On 4-dimensional defects of 6d N=(2,0) theory

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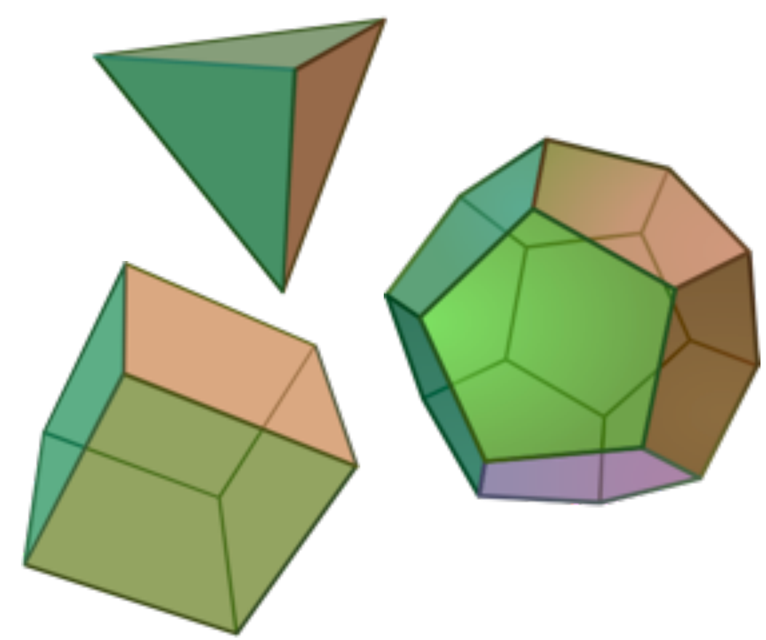
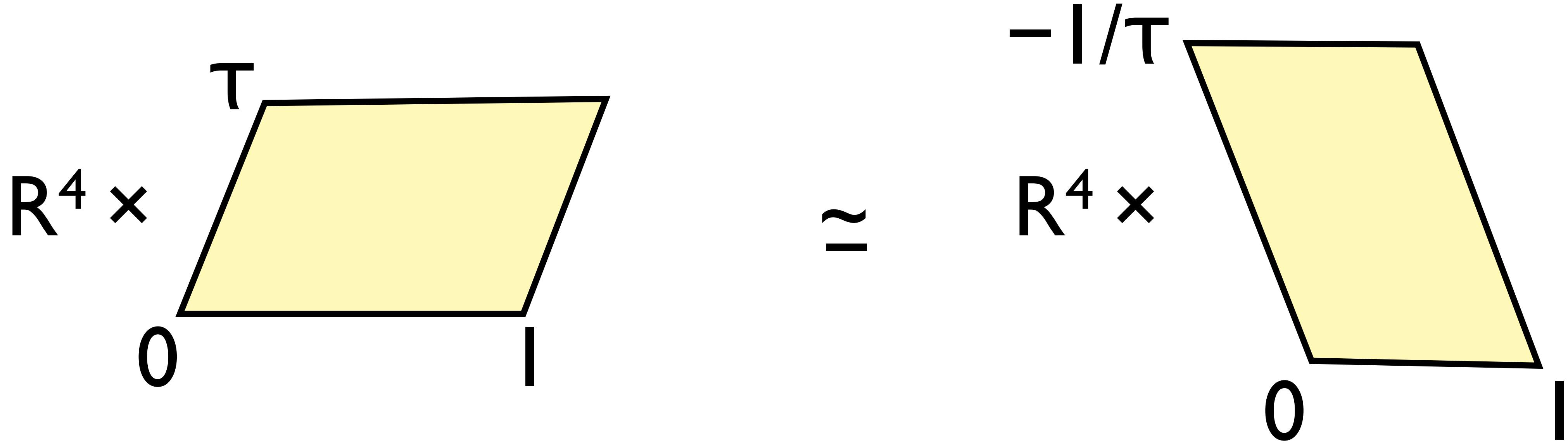
A 6d $\mathcal{N}$=(2,0) theory is one of :

X=	A_{K-1}	D_K	$E_{6,7,8}$
Type IIB on	C^2/Z_K	C^2/dihed	C^2/tetra C^2/octa C^2/icosa



6d theory of type X on T^2
= 4d $\mathcal{N}$=4 SYM with gauge group X

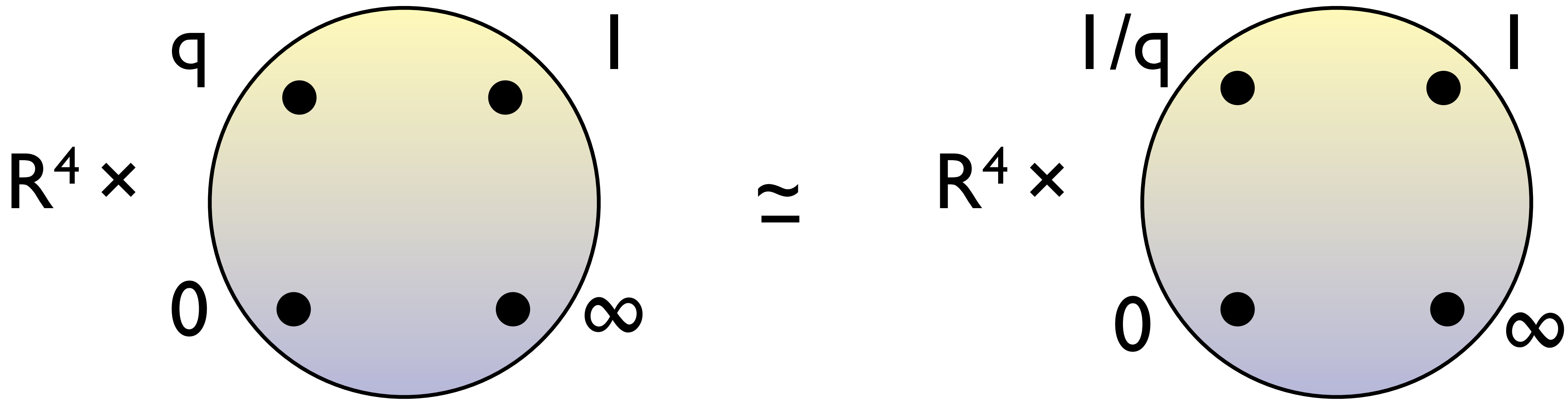
[Witten '96]



➡ 4d $\mathcal{N}$=4 SYM with gauge group X at coupling τ
= 4d $\mathcal{N}$=4 SYM with gauge group X at coupling $-1/\tau$

[Montonen-Olive '77] [Olive-Witten '78]

6d theory of type SU(2) on [Gaiotto '09]

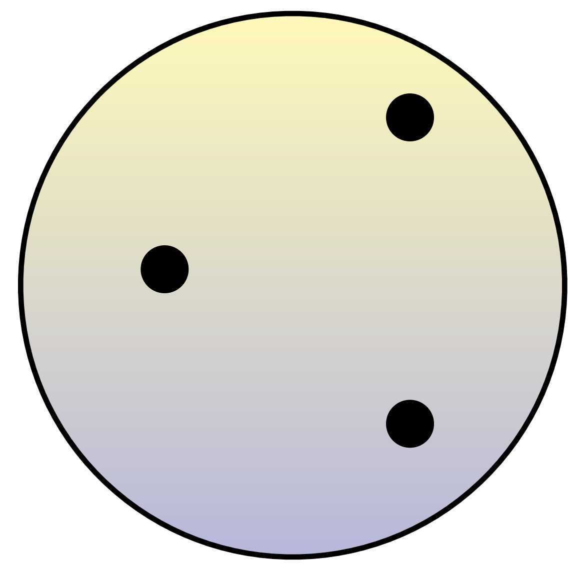


4d N=2 SU(2) theory with 4 hypers at coupling τ

= 4d N=2 SU(2) theory with 4 hypers at coupling $-1/\tau$

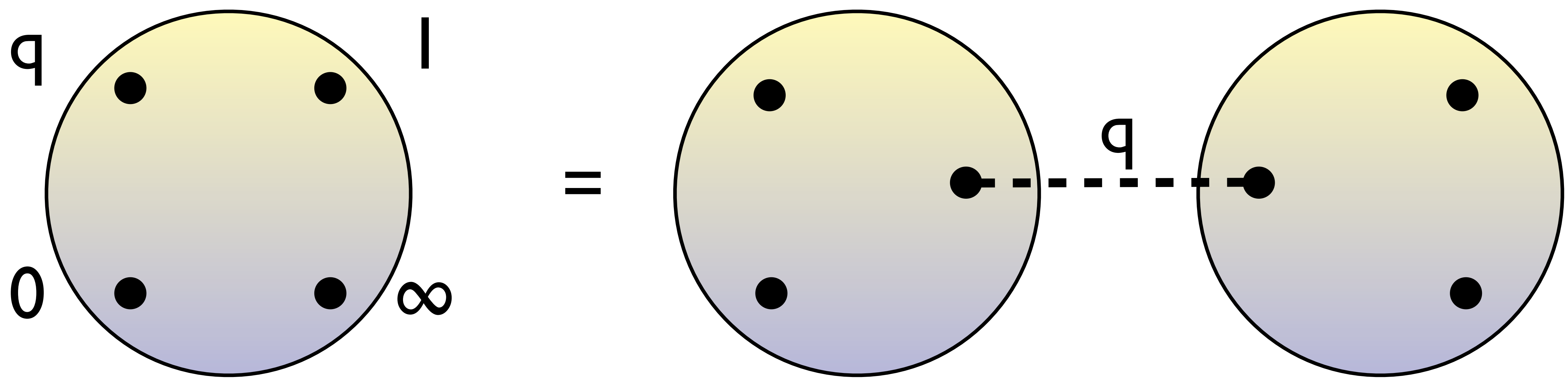
[Seiberg-Witten '94]

6d theory of type SU(2) on

$R^4 \times$ 

= 2 fundamental hypers of SU(2)

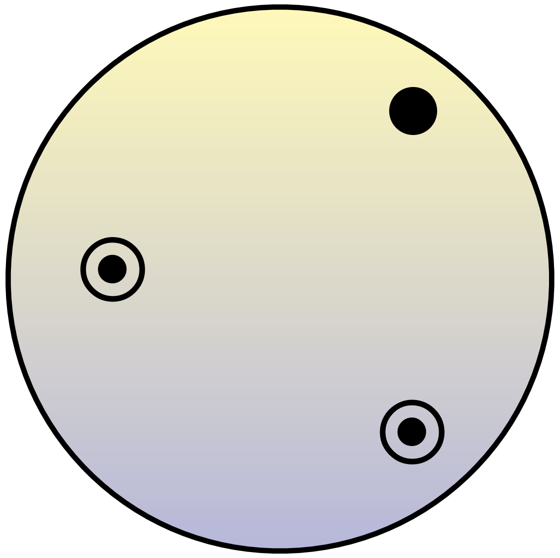
6d theory of type SU(2) on



➡ 2 hypers of SU(2) + 2 hypers of SU(2)
+ SU(2) gauge group at coupling τ

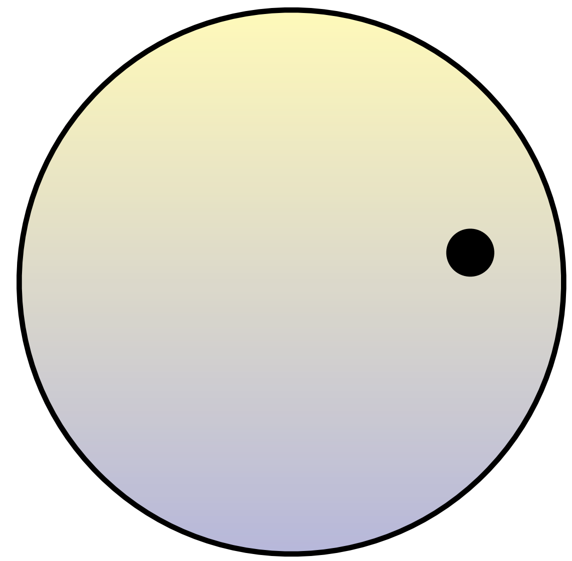
6d theory of type SU(3) on

$R^4 \times$



= 3 fundamental hypers of SU(3)

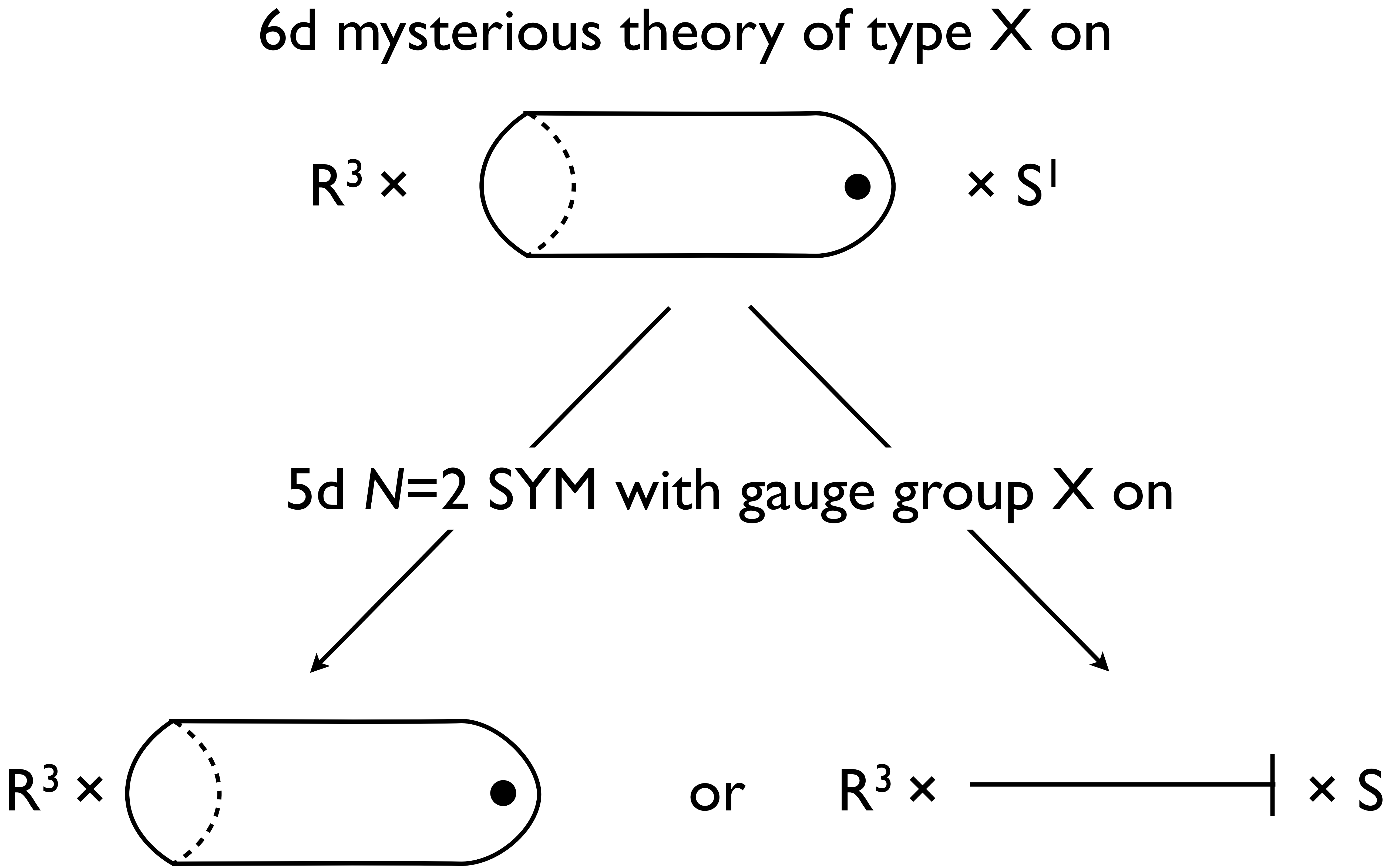
What are these • or ⊙ ?

$R^4 \times$ 

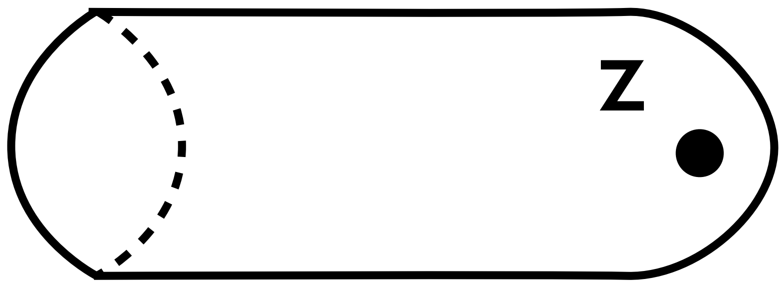
$\cup$

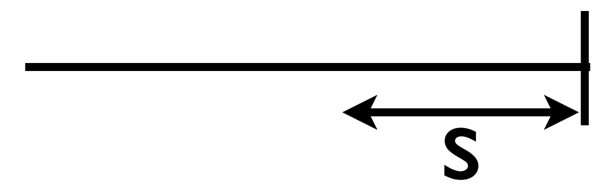
$R^4 \times$ •

Codimension-2, half-BPS defects


6d mysterious theory of type X on
R³ × × S¹
5d N=2 SYM with gauge group X on
R³ ×
or
R³ × × S¹

5d N=2 SYM with gauge group X on

$R^3 \times$ 

or $R^3 \times$ 

$\times S^1$

Hitchin pole

$\Phi_4 + i\Phi_5 \sim \tilde{\rho}(\sigma^+)/z$

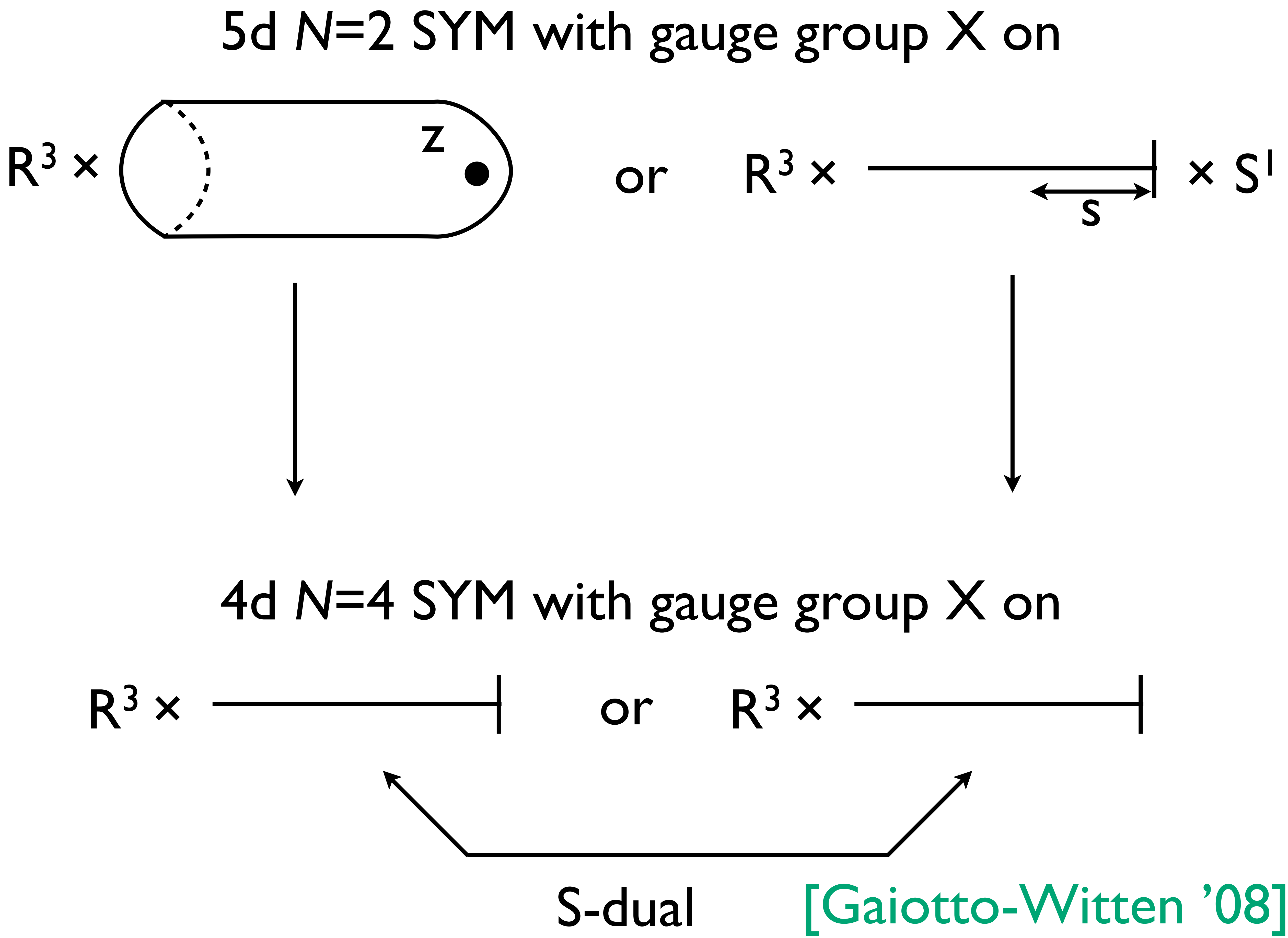
$\tilde{\rho} : SU(2) \to X$

Nahm pole

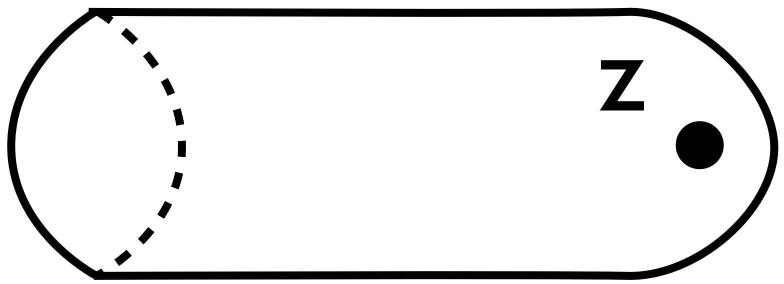
$d\Phi_1/ds = [\Phi_2, \Phi_3]$

$\Phi_a \sim \rho(\sigma_a)/s$

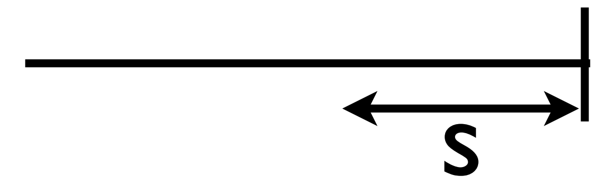
$\rho : SU(2) \to X$


5d N=2 SYM with gauge group X on
R³ × z
or R³ × s × S¹
4d N=4 SYM with gauge group X on
R³ ×
or R³ ×
S-dual
[Gaiotto-Witten '08]

5d $\mathcal{N}$=2 SYM with gauge group X on

$R^3 \times$ 

or

$R^3 \times$ 

$\times S^1$

Hitchin pole

Nahm pole

$d\Phi_1/ds=[\Phi_2, \Phi_3]$

$\Phi_4 + i\Phi_5 \sim \tilde{\rho}(\sigma^+)/z$

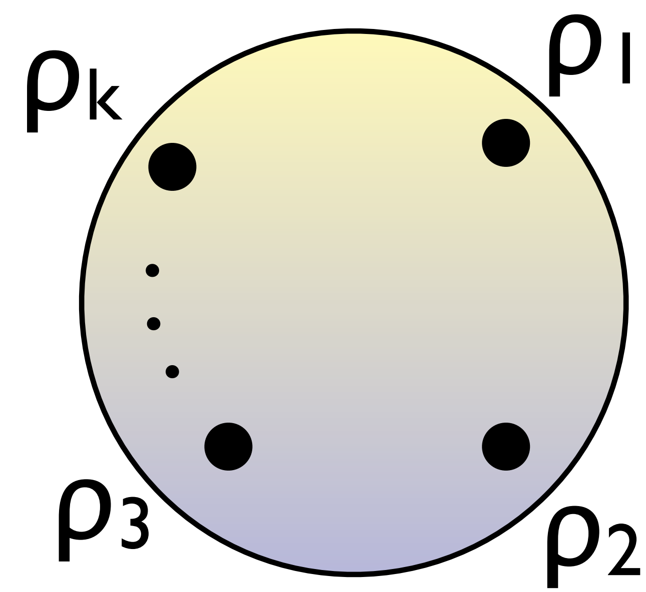
$\Phi_a \sim \rho(\sigma_b)/s$

$\tilde{\rho} : SU(2) \rightarrow X$

$\rho : SU(2) \rightarrow X$

can be analyzed by extending [Gaiotto-Witten '08]

6d theory of type X on $R^4 \times$



Seiberg-Witten curve
= the spectral curve of the Hitchin system
with poles given by $\tilde{\rho}$

[Gaiotto-Moore-Neitzke '10]

dim. of Coulomb branch

$$= [\sum \dim O(\tilde{\rho}(\sigma^+))]/2 - \dim X$$

dim. of Higgs branch

$$= [\sum \dim X - \mathrm{rank}\, X - \dim O(\rho(\sigma^+))]/2 + \mathrm{rank}\, X$$

[Benini-YT-Xie '10]

- $\rho(\sigma^+)$ is a nilpotent element in the Lie algebra of X.
  When X=SU(N), it's just a direct sum of Jordan blocks with zeros along the diagonal.

- $O(\rho(\sigma^+))$ is the subset of the Lie algebra of X conjugate to $\rho(\sigma^+)$ called a nilpotent orbit.
  When X=SU(N), it contains elements whose Jordan normal form has the specified type.

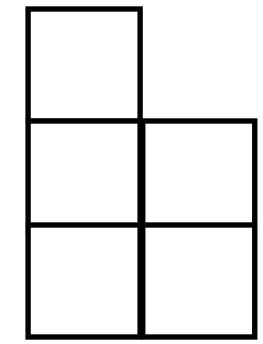
- Classifying ρ is equal to classifying O.

[Jacobson-Morozov, early '50]

When X=SU(N),

Nahm pole $\rho : SU(2) \to SU(N)$

determines $N = n_1 + n_2 + \ldots + n_k$



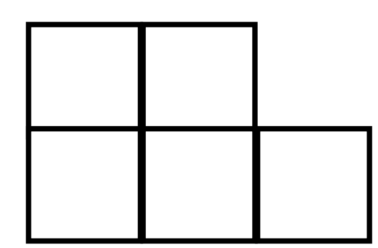
5=3+2

N-dim. irrep of SU(N) dim. of irrep of SU(2)

Hitchin pole $\tilde{\rho} : SU(2) \to SU(N)$ is then given by

the dual partition $N = m_1 + m_2 + \ldots + m_s$.

$\rho \Leftrightarrow \tilde{\rho}$ is one-to-one.



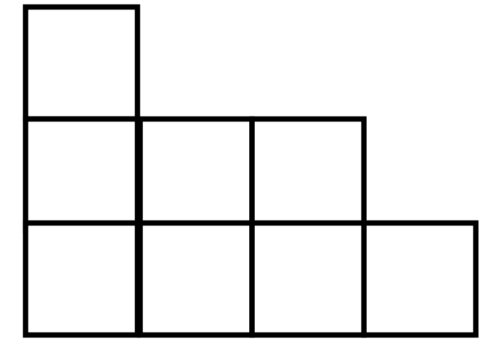
This is a happy accident for SU(N).

5=2+2+1

When X=SO(2N),

Nahm pole $\rho : SU(2) \to SO(2N)$

determines $2N=n_1+n_2+\ldots+n_k$



8=3+2+2+1

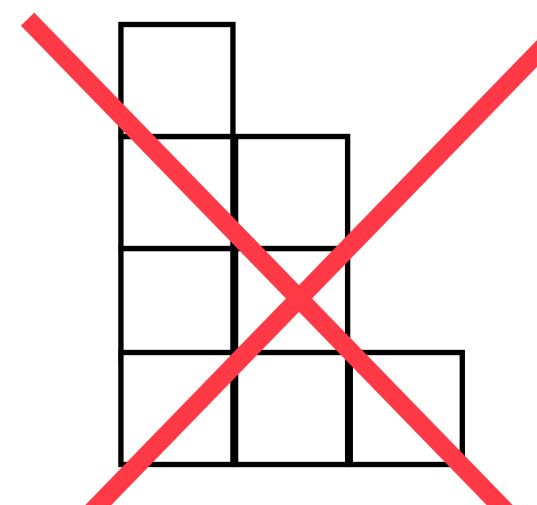
2N-dim. irrep of SO(2N)   dim. of irrep of SU(2)

s.t. each even n appears even times = a D-partition

Hitchin pole $\tilde{\rho} : SU(2) \to SO(2N)$ is then given by

the dual partition $2N=m_1+m_2+\ldots+m_s$.

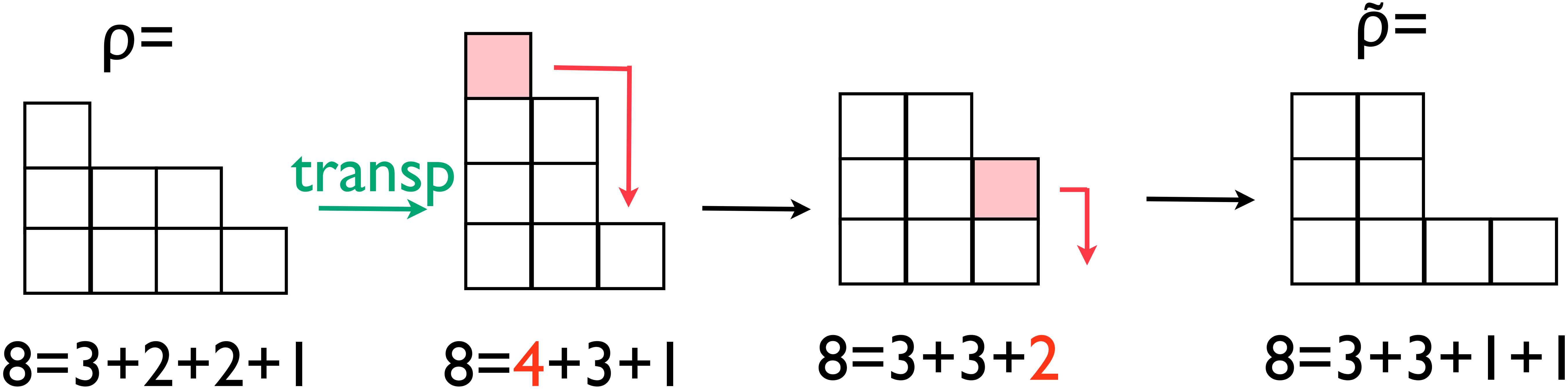
if the dual partition is a D-partition.

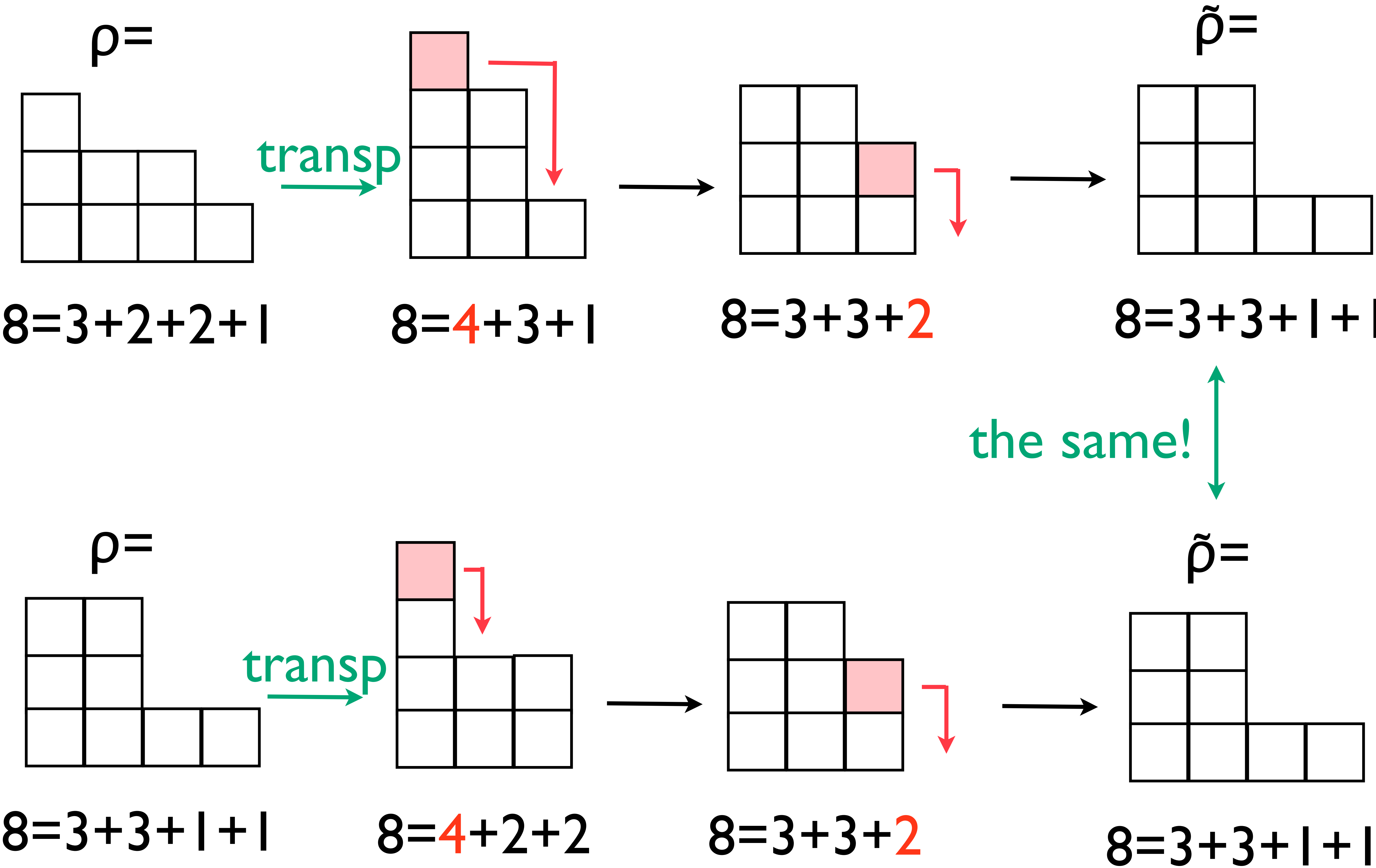


8=4+3+1

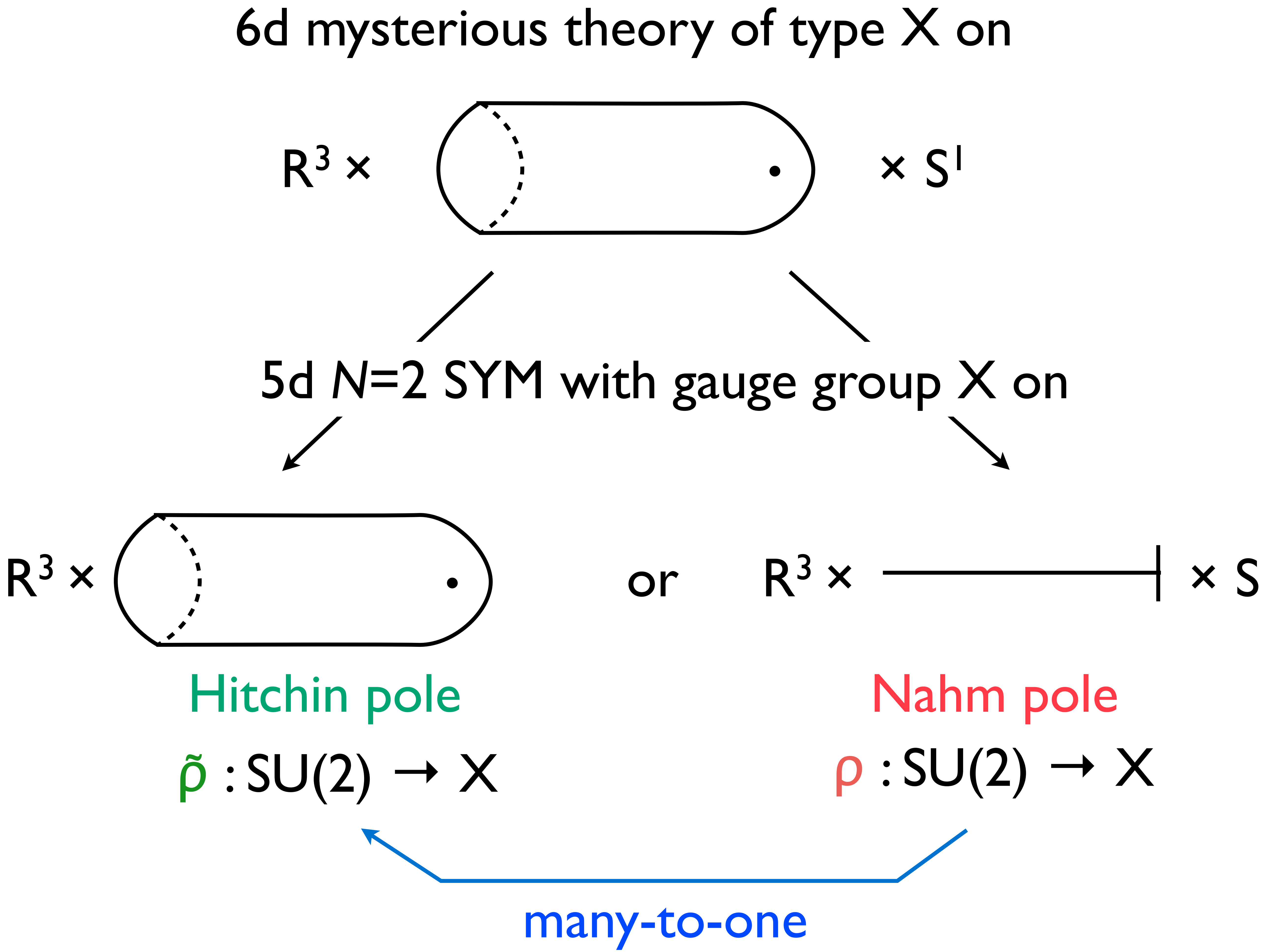
Hitchin pole $\tilde{\rho}: SU(2) \to SO(2N)$ is given by the D-collapse of the dual partition

$$2N = m_1 + m_2 + \ldots + m_s.$$

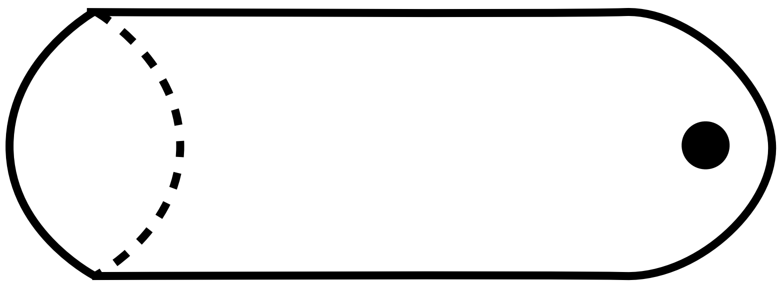




ρ=
transp
8=3+2+2+1
8=4+3+1
8=3+3+2
ρ̃=
8=3+3+1+1
the same!
ρ=
transp
8=3+3+1+1
8=4+2+2
8=3+3+2
ρ̃=
8=3+3+1+1


6d mysterious theory of type X on
R3 ×
× S1
5d N=2 SYM with gauge group X on
R3 ×
or
R3 ×
× S1
Hitchin pole
ρ̃ : SU(2) → X
Nahm pole
ρ : SU(2) → X
many-to-one

5d $\mathcal{N}$=2 SYM with gauge group SO(8) on

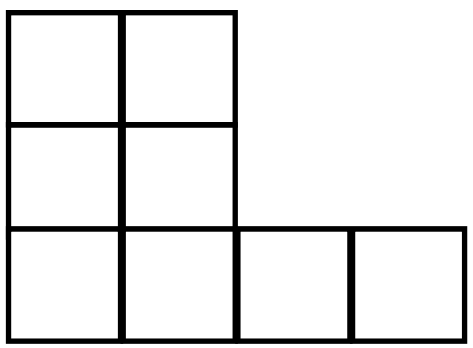
$R^3 \times$  or $R^3 \times$  $\times S^1$

Hitchin pole

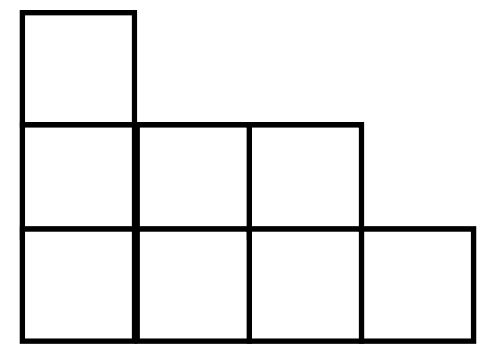
$\tilde{\rho}$: SU(2) → SO(8)

Nahm pole

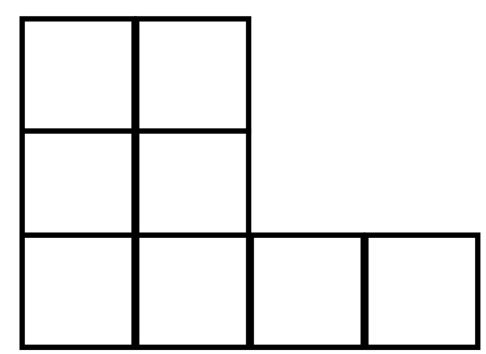
ρ : SU(2) → SO(8)



8=3+3+1+1



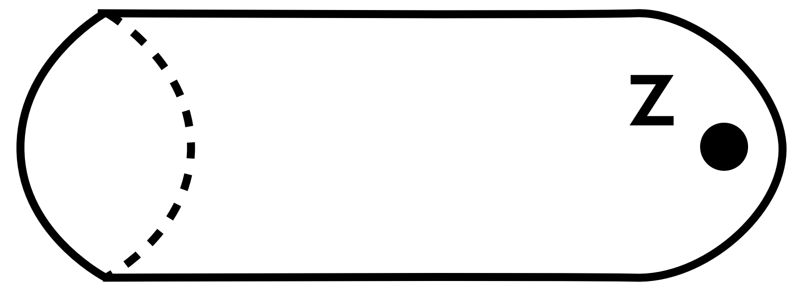
8=3+2+2+1



8=3+3+1+1

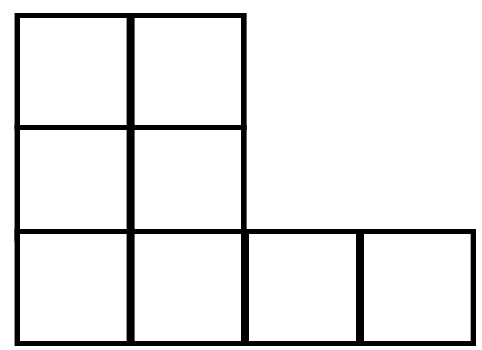
There is a subtle additional datum on the Hitchin side.

5d $\mathcal{N}$=2 SYM with gauge group SO(8) on

$\mathbb{R}^3 \times$ 

Hitchin pole

$\Phi_4 + i\Phi_5 \sim \tilde{\rho}(\sigma^+)/z$, $\tilde{\rho}$: $SU(2) \to SO(8)$

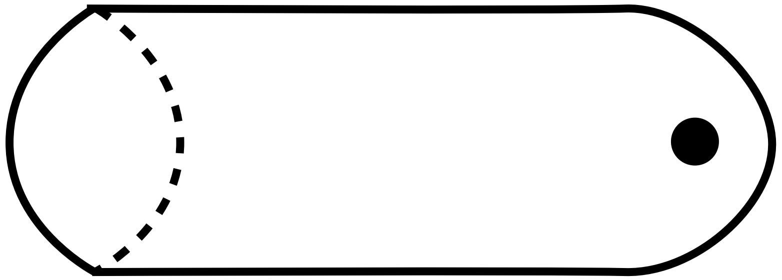


8=3+3+1+1

breaks SO(8) to $S[O(2)\times O(2)] \simeq \mathbb{Z}_2$

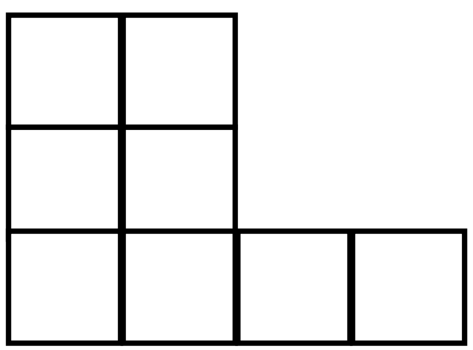
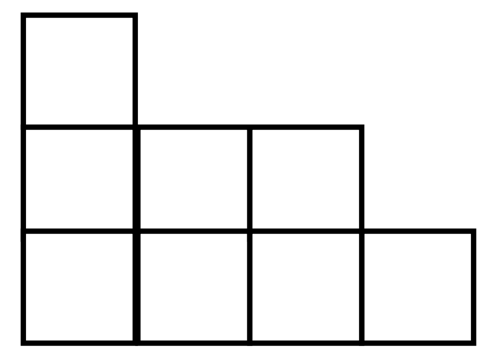
We have a choice to gauge or not to gauge this $\mathbb{Z}_2$.

5d $\mathcal{N}$=2 SYM with gauge group SO(8) on

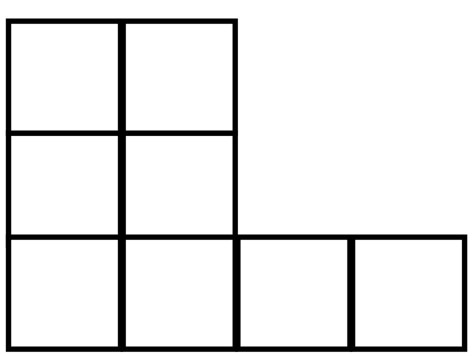
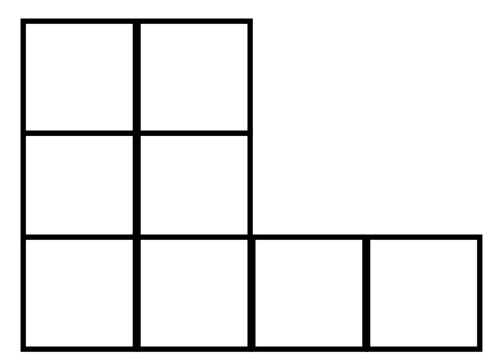
$\mathbb{R}^3 \times$  or $\mathbb{R}^3 \times$  $\times S^1$

Hitchin pole | Nahm pole

$\tilde{\rho} : SU(2) \to SO(8)$ | $\rho : SU(2) \to SO(8)$

 8=3+3+1+1 ⟵  8=3+2+2+1

do gauge $\mathbb{Z}_2$.

 8=3+3+1+1 ⟵  8=3+3+1+1

don't gauge $\mathbb{Z}_2$.

Nahm pole $\rho : SU(2) \to X$

d_{LS} ↓ 'duality' map of Lusztig-Spaltenstein ['82]

Hitchin pole $\tilde{\rho} : SU(2) \to X$

$d_{LS}^2 = 1$ for $X = SU(N)$,

$d_{LS}^3 = d_{LS}$ in general

not a bijection.

Nahm pole $\rho : SU(2) \rightarrow X$

$$\Big\downarrow d_{SA} \qquad \text{Sommers-Achar ['02]}$$

Hitchin pole $\tilde{\rho} : SU(2) \rightarrow X$

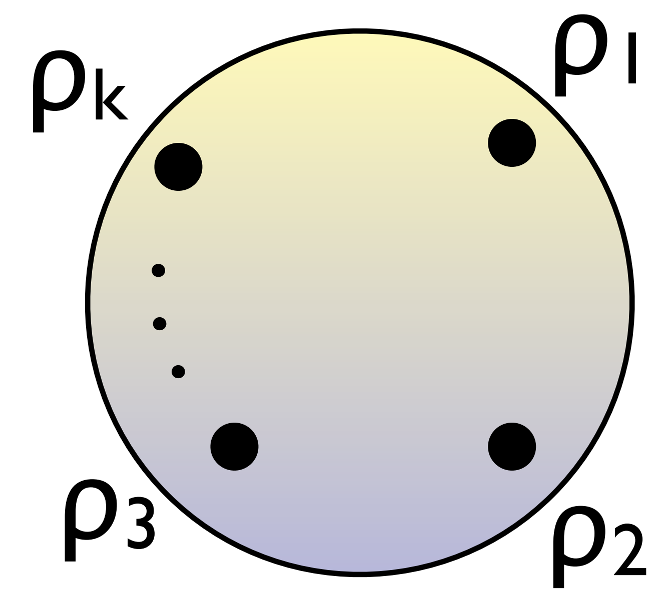
together with a discrete subgroup of X commuting with $\tilde{\rho}(SU(2))$

It is an injection.

- For X=SU(N), SO(2N), M-theoretic analysis giving $\tilde{\rho}$ and the discrete group in terms of ρ agrees with the mathematical map by Spaltenstein, Sommers and Achar.

- We propose that for $X=E_n$, it's also given by their map.

- To test the proposal, we can do the following ...

6d theory of type X on $R^4 \times$ 

Seiberg-Witten curve
= the spectral curve of the Hitchin system
with poles given by $\tilde{\rho}$

dim. of Coulomb branch

$$= [\sum \dim O(\tilde{\rho}(\sigma^+))]/2 - \dim X$$

dim. of Higgs branch

$$= [\sum \dim X - \operatorname{rank} X - \dim O(\rho(\sigma^+))]/2 + \operatorname{rank} X$$

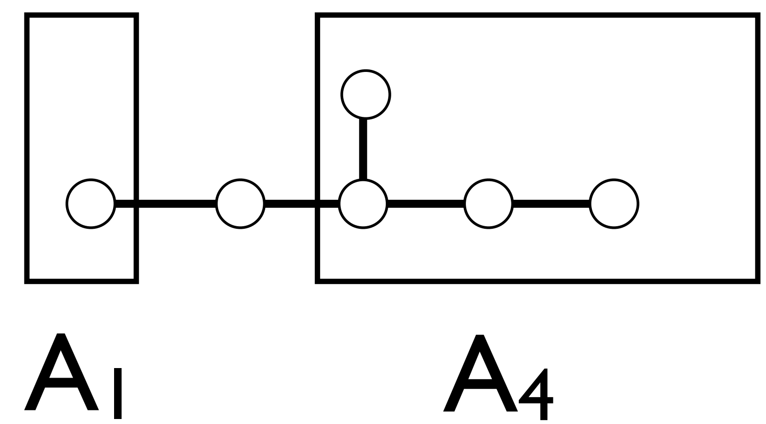
The Seiberg-Witten curve of
2 hypermultiplets of 27 of E_6 [Terashima-Yang '98]
(without dynamical gauge fields) is known.

dim Coulomb = 0
dim Higgs = 54

It can be rewritten as a spectral curve
of an E_6 -Hitchin system with [YT-Terashima '11]
$\tilde{\rho}_1=E_6$, $\tilde{\rho}_2=A_1$, $\tilde{\rho}_3=A_4+A_1$

the Bala-Carter label

$\tilde{\rho}_3 = A_4 + A_1$



$$\tilde{\rho}_3 : SU(2) \begin{cases} \xrightarrow{2\times 2} SU(2) = A_1 \\ \quad + \\ \xrightarrow{5\times 5} SU(5) = A_4 \end{cases} \hookrightarrow E_6$$

$\tilde{\rho}_1 = E_6$, $\quad$ $\dim(O(\tilde{\rho}_1)) = 72$,

$\tilde{\rho}_2 = A_1$, $\quad$ $\dim(O(\tilde{\rho}_2)) = 22$,

$\tilde{\rho}_3 = A_4 + A_1$ $\quad$ $\dim(O(\tilde{\rho}_3)) = 62$

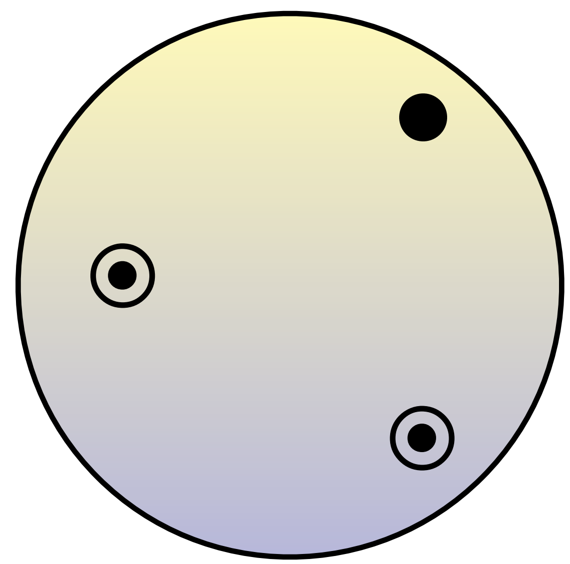
$\updownarrow$ Lusztig-Spaltenstein

$\rho_1 = 0$, $\quad$ $\dim(O(\rho_1)) = 0$,

$\rho_2 = E_6(a_1)$, $\quad$ $\dim(O(\rho_2)) = 70$,

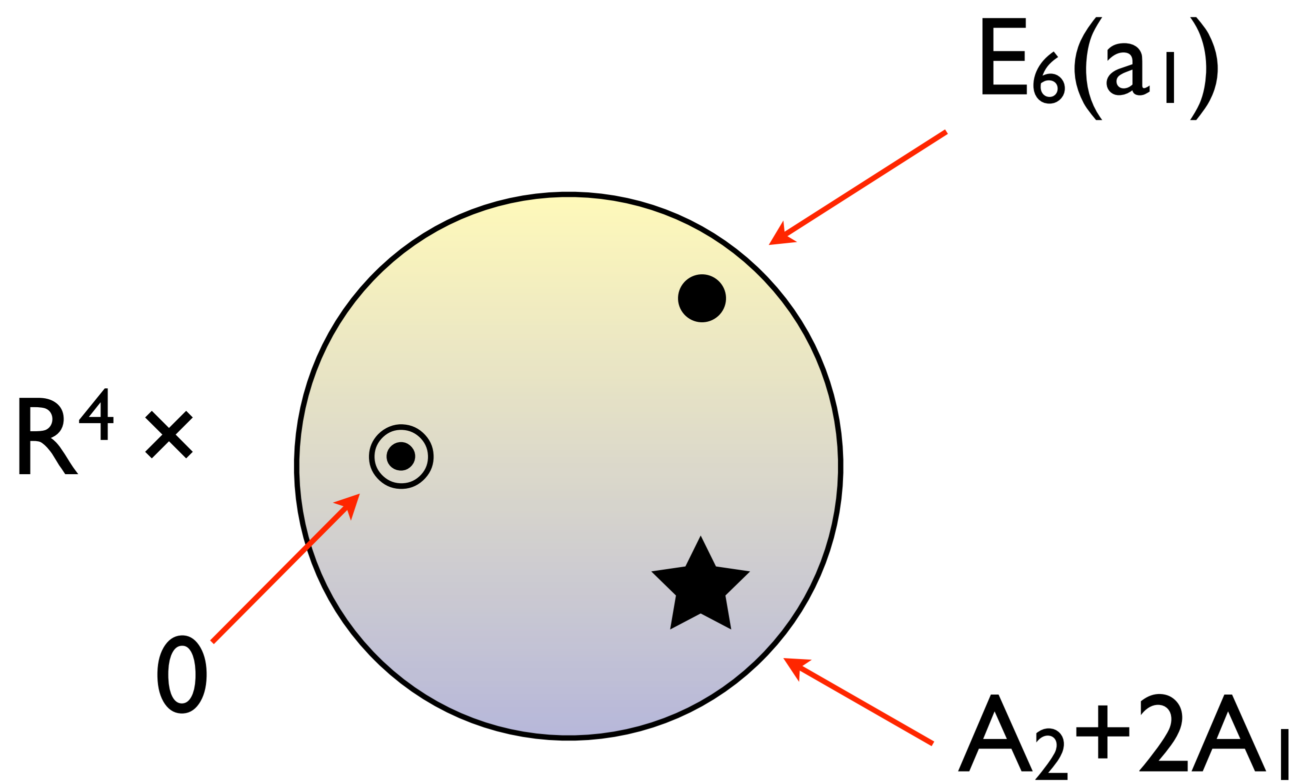
$\rho_3 = A_2 + 2A_1$ $\quad$ $\dim(O(\rho_3)) = 50$,

6d theory of type SU(3) on

$R^4 \times$ 

= 3 fundamental hypers of SU(3)

6d theory of type E_6 on



= 2 hypers in 27 of E_6

$\tilde{\rho}_1 = E_6$, $\dim(O(\tilde{\rho}_1)) = 72$,

$\tilde{\rho}_2 = A_1$, $\dim(O(\tilde{\rho}_2)) = 22$,

$\tilde{\rho}_3 = A_4 + A_1$ $\dim(O(\tilde{\rho}_3)) = 62$

dim. of Coulomb branch

$$= [\sum \dim O(\tilde{\rho}(\sigma^+))]/2 - \dim X$$

$$= 36 + 11 + 31 - 78$$

$$= 0$$

$\rho_1=0,$ $\dim(O(\rho_1))=0,$

$\rho_2=E_6(a_1),$ $\dim(O(\rho_2))=70,$

$\rho_3=A_2+2A_1$ $\dim(O(\rho_3))=50,$

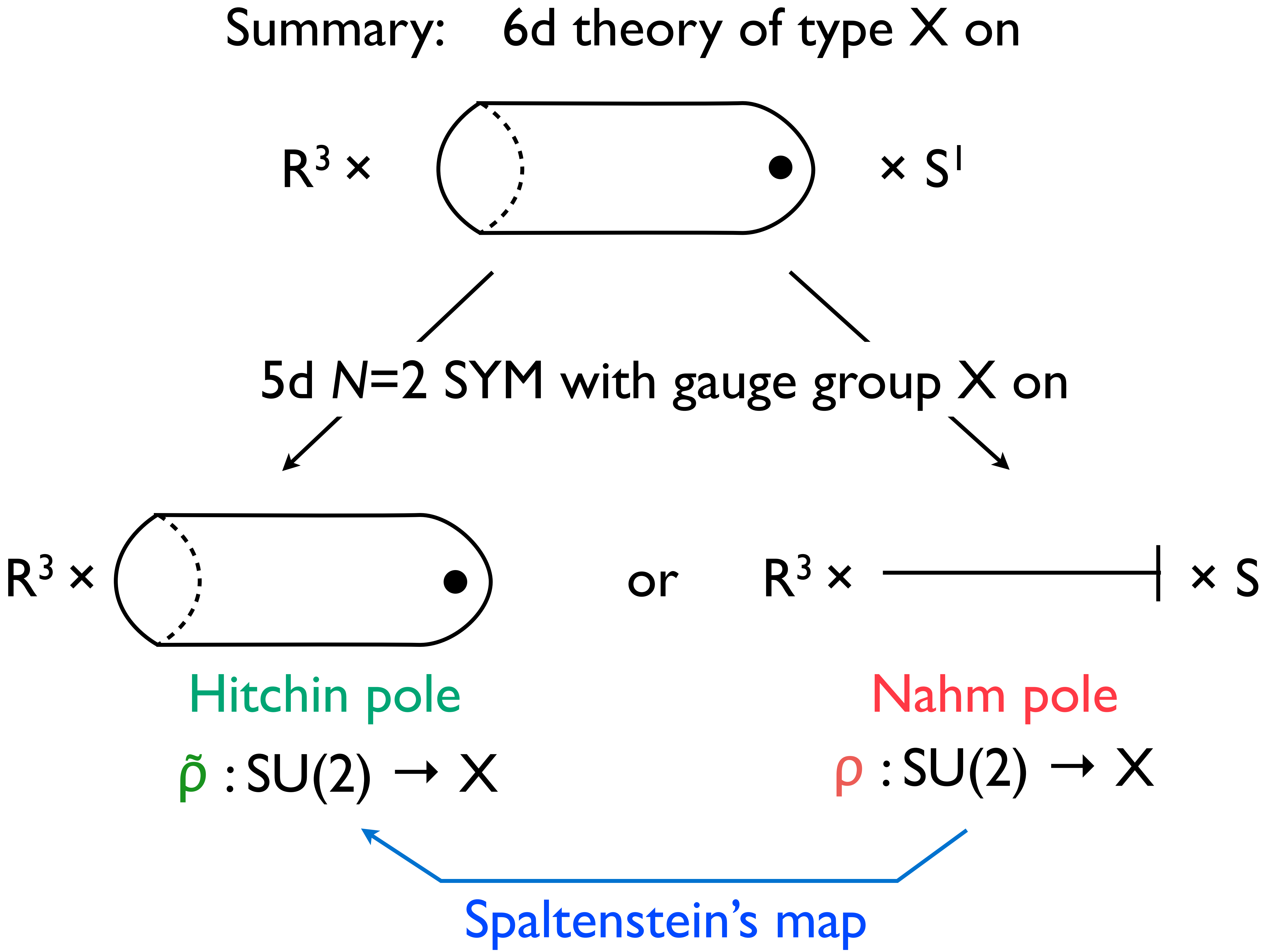
dim. of Higgs branch

$$= [\sum \dim X - \text{rank } X - \dim O(\rho(\sigma^+))]/2 + \text{rank } X$$

$$= [(78-6-0)+ (78-6-70)+ (78-6-50)]/2 + 6$$

$$= 36+1+11+6$$

$$= 54$$


Summary: 6d theory of type X on
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Spaltenstein's map