Emergent bubbling geometries in the plane wave matrix model

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Gauge/Gravity Correspondence for $SU(2|4)$ symmetric theories

Plane Wave Matrix Model
(1D Quantum Mechanics)

$\Longleftrightarrow$

IIA SUGRA solution

This gravity solution was constructed by Lin and Maldacena in '05.

Q: How does the space-time geometry emerge in the framework of the corresponding gauge theory?

Gauge/Gravity Correspondence for $SU(2|4)$ symmetric theories

Plane Wave Matrix Model (1D Quantum Mechanics)	$\Longleftrightarrow$	IIA SUGRA solution

Eigenvalue dist. of matrices $\longleftrightarrow$ Geometry

Range of the eigenvalues $\longleftrightarrow$ Typical geometric scale

I'm going to show

how the geometry can be constructed
from the Plane Wave Matrix Model!

Plan of Talk

1) Gravity Side

2) PWMM (Gauge Theory)

3) Geometry from PWMM

4) Summary, Some Notes and Future Works

Gravity Side

The IIA SUGRA solutions dual to $SU(2|4)$ symmetric theories have $R\times SO(3)\times SO(6)$ isometry.

The metric can be written by a single function $V(r,z)$:

[Lin-Lunin-Maldacena '04, Lin-Maldacena '05]

$$ds_{10}^2 = \left(\frac{\ddot{V}-2\dot{V}}{-V''}\right)^{1/2}\left\{-4\frac{\ddot{V}}{\ddot{V}-2\dot{V}}dt^2 - 2\frac{V''}{\dot{V}}(dr^2+dz^2) + 4d\Omega_5^2 + 2\frac{V''\dot{V}}{\Delta}d\Omega_2^2\right\},$$

$$C_3 = -4\frac{\dot{V}^2V''}{\Delta}dt\wedge d\Omega_2, \quad B_2 = \left(\frac{(\dot{V}^2)'}{\Delta}+2z\right)d\Omega_2, \cdots \quad \left(\Delta = (\ddot{V}-2\dot{V})V'' - (\dot{V}')^2\right)$$

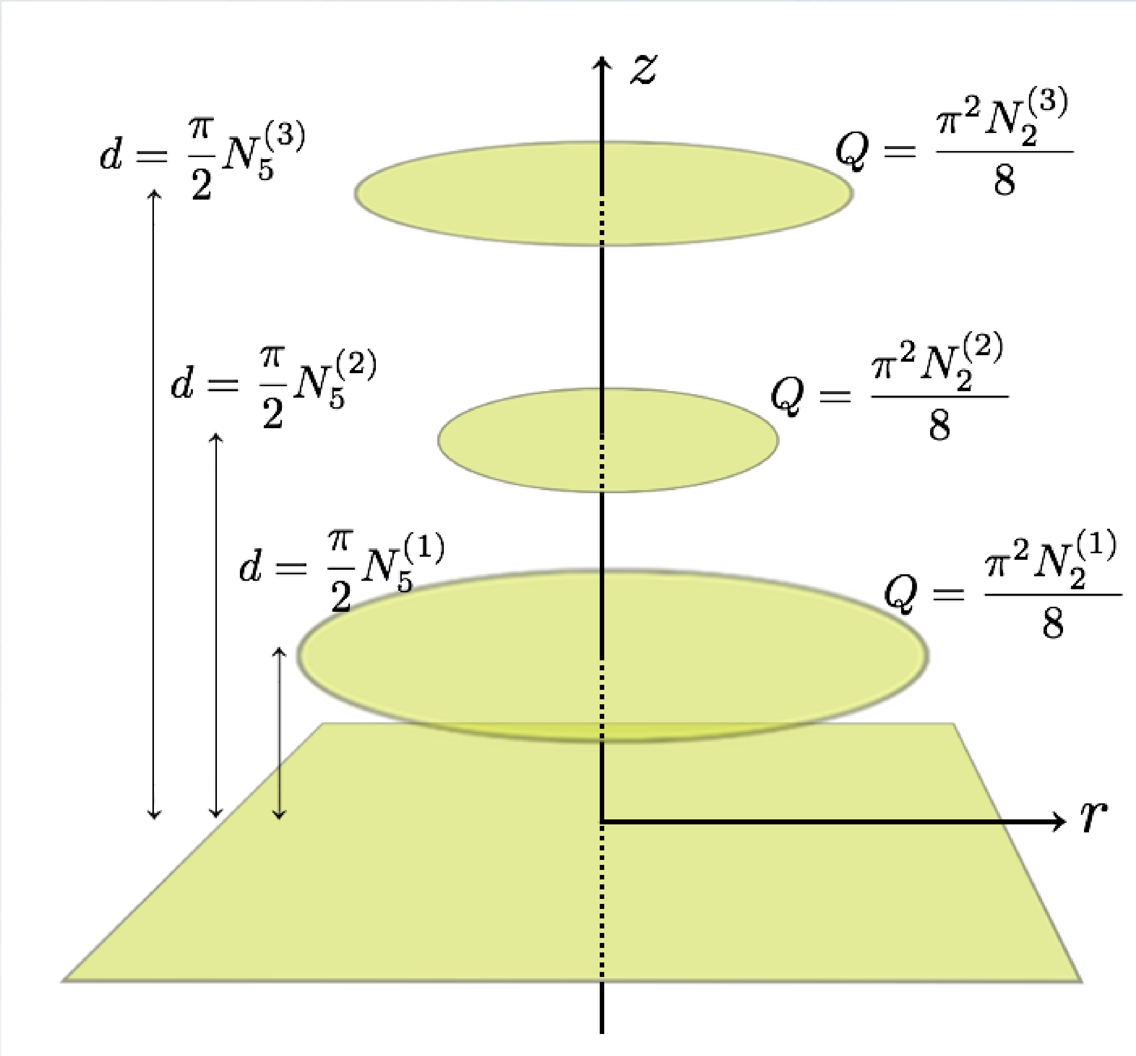
EoM and Killing spinor eq. for the geometry

→ V satisfies the Laplace eq. in a 3D axisymmetric system.

→ Reducing to axisymmetric electrostatic problems with "conducting disks."

Gravity Side

General geometry:



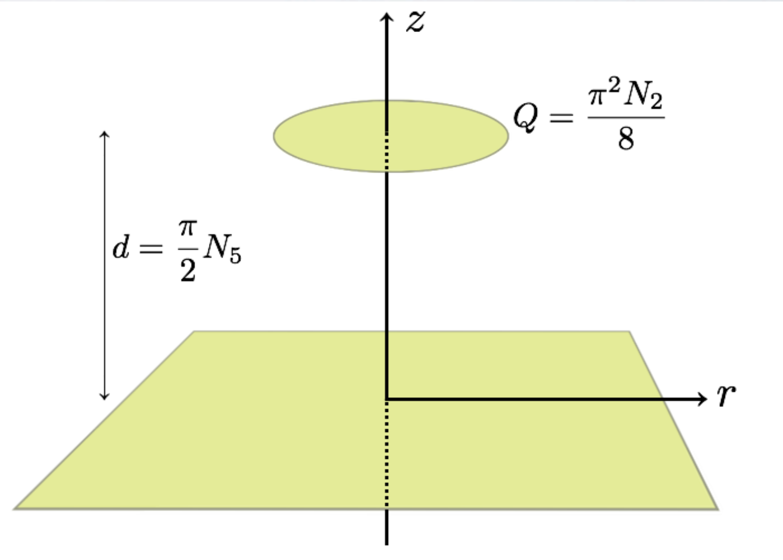
The geometries are labeled by $\{N_2{}^{(s)}, N_5{}^{(s)}\}_{s=1,\dots,\Lambda}$

Λ=1 case:

$$V(r,z) = V_0\left(r^2 z - \frac{2}{3}z^3\right) + \beta(\kappa) V_0 R^3 \int_{-R}^{R} dt \left(-\frac{1}{\sqrt{r^2+(z+d+it)^2}} + \frac{1}{\sqrt{r^2+(z-d+it)^2}}\right)\frac{f_\kappa(t)}{\pi}.$$

V_0: constant, R: radius of disk, $\kappa = d/R$, $\beta(\kappa)$: function of κ

$$f_\kappa(x) - \frac{1}{\pi}\int_{-1}^{1} dy \frac{2\kappa}{4\kappa^2 + (x-y)^2} f_\kappa(y) = 1 - \frac{2\kappa}{\beta(\kappa)} x^2$$



PWMM(Gauge Theory Side)

PWMM is a 1D Matrix Theory with $SU(2|4)$ symmetry.

[Berenstein-Maldacena-Nastase '02]

$$S = \frac{1}{g^2}\int d\tau\, \mathrm{Tr}\Big(\frac{1}{2}(D_\tau X_a)^2 + \frac{1}{2}(D_\tau X_m)^2 + \frac{1}{4}(2\epsilon_{abc}X_c - i[X_a, X_b])^2 - \frac{1}{2}[X_a, X_m]^2$$

$$-\frac{1}{4}[X_m, X_n]^2 + \frac{1}{2}X_m X^m + \text{fermions}\Big)$$

$$D_\tau = \partial_\tau - i[A_\tau, *]$$

$a,b=1,2,3,\quad m,n=4,\dots,9$

The vacua are given by $SU(2)$ rep.

$$\hat{X}_a = -2\bigoplus_{s=1}^{\Lambda}(\mathbf{1}_{N_2^{(s)}} \otimes L_a^{[N_5^{(s)}]})$$

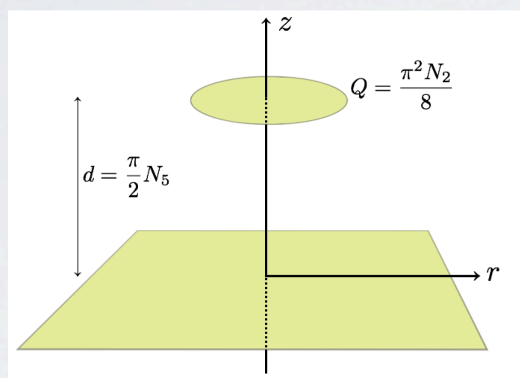
${L_a}^{[N_5^{(s)}]}$: spin $(N_5^{(s)} - 1)/2$ rep. mat.

$N_2^{(s)}$: multiplicity of the spin $(N_5^{(s)} - 1)/2$ rep.

・・・labeled by $\{N_2^{(s)}, N_5^{(s)}\}_{s=1,\dots,\Lambda}$

So as to construct geometries

Consider simple vacua (Λ=1) from now on.



Then apply the localization method to PWMM
and get the same equation as that obtained on the gravity side:

$$f_\kappa(x) - \frac{1}{\pi}\int_{-1}^{1} dy \frac{2\kappa}{4\kappa^2 + (x-y)^2} f_\kappa(y) = 1 - \frac{2\kappa}{\beta(\kappa)} x^2$$

Localization

Let us apply the localization method to PWMM. [Y.A.-Ishiki-Okada-Shimasaki '12]

- SUSY: quarter BPS sector such that

$$\phi(\tau) = 2(-X_3(\tau) + \sinh(\tau) X_8(\tau) + i\cosh(\tau) X_9(\tau))$$ is invariant.

- B.C.: all fields approach to the vacuum configurations at $\tau \to \infty$.

- $\mathcal{V}$: $\delta_s \mathcal{V}$ is SUSY-invariant and positive-definite. $\mathcal{V} = \int d\tau \, \mathrm{Tr}[\Psi \overline{\delta_s \Psi} + \mathcal{V}_{ghost}]$

Then,

$$Z(t) := \int \mathcal{D}X e^{-S[X] - t\delta_s \mathcal{V}} \Rightarrow \frac{dZ(t)}{dt} = 0 \Rightarrow \int \mathcal{D}X e^{-S[X]} = Z(0) = Z(\infty)$$

Localized around $\delta_s \mathcal{V}$=0 !

$$\hat{X}_a(\tau) = -2 \bigoplus_{s=1}^{\Lambda} (\mathbf{1}_{N_2^{(s)}} \otimes L_a^{[N_5^{(s)}]}) \qquad \hat{X}_9(\tau) = \frac{M}{\cosh(\tau)} \qquad ([L_a, M] = 0)$$

Localization

Hence, expectation values of any supersymmetric operators can be obtained exactly by 1-loop calculations.

$$\langle \prod_a \mathrm{Tr} f_a(\phi(\tau_a)) \rangle = \langle \prod_a \mathrm{Tr} f_a(4L_3 + 2iM) \rangle_{MM}$$

When Λ=1, $\langle \ \rangle_{MM}$ is evaluated by the following partition function:

$$Z = \int \prod_i dq_i \prod_{J=0}^{N_5-1} \prod_{i>j}^{N_2} \frac{\{(2J+2)^2 + (q_i - q_j)^2\}\{(2J)^2 + (q_i - q_j)^2\}}{\{(2J+1)^2 + (q_i - q_j)^2\}^2} e^{-\frac{2N_5}{g^2}\sum_i {q_i}^2}.$$

q_i: eigenvalues of Q, where $M = Q \otimes \mathbf{1}_{N_5}$

Interacting Fermi Gas System

In the large-N_5 limit, the partition fn. can be written as

$$Z=\int\prod_i dq_i\prod_{i>j}^{N_2}\tanh^2\frac{\pi(q_i-q_j)}{2}\exp\left\{-\frac{2N_5}{g^2}\sum_i q_i^2+\frac{2N_5}{(2N_5)^2+(q_i-q_j)^2}-\cdots\right\}$$

Repulsive force Central force Attractive force

By using the Cauchy id.,

$$\prod_{i\neq j}^{N_2}\tanh\frac{\pi(q_i-q_j)}{2}=\sum_{\sigma\in S_{N_2}}(-)^{\epsilon(\sigma)}\prod_{i=1}^{N_2}\frac{1}{\cosh\frac{\pi(q_i-q_{\sigma(i)})}{2}}$$

[Marino-Putrov '12]

Interacting Fermi gas system with hamiltonian

$$H=\sum_{i=1}^{N_2}\log\cosh p_i+\frac{2N_5}{g^2}\sum_{i=1}^{N_2}q_i^2-\frac{1}{2}\sum_{i\neq j}^{N_2}\frac{2N_5}{(2N_5)^2+(q_i-q_j)^2}$$

Interacting Fermi Gas System

We consider the limit in which the SUGRA approx. is valid:

large-N_2 , large-N_5 , $\lambda := g^2 N_2 \gg N_5$

・・・Thomas-Fermi approximation at zero temperature

The system is described by mean-field density $\underline{\rho(q)}$.

=eigenvalue density

It is determined by the following saddle point eq.:

$$\mu = \pi\rho(q) + \frac{2N_5}{g^2}q^2 - \int_{-q_m}^{q_m} dq' \, \frac{2N_5}{(2N_5)^2 + (q-q')^2}\rho(q')$$

μ : chemical potential

q_m: upper limit of the support of $\rho(q)$

Construction of geometry

$$\frac{\pi}{\mu}\rho(q_m x) - \int_{-1}^{1} dy\, \frac{2N_5/q_m}{(2N_5/q_m)^2 + (x-y)^2}\frac{\rho(q_m y)}{\mu} = 1 - \frac{2N_5 q_m^2}{\mu g^2}x^2$$

$$\Updownarrow$$

$$f_\kappa(x) - \frac{1}{\pi}\int_{-1}^{1} dy \frac{2\kappa}{4\kappa^2 + (x-y)^2} f_\kappa(y) = 1 - \frac{2\kappa}{\beta(\kappa)}x^2$$

Construction of geometry

$$\frac{\pi}{\mu}\rho(q_m x) - \int_{-1}^{1} dy\, \frac{2N_5/q_m}{(2N_5/q_m)^2 + (x-y)^2}\frac{\rho(q_m y)}{\mu} = 1 - \frac{2N_5 q_m^2}{\mu g^2}x^2$$

Identification to obtain the gravity picture

$\rho(q) = \frac{\mu}{\pi} f_\kappa(q/q_m)$ $\propto$ charge density on the disk

By this identification, the range of the eigenvalue dist. can be written by the quantities on the gravity side:

$q_m = \frac{2}{\pi}R$ so $R_{S^5}^2 = 2\pi q_m$

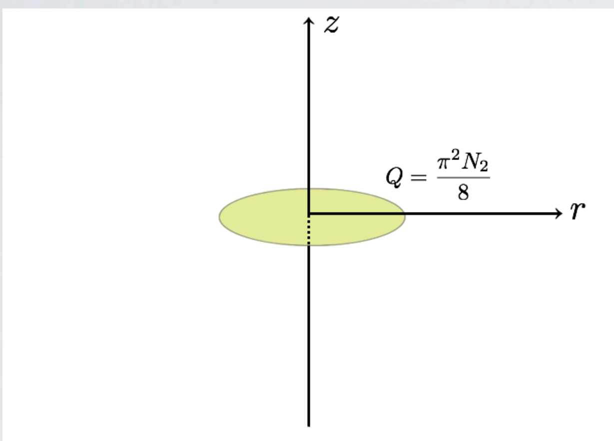
The gravity solution corresponding to a simple vacuum in PWMM has been constructed from the gauge theory side!

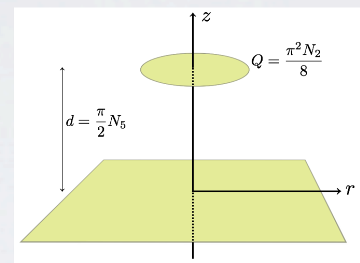
D2-and NS5-brane limit

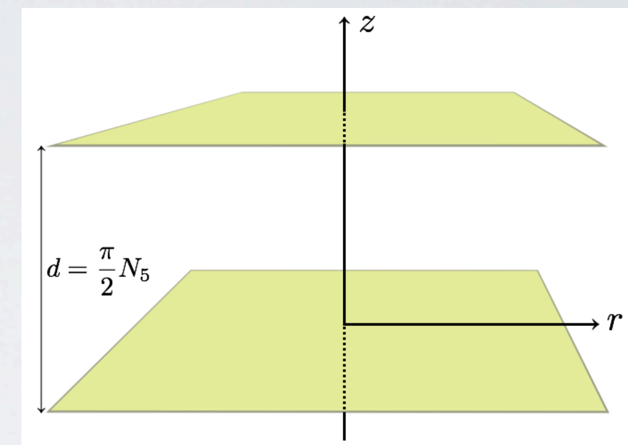
On the gravity side, [Ling-Mohazab-Shieh-Anders-Raamsdonk '06]

these two double scaling limits are known:

D2-brane limit | NS5-brane limit







$\kappa \to \infty$ $\qquad \kappa = \dfrac{d}{R}$ $\qquad \kappa \to 0$

$$R_{S^5}^2 = \pi (3 g_{S^2}^2 N_2)^{\frac{1}{3}} \qquad\qquad R_{S^5}^2 = 2\pi (8\lambda)^{\frac{1}{4}}$$

On the gauge theory side, solutions in the corresponding limits are

$$\rho(q) = \frac{\mu}{\pi}\left[1 - \left(\frac{q}{q_m}\right)^2\right], \quad q_m = \left(\frac{3\pi\lambda}{8N_5}\right)^{\frac{1}{3}} \qquad \rho(q) = \frac{\mu q_m}{3\pi N_5}\left[1 - \left(\frac{q}{q_m}\right)^2\right]^{\frac{3}{2}}, \quad q_m = (8\lambda)^{\frac{1}{4}}$$

Then, $R_{S^5}^2 = 2\pi q_m$

reproduce the same results as those on the gravity side.

Summary

- The mean-field density, that is, the eigenvalue distribution in the matrix integral determines the geometry.

- The range of the eigenvalue dist. corresponds to S_5 radius.

Some Notes and Future Works

- We can also reconstruct geometries dual to general vacua ($\Lambda \neq 1$) in the same way.

- This method also works for other $SU(2|4)$ symmetric theories such as SYM on $R \times S^2$ and $R \times S^3/Z_k$.

- Exact proof for the NS5-brane limit. (Work in progress)