Exact spectrum of tachyons in AdS/CFT

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Tachyon and instability

Lagrangian density of a complex-scalar QFT

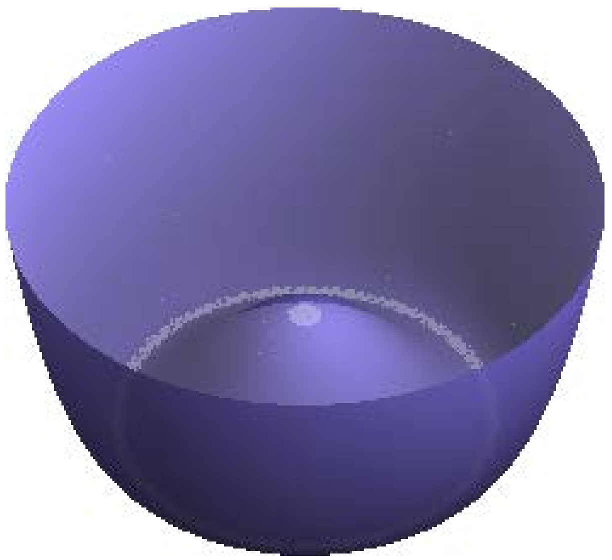
$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi, \bar{\phi})$$

The 1st derivative defines the vacuum, the 2nd the mass

$$0 = \partial^2 \phi + \partial_{\bar{\phi}} V(\phi, \bar{\phi}) \Big|_{(\phi,\bar{\phi})=(\phi_0,\bar{\phi}_0)} = \text{c.c.}$$

$$(\text{mass})^2 = \frac{\partial^2 V(\phi, \bar{\phi})}{\partial\phi\, \partial\bar{\phi}} \Big|_{(\phi,\bar{\phi})=(\phi_0,\bar{\phi}_0)}, \quad \begin{cases} (\text{mass})^2 > 0 & \text{stable} \\ (\text{mass})^2 < 0 & \text{unstable} \end{cases}$$

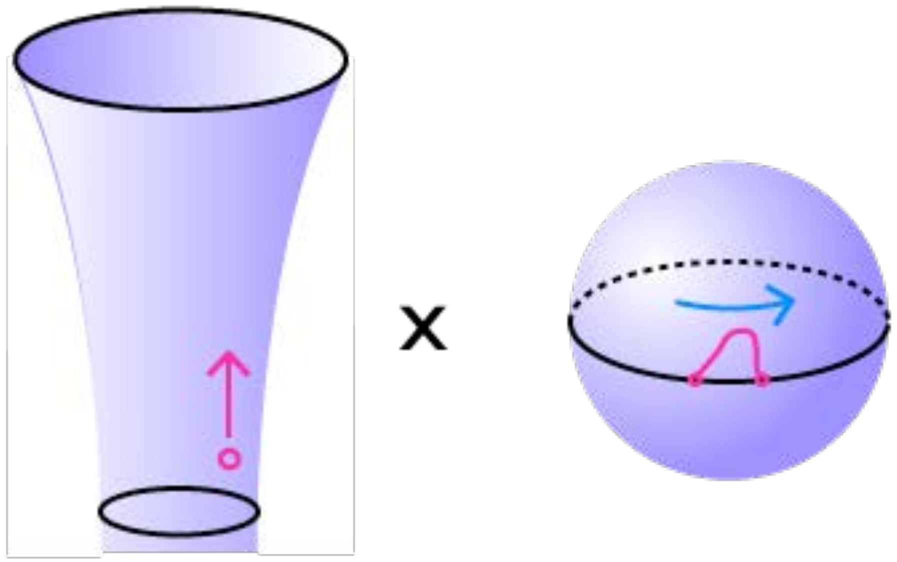
When the mass is pure imaginary,
the corresponding particle is called a tachyon,
and the extremum is unstable



Brane-antibrane system

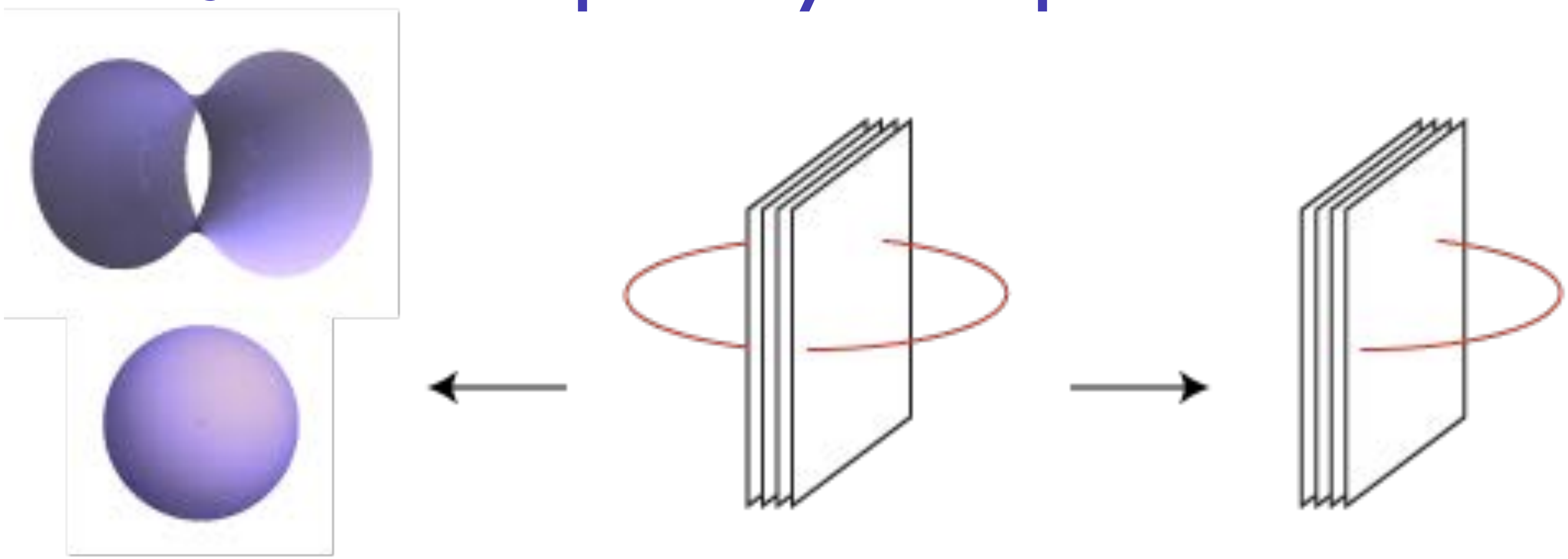
D-brane & D-antibrane (D-$\bar{\text{D}}$) system in the flat spacetime is an example of unstable state in string theory

D-brane & D-antibrane and open strings in between in the curved spacetime ($AdS_5 \times S^5$) are less well-understood



AdS/CFT correspondence

$AdS_5 \times S^5$ is the primary example of AdS/CFT



Superstring theory on $AdS_5 \times S^5$

$N \to \infty,\ g_s \to 0$

$\lambda = N g_s$

Stack of N D3-branes

$\stackrel{?}{=}$

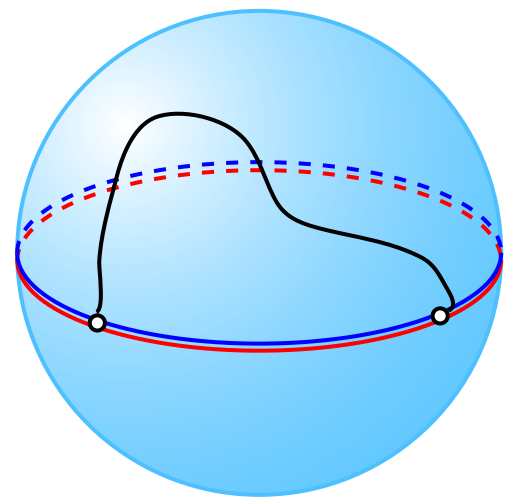
$\mathcal{N}$=4 4dim SU(N) super Yang-Mills

$N \to \infty,\ g_{\rm YM} \to 0$

$\lambda = N g_{\rm YM}^2$

AdS/CFT correspondence

The energy of an open string in $AdS_5 \times S^5$ ending on a pair of "giant-graviton" D-$\overline{\text{D}}$ branes



should be dual to the dimension of a determinant-like operator in 4D $SU(N)$ $\mathcal{N}$=4 super Yang-Mills theory

$$\mathcal{O}_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,\epsilon^{k_1\cdots k_N}\,\epsilon_{l_1\cdots l_N}\times$$
$$Y^{j_1}_{i_1}\cdots Y^{j_{N-1}}_{i_{N-1}}\,V^{l_N}_{i_N}\,\overline{Y}^{l_1}_{k_1}\cdots\overline{Y}^{l_{N-1}}_{k_{N-1}}\,W^{j_N}_{k_N}$$

Hope: demonstrate this duality using integrability

[Balasubramanian, Berkooz, Naqvi, Strassler (2001)] [Balasubramanian, Huang, Levi, Naqvi (2002)]

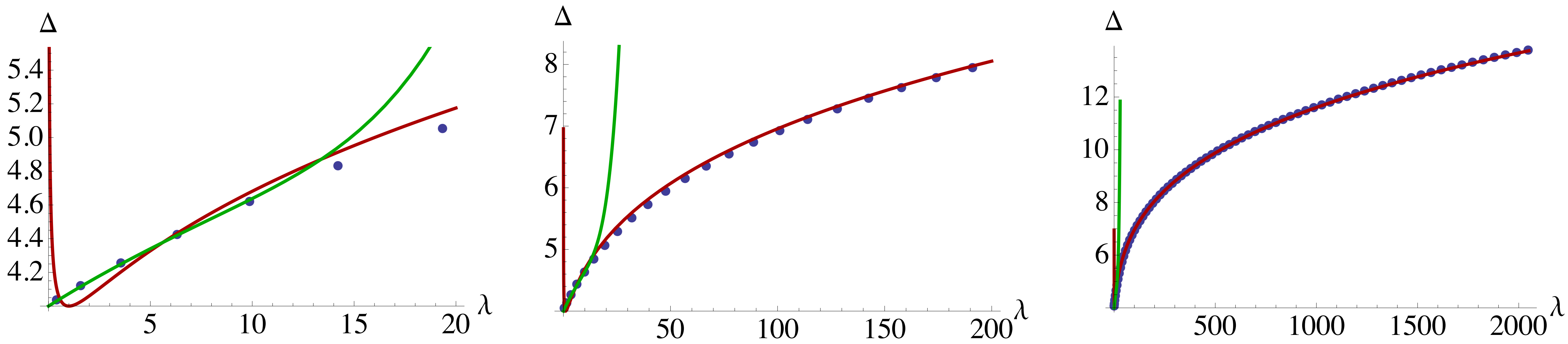
Integrability Methods

The spectral problem at large N is now "solvable" through (Asymptotic/Thermodynamic) Bethe Ansatz

$$E_{\text{string}}(\lambda) \overset{\sim}{\longleftarrow} E_{\text{ABA}}(\lambda) \;\text{ or }\; E_{\text{TBA}}(\lambda) \overset{\sim}{\longrightarrow} \Delta_{\text{SYM}}(\lambda)$$

We want to solve TBA; i.e. obtain $E_{\text{TBA}}(\lambda)$

Example: the exact dimension of Konishi operator



Green: SYM, weak 5-loop Blue: TBA, numerics Red: String, strong 1-loop

[Gromov, Kazakov, Vieira (2009)] [Frolov (2010)] and others

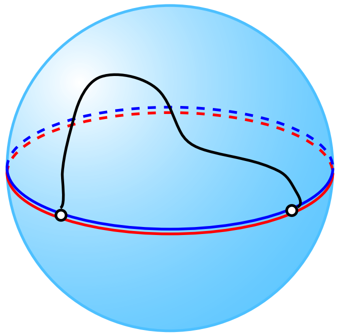
To do

$$E_{\text{string}}(\lambda) \stackrel{\sim}{\longleftarrow} E_{\text{ABA}}(\lambda) \ \text{or} \ E_{\text{TBA}}(\lambda) \stackrel{\sim}{\longrightarrow} \Delta_{\text{SYM}}(\lambda)$$

We propose BTBA equations

(Boundary Thermodynamic Bethe Ansatz)

and solve them numerically



$$\mathcal{O}_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,\epsilon^{k_1\cdots k_N}\,\epsilon_{l_1\cdots l_N}\,Y_{i_1}^{j_1}\cdots Y_{i_{N-1}}^{j_{N-1}}\,V_{i_N}^{l_N}\,\overline{Y}_{k_1}^{l_1}\cdots\overline{Y}_{k_{N-1}}^{l_{N-1}}\,W_{k_N}^{j_N}$$

However,

Integrability vs Instability

Can we apply integrability methods to unstable systems?

<u>Problem 1</u>

- Even in $AdS_5 \times S^5$, there are unstable strings with complex energy whereas the energy is always real in integrability methods

<u>Problem 2</u>

- TBA energy may diverge in the system with closed string tachyons
eg. non-susy TsT-transformed $AdS_5 \times S^5$ vs γ-deformed SYM

What happens in the open string sector?

cf. similar question [Rastelli, Pomoni] 1002.0006

We encounter *apparent* singularities

Plan of Talk

✓ Introduction

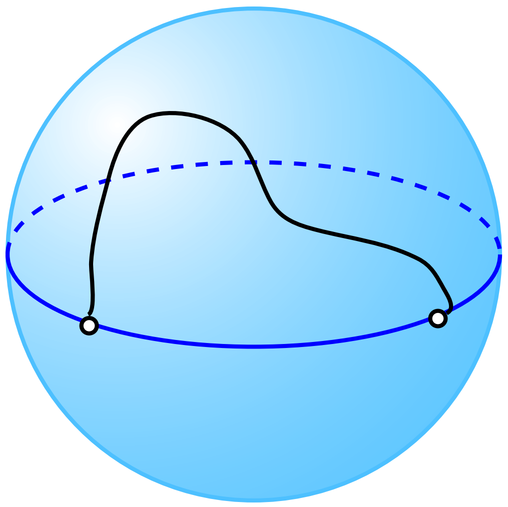
- Determinants and giant-gravitons
- BTBA equations and energy bound
- Summary and outlook

Determinants and giant-gravitons

Spherical Maximal Giant Gravitons (SMGG's)

Giant graviton = Half-BPS, D3-brane solution on $\mathrm{AdS}_5 \times \mathrm{S}^5$

carrying a large angular momentum $J = \mathcal{O}(N)$



Spherical $\Leftrightarrow$ **"wrap" on $\mathrm{S}^3 \subset \mathrm{S}^5$**

with the angular momentum bound $J \leq N$

Maximal $\Leftrightarrow$ $J = N$

SMGG's are classified by the choice:

$$\mathrm{S}^3 \subset \mathrm{S}^5 = \{|X|^2 + |Y|^2 + |Z|^2 = R^2_{\text{sphere}}\}$$

$$X = 0 \text{ or } Y = 0 \text{ or } Z = 0 \cdots$$

$\overline{Y} = 0$ **brane** $\Leftrightarrow$ **Carrying negative angular momentum compared to $Y = 0$**

[McGreevy, Susskind, Toumbas (2000)]

Giant graviton is determinant

SMGG's are dual to determinants

$$\det \Phi^N = \epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} \Phi^{j_1}_{i_1} \cdots \Phi^{j_N}_{i_N}$$

Open strings on the Y=0 brane are dual to det-like operator

$$\det \left(Y^{N-1} V \right) = \epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} Y^{j_1}_{i_1} \cdots Y^{j_{N-1}}_{i_{N-1}} V^{j_N}_{i_N}$$

A pair of open strings on Y=0 and Ybar=0 should be dual to:

$$\mathcal{O}_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} \epsilon^{k_1 \cdots k_N} \epsilon_{l_1 \cdots l_N} Y^{j_1}_{i_1} \cdots Y^{j_{N-1}}_{i_{N-1}} V^{l_N}_{i_N} \overline{Y}^{l_1}_{k_1} \cdots \overline{Y}^{l_{N-1}}_{k_{N-1}} W^{j_N}_{k_N}$$

[Balasubramanian, Berkooz, Naqvi, Strassler (2001)] [Balasubramanian, Huang, Levi, Naqvi (2002)]

GG as boundary condition

GG is a boundary condition for an asymptotic open spin chain

Y=0 brane: $\epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} Y^{j_1}_{i_1} \cdots Y^{j_{N-1}}_{i_{N-1}} (ZZ \ldots ZZ)^{j_N}_{i_N}$

Dilatation operator of $\mathcal{N}$=4 SYM = Spin chain Hamiltonian

- Ground state $\quad (ZZ \ldots ZZ) \sim |0\rangle$
- One-particle state

$$\sum_x e^{ipx} (Z \ldots Z \chi Z \ldots Z) \sim A^\dagger_\chi(p)\,|0\rangle$$

- Two-particle state

$$\sum_{x<x'} e^{ip_1 x + ip_2 x'} (Z \ldots Z \chi Z Z \chi' Z \ldots Z) \sim A^\dagger_\chi(p_1) A^\dagger_\chi(p_2)\,|0\rangle$$

GG as boundary condition

GG is a boundary condition for an asymptotic open spin chain

Y=0 brane: $\epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} Y^{j_1}_{i_1} \cdots Y^{j_{N-1}}_{i_{N-1}} (ZZ \ldots ZZ)^{j_N}_{i_N}$

The Y=0 preserves the symmetry psu(1|2)^2

which determines *the reflection matrix*,

a solution of the boundary Yang-Baxter relation

$$\mathbb{S}(-p_2, -p_1)\mathbb{R}_Y(p_1)\mathbb{S}(p_1, -p_2)\mathbb{R}_Y(p_2) = \mathbb{R}_Y(p_2)\mathbb{S}(p_2, -p_1)\mathbb{R}_Y(p_1)\mathbb{S}(p_1, p_2)$$

$$\mathbb{R}^-_Y(p) = R^-_0(p)^2 \begin{pmatrix} e^{-ip/2} & & & \\ & -e^{ip/2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\otimes 2}$$

$$R^-_0(p)^2 = -e^{-ip}\sigma(p, -p)$$ **obeys boundary crossing relation**

[Hofman, Maldacena (2007)] [Chen, Correa (2007)]

The Y_θ=0 brane

New reflection amplitudes can be found by rotating $\boldsymbol{R_Y}$

- $\mathcal{N}$=4 SYM: **Field redefinition:** $\det Y^N \to \det\left(Y\cos\theta + \overline{Y}\sin\theta\right)^N$
- Integrable system:

$$\textbf{Rotation } T: \begin{pmatrix}1\\2\end{pmatrix} \to \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}, \quad \textbf{same for } (\dot{1},\dot{2})$$

$$\mathbb{R}_\theta^-(p) \equiv T R_Y^- T^{-1} = R_0^-(p)^2 \begin{pmatrix} \cos^2\theta\, e^{-ip/2} - \sin^2\theta e^{ip/2} & \sin\theta\cos\theta\left(e^{-ip/2}+e^{ip/2}\right) & & \\ \sin\theta\cos\theta\left(e^{-ip/2}+e^{ip/2}\right) & \sin^2\theta\, e^{-ip/2} - \cos^2\theta e^{ip/2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\otimes 2}$$

- $\boldsymbol{R_\theta}$ still preserves integrability

(solutions of boundary Yang-Baxter relation)

$$\mathbb{S}(-p_2,-p_1)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,-p_2)\,\mathbb{R}(p_2) = \mathbb{R}(p_2)\,\mathbb{S}(p_2,-p_1)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,p_2)$$

- θ = π/2 corresponds to the Ybar=0 brane

YbarY determinant-like operator

A pair of open strings on Y=0 and Ybar=0 should be dual to:

$$\mathcal{O}_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,\epsilon^{k_1\cdots k_N}\,\epsilon_{l_1\cdots l_N}\,Y_{i_1}^{j_1}\cdots Y_{i_{N-1}}^{j_{N-1}}\,V_{i_N}^{l_N}\,\overline{Y}_{k_1}^{l_1}\cdots\overline{Y}_{k_{N-1}}^{l_{N-1}}\,W_{k_N}^{j_N}$$

In the 't Hooft limit, its dimension should take the factorized form

$$\Delta = 2N - 2 + \Delta[V] + \Delta[W]$$

The simplest case is $V = Z^L,\ W = Z^{L'}$

$$\Delta[V] = L + \text{wrapping}, \quad \Delta[W] = L' + \text{wrapping}$$

The energy of a corresponding open string should be

$$E = 2N + E_{\text{open}}[V] + E_{\text{open}}[W]$$

$$E_{\text{open}}[V] = -1 + L + \text{wrapping}$$

Caveat!

Actually the representative state is not a dilatation eigenstate

$$\mathcal{O}_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,\epsilon^{k_1\cdots k_N}\,\epsilon_{l_1\cdots l_N}\,Y^{j_1}_{i_1}\cdots Y^{j_{N-1}}_{i_{N-1}}\,V^{l_N}_{i_N}\,\overline{Y}^{l_1}_{k_1}\cdots\overline{Y}^{l_{N-1}}_{k_{N-1}}\,W^{j_N}_{k_N}$$

An example of potential mixings

$$\mathcal{O}'_{Y,\overline{Y}}[V,W] \sim \epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,\epsilon^{k_1\cdots k_N}\,\epsilon_{l_1\cdots l_N}\,Y^{j_1}_{i_1}\cdots (Y\overline{Y})^{j_{N-1}}_{i_{N-1}}\,V^{l_N}_{i_N}\,\overline{Y}^{l_1}_{k_1}\cdots\delta^{l_{N-1}}_{k_{N-1}}\,W^{j_N}_{k_N}$$

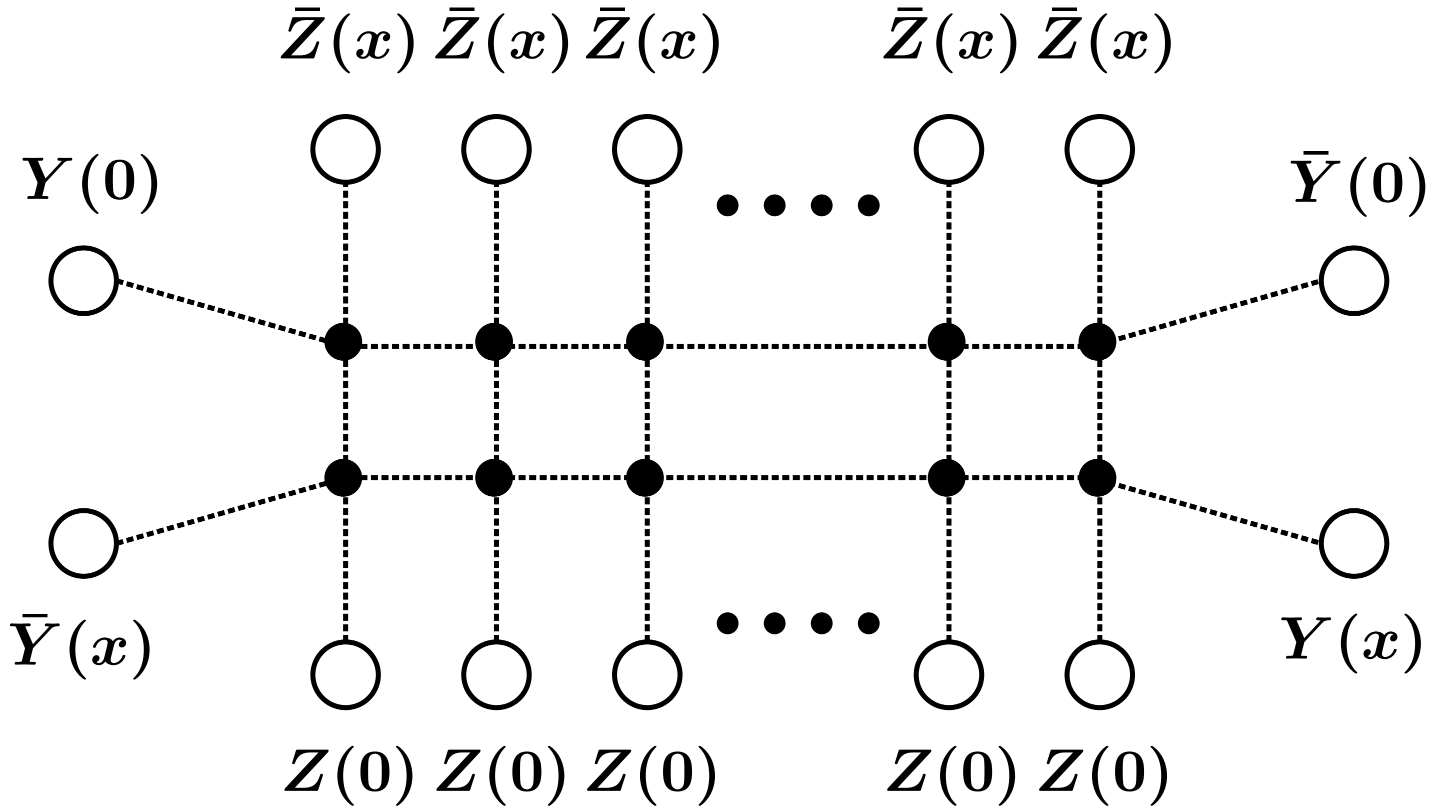
We must use the true eigenstate before computing wrapping diagram

- Nevertheless, the wrapping computation seem to be insensitive to the details
- The classification of dilatation eigenstate at finite *N* is difficult particularly when the length of operator exceeds *N*

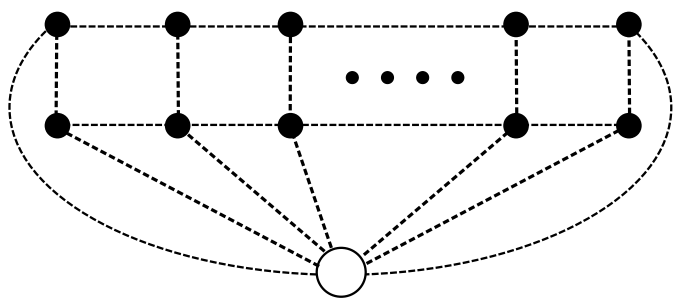
(cf. to classify all representations of non-semi-simple Brauer algebra)

Wrapping diagram

After a lot of tree-level contractions between $\boldsymbol{Y}$-$\overline{\boldsymbol{Y}}$, we obtain



Spacetime structure
(amputated)

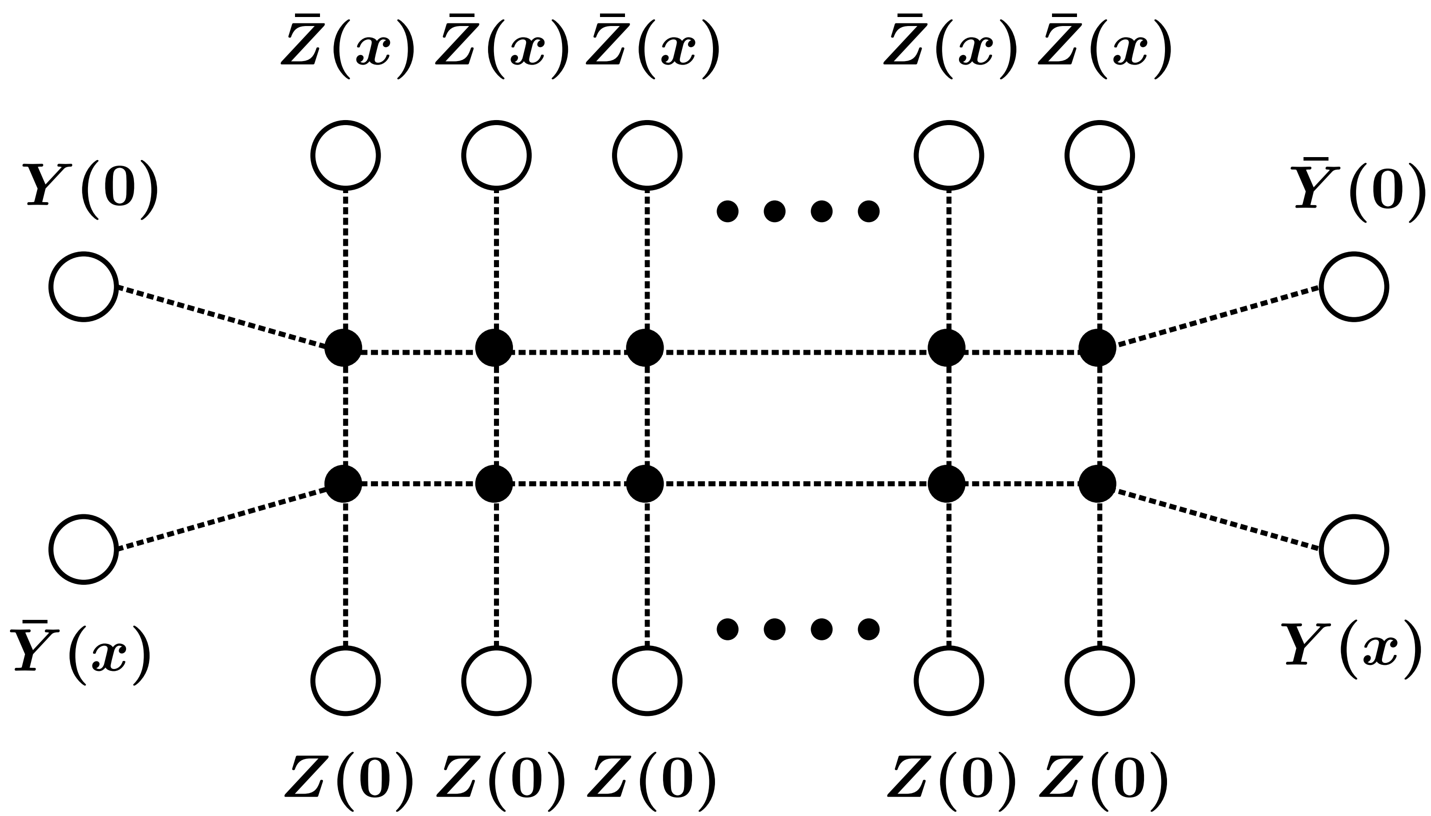


this is same as the so-called zig-zag diagram

cf. [Brown, Schnetz] arXiv:1208.1890, [Schnetz] arXiv:1210.5376

Wrapping diagram

After a lot of tree-level contractions between $\boldsymbol{Y}$-$\bar{\boldsymbol{Y}}$, we obtain



The result is

$$\delta\Delta_L = -\frac{4(g/2)^{4L}}{4L-1}\binom{4L}{2L}\zeta(4L-3) + \mathcal{O}(g^{4L+2}), \quad g \ll 1$$

Agree with the boundary Lüscher formula for $L = J > 1$

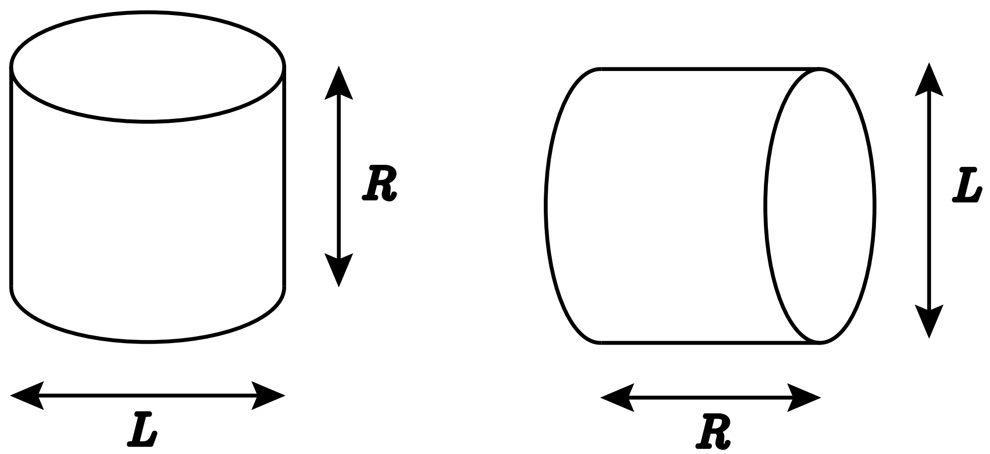
cf. [Brown, Schnetz] arXiv:1208.1890, [Schnetz] arXiv:1210.5376

BTBA equations and energy bound

Exact dimension/energy

Begin with the equivalence of Euclidean worldsheet partition functions

[Zamolodchikov (1990)]
[Arutyunov, Frolov (2007)]



$$Z_E(L,R) = \int [dX]\, e^{-S_E} = \int [d\tilde{X}]\, e^{-\tilde{S}_E} = \tilde{Z}(R,L)$$

In Hamiltonian formalism, $\mathrm{tr}\, e^{-RH(L)} = \mathrm{tr}\, e^{-L\tilde{H}(R)}$

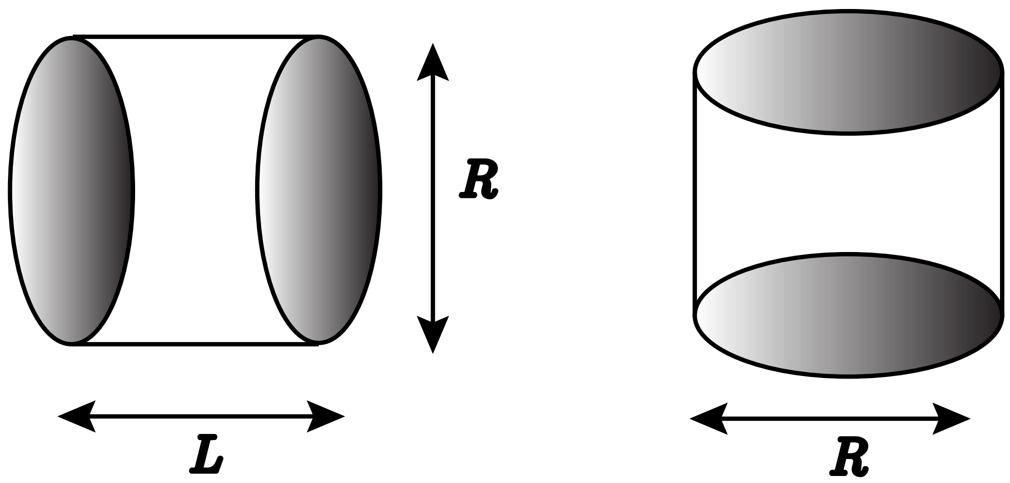
Take the large R limit, $e^{-RE_0(L)} = \lim_{R\to\infty} e^{-\tilde{\mathcal{F}}(\mathcal{R})}$

The "mirror" free energy can be computed by the "mirror" asymptotic Bethe Ansatz equations in the thermodynamic limit

⇨ Thermodynamic Bethe Ansatz equations (TBA)

Mirror trick with boundary

A simple generalization is to change boundary conditions



$(R \to \infty)$

Ground-state BTBA

$$\log \frac{Y_a}{Y_a^\circ} = \log \left(\frac{1 \pm Y_b}{1 \pm Y_b^\circ} \right) \star K_{ba} \quad \textbf{for auxiliary Y}$$

$$\log \frac{Y_Q}{Y_Q^\bullet} = \log \left(\frac{1 \pm Y_b}{1 \pm Y_b^\circ} \right) \star K_{bQ}$$

$$Y_{\text{aux}}^\circ = \text{asymptotic Y-functions}, \quad Y_Q^\circ = 0$$

Exact ground-state energy

$$E_{\text{BTBA}}(L,g) = -\sum_{Q=1}^{\infty} \int_0^\infty \frac{d\tilde{p}_Q}{2\pi} \log(1 + Y_Q)$$

Summary of YbarY energy

YbarY BTBA: $$\log \frac{Y_a}{Y_a^\circ} = \log\left(\frac{1 \pm Y_b}{1 \pm Y_b^\circ}\right) \star K_{ba}$$

(∞ nonlinear integral equations can be solved by numerical iteration)

BTBA energy: $$E_{\mathrm{BTBA}}(L,g) = -\sum_{Q=1}^{\infty} \int_0^\infty \frac{d\tilde{p}_Q}{2\pi} \log(1 + Y_Q)$$

Our BTBA describes Δ of the determinant-like operator:

$$\mathcal{O}_{Y,\overline{Y}}[Z^L, Z^{L'}] \sim \epsilon^{i_1\cdots i_N}\, \epsilon_{j_1\cdots j_N}\, \epsilon^{k_1\cdots k_N}\, \epsilon_{l_1\cdots l_N} \times$$
$$Y_{i_1}^{j_1} \cdots Y_{i_{N-1}}^{j_{N-1}} (Z^L)_{i_N}^{l_N}\, \overline{Y}_{k_1}^{l_1} \cdots \overline{Y}_{k_{N-1}}^{l_{N-1}} (Z^{L'})_{k_N}^{j_N}$$

$$\Delta = 2N - 2 + L + L' + \underline{E_{\mathrm{BTBA}}(L,g) + E_{\mathrm{BTBA}}(L',g)}$$

all wrapping corrections, negative values

Interestingly, there exists a lower bound for the (B)TBA energy

YQ(v) at large v

BTBA equation for YQ in the large v limit

$$\log\frac{Y_Q(v)}{Y_Q^{\bullet}(v)} = -2\int_{-\infty}^{\infty} dt\,\log(1+Y_{Q'}(t))\,K_{\Sigma}^{Q'Q}(t,v) + \dots$$

$$\sim -4E_{BTBA}\log(v), \quad v \gg 1$$

$$\Leftrightarrow \quad \log Y_Q(v) \sim -(4L+4E_{\mathrm{BTBA}})\log(v)$$

$$K^{\Sigma}_{Q'Q}(t,v) = \frac{1}{2\pi i}\frac{\partial}{\partial t}\log\Sigma^{Q'Q}(t,v)$$

$$\begin{aligned}\frac{1}{i}\log\Sigma^{Q'Q}(t,v) &= \Phi(y_1^+,y_2^+) - \Phi(y_1^+,y_2^-) - \Phi(y_1^-,y_2^+) + \Phi(y_1^-,y_2^-)\\ &+\frac{1}{2}\Big(\Psi(y_2^+,y_1^+)+\Psi(y_2^-,y_1^+)-\Psi(y_2^+,y_1^-)-\Psi(y_2^-,y_1^-)\Big)\\ &-\frac{1}{2}\Big(\Psi(y_1^+,y_2^+)+\Psi(y_1^-,y_2^+)-\Psi(y_1^+,y_2^-)-\Psi(y_1^-,y_2^-)\Big)\\ &+\frac{1}{i}\log\frac{i^{Q'}\,\Gamma\big[Q-\frac{i}{2}g\big(y_1^+ + \frac{1}{y_1^+} - y_2^+ - \frac{1}{y_2^+}\big)\big]}{i^{Q}\,\Gamma\big[Q'+\frac{i}{2}g\big(y_1^+ + \frac{1}{y_1^+} - y_2^+ - \frac{1}{y_2^+}\big)\big]}\frac{1-\frac{1}{y_1^+y_2^-}}{1-\frac{1}{y_1^-y_2^+}}\sqrt{\frac{y_1^+y_2^-}{y_1^-y_2^+}}\end{aligned}$$

$$\Phi(x_1,x_2) = i\oint\frac{dw_1}{2\pi}\oint\frac{dw_2}{2\pi}\frac{1}{(w_1-x_1)(w_2-x_2)}\log\frac{\Gamma[1+\frac{ig}{2}\left(w_1+\frac{1}{w_1}-w_2-\frac{1}{w_2}\right)]}{\Gamma[1-\frac{ig}{2}\left(w_1+\frac{1}{w_1}-w_2-\frac{1}{w_2}\right)]}$$

$$\Psi(x_1,x_2) = i\oint\frac{dw}{2\pi}\frac{1}{w-x_2}\log\frac{\Gamma[1+\frac{ig}{2}\left(x_1+\frac{1}{x_1}-w-\frac{1}{w}\right)]}{\Gamma[1-\frac{ig}{2}\left(x_1+\frac{1}{x_1}-w-\frac{1}{w}\right)]}$$

$$x(v) = \frac{1}{2}\left(v - i\sqrt{4-v^2}\right), \quad y_1^{\pm} = x\Big(t \pm \frac{iQ'}{g}\Big), \quad y_2^{\pm} = x\Big(v \pm \frac{iQ}{g}\Big)$$

YQ(v) at large v

BTBA equation for YQ in the large v limit

$$\log \frac{Y_Q(v)}{Y_Q^{\bullet}(v)} = -2 \int_{-\infty}^{\infty} dt \, \log(1 + Y_{Q'}(t)) \, K_{\Sigma}^{Q'Q}(t, v) + \dots$$

$$\sim -4 E_{BTBA} \log(v), \quad v \gg 1$$

$$\Leftrightarrow \quad \log Y_Q(v) \sim -(4L + 4E_{\rm BTBA}) \log(v)$$

However, the integrals in BTBA energy diverges if $Y_Q(v) \sim 1/v$

$$\int_0^{\infty} \frac{dv}{2\pi} \frac{d\widetilde{p}_Q}{dv} \log(1 + Y_Q(v)) \sim ({\rm const}) \int^{\infty} dv \, v^{-4L - 4E_{\rm BTBA}}$$

The BTBA energy cannot be negative and large

$$4L + 4E_{\rm BTBA} > 1 \quad \Leftrightarrow \quad E_{\rm BTBA} > 1/4 - L$$

YQ(v) at large Q

BTBA equation for YQ in the large Q limit

$$\Leftrightarrow \quad \log Y_Q(v) \sim (3 - 4L - 4E_{\rm BTBA}) \log(Q)$$

However, the sum in BTBA energy diverges if $Y_Q(v) \sim 1/Q$

$$E_{\rm BTBA} = - \sum_{Q=1}^{\infty} \int_0^{\infty} \frac{d\tilde{p}_Q}{2\pi} \log(1 + Y_Q)$$

$$\sim \sum_{Q=1}^{\infty} ({\rm const})\, Q^{3-4L-4E_{\rm BTBA}}$$

The BTBA energy cannot be negative and large

$$4L + 4E_{\rm BTBA} > 4 \quad \Leftrightarrow \quad E_{\rm BTBA} > 1 - L$$

Closer look at the bound

The stronger bound is

$$E_{\text{open}}[Z^L] = L - 1 + E_{\text{BTBA}}(L, g) > 0$$

It is impossible to saturate this lower bound.

Suppose $E_{\text{BTBA}} = 1 - L$

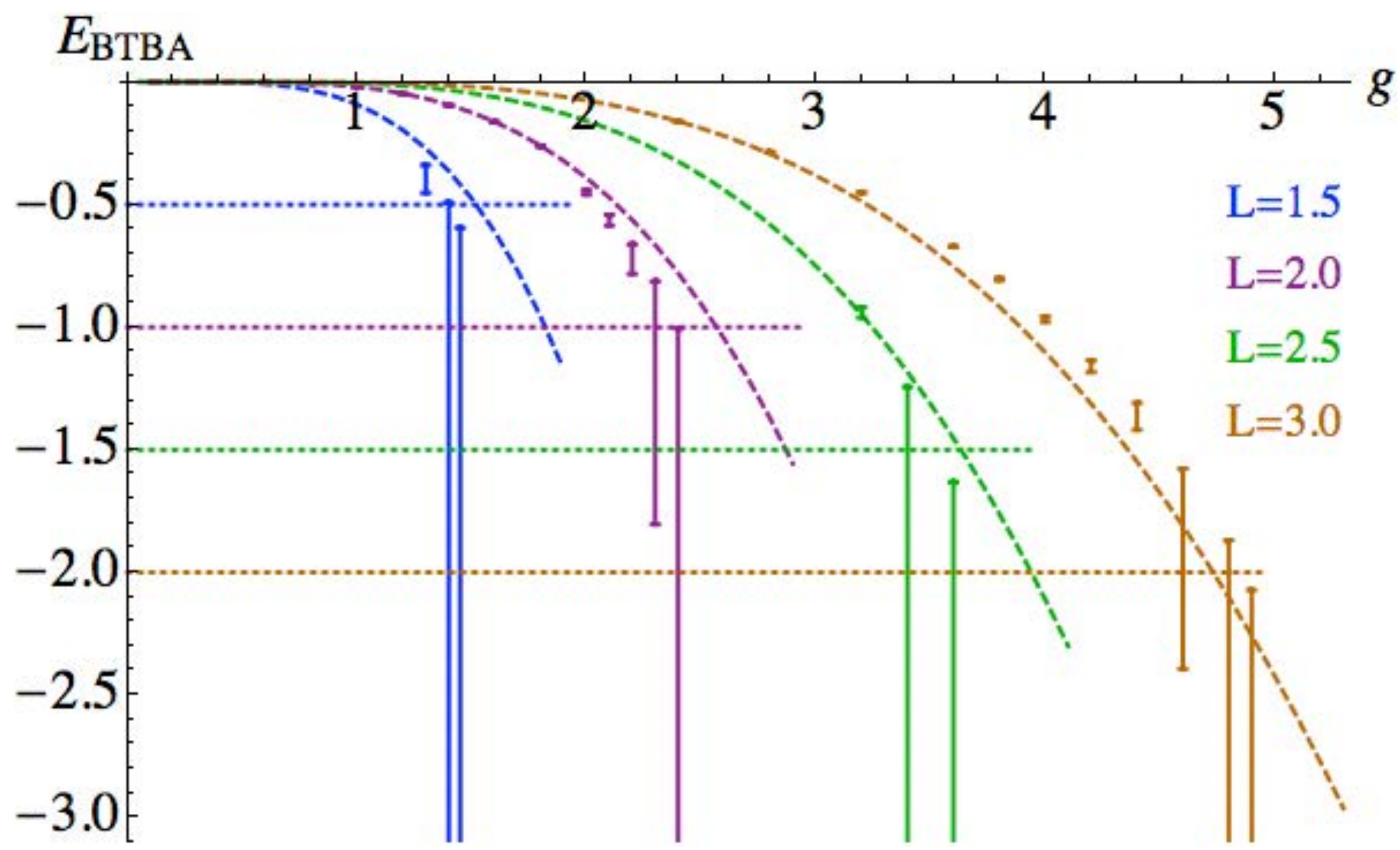
then BTBA dictates $Y_Q(v) \sim 1/Q$

This implies E_{BTBA} diverges, which is a contradiction

A sign of divergences can also be seen at numerical analysis

(ie. indeed TBA energy seems to "hit" the bound)

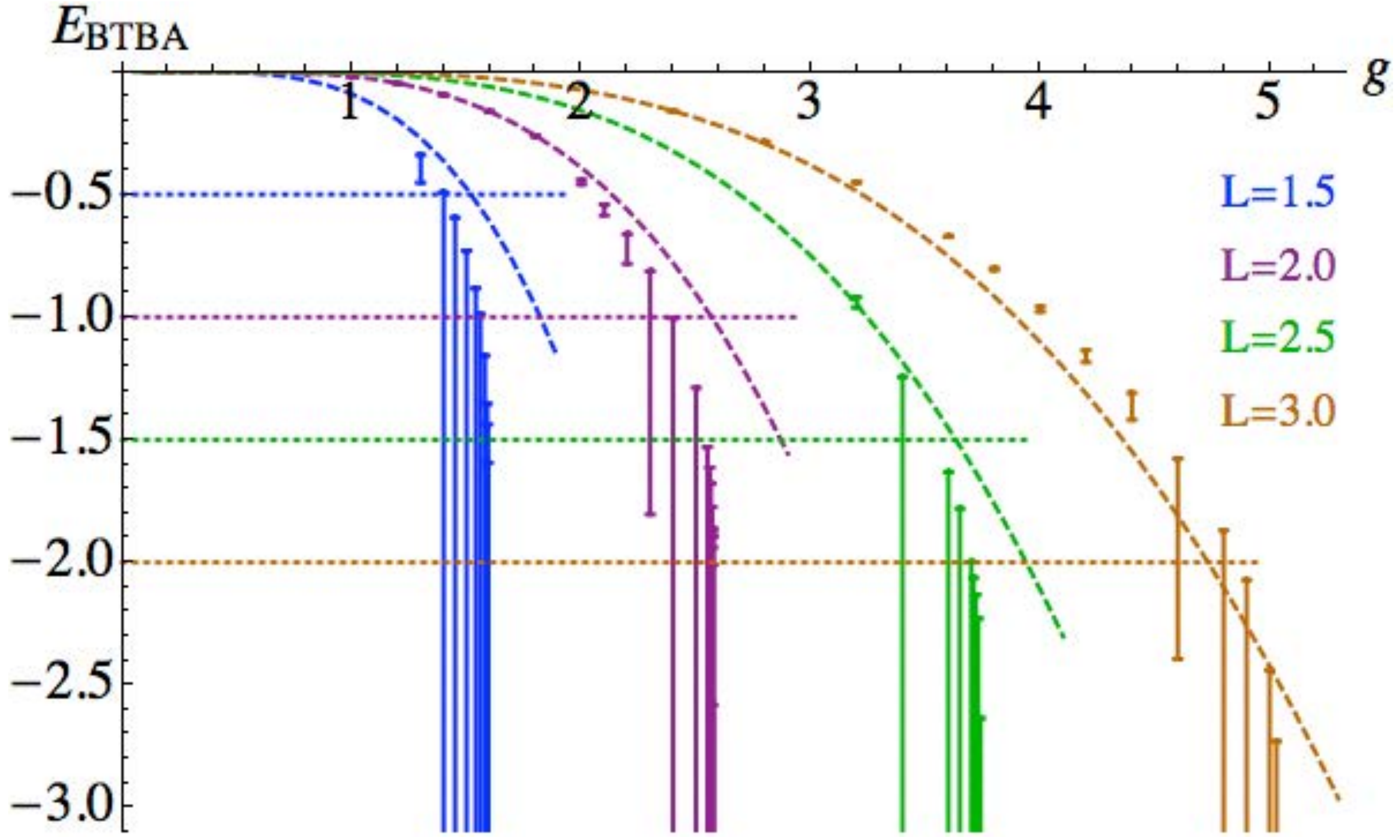
Numerical Results



Solid: BTBA solution,   Dashed: Lüscher formula,   Dotted: Lower bound

$$E_{\rm BTBA}^{\rm (num)}(J,g) = -\sum_{Q=1}^{Q_{\max}} \int_0^\infty \frac{d\widetilde{p}_Q}{2\pi}\, \log(1+Y_Q) - \sum_{Q=Q_{\max}+1}^{100} \int_0^\infty \frac{d\widetilde{p}_Q}{2\pi}\, \log(1+Y_Q^\bullet)$$

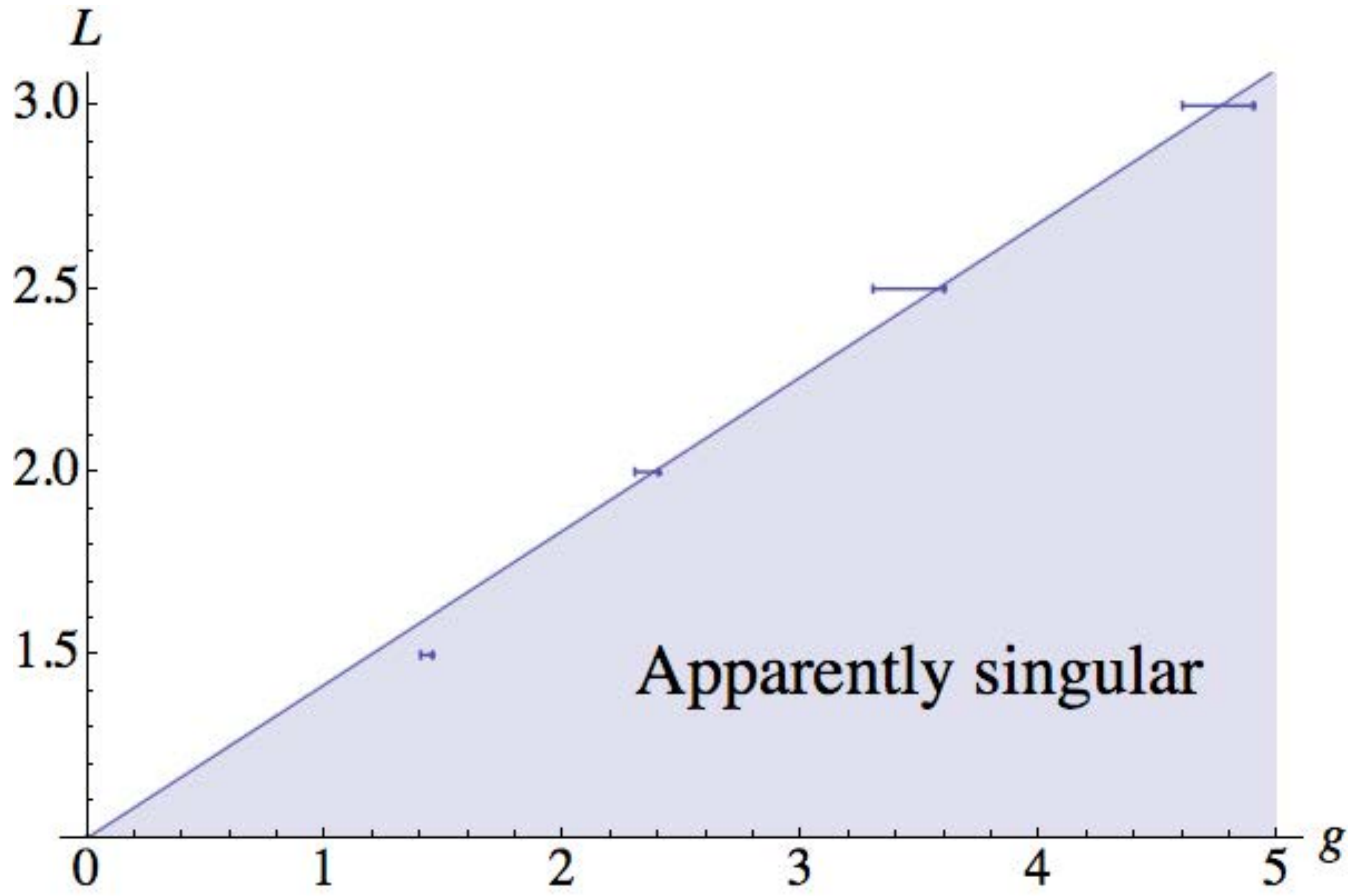
Numerical Results



Solid: BTBA solution, Dashed: Lüscher formula, Dotted: Lower bound

$$E^{(\rm num)}_{\rm BTBA}(J,g) = -\sum_{Q=1}^{Q_{\max}} \int_0^\infty \frac{d\tilde{p}_Q}{2\pi} \log(1+Y_Q) - \sum_{Q=Q_{\max}+1}^{100} \int_0^\infty \frac{d\tilde{p}_Q}{2\pi} \log(1+Y_Q^\bullet)$$

Phase diagram



under the assumption that the $L = 1$ energy diverges at $g = 0$

Summary and outlook

Summary

- Studied the spectrum of determinant-like operators
  dual to open strings ending on giant gravitons
- Wrapping corrections from $\mathcal{N}$=4 SYM agree with the Lüscher formula
- Proposed and solved BTBA equations for Y=0 & Ybar=0
- Found the lower-bound for the (B)TBA energy

Future works

- Beyond the critical coupling? Compare with string theory?
- How to compute the dimension of the L=1 state?
- AdS/CFT for unstable systems?

Thank you for attention

Спасибо за внимание