Non-perturbative Study on Duality and Phase Structure in Minimal String Theory

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based on collaborations with

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Main Reference

[CIY4], "Analytic Study for the String Theory Landscapes via Matrix Models,"
Phys.Rev. D86 (2012) 126001 [arXiv:1206.2351 [hep-th]]

[CIY5], "Duality Constraints on String Theory I: spectral networks and instantons,"
arXiv:1308.6603 [hep-th]

[CIY6], "Duality Constraints on String Theory II: Vacuum Connection Formula and Bootstrap for String Theory," **in progress**

"Non-perturbative Study of Strings"

Non-perturbative Completions

a number of completions

$$\mathcal{F}(g) \simeq \sum_{n=0}^{\infty} g^{2n-2}\mathcal{F}_n + \theta \times \sqrt{g}\exp\left[\frac{1}{g}\sum_{n=0}^{\infty}\mathcal{F}_n^{(\mathrm{inst})}\right] + \cdots$$

Information of Exact Theory

perturbative string theory

1) Instanton spectrum to guarantee the exact theory

2) Behavior of $\mathcal{F}(g)$ $\left(g \in \mathbb{C}\right)$ (Phase Structure in g)

3) Physics of Dualities at a Full Non-perturbative Level

4) Capturing the Landscape of (Meta-)stable Vacua

“Non-perturbative Study of Strings”

Non-perturbative Completions

a number of completions

$$\mathcal{F}(g) \simeq \sum_{n=0}^{\infty} g^{2n-2}\mathcal{F}_n + \theta \times \sqrt{g}\exp\Big[\frac{1}{g}\sum_{n=0}^{\infty}\mathcal{F}_n^{(\mathrm{inst})}\Big] + \cdots$$

Information of Exact Theory

perturbative string theory

Additional Principles are Necessary

(# several principles)

Non-perturbative Duality ← Duality Constraints [CIY5 ‘13]

cf) also [Imamura-san’s talk]

Today, we discuss duality constraints and its beyond
(especially in (2,3); (2,5) minimal strings, as examples)

Plan of the talk

1) What is “Duality Constraints”?

2) Stokes phenomena in Isomonodromy Systems

3) Phase Structure in g, (and the landscapes)

4) Summary

Non-perturbative Duality

Two-matrix model

$$\mathcal{Z} = \int dX dY \, e^{-N \mathrm{tr}[V_1(X) + V_2(Y) - XY]}$$

Integrate Y → **X-system**

Integrate X → **Y-system**

X-system ⟷ Y-system: spectral (p-q) dual

These two systems are generally **not the same**
<u>at the level of non-perturbative completions</u>

Spectral Curves

X-system ⟷ Y-system

Resolvent of X

$$R(x) = \left\langle \frac{1}{N}\mathrm{tr}\frac{1}{x-X} \right\rangle$$

Resolvent of Y

$$\widetilde{R}(y) = \left\langle \frac{1}{N}\mathrm{tr}\frac{1}{y-Y} \right\rangle$$

They are given by **the same algebraic equation**:

$$F(x,R) = f\left(x, R - V_1'(x)\right) = 0$$

$$\widetilde{F}(y,\widetilde{R}) = f\left(\widetilde{R} - V_2'(y), y\right) = 0$$

We can identify

That is,

$$f(x,y) = 0$$

if x is spacetime → X-system

if y is spacetime → Y-system

X-system ⟷ p-q dual ⟷ Y-system

From spectral curve to full amplitudes

if x is spacetime

$$f(x, y) = 0$$

X-system

Isomonodromy descriptions

$$g\frac{\partial \Psi}{\partial x} = \mathcal{Q}(t;x)\Psi$$
$$g\frac{\partial \Psi}{\partial t} = \mathcal{B}(t;x)\Psi$$

p × p matrix-valued rational function in x

(y has p branches)

1. $\det\big(y - \mathcal{Q}(t;x)\big) \underset{g\to 0}{\propto} f(x,y) = 0$

2. isomonodromic τ = MM partition function

$$\mathcal{Z}(t) \leftarrow \tau_X(t)$$

(But, we have to be careful!)

Sequence of models

Iso-Monodromy Systems (IMS) are given by **spectral curves**

We focus on the case

$$f(x,y)=0$$ a polynomial in (x,y)

(p,q) minimal string theory

$$f(x,y)=y^{p}+a_1y^{p-1}+\cdots+b_0x^{q}+b_1x^{q-1}+\cdots$$

(p,q)=(2,3) ↔ (3,2): Pure-gravity ⇔ Painleve I (P_I)

(p,q)=(2,5) ↔ (5,2): Yang-Lee edge, ⋯

Other IMS's: (p,q) MST = a single (but higher) singularity at x,y=∞

- Multiple singularities → [Marshakov-san,Gavrylenko-san]
- Multiple singularities + qIMS → [Yamada-san]
- System of Toda equations → [Ito-san,Locke-san]

Universal framework for Matrix models/Gauge theory

Duality in Isomonodromy Systems

$$f(x,y)=0$$

if x is spacetime

if y is spacetime

X-system

$$g\frac{\partial\Psi}{\partial x}=\mathcal{Q}(t;x)\Psi$$

$$g\frac{\partial\Psi}{\partial t}=\mathcal{B}(t;x)\Psi$$

$$\det\big(y-\mathcal{Q}(t,x)\big)\underset{g\to 0}{\propto} f(x,y)=0$$

p-q dual

Y-system

$$g\frac{\partial\widetilde{\Psi}}{\partial y}=\widetilde{\mathcal{Q}}(t;y)\widetilde{\Psi}$$

$$g\frac{\partial\widetilde{\Psi}}{\partial t}=\widetilde{\mathcal{B}}(t;y)\widetilde{\Psi}$$

$$\det\big(x-\widetilde{\mathcal{Q}}(t,y)\big)\underset{g\to 0}{\propto} f(x,y)=0$$

p-q duality and T-duality: References

- [Fukuma-Kawai-Nakayama '92]
  [P,Q]=1 <=> [Q,-P]=1
- [Kharchev-Marshakov '92]
  Kontsevich MM: from (p,1) to (p,q)
- [Bertola-Eynard-Harnad '01-04]
  Two-matrix models, saddle point analysis
- [Eynard-Orantin '07 - …]
  x-y symmetry: All-order perturbative check
- [Asatani-Kuroki-Okawa-Sugino-Yoneya '96]
  Kramers-Wannier duality in Random Surfacs
- [Kuroki-Sugino '07]
  D-instanton fugacity

“Equivalence” of the duality in the literature

$$g\frac{\partial\Psi}{\partial x} = \mathcal{Q}(t;x)\Psi \qquad g\frac{\partial\Psi}{\partial t} = \mathcal{B}(t;x)\Psi$$

p-q dual

$$g\frac{\partial\widetilde{\Psi}}{\partial y} = \widetilde{\mathcal{Q}}(t;y)\widetilde{\Psi} \qquad g\frac{\partial\widetilde{\Psi}}{\partial t} = \widetilde{\mathcal{B}}(t;y)\widetilde{\Psi}$$

string equation (diff. eqn for $\tau(t)$) [Fukuma-Kawai-Nakayama '92]

$$\big[g\partial_x - \mathcal{Q}(t;x), g\partial_t - \mathcal{B}(t;x)\big] = 0$$

(equivalent equations)

$$\Leftrightarrow \quad \big[g\partial_y - \widetilde{\mathcal{Q}}(t;y), g\partial_t - \widetilde{\mathcal{B}}(t;y)\big] = 0$$

E.g) pure-gravity (p,q)=(2,3) ↔ (3,2)

$$u(t) = \partial_t^2 \ln \tau(t)$$

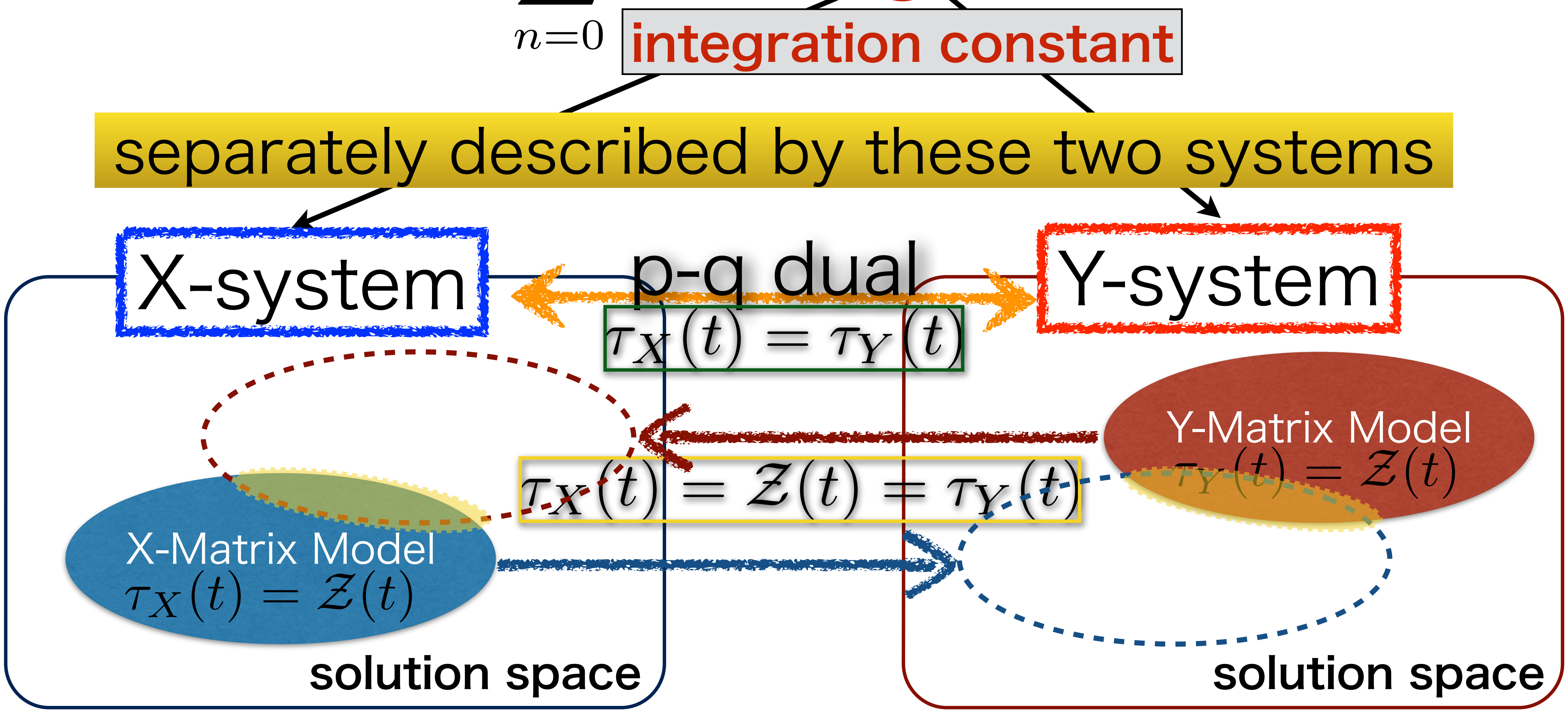
Both systems give $P_I:\ u''(t) = 6(u^2 + t)$

However, this only guarantees the equivalence under formal (asymptotic) solutions (i.e. perturbation theory)

Non-perturbative duality v.s. perturbative equivalence

$$P_I\,:\;u''(t)=6(u^2+t)$$

$$\longrightarrow\quad u(t)\simeq\sqrt{-t}\sum_{n=0}^{\infty}u_n t^{-\frac{4n}{5}}+\theta\times(-t)^{\frac{1}{8}}e^{-\frac{8\sqrt{3}}{5g}(-t)^{\frac{5}{4}}}+\cdots$$



Only a part of solutions are realized by matrix models

Duality can even give a constraint on matrix models [CIY5'13]

So, we will briefly discuss

1) What is “solution space” in IMS

2) What is the matrix-model solutions

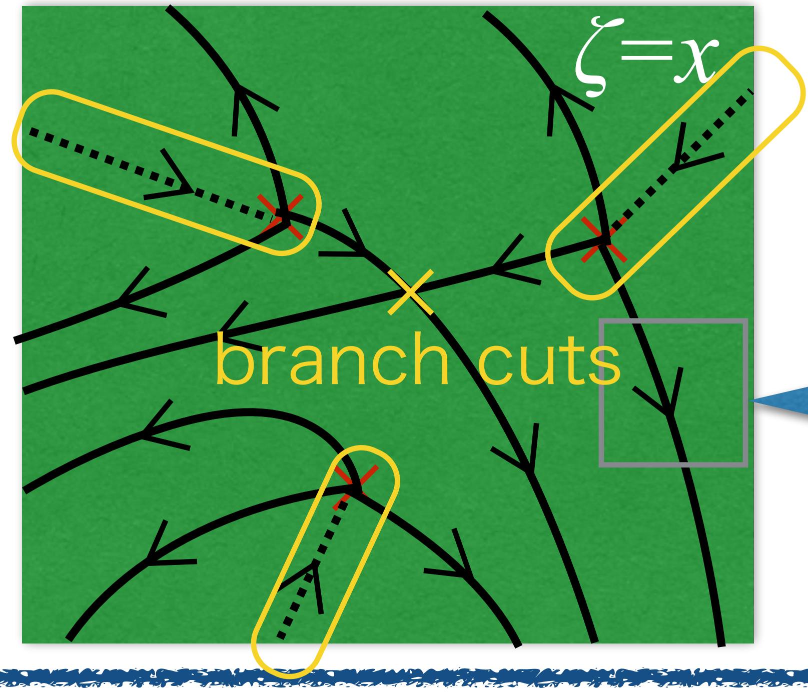
1) Solution Space and Stokes Phenomena

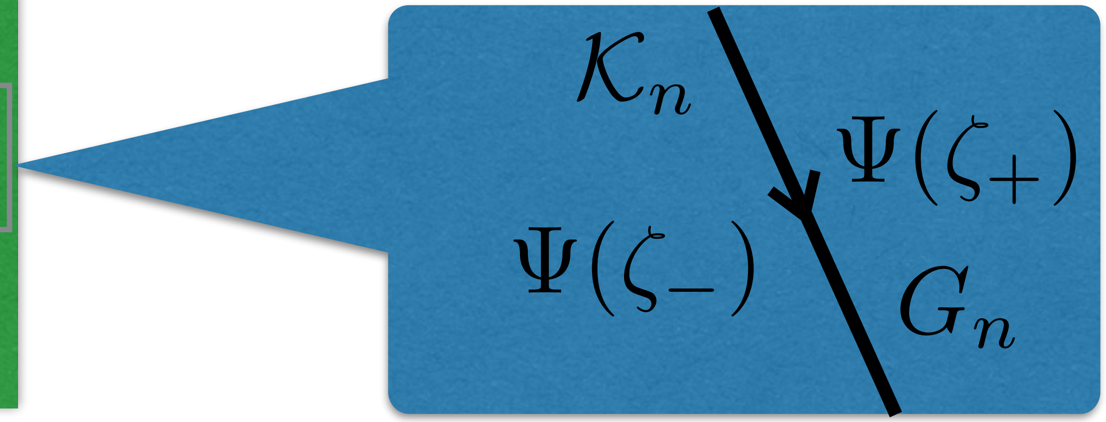
Solution spaces of String Equation ↔ Stokes Phenomena of Ψ

Spectral Networks [Deift-Zhou'93,⋯]

$$\Psi(\zeta_+) = \Psi(\zeta_-) G_n \qquad \zeta \in \mathcal{K}_n$$

⟶ $\mathcal{K} =$





Stokes jump graph

$$\Psi(g;\zeta) \simeq \Big[I_p + \sum_{n=1}^{\infty} g^n Z^{(n)}(\zeta) \Big] Z_{\rm cl}(\zeta) e^{\frac{1}{g}\varphi(\zeta)} \qquad (g \to 0;\ \text{fixed } \zeta \in \mathbb{C} \setminus \mathcal{K})$$

- Every line $\mathcal{K}$n is associated with a pxp matrix $\big\{ G_n \big\}_{n=1}^{\#\mathcal{K}}$
- Junction satisfies the matrix conservation law $G_{n_1} G_{n_2} \cdots G_{n_L} = Id$

If G is properly chosen, Ψ satisfies this expansion in $\zeta \in \mathbb{C} \setminus \mathcal{K}$

Spectral Networks in (p,q) Minimal String Theory [CIY4 '12; CIY5 '13]

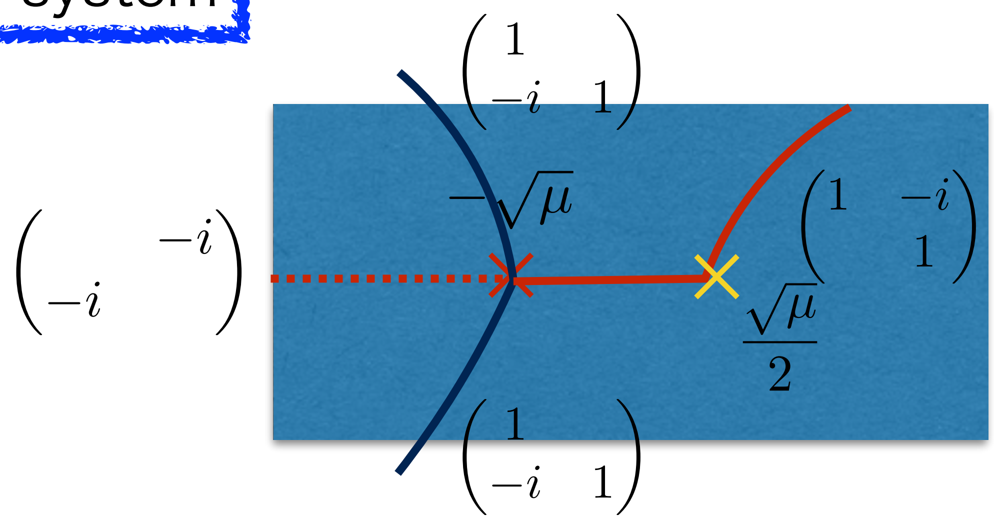
(there are several consistency conditions)

Results **(Here we only show the two cases)**

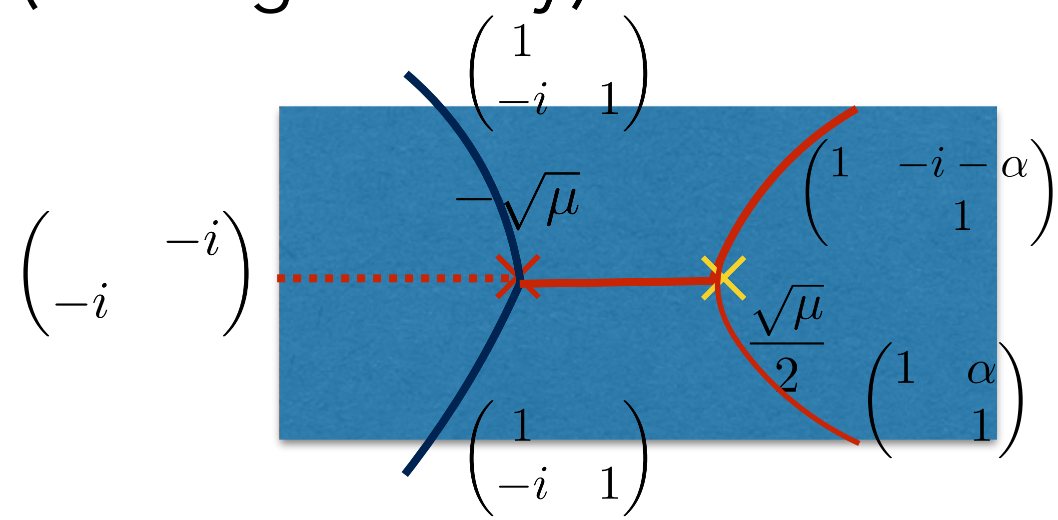
(p,q)=(2,3)

X-system

$$d\varphi(\zeta) = \pm\frac{1}{\sqrt{2}}(2\zeta - \sqrt{\mu})\sqrt{\zeta + \sqrt{\mu}}$$



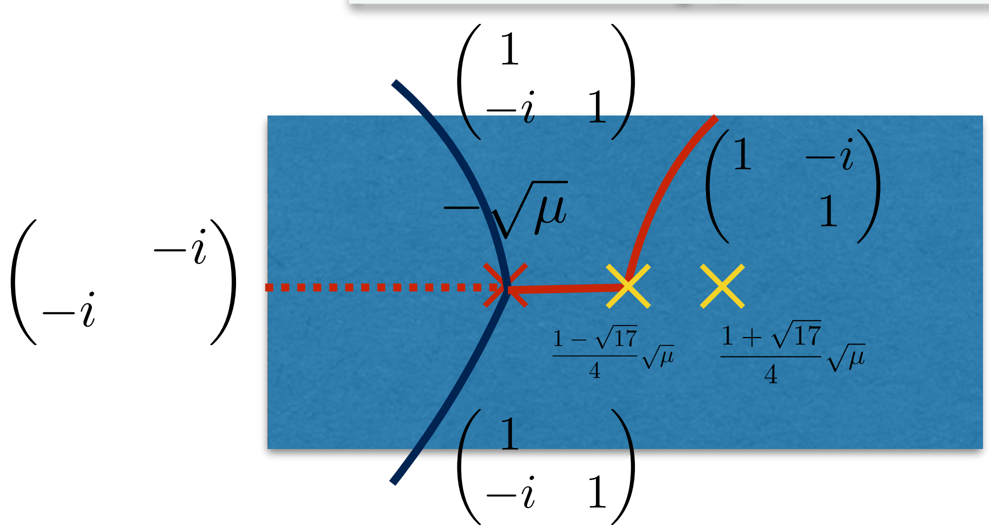
(more generally)

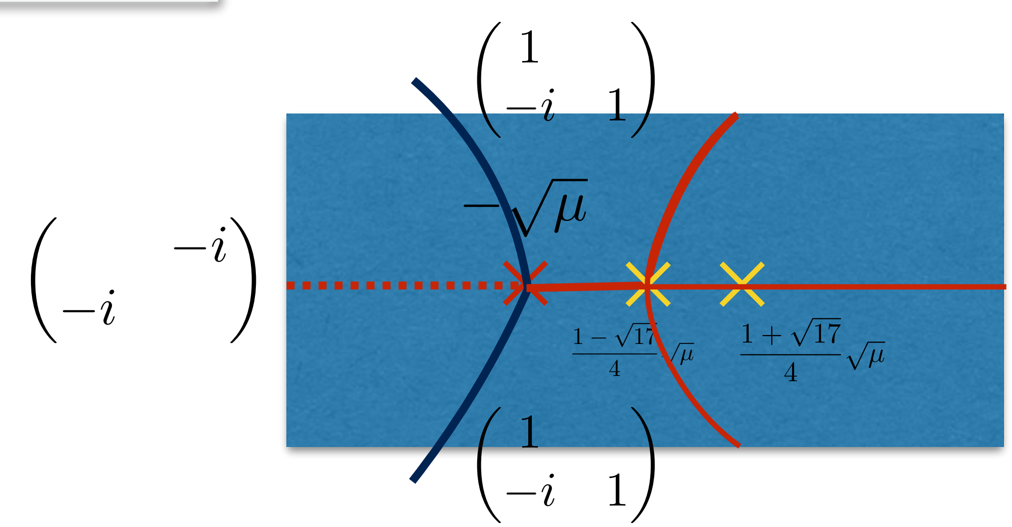


(p,q)=(2,5)

X-system

$$d\varphi(\zeta) = \pm\frac{1}{\sqrt{2}}(4\zeta^2 - 2\sqrt{\mu}\zeta - \mu)\sqrt{\zeta + \sqrt{\mu}}$$



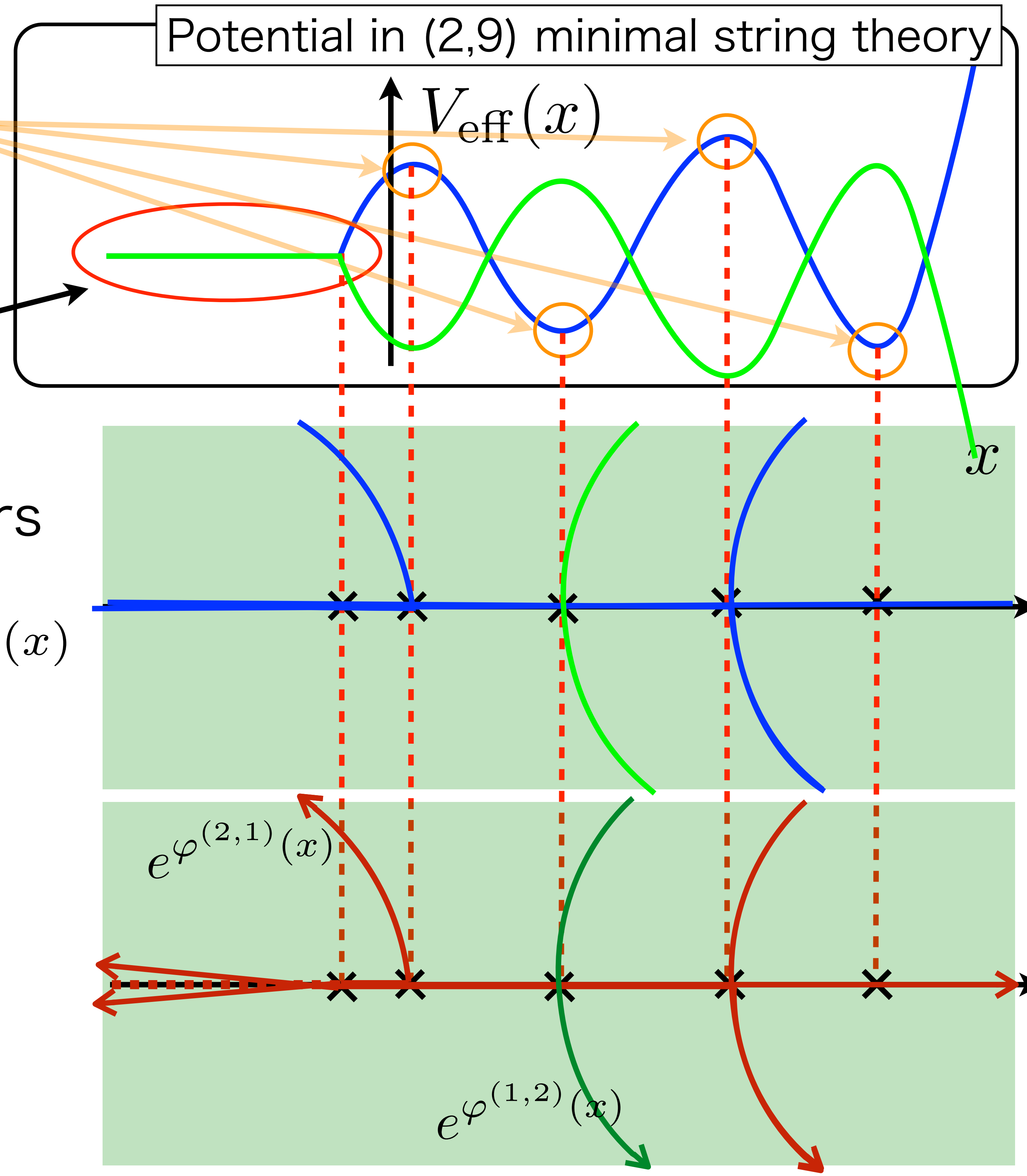


(more generally)

2) "Spectral Networks" = Matrix-Model Contours [CIY4 '12]

Instantons

Condensation of other eigenvalues

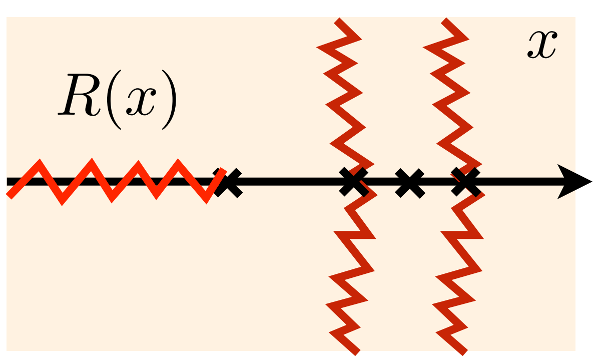


Matrix-model contours

$$\mathcal{Z}(t) \simeq \int_{\mathcal{C}} dx e^{-\frac{1}{g}V_{\rm eff}(x)}$$

Spectral Networks

(Stokes Phenom)



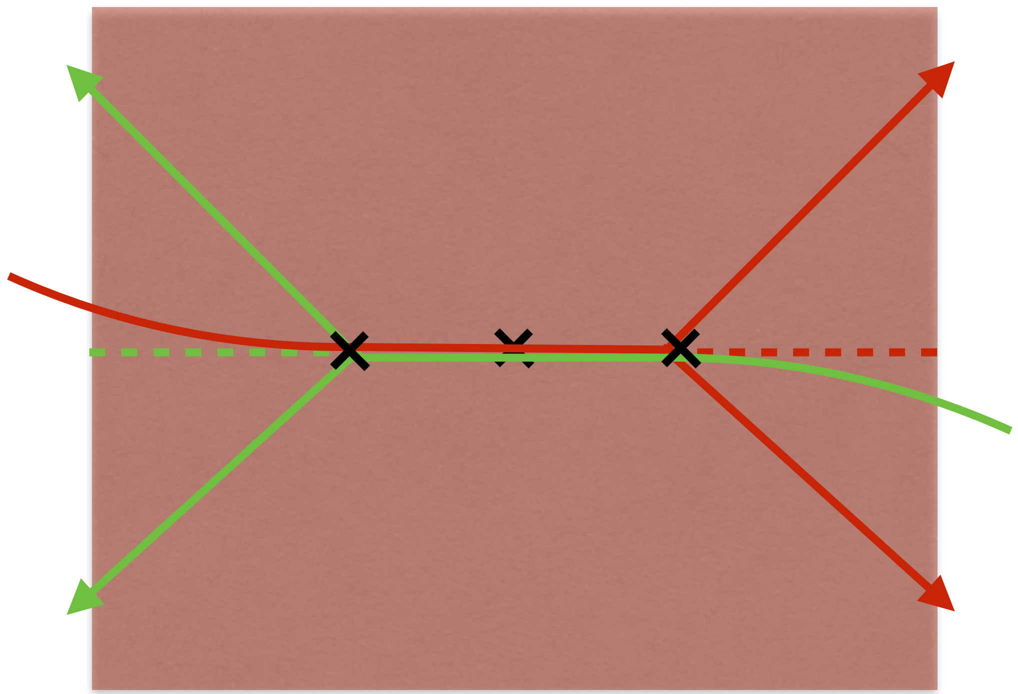
Spectral Networks in (p,q) Minimal String Theory [CIY4 '12; CIY5 '13]

(there are several consistency conditions)

Results **(Here we only show the two cases)**

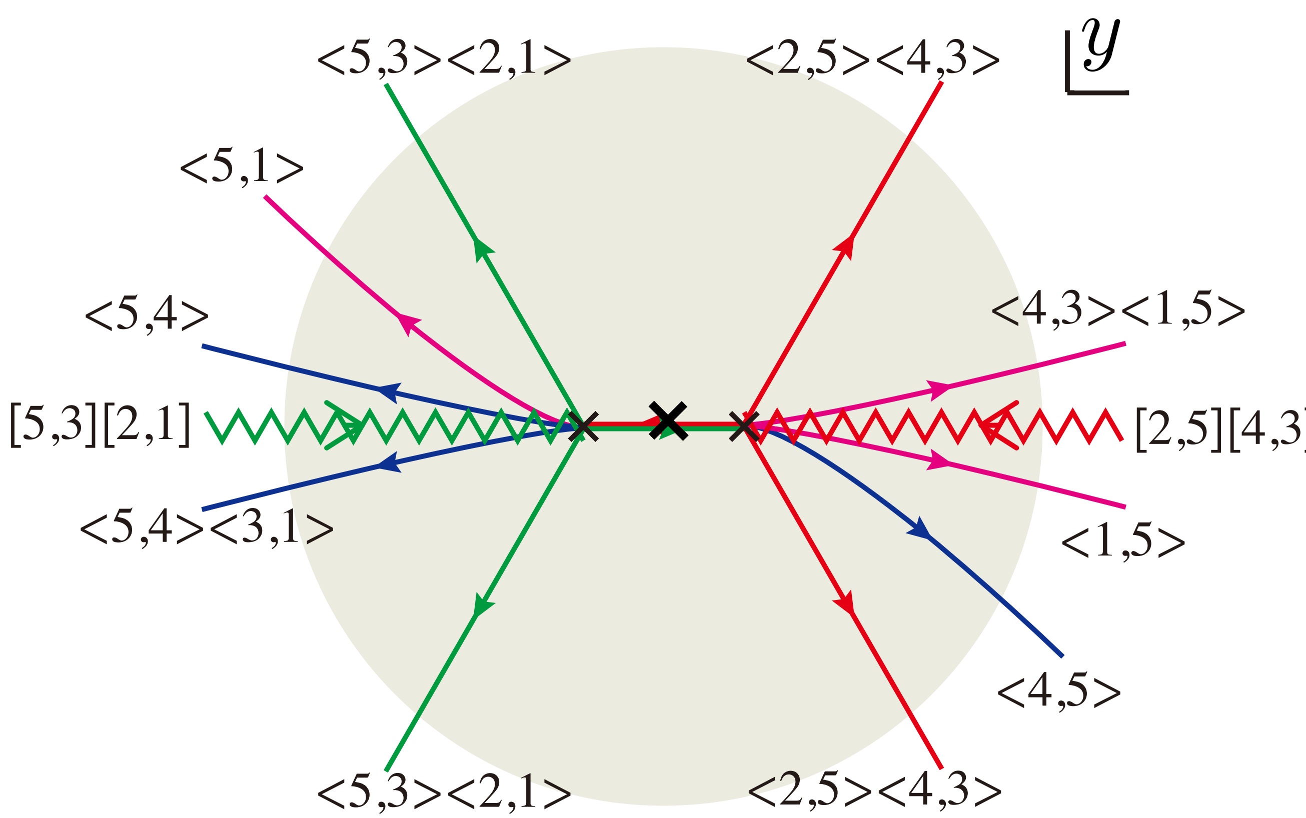
(p,q)=(3,2)

Y-system



(p,q)=(5,2)

Y-system

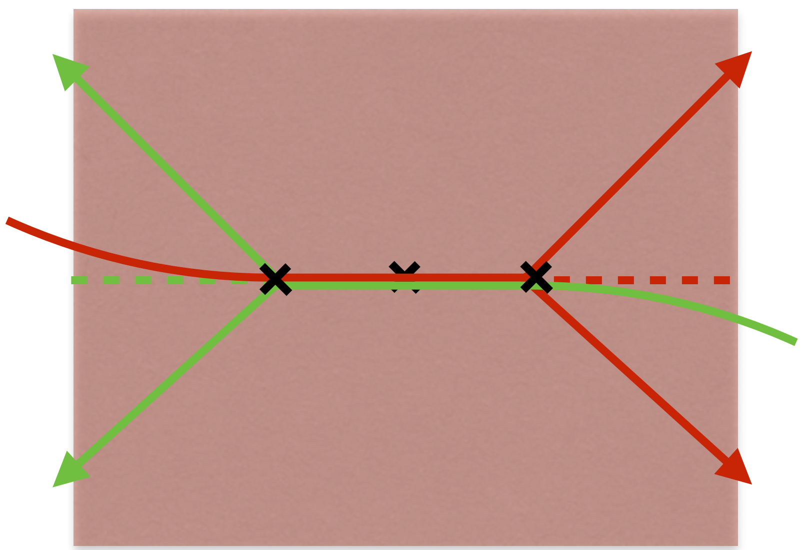


Duality Check of (3,2) ↔ (2,3) Models [CIY6 '14]

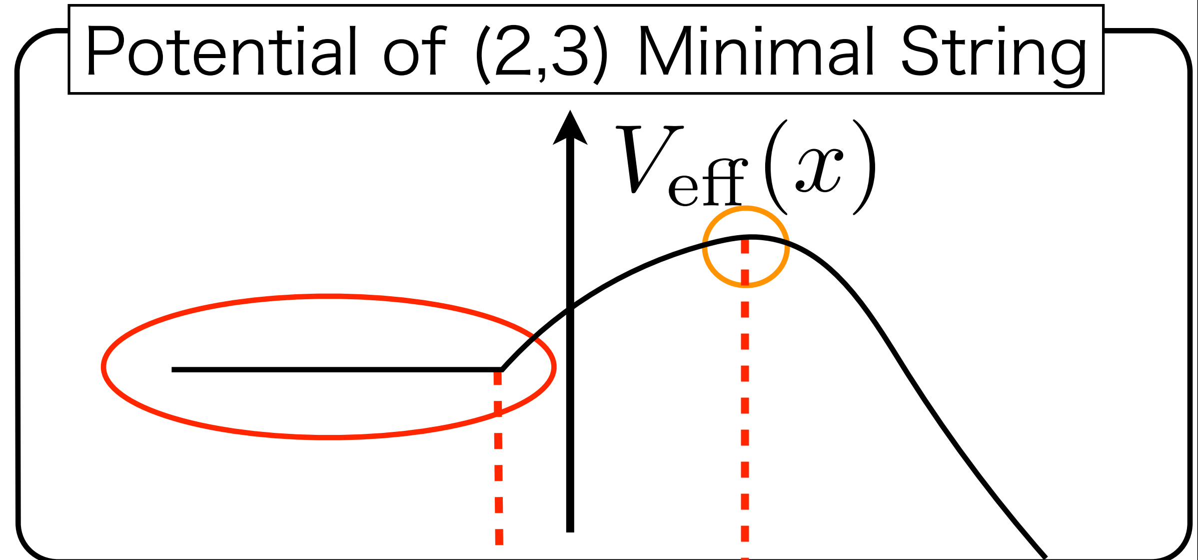
We can check the phase structure in g -> 0 (or μ -> ∞)

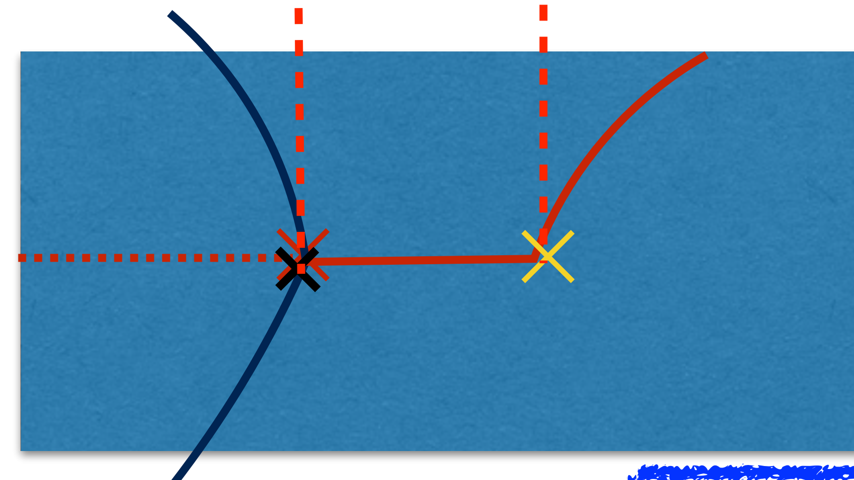
$(\mu = -t)$

Y-system

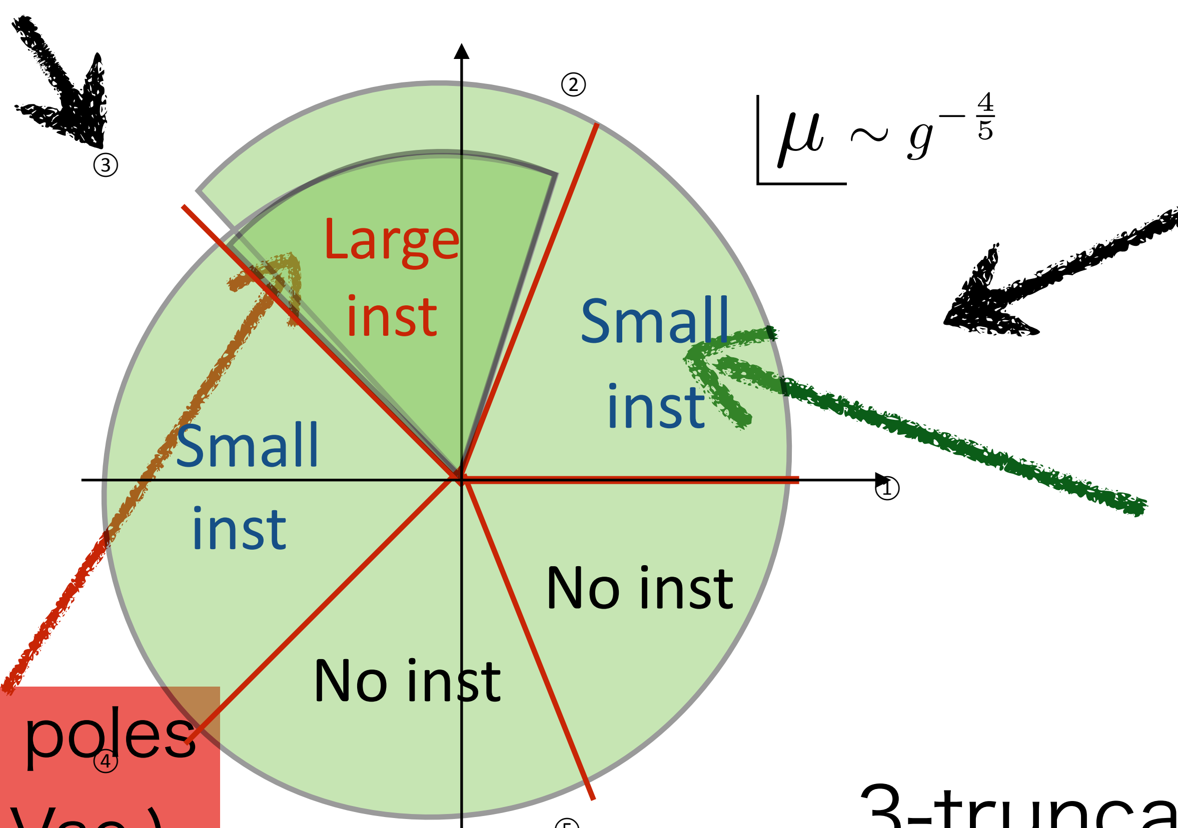


D-instanton





X-system



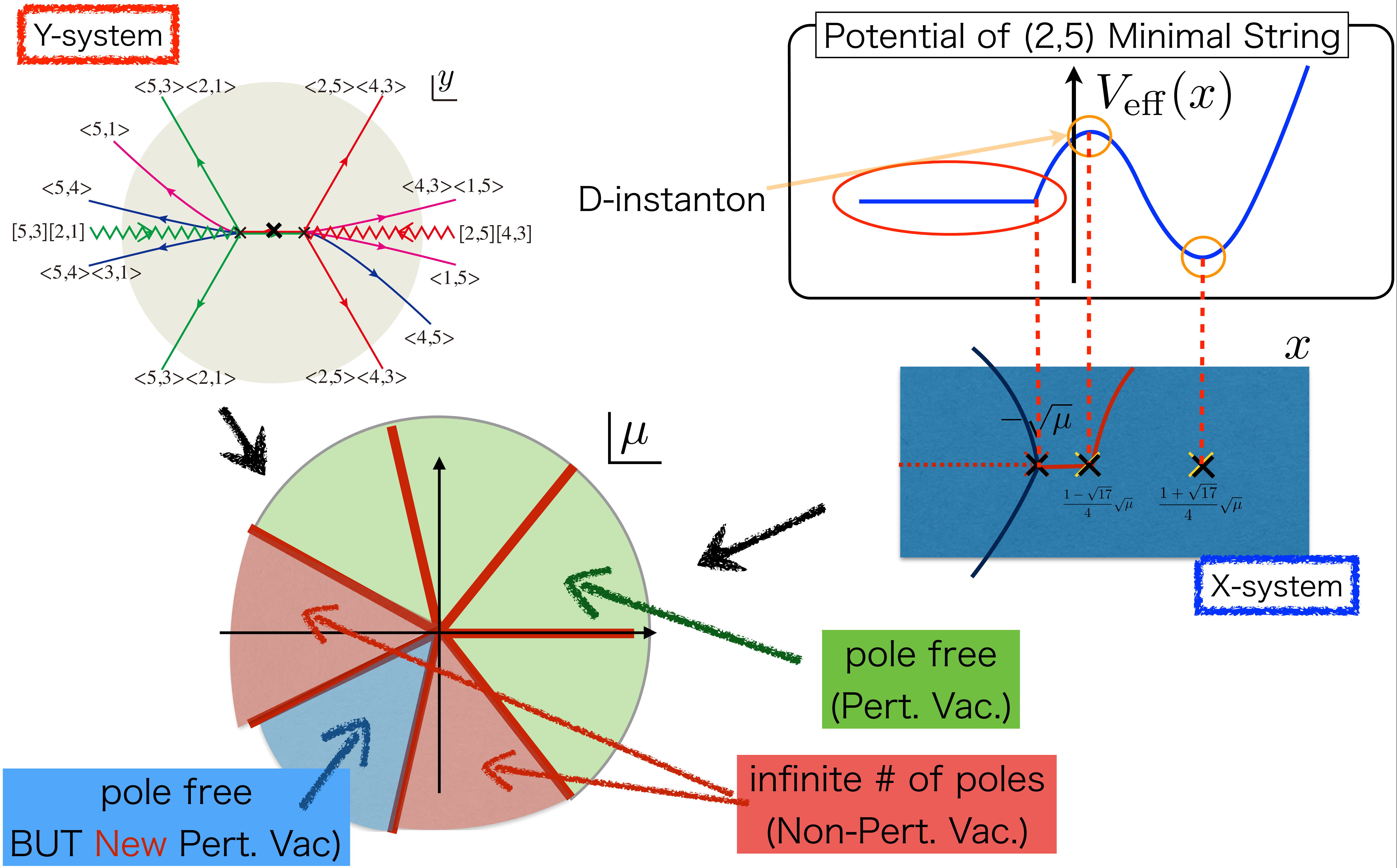
pole free
(Pert. Vac.)

infinite # of poles
(Non-Pert. Vac.)

3-truncated solutions [Its-et.al.,⋯]

Duality Check of (5,2) ↔ (2,5) Models [CIY6 '14]

We can check the phase structure in g -> 0 (or μ -> ∞)

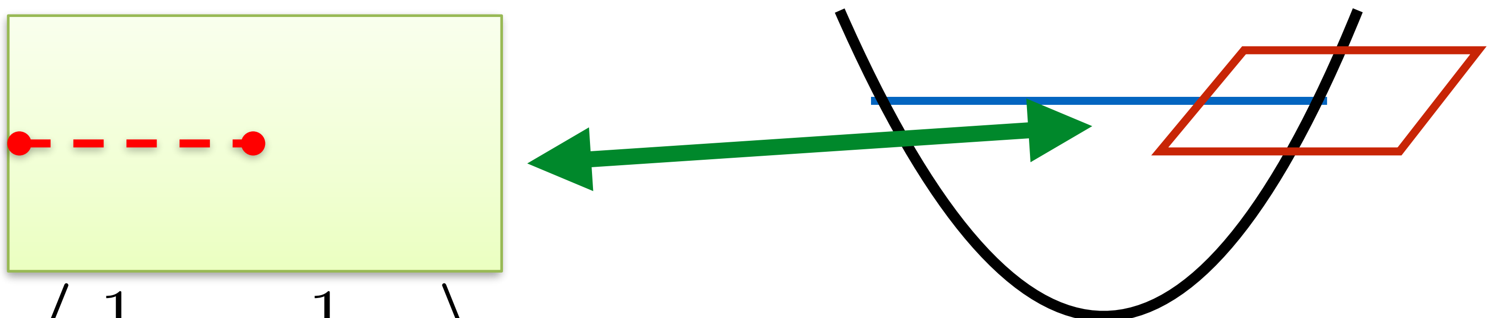


2) Condition for matrix-model solutions

One-cut Boundary Condition [CIY2 ‘11]

Resolvent

matrix models



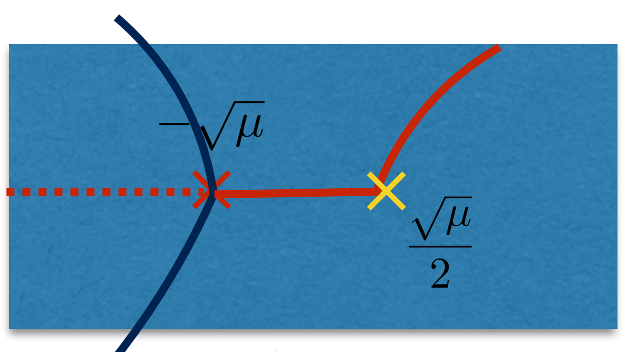
$$\left\langle \frac{1}{N}\mathrm{tr}\frac{1}{x-X} \right\rangle$$

There should be a single cut

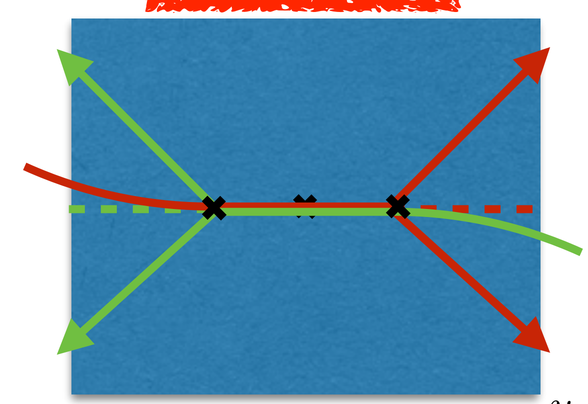
So far, there is no inconsistency with this

Y-system

(p,q)=(2,3)

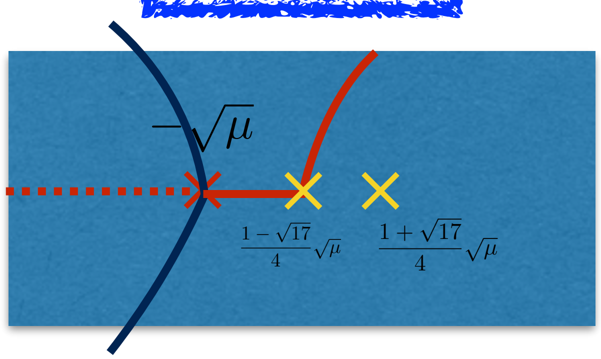


p-q dual

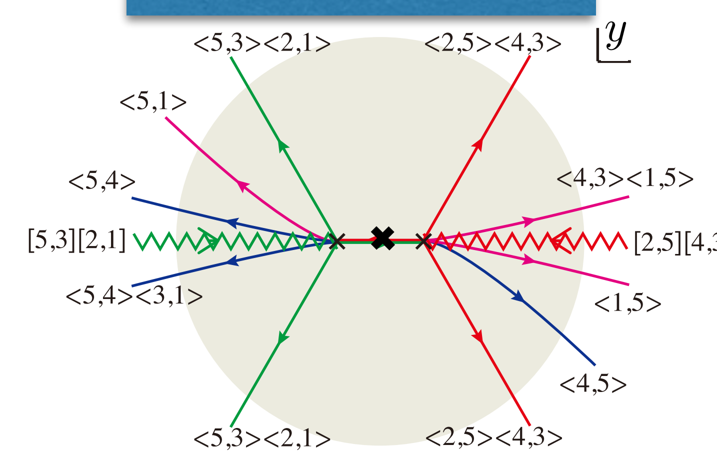


X-system

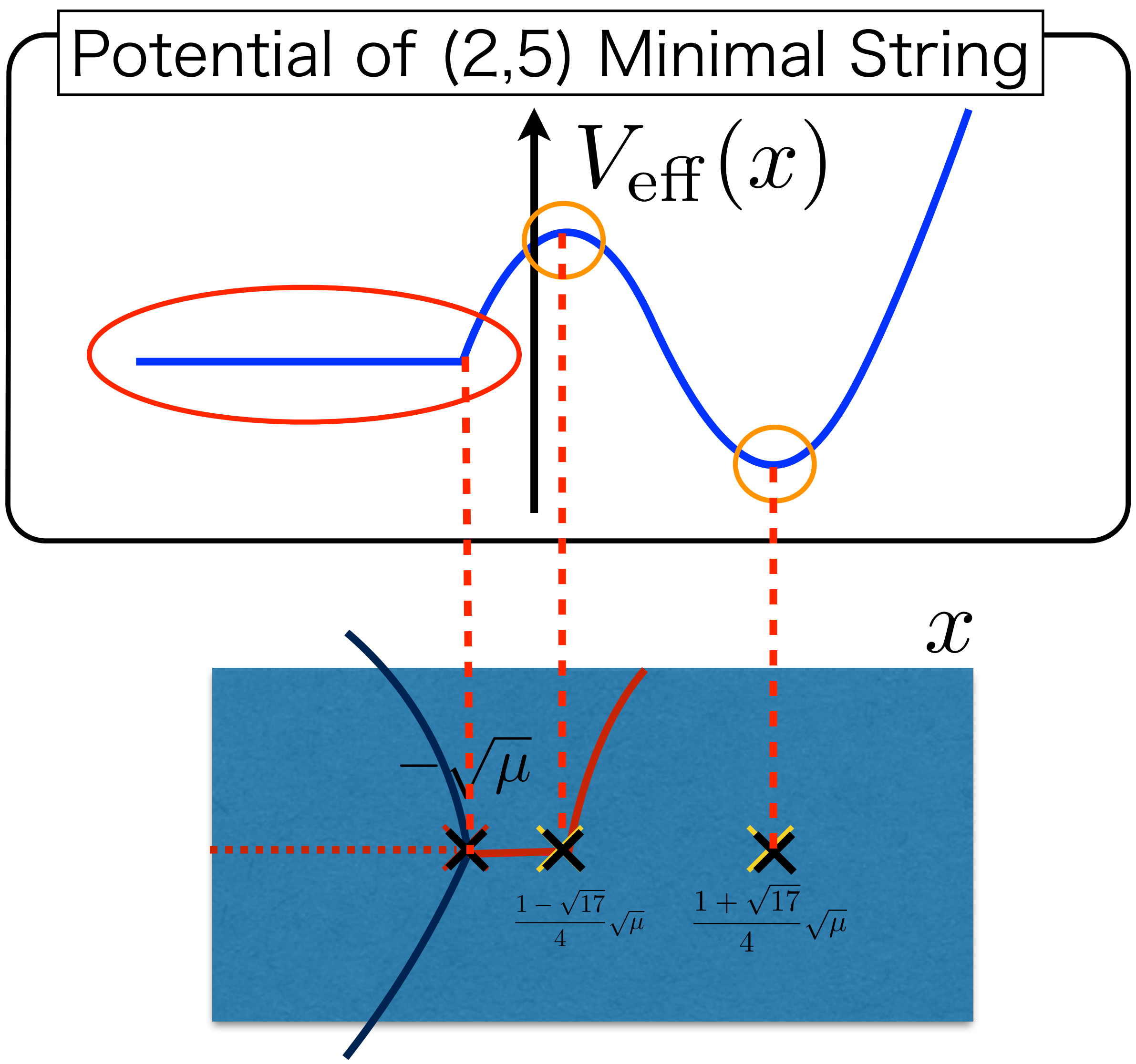
(p,q)=(2,5)



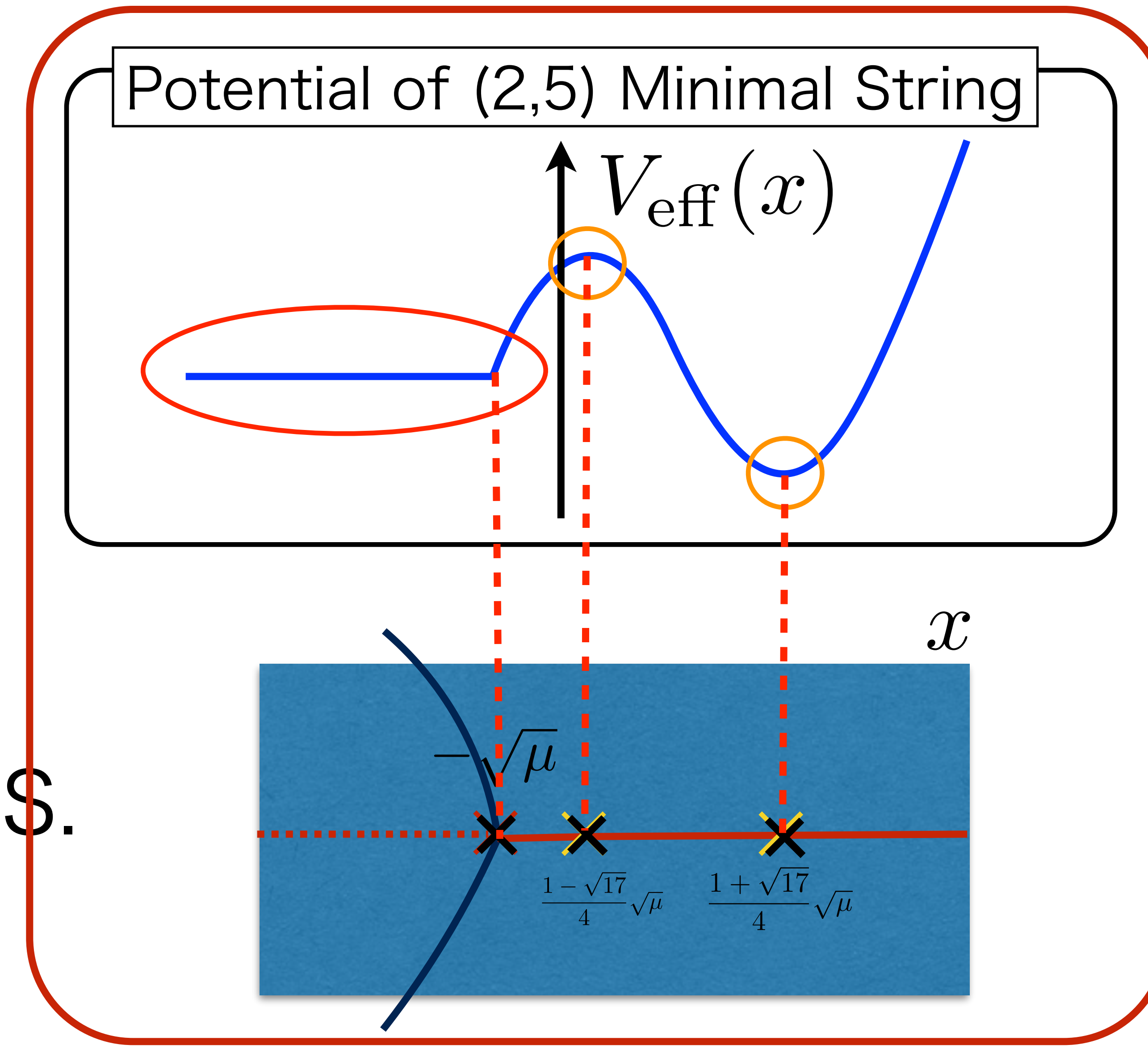
p-q dual



BUT, a non-trivial situation happens if



V.S.

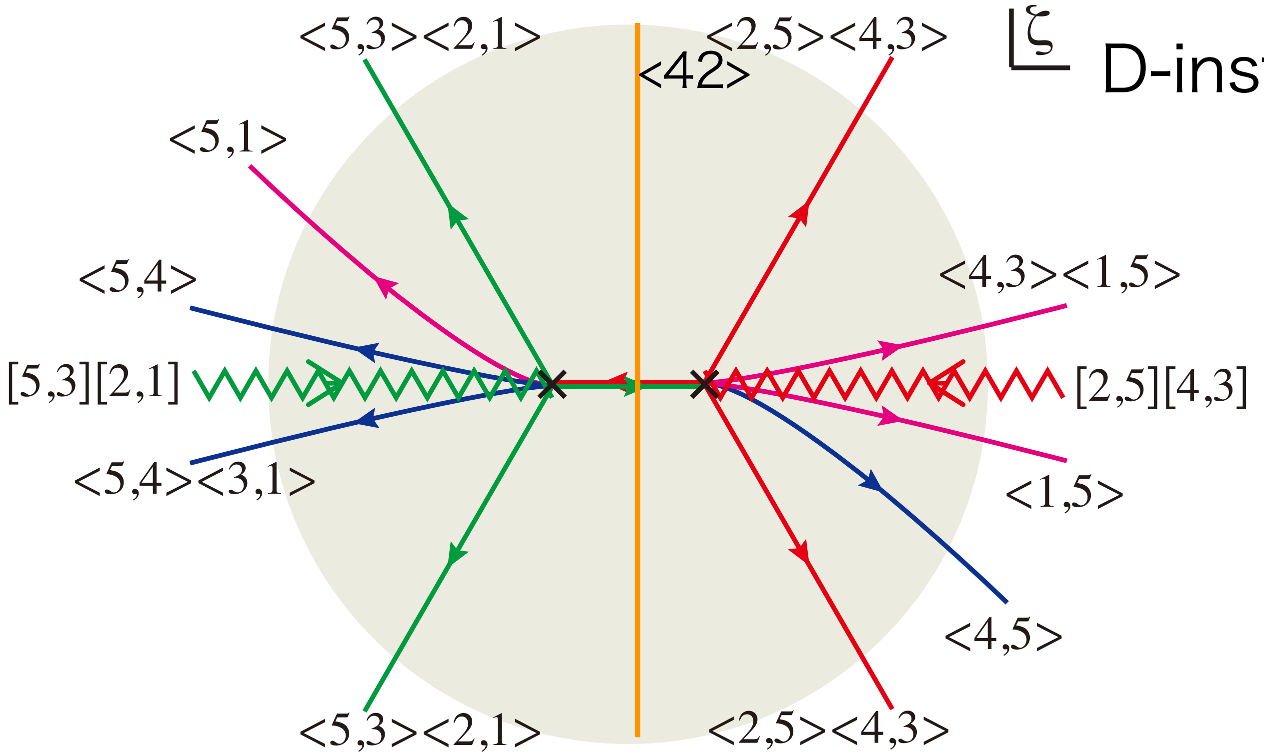


See large instanton modes

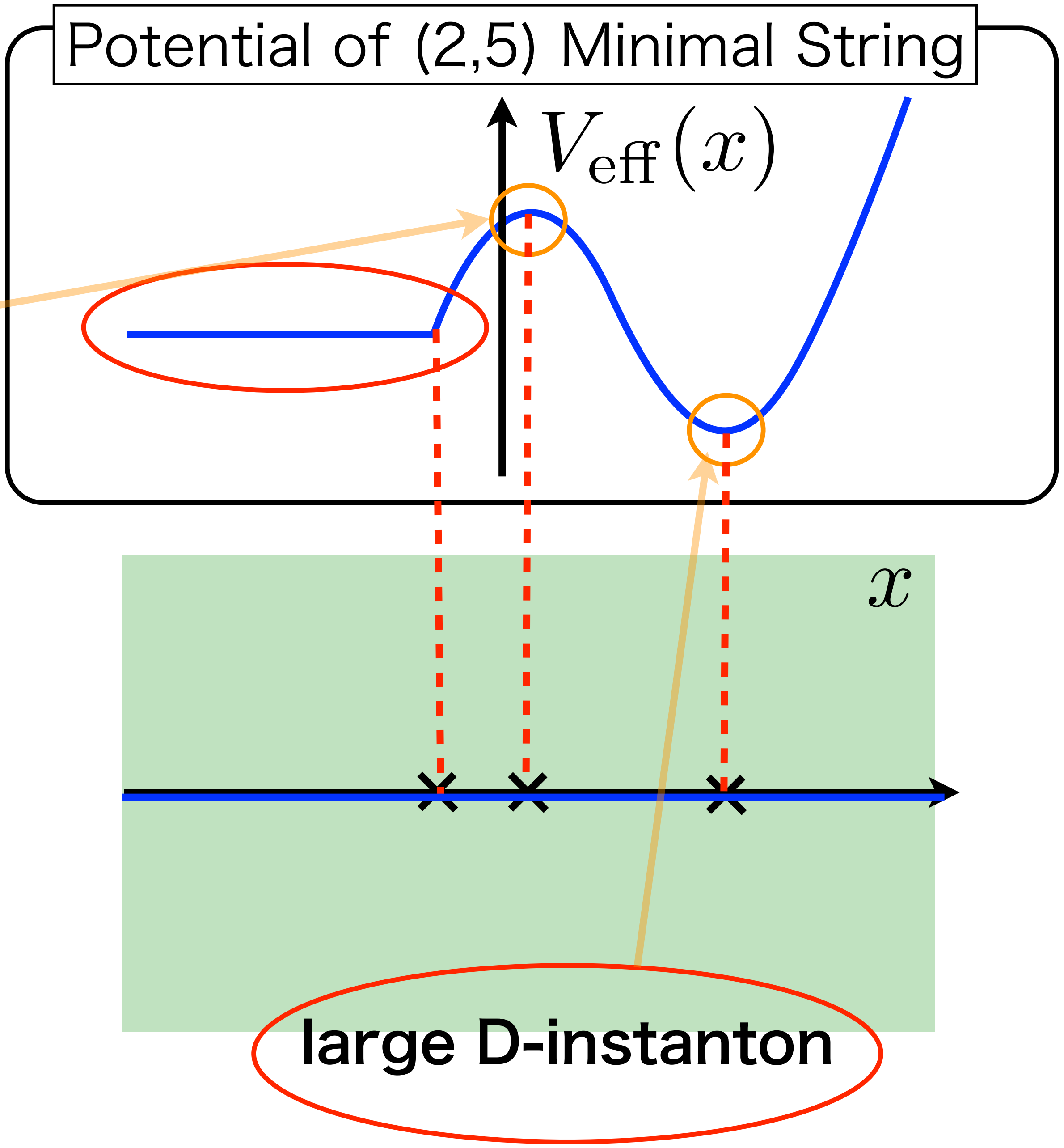
[CIY5 ‘13]

$$\mathcal{F} \simeq \mathcal{F}_{\rm pert} + e^{+\frac{1}{g}S_I} + \cdots$$

Large instantons → observed on both sides

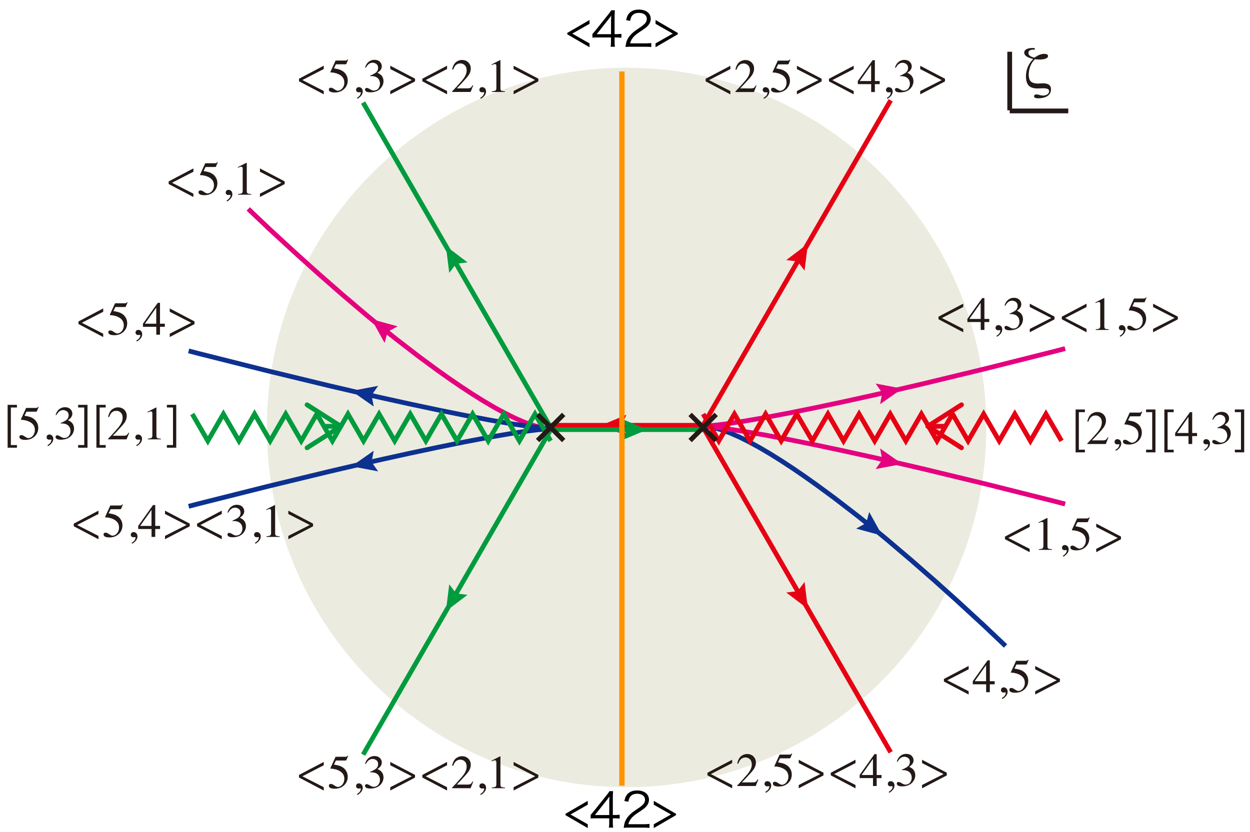


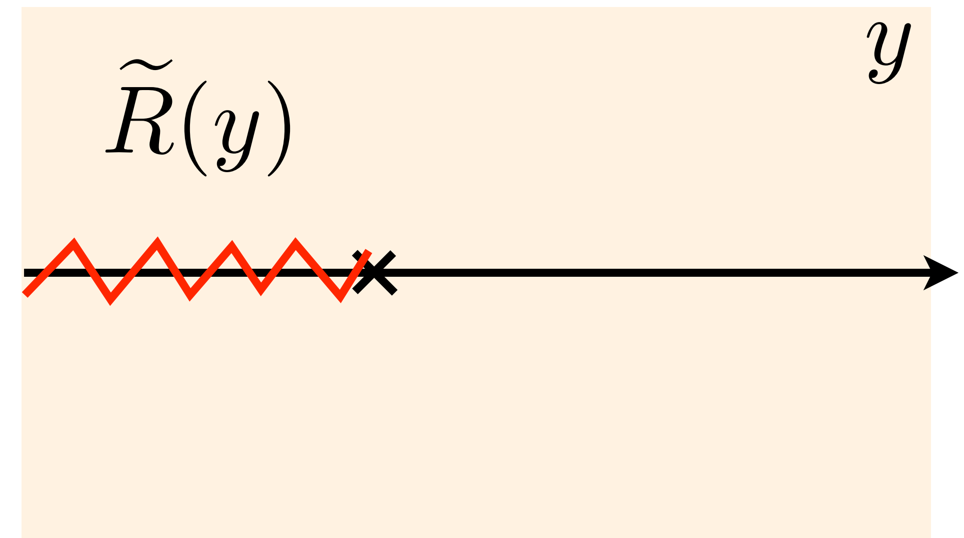
picked up by vertical contour



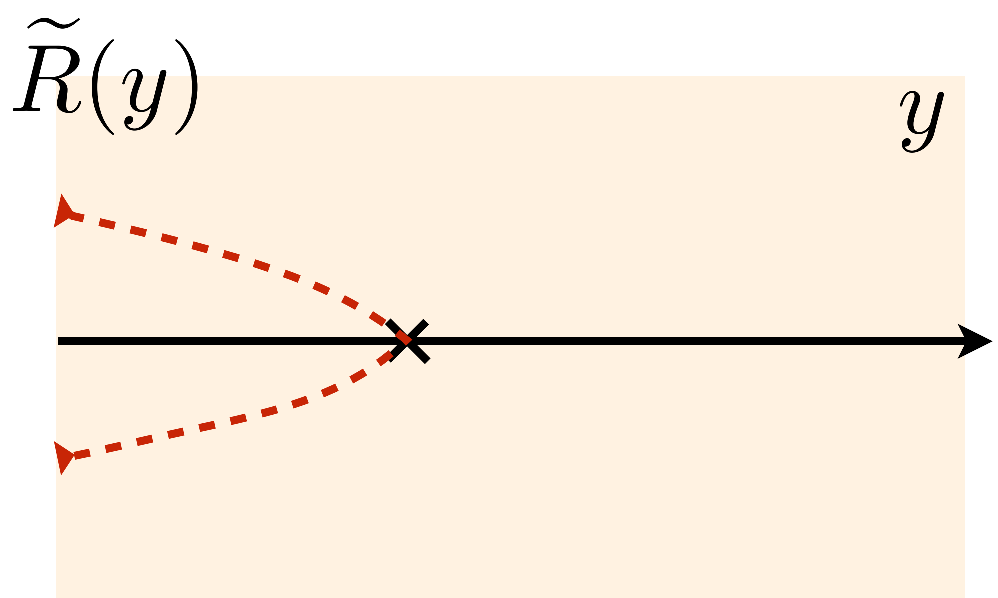
One-cut Boundary condition

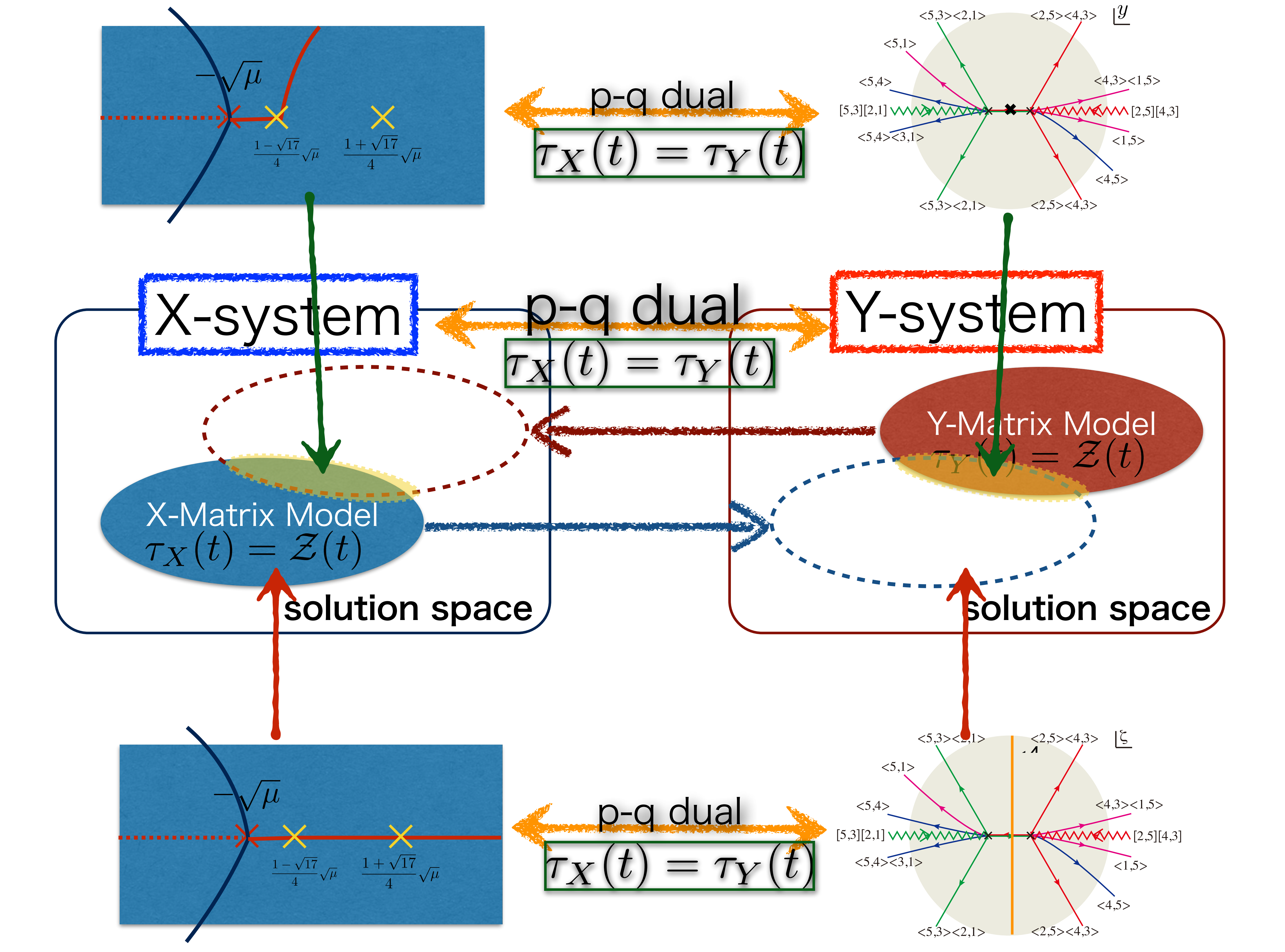
[CIY5 ‘13]





This model cannot satisfy One-cut BC





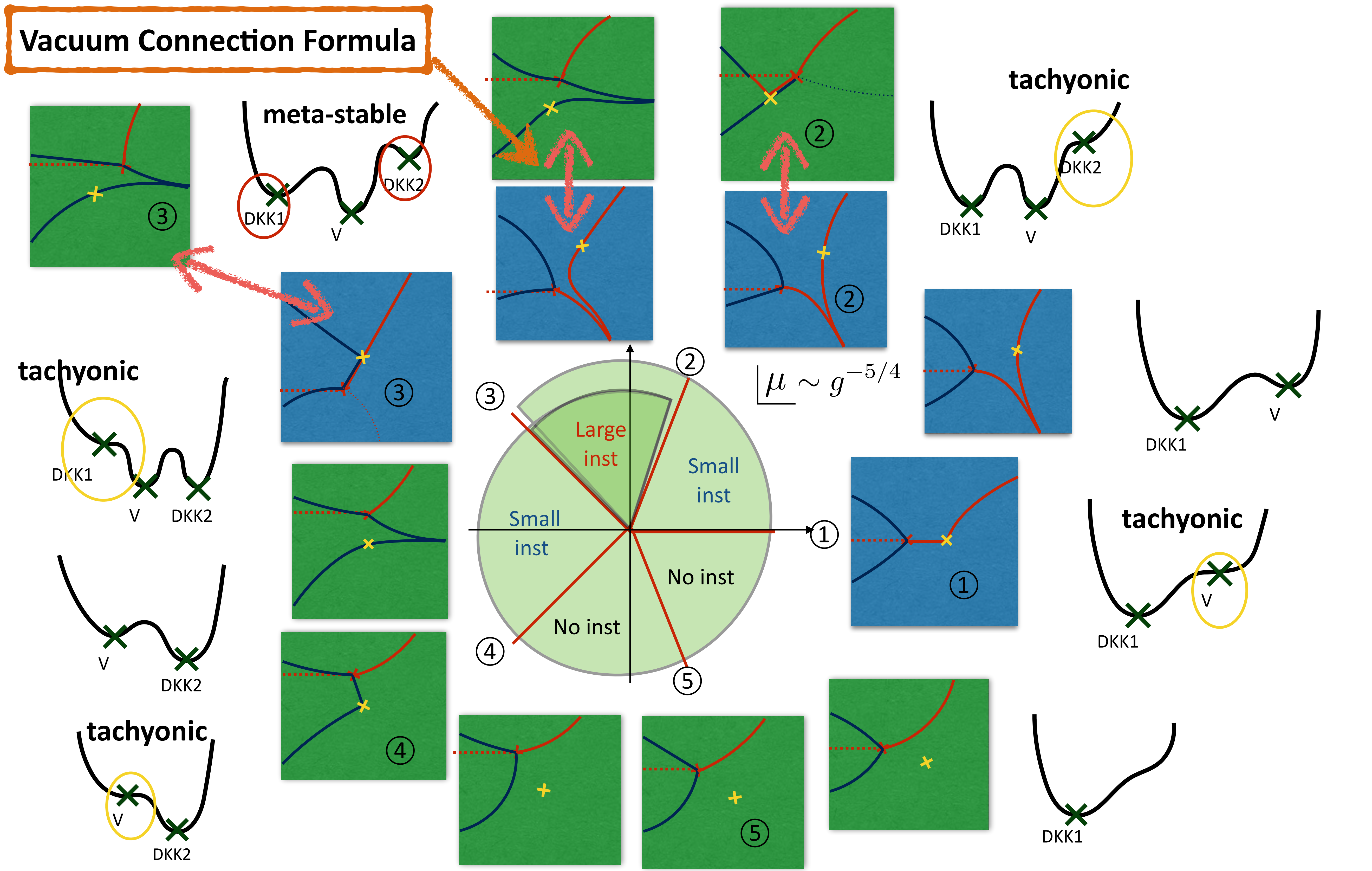
$-\sqrt{\mu}$
$\frac{1-\sqrt{17}}{4}\sqrt{\mu}$
$\frac{1+\sqrt{17}}{4}\sqrt{\mu}$
p-q dual
$\tau_X(t) = \tau_Y(t)$
<5,3><2,1>
<2,5><4,3>
y
<5,1>
<5,4>
<4,3><1,5>
[5,3][2,1]
[2,5][4,3]
<5,4><3,1>
<1,5>
<4,5>
X-system
p-q dual
$\tau_X(t) = \tau_Y(t)$
Y-system
Y-Matrix Model
$\tau_Y(t) = \mathcal{Z}(t)$
X-Matrix Model
$\tau_X(t) = \mathcal{Z}(t)$
solution space
solution space
$-\sqrt{\mu}$
$\frac{1-\sqrt{17}}{4}\sqrt{\mu}$
$\frac{1+\sqrt{17}}{4}\sqrt{\mu}$
p-q dual
$\tau_X(t) = \tau_Y(t)$
<5,3><2,1>
<2,5><4,3>
ζ
<5,1>
<5,4>
<4,3><1,5>
[5,3][2,1]
[2,5][4,3]
<5,4><3,1>
<1,5>
<4,5>

Summary

- We consider Duality in Non-perturbative Completion
- Matrix models are known to possess non-perturbative [contour] ambiguity (which is the ambiguity of D-instanton chemical potentials θ).
- Duality may be broken non-perturbatively
- Therefore, if one requires "*string duality acts non-perturbatively,*" as a principle, then it provides a constraint on non-perturbative ambiguity of string theory

This is the first quantitative observation
on non-perturbative principle of string theory

Non-perturbative Phase Structure in complex g [CIY6 '14]
and change of Scenery in String Theory Landscapes



Thank you for your attention!