^4(a-m_r)}{2a(2a-\epsilon)},$$

$$Z_{(2),(0)} = \frac{1}{2!\epsilon_1\epsilon_2^2(\epsilon_1-\epsilon_2)}\frac{\prod_{r=1}^4(a+m_r)(a+m_r+\epsilon_2)}{2a(2a+\epsilon_2)(2a+\epsilon+\epsilon_2)},$$

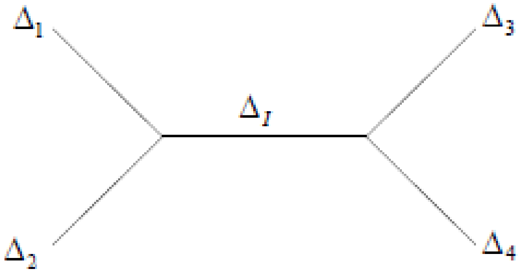
$$Z_{(0),(2)} = \frac{1}{2!\epsilon_1\epsilon_2^2(\epsilon_1-\epsilon_2)}\frac{\prod_{r=1}^4(a-m_r)(a-m_r-\epsilon_2)}{2a(2a-\epsilon_2)(2a-\epsilon)(2a-\epsilon-\epsilon_2)},$$

$$Z_{(11),(0)} = -\frac{1}{2!\epsilon_1^2\epsilon_2(\epsilon_1-\epsilon_2)}\frac{\prod_{r=1}^4(a+m_r)(a+m_r+\epsilon_1)}{2a(2a+\epsilon_1)(2a+\epsilon)(2a+\epsilon+\epsilon_1)},$$

$$Z_{(0),(11)} = -\frac{1}{2!\epsilon_1^2\epsilon_2(\epsilon_1-\epsilon_2)}\frac{\prod_{r=1}^4(a-m_r)(a-m_r+\epsilon_1)}{2a(2a-\epsilon_1)(2a-\epsilon)(2a-\epsilon-\epsilon_1)},$$

$$Z_{(1),(1)} = \frac{1}{\epsilon_1^2\epsilon_2^2}\frac{\prod_{r=1}^4(a+m_r)(a-m_r)}{(4a^2-\epsilon_1^2)(4a^2-\epsilon_2^2)}.$$

2d: conformal block ←model independent (representation theoretic) quantity



$$\phi(z)\phi(w) \sim (1/2)\log(z-w).$$

Free field representation of the conformal block

$$\mathcal{B}(q) \sim \langle 0| \mathrm{e}^{\alpha_1\phi(0)}\mathrm{e}^{\alpha_2\phi(q)}\mathrm{e}^{\alpha_3\phi(1)}\mathrm{e}^{\alpha_4\phi(\infty)} \times \left(\int_0^q \mathrm{d}z \mathrm{e}^{2\sqrt{\beta}\phi(z)}\right)^{N_L}\left(\int_1^\infty \mathrm{d}w \mathrm{e}^{2\sqrt{\beta}\phi(w)}\right)^{N_R}|0\rangle.$$

screening charges

$$Q_E = \sqrt{\beta} - \frac{1}{\sqrt{\beta}}$$

$$\Delta_i = \frac{1}{4}\alpha_i(\alpha_i - 2Q_E), \qquad \Delta_I = \frac{1}{4}\alpha_I(\alpha_I - 2Q_E).$$

$$c = 1 - 6\left(\sqrt{\beta} - \frac{1}{\sqrt{\beta}}\right)^2$$

$$\alpha_I = \alpha_1 + \alpha_2 + 2\sqrt{\beta}N_L = -\alpha_4 - \alpha_4 - 2\sqrt{\beta}N_R + 2Q_E.$$

- 0d: β -deformed matrix model of Selberg type

$$\langle\langle \mathcal{O} \rangle\rangle = \frac{1}{\mathcal{N}} \int_0^1 d^{N_L}x \prod_{I=1}^{N_L} x_I^{\sqrt{\beta}\alpha_1}(1-x_I)^{\sqrt{\beta}\alpha_2} \prod_{1\le I<J\le N_L} |x_I - x_J|^{2\beta} \times \int_0^1 d^{N_R}y \prod_{J=1}^{N_R} y_J^{\sqrt{\beta}\alpha_4}(1-y_J)^{\sqrt{\beta}\alpha_3} \prod_{1\le I<J\le N_R} |y_I - y_J|^{2\beta} \mathcal{O}.$$

$$\widehat{M}(x,y;q) = \prod_{I=1}^{N_L}(1-qx_I)^{\sqrt{\beta}\alpha_3} \prod_{J=1}^{N_R}(1-qy_J)^{\sqrt{\beta}\alpha_2} \prod_{I=1}^{N_L}\prod_{J=1}^{N_R}(1-qx_I y_J)^{2\beta}.$$

$$\langle\langle 1 \rangle\rangle = 1.$$

$\mathcal{N}=2$, $SU(2)$, $N_f=4$: six parameters

$$\frac{\epsilon_1}{g_s}, \quad \frac{a}{g_s}, \quad \frac{m_1}{g_s}, \quad \frac{m_2}{g_s}, \quad \frac{m_3}{g_s}, \quad \frac{m_4}{g_s}$$

By examining the explicit form of the first order q-expansion coefficients, we get

0d-4d relation

$$\sqrt{\beta}N_L = \frac{a-m_2}{g_s}, \qquad \sqrt{\beta}N_R = -\frac{a+m_3}{g_s},$$

$$\alpha_1 = \frac{1}{g_s}(m_2 - m_1 + \epsilon), \qquad \alpha_2 = \frac{1}{g_s}(m_2 + m_1),$$

$$\alpha_3 = \frac{1}{g_s}(m_3 + m_4), \qquad \alpha_4 = \frac{1}{g_s}(m_3 - m_4 + \epsilon).$$

$$\epsilon = \epsilon_1 + \epsilon_2 \qquad \frac{\epsilon_1}{g_s} = \sqrt{\beta}, \quad \frac{\epsilon_2}{g_s} = -\frac{1}{\sqrt{\beta}}.$$

4. On a basis of q-expansion

$$\widehat{M}(x,y;q) = \prod_{I=1}^{N_L}(1-qx_I)^{\sqrt{\beta}\alpha_3}\prod_{J=1}^{N_R}(1-qy_J)^{\sqrt{\beta}\alpha_2} \quad \prod_{I=1}^{N_L}\prod_{J=1}^{N_R}(1-qx_Iy_J)^{2\beta}.$$

- There is a bais of q-expansion

$$\widehat{M}(x,y;q) = \sum_{\lambda_1,\lambda_2} q^{|\lambda_1|+|\lambda_2|}\widehat{M}_{\lambda_1,\lambda_2}(x,y).$$

such that

$$Z_{\lambda_1,\lambda_2} = \langle\langle \widehat{M}_{\lambda_1,\lambda_2}(x,y)\rangle\rangle.$$

- Factorization (conjecture)

Itoyama-T.O. [1003.29292]

$$\widehat{M}_{\lambda_1,\lambda_2}(x,y) = M_{\lambda_1,\lambda_2}(x)\widetilde{M}_{\lambda_1,\lambda_2}(y).$$

Explicit forms

$$M_{(1),(0)}(x) = -\alpha_2 - \left(\frac{\alpha_I - 2Q_E}{\alpha_I - Q_E}\right)\sqrt{\beta}p_1(x),$$

$$\widetilde{M}_{(1),(0)}(y) = \sqrt{\beta}p_1(y),$$

$$p_k(x) = \sum_{I=1}^{N_L} x_I^k,$$

$$p_k(y) = \sum_{J=1}^{N_R} y_J^k.$$

$$M_{(0),(1)}(x) = \sqrt{\beta}p_1(x),$$

$$\widetilde{M}_{(0),(1)}(y) = -\alpha_3 - \left(\frac{\alpha_I}{\alpha_I - Q_E}\right)\sqrt{\beta}p_1(y),$$

$$\begin{aligned} M_{(2),(0)}(x) = &\frac{1}{2}\alpha_2^2 - \frac{\alpha_2}{2\sqrt{\beta}} + \frac{(2a-\epsilon_1)\alpha_2}{(2a+\epsilon_2)}\sqrt{\beta}P^{(1/\beta)}_{(1)}(x) \\ &- \frac{2(2a-\epsilon-\epsilon_2)}{\epsilon_2(\epsilon_1-\epsilon_2)(2a+\epsilon_2)}\beta P^{(1/\beta)}_{(1^2)}(x) \\ &+ \frac{(\epsilon_1+\epsilon_2)(2a-\epsilon_1+\epsilon_2)(2a-\epsilon-\epsilon_2)}{4ag_s(2a+\epsilon_2)}\sqrt{\beta}P^{(1/\beta)}_{(2)} \end{aligned}$$

$$\widetilde{M}_{(2),(0)}(y) = \beta P^{(1/\beta)}_{(2)}(y).$$

Some properties of M

(1) In general, they are inhomogeneous polynomials

(2)

$$\widetilde{M}_{\lambda_1,(0)}(y) = \text{const} \times P^{(1/\beta)}_{\lambda_1}(y).$$

(3) They are greatly simplified at $\beta = 1$

Cauchy identity

$$\prod_I \prod_J \frac{1}{1 - q x_I y_J} = \exp\left(\sum_{k=1}^{\infty} \frac{q^k}{k} p_k(x) p_k(y)\right)$$
$$= \sum_\lambda q^{|\lambda|} s_\lambda(x) s_\lambda(y)$$
$$= \sum_\lambda q^{|\lambda|} s_\lambda(\{p_k(x)\}) s_\lambda(\{p_k(y)\}).$$

$$\widehat{M}(x, y; q) = \prod_{I=1}^{N_L} (1 - q x_I)^{\sqrt{\beta}\alpha_3} \prod_{J=1}^{N_R} (1 - q y_J)^{\sqrt{\beta}\alpha_2} \prod_{I=1}^{N_L} \prod_{J=1}^{N_R} (1 - q x_I y_J)^{2\beta}$$

$$= \exp\left(-\sum_{k=1}^{\infty} \frac{q^k}{k} (\alpha_2 + \sqrt{\beta} p_k(x)) \sqrt{\beta} p_k(y)\right) \exp\left(-\sum_{k=1}^{\infty} \frac{q^k}{k} \sqrt{\beta} p_k(x) (\alpha_3 + \sqrt{\beta} p_k(y)\right).$$

$$\exp\left(-\sum_{k=1}^{\infty} \frac{q^k}{k} (\alpha_2 + \sqrt{\beta} p_k(x)) \sqrt{\beta} p_k(y)\right) = \sum_{\lambda_1} q^{|\lambda_1|} s_{\lambda_1}(\{-\alpha_2 - \sqrt{\beta} p_k(x)\}) s_{\lambda_1}(\{\sqrt{\beta} p_k(y)),$$

$$\exp\left(-\sum_{k=1}^{\infty} \frac{q^k}{k} \sqrt{\beta} p_k(x) (\alpha_3 + \sqrt{\beta} p_k(y)\right) = \sum_{\lambda_2} q^{|\lambda_2|} s_{\lambda_2}(\{\sqrt{\beta} p_k(x)) s_{\lambda_2}(\{-\alpha_3 - \sqrt{\beta} p_k(y)\}).$$

q-expansion in Schur polynomial basis

$$\widehat{M}(x,y;q)$$
$$= \sum_{\lambda_1,\lambda_2} q^{|\lambda_1|+|\lambda_2|} s_{\lambda_1}\left(\{-\alpha_2 - \sqrt{\beta}p_k(x)\}\right) s_{\lambda_2}\left(\{\sqrt{\beta}p_k(x)\}\right) \;\; s_{\lambda_1}\left(\{\sqrt{\beta}p_k(y)\}\right) s_{\lambda_2}\left(\{-\alpha_3 - \sqrt{\beta}p_k(y)\}\right).$$

But for generic β ,

$$M_{\lambda_1,\lambda_2}(x) \neq s_{\lambda_1}\left(\{-\alpha_2 - \sqrt{\beta}p_k(x)\}\right) s_{\lambda_2}\left(\{\sqrt{\beta}p_k(x)\}\right),$$

$$\widetilde{M}_{\lambda_1,\lambda_2}(y) \neq s_{\lambda_1}\left(\{\sqrt{\beta}p_k(y)\}\right) s_{\lambda_2}\left(\{-\alpha_3 - \sqrt{\beta}p_k(y)\}\right).$$

Only for β =1,

Mironov-Morozov-Shakirov [1012.3137]

$$M_{\lambda_1,\lambda_2}(x) = s_{\lambda_1}\left(\{-\alpha_2 - p_k(x)\}\right) s_{\lambda_2}\left(\{p_k(x)\}\right),$$
$$\widetilde{M}_{\lambda_1,\lambda_2}(y) = s_{\lambda_1}\left(\{p_k(y)\}\right) s_{\lambda_2}\left(\{-\alpha_3 - p_k(y)\}\right).$$

(β =1 for SU(N): Zhang-Matsuo [1110.5255])

5. Summary

- ✓ β -deformed matrix model of Selberg type
- ✓ calculation method by using the Jack polynomials and Kadell integral
- ✓ Application to q-expansion of the Nekrasov partition function

- Characterization of $M_{\lambda_1,\lambda_2}(x)$ and $\widetilde{M}_{\lambda_1,\lambda_2}(y)$ for generic β ?
- generalization to other gauge theories?