5d SCFTs, 5-brane webs & Their Duality

Masato Taki RIKEN, Hashimoto Lab

based on
[MT, arXiv:1401.7200]
[MT, arXiv:1310.7509]
[L.Bao-V.Mitev-E.Pomoni-MT-F.Yagi, arXiv:1310.3841]
c.f.
[Hayashi-Kim-Nishinaka, arXiv:1310.3854]

2014.3/6
@ Biwako ws

What I will show in this talk:

duality between various CY_3 compactification of M-thy.

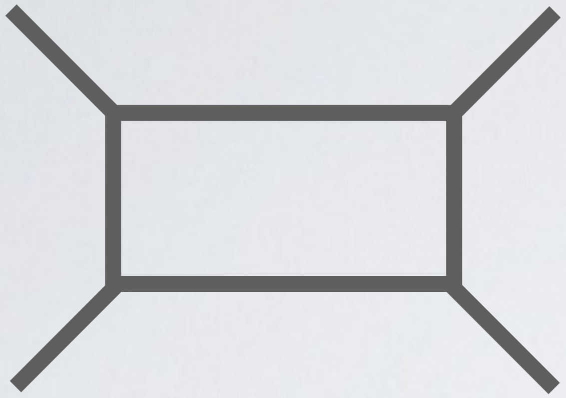
||

5-brane web configurations in Type IIB

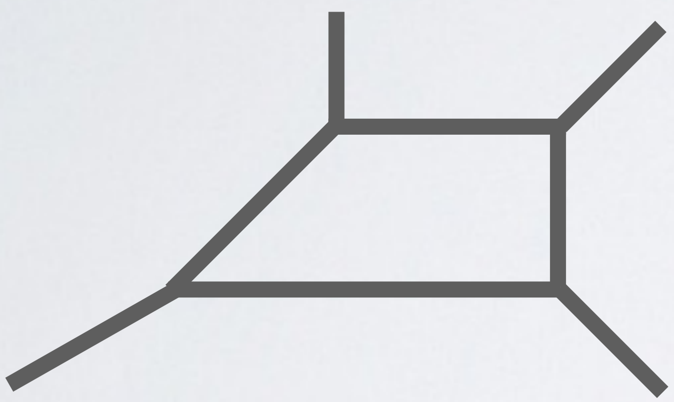
↓

unique $\mathcal{N}$=1 SCFT in 5d

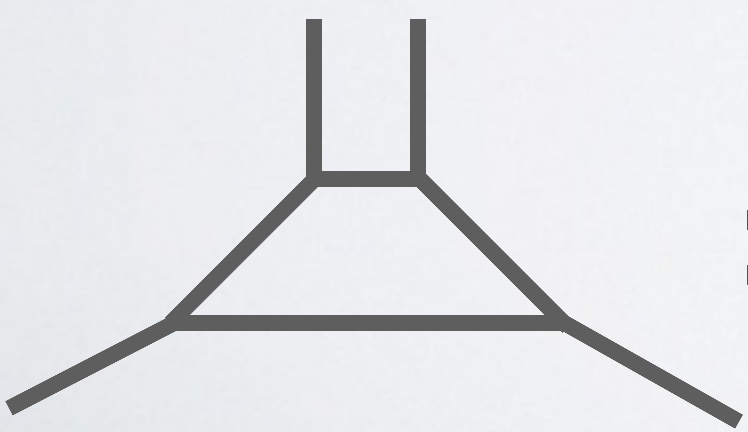
Example: local Hirzebruch (toric CY 3-fold)



$\mathbb{F}_0$



$\mathbb{F}_1$

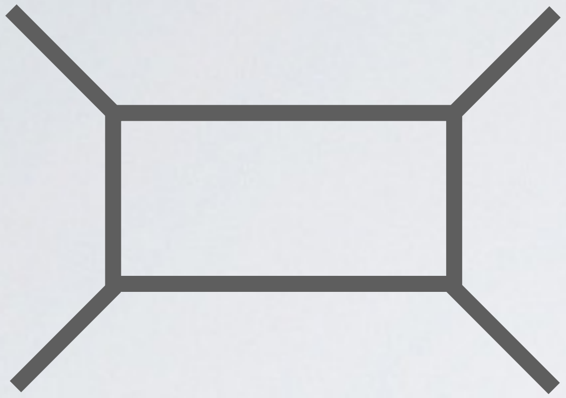


$\mathbb{F}_2$

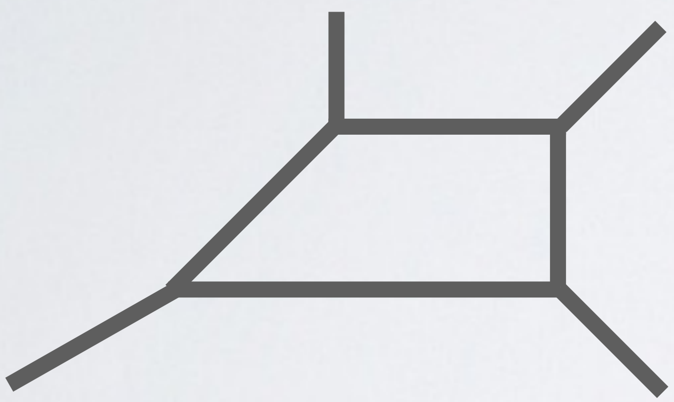
only TWO candidates for the 5d $\mathcal{N}$=1 SCFTs

[Seiberg, '96] etc

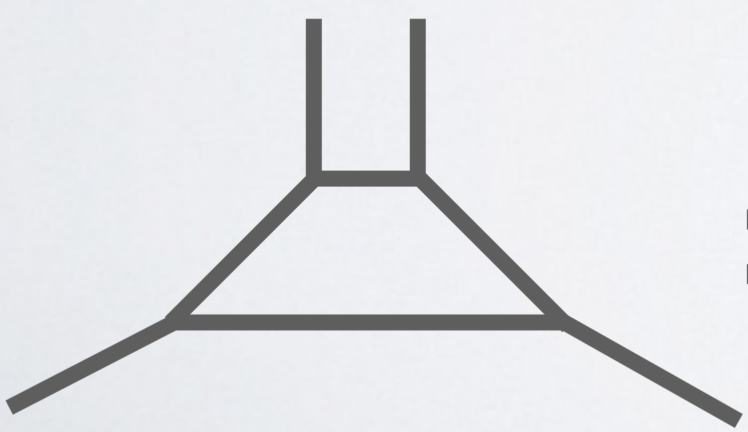
Example: local Hirzebruch (toric CY 3-fold)



$\mathbb{F}_0$ → E_1 SCFT

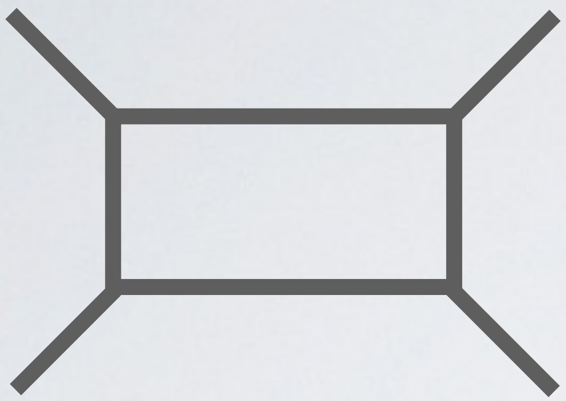


$\mathbb{F}_1$ → E_1 SCFT



$\mathbb{F}_2$ → E_1 SCFT

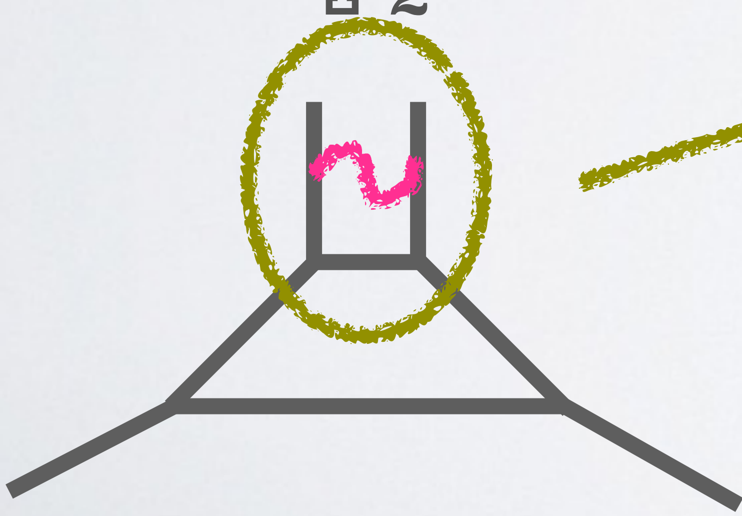
Example: local Hirzebruch (toric CY 3-fold)



$\mathbb{F}_0$

$\|$

$\mathbb{F}_2$



Equivalence between Nekrasov (topological string) partition functions

$$Z_{\mathbb{F}_0} = Z_{\text{extra}} \cdot Z_{\mathbb{F}_2}$$

[MT,13]
[Bao-Mitev-Pomoni-MT-Yagi, 13]
[Hayashi-Kim-Nishinaka,13]
[Bergman-Gomez-Zafrir,'13]

1. 5D SCFTs

Question:

Why 5d gauge theory?

Question:

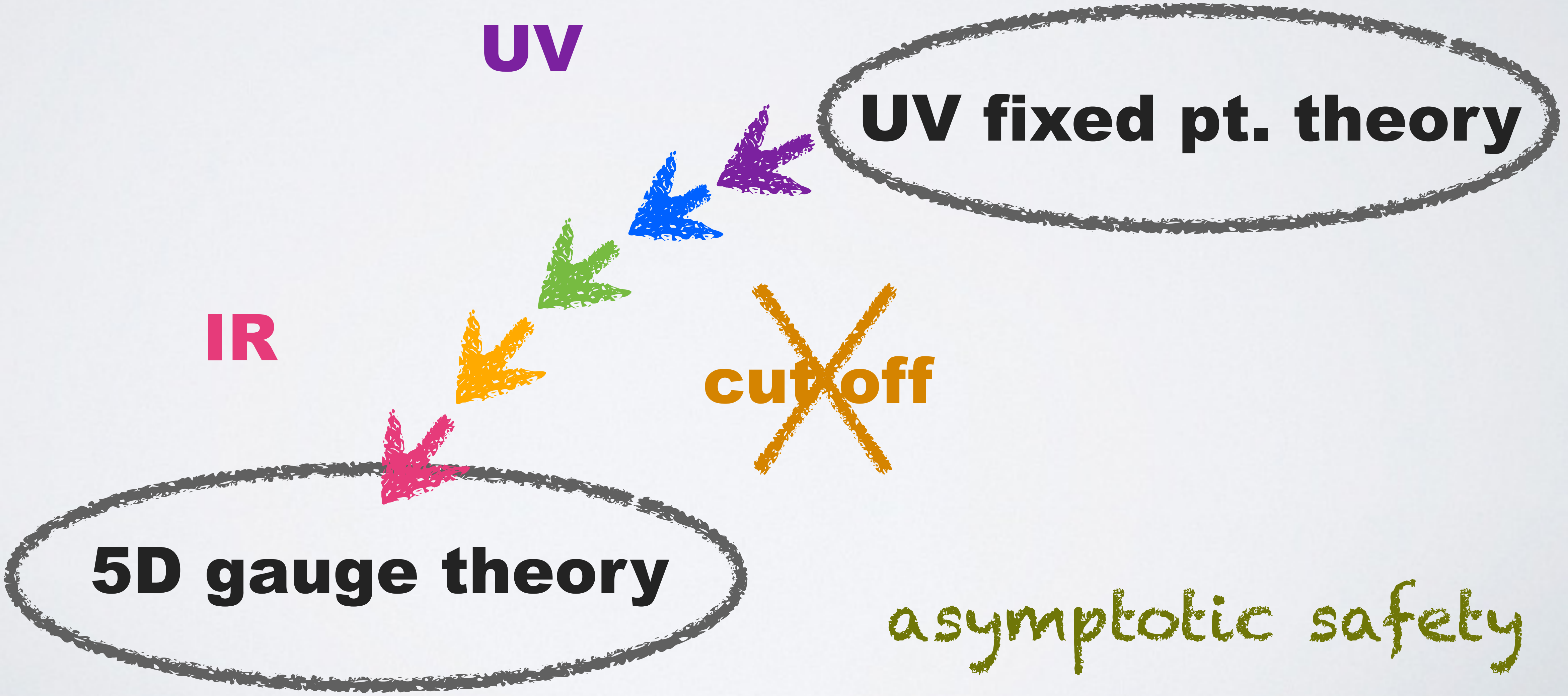
Why 5d gauge theory?

- non-renormalizable & trivial
- only cut-off theory?
- some susy theories are well-defined via UV fixed points [Seiberg, '96]

5D minimal SUSY gauge theory

[Seiberg, '96]

SU(2) gauge theory with 0, 1, 2, ... , 7 quarks has UV fixed point.



5D SU(2) theory & flavor sym. of UV SCFT

flavor symmetry of gauge theory

$$SO(2N_f) \times U(1)$$

↑

subgroup of the global symmetry of UV SCFT because relevant deformation breaks it.

5D SU(2) theory & flavor sym. of UV SCFT

$$N_f = 0 \qquad E_1 = SU(2)$$

$$N_f = 1 \qquad E_2 = SU(2) \times U(1)$$

$$N_f = 2 \qquad E_3 = SU(3) \times SU(2)$$

$$N_f = 3 \qquad E_4 = SU(5)$$

$$N_f = 4 \qquad E_5 = SO(10)$$

$$N_f = 5 \qquad E_6$$

$$N_f = 6 \qquad E_7$$

$$N_f = 7 \qquad E_8$$

Stringy Analysis

5d theory via (p,q) 5-brane web

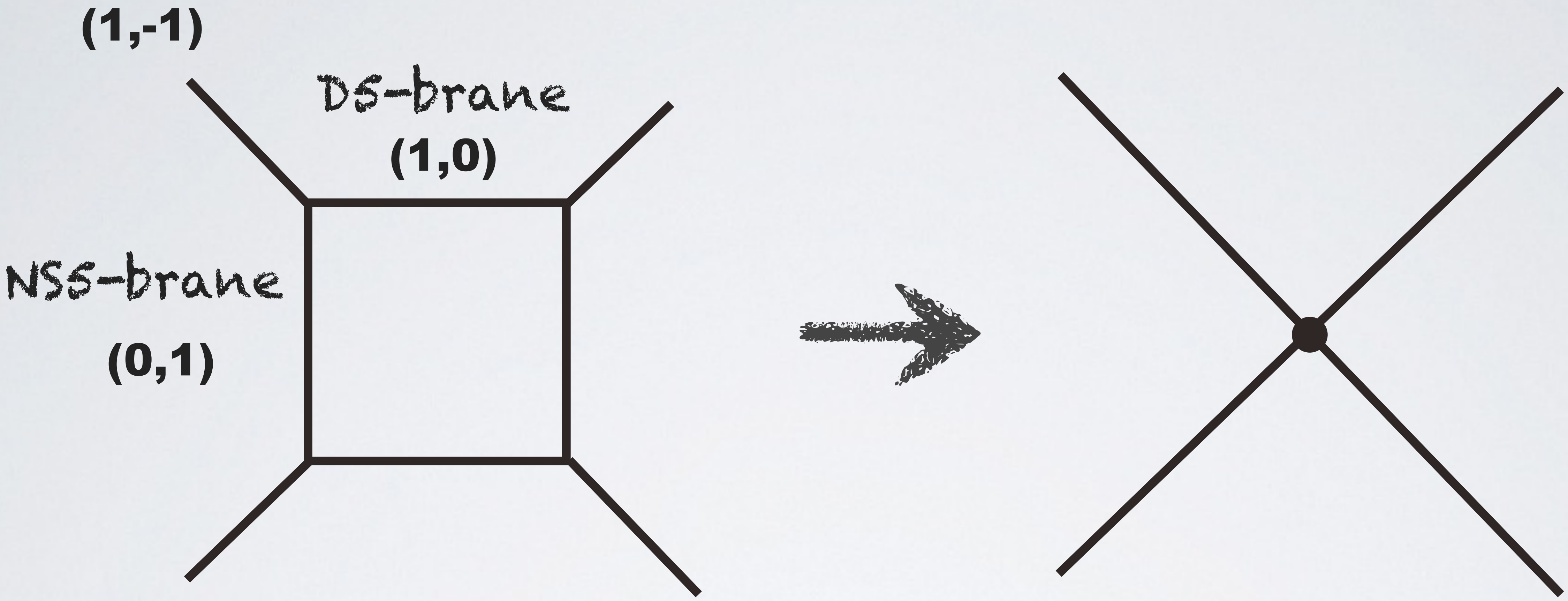
[Aharony-Hanany, '97]

We can construct these 5d thys by using (p,q) 5-brane web in Type IIB string

(1,0) 5-brane : D5-brane

(0,1) 5-brane : NS5-brane

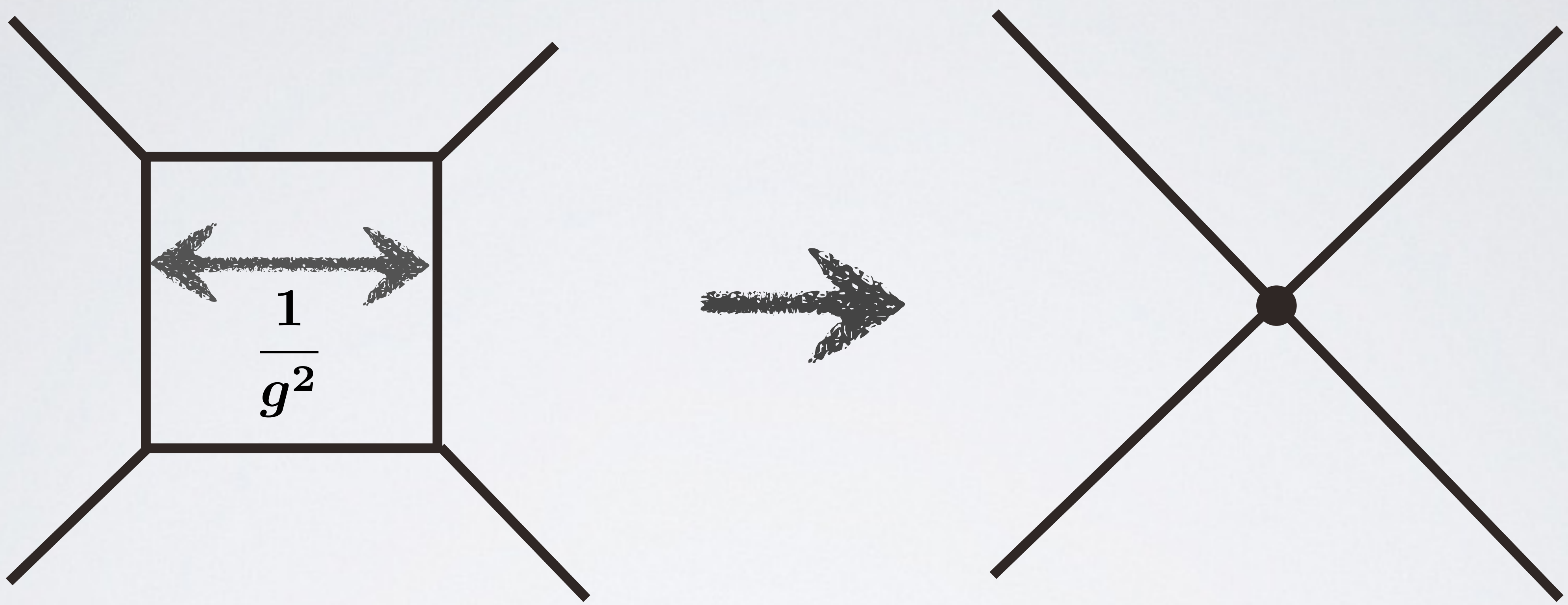
pure SU(2) YM [Aharony-Hanany, '97]



5d UV fixed pt thy

E_1 SCFT

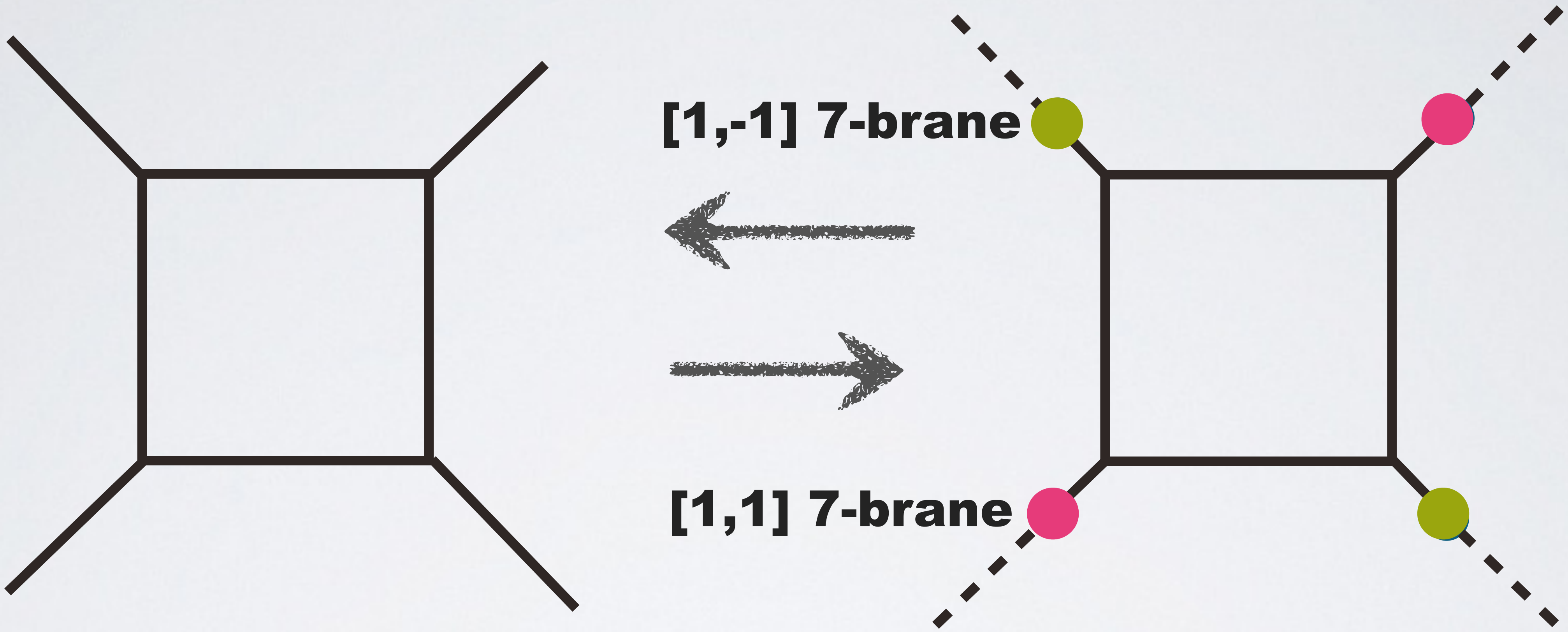
pure SU(2) YM [Aharony-Hanany, '97]



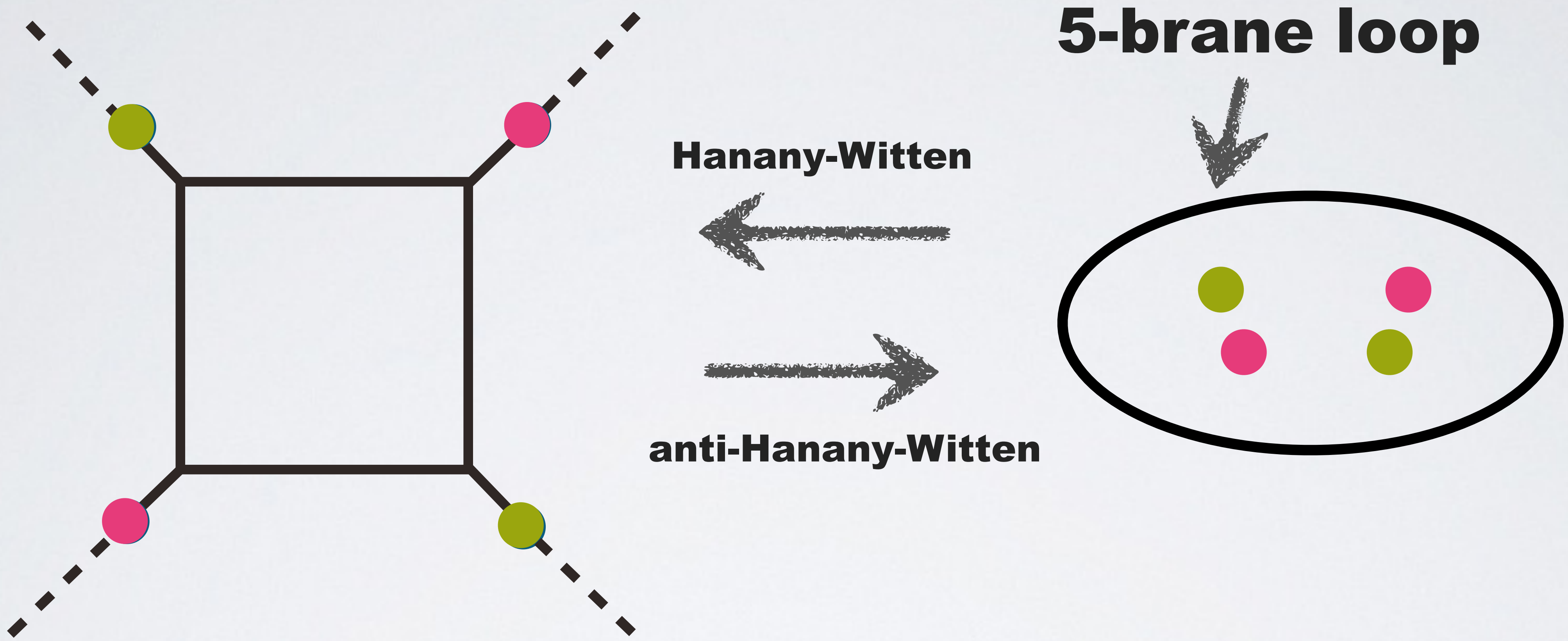
strongly-coupled

E_1 SCFT

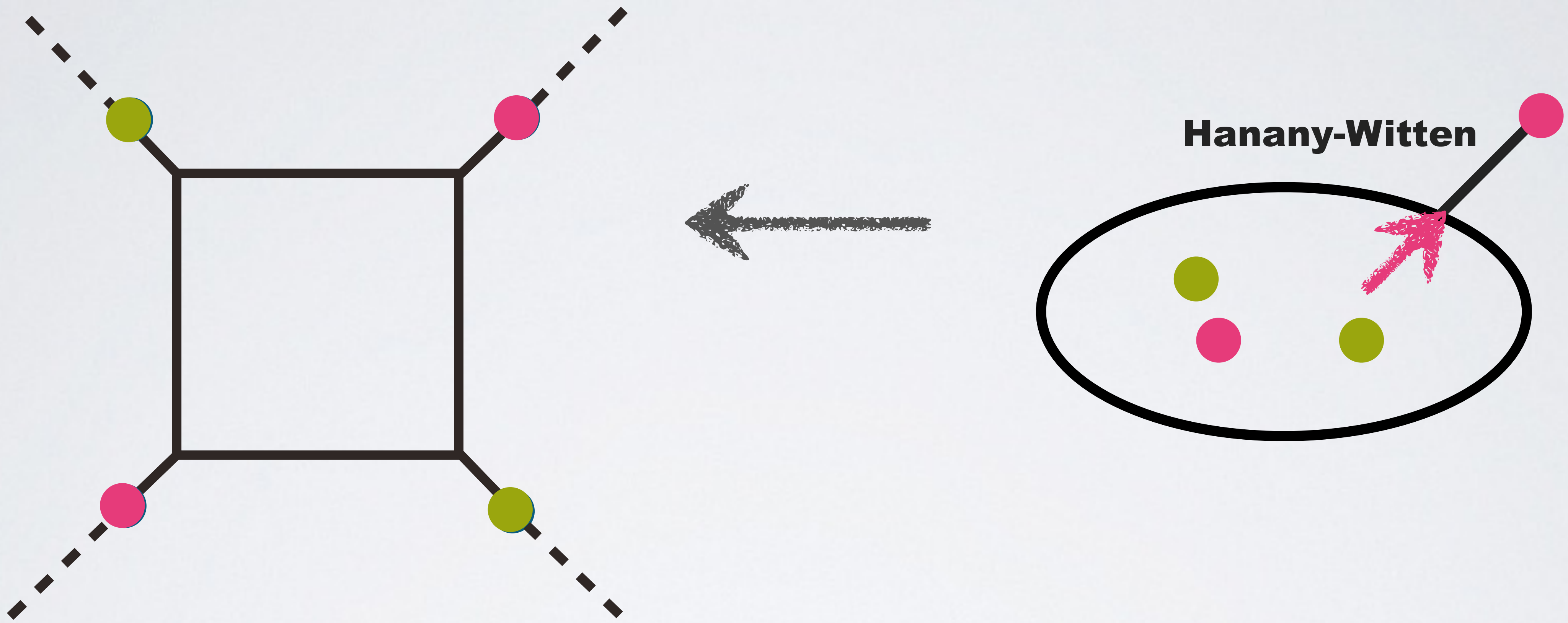
pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, '99]



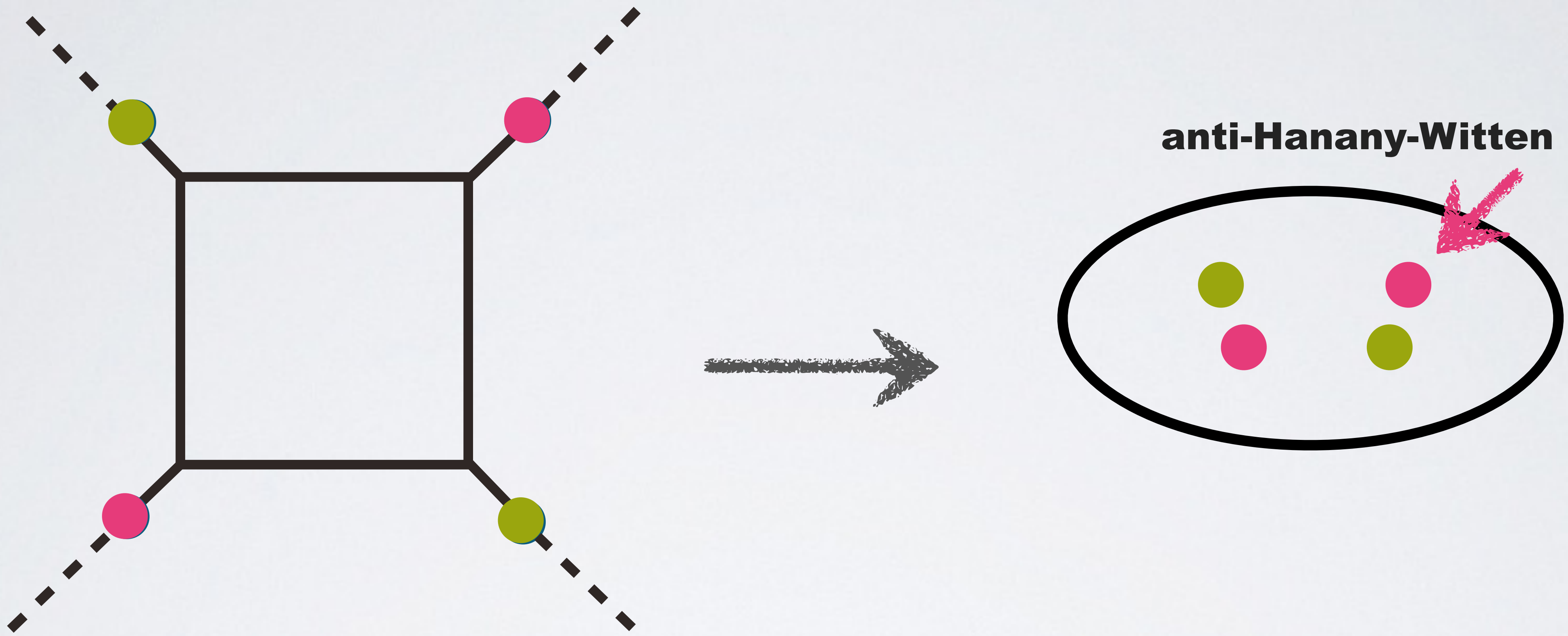
pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]



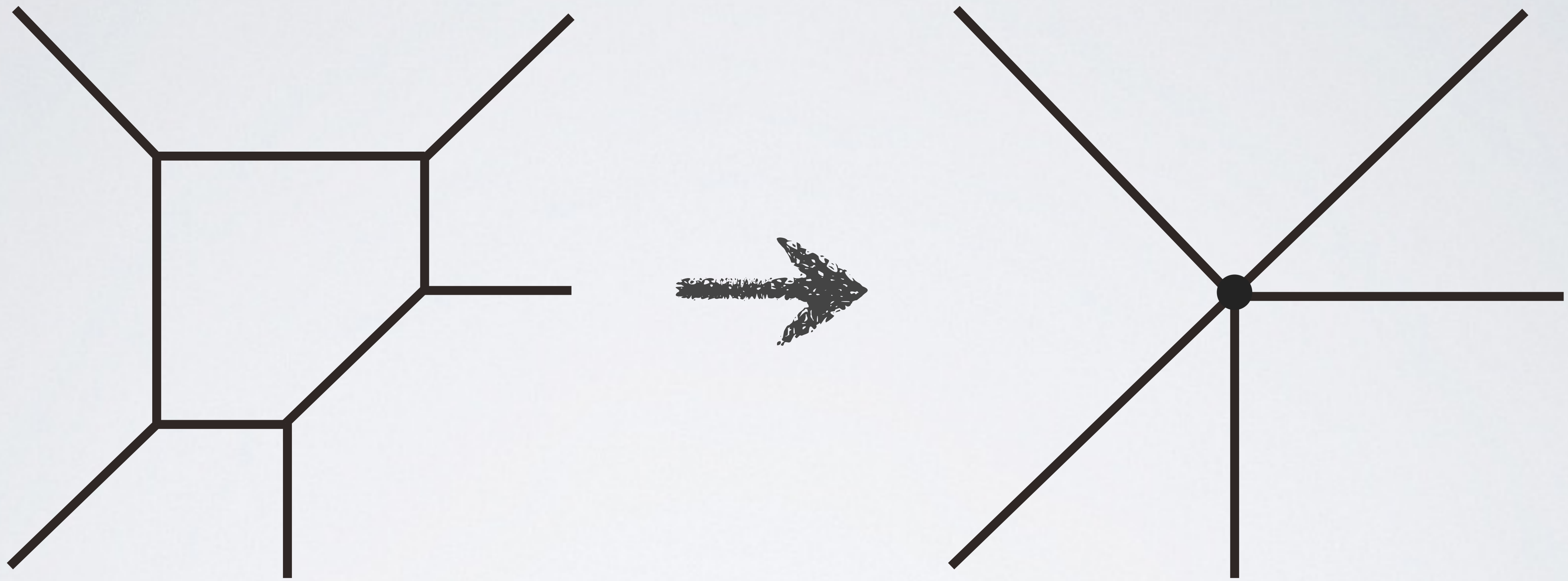
pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]



pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, '99]



N_f=1 SU(2) SQCD



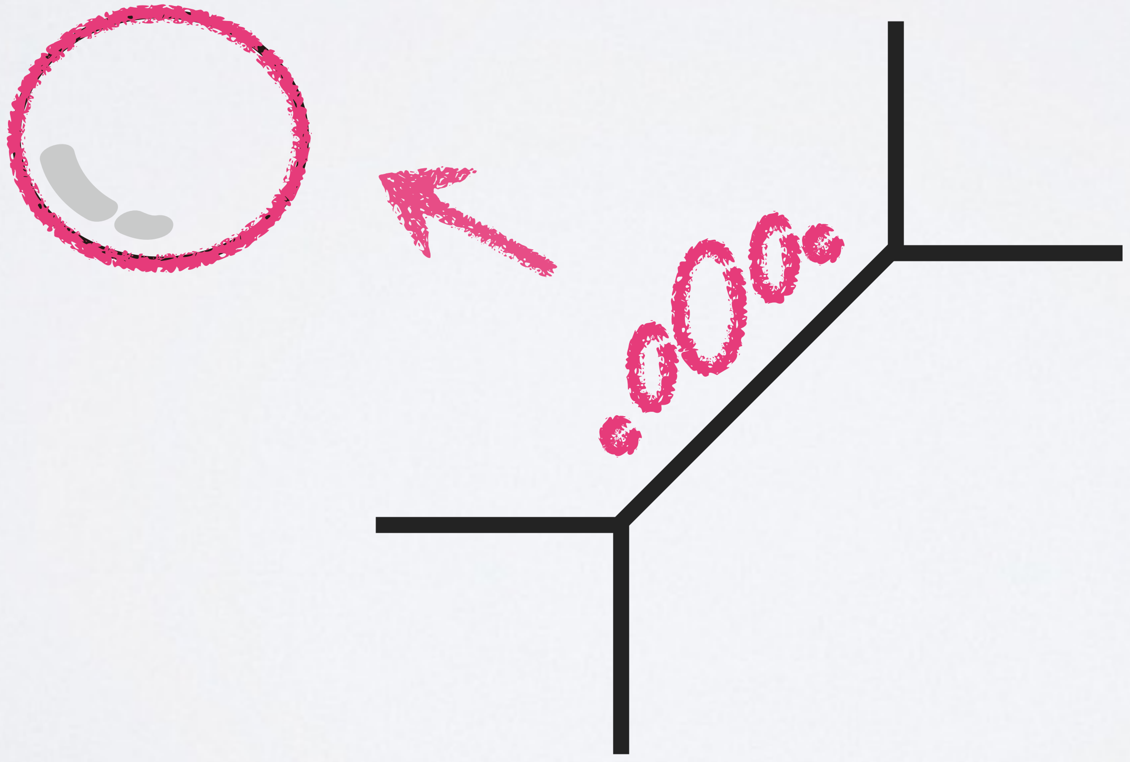
5d UV fixed pt thy

E_2 SCFT

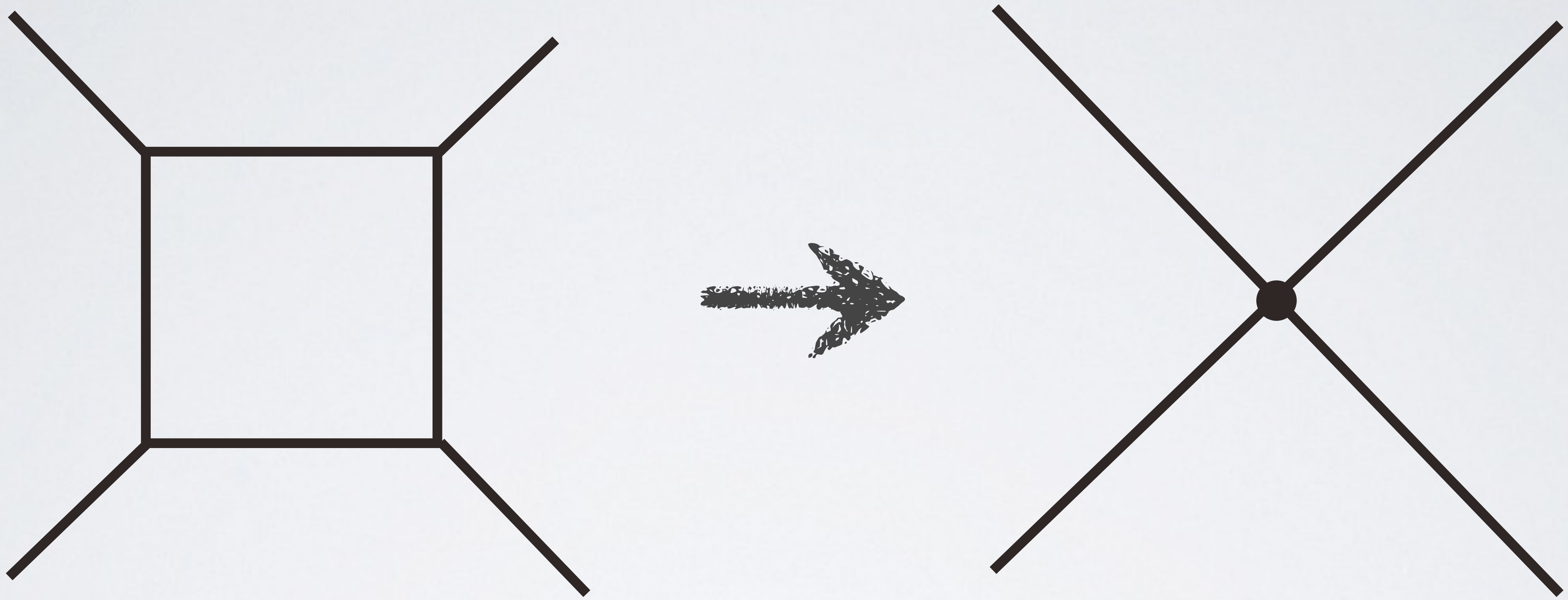
5d theory via Calabi-Yau compactification

We can also construct them by using M-thy on toric geometry (web)

resolved conifold $A_1 B_2 - A_2 B_2 = 0$

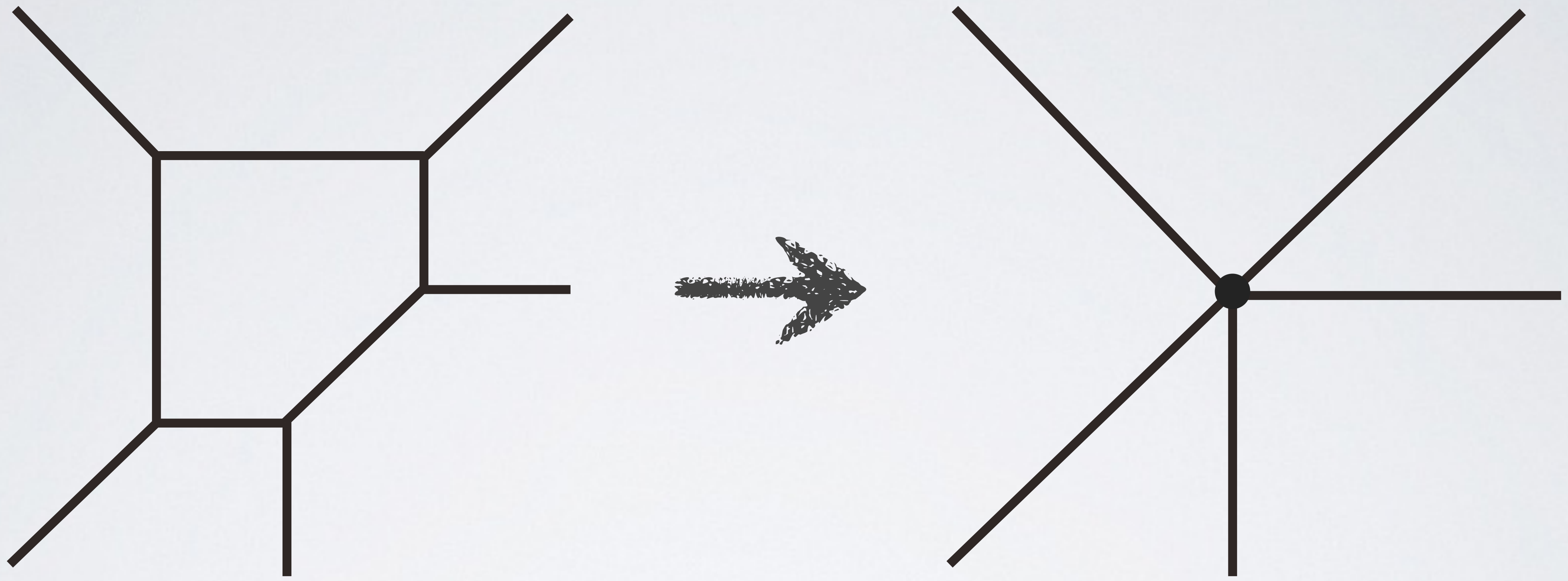


pure SU(2) YM



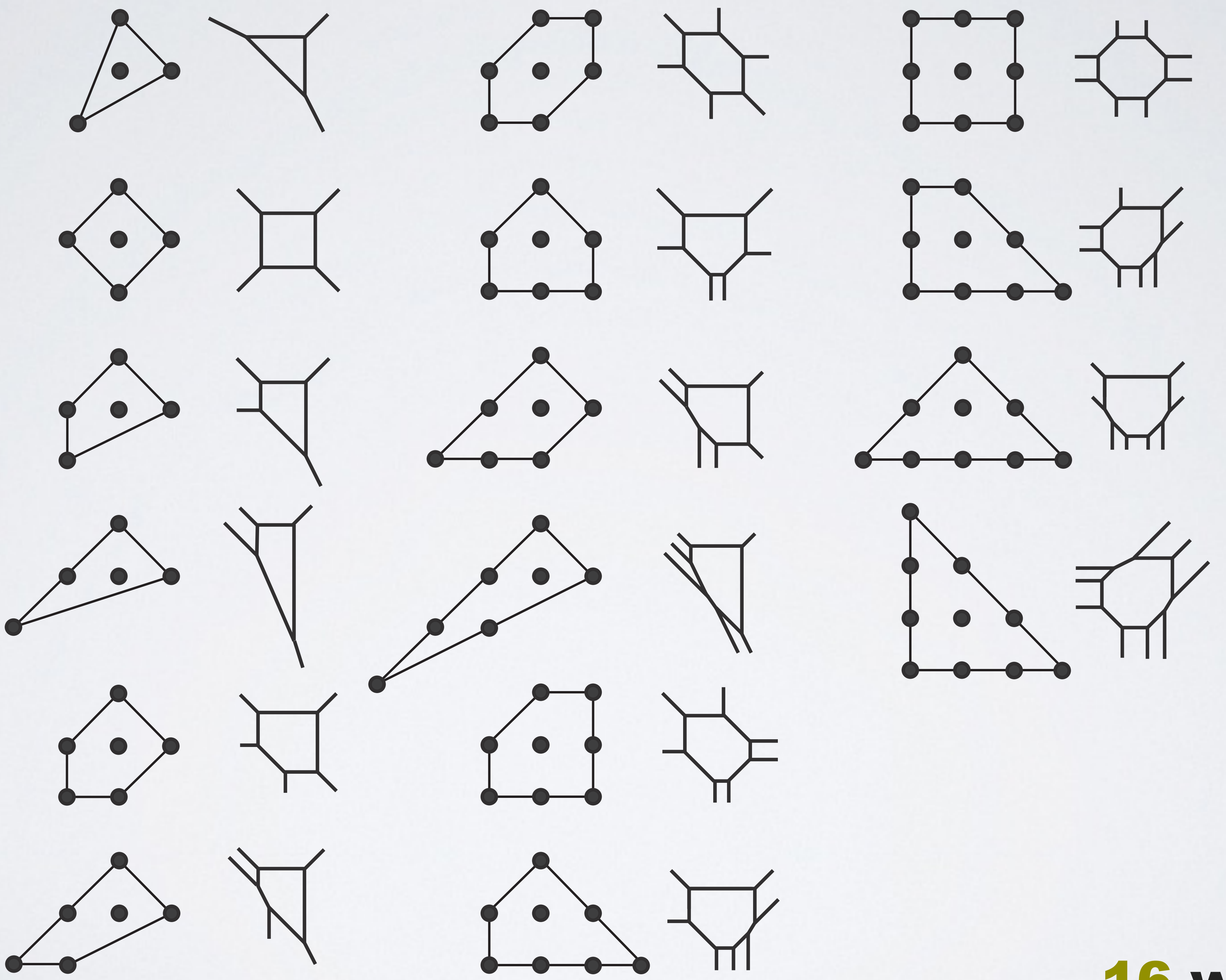
E_1 SCFT

$N_f = 1$ SU(2) SQCD



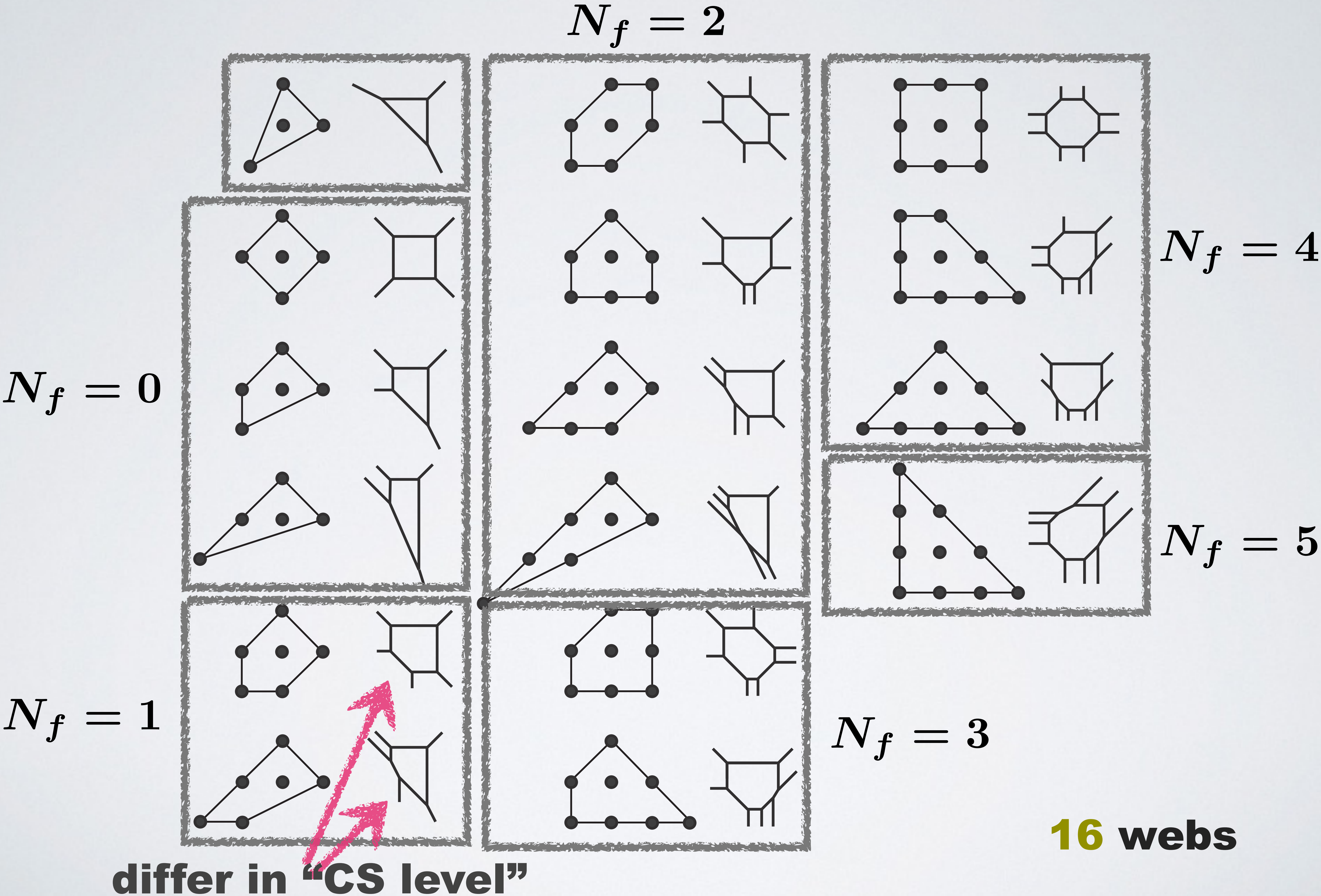
E_2 SCFT

Possible SU(2) theories (grids & webs)



16 webs

Possible SU(2) theories (grids & webs)



Topological vertex

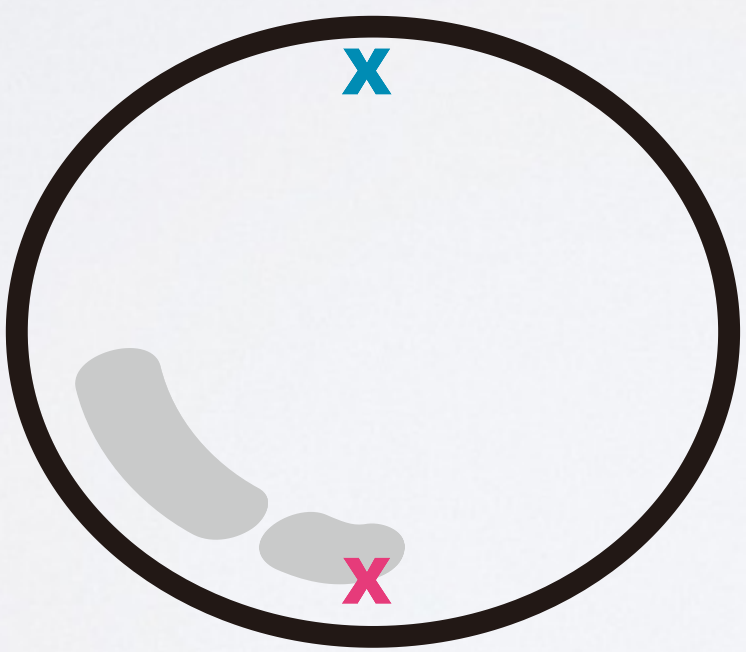
Nekrasov partition functions are computed by topological vertex formalism exactly!

[Aganagic-Klemm-Marino-Vafa, '03]

[Awata-Kanno] [Iqbal-Kozcaz-Vafa]

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\text{5d}}_{\text{Nek}}(t,q,m,u,a) \right|^2$$

$S^1 \times$ 

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

$$I(x,y,m,u) = \int [da] \left| Z^{\rm 5d}_{\rm Nek}(t,q,m,u,a) \right|^2$$

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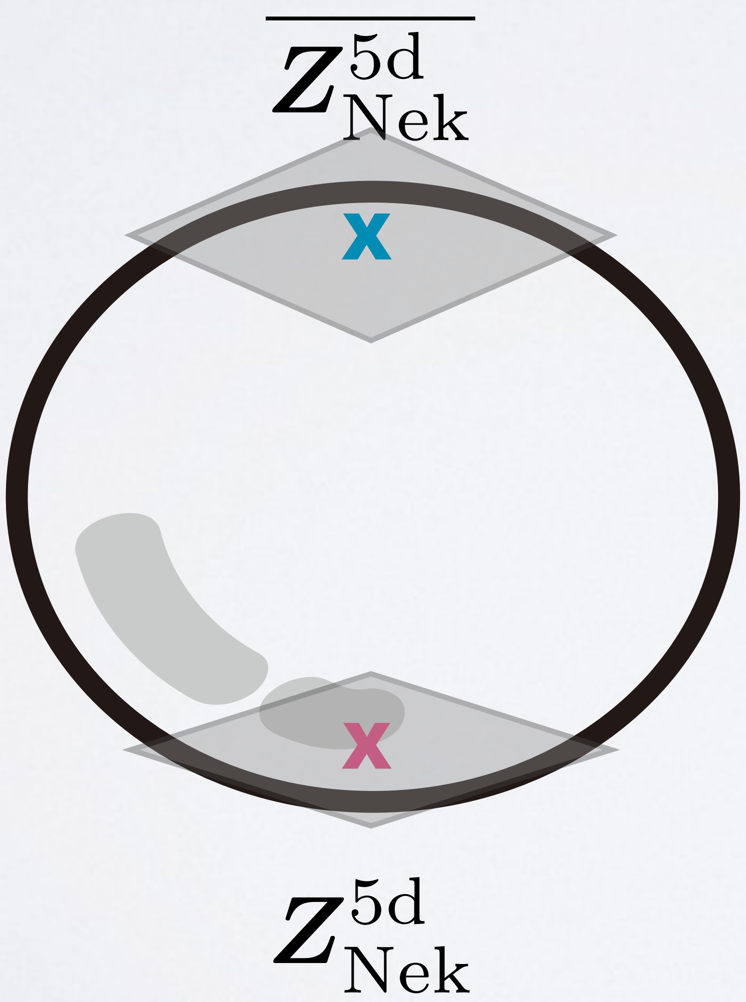


Topological vertex & KKL formula

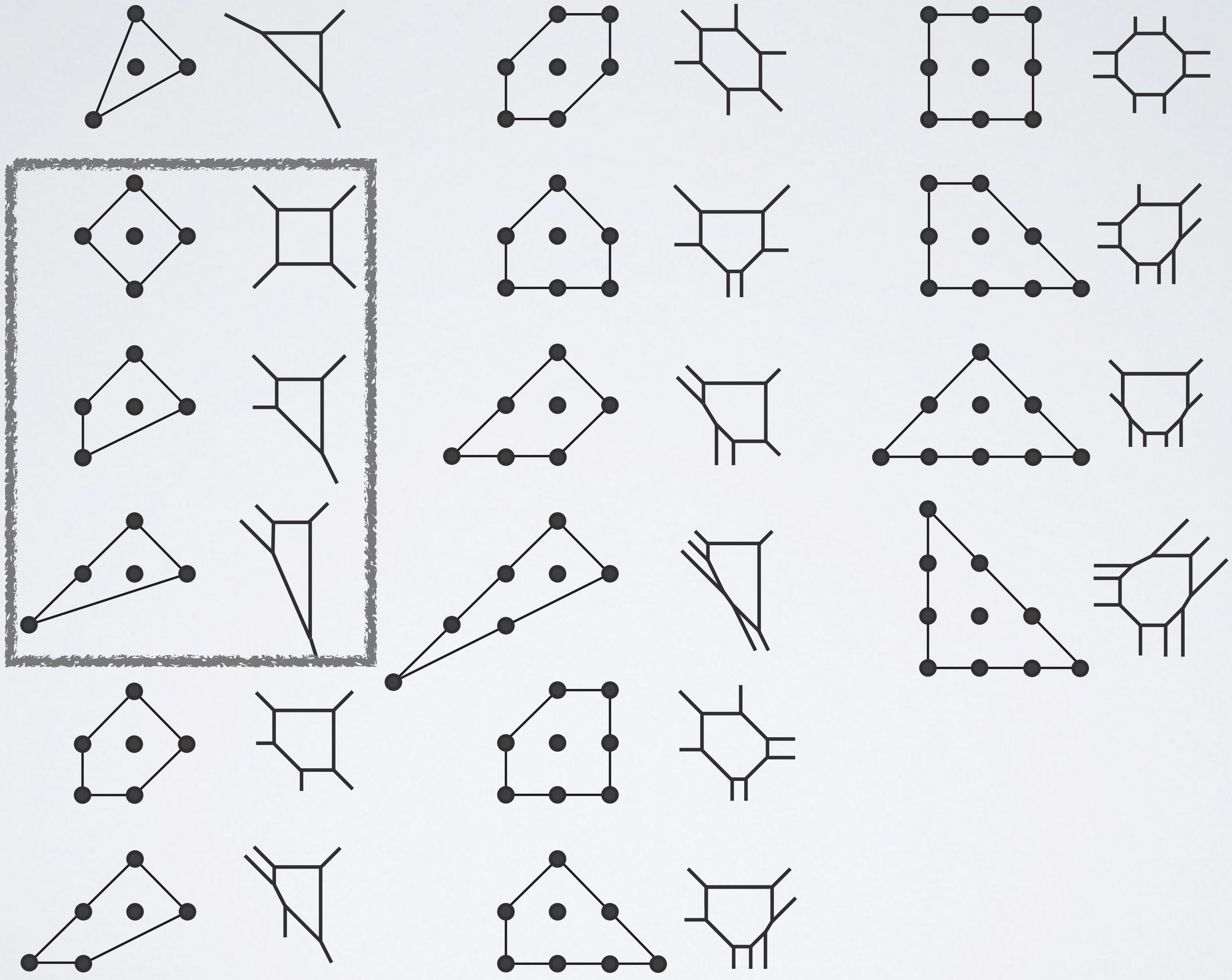
a web diagram → **Nekrasov partition function** & **superconformal index** of the corresponding 5d QFT

2. “Seiberg duality” via 7-brane move

[MT, ’14]

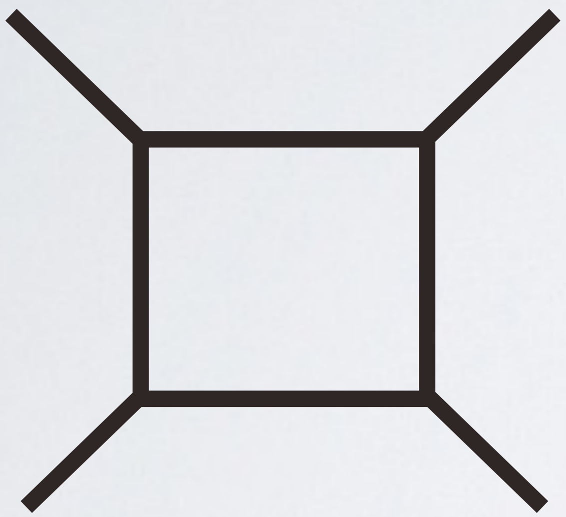
Possible SU(2) theories (grids & webs)

$N_f = 0$



rank-one theories

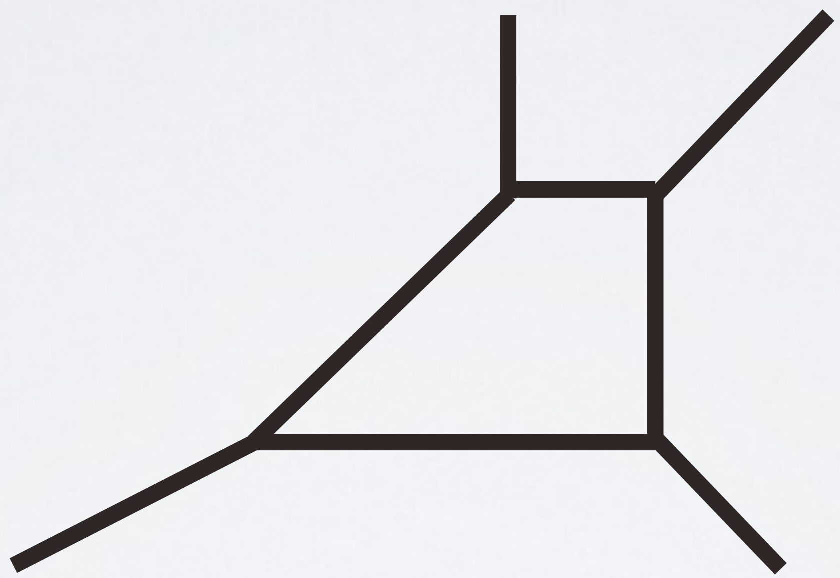
discussion so far : [Douglas-Katz-Vafa,’97]
[Aharony-Hanany,’97]
[Aharony-Hanany-Kol,’97] ...



$\mathbb{F}_0$

E_1

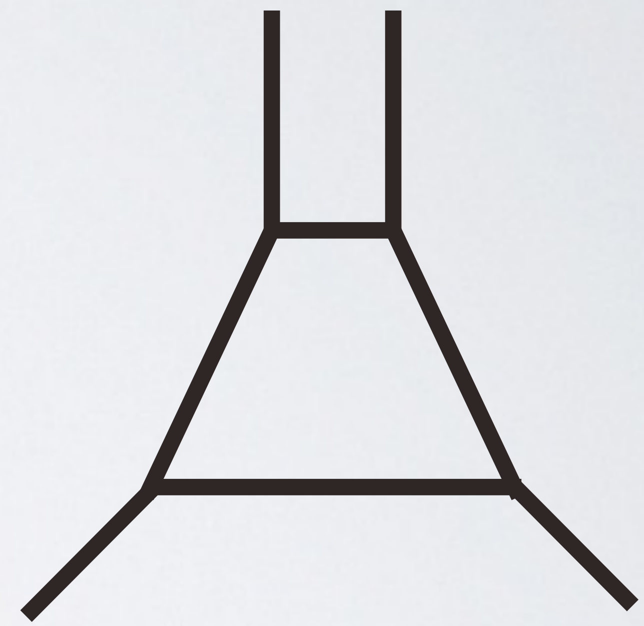
SU(2) YM θ=0



$\mathbb{F}_1$

$\tilde{E}_1$

SU(2) YM θ=π : discrete theta angle



$\mathbb{F}_2$

??

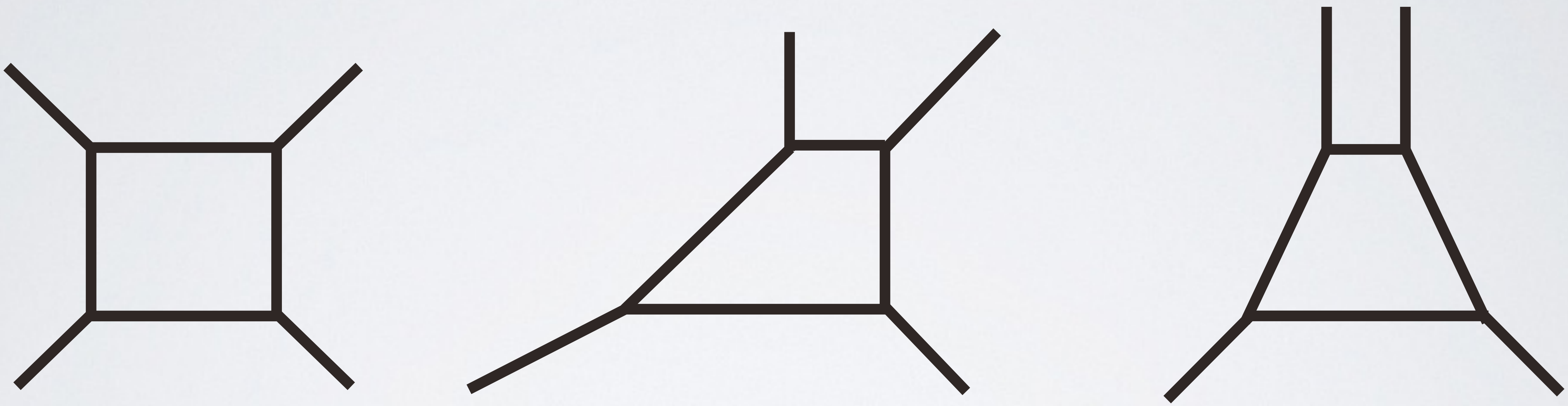
rank-one theories

$$\boldsymbol{\pi_4(SU(2)) = \mathbb{Z}_2}$$

Only two theories !

rank-one theories [MT, arXiv:1310.7509]

7-brane pict. works



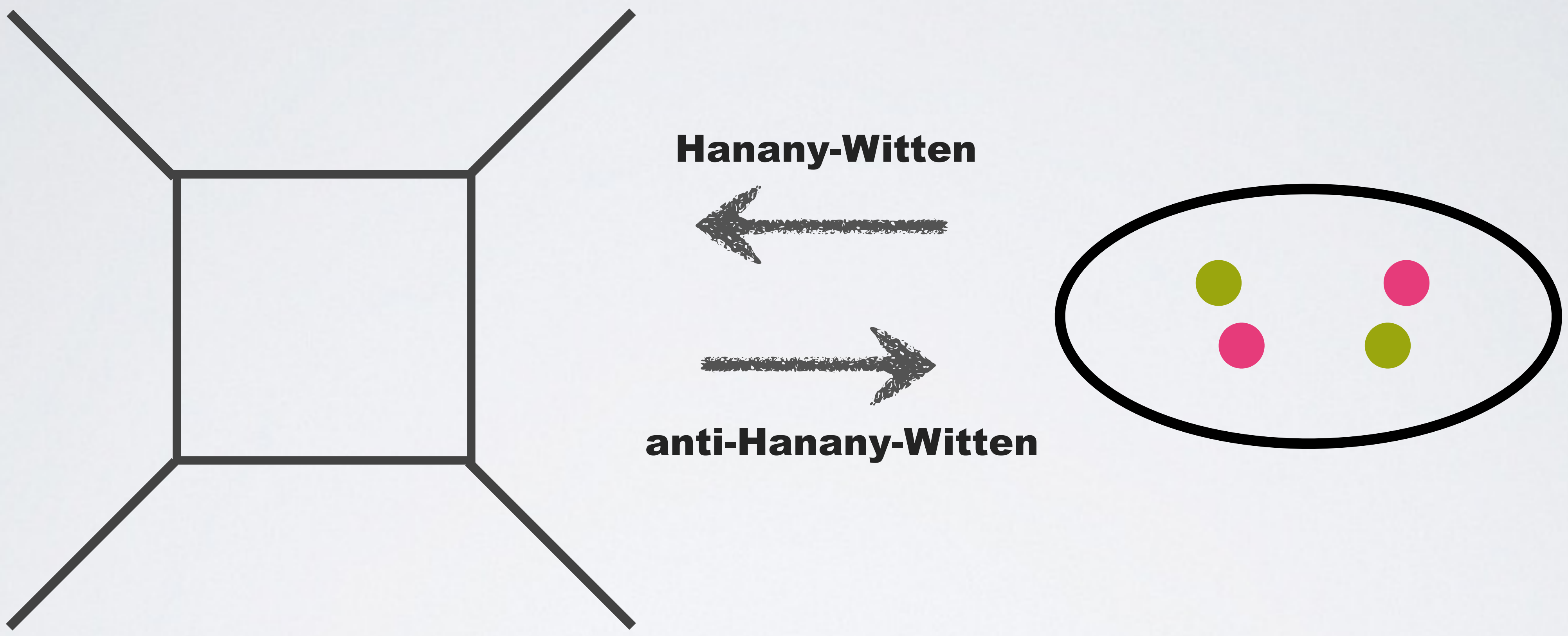
$\mathbb{F}_0$ $\mathbb{F}_1$ $\mathbb{F}_2$

E_1 **$\tilde{E}_1$** ~~??~~

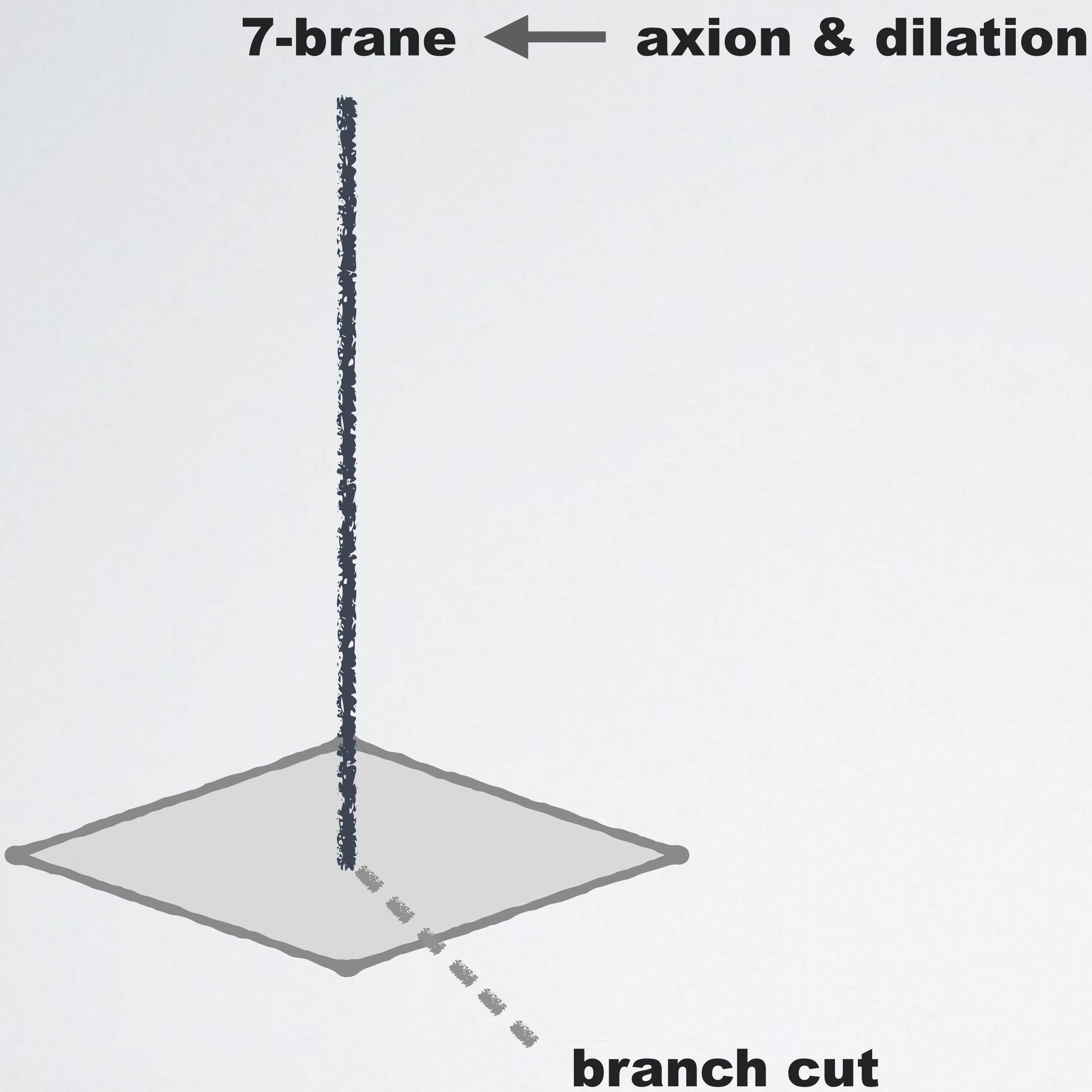
SU(2) YM θ=0 **SU(2) YM θ=π**

rank-one theory $\mathbb{F}_0$

- we will use 7-brane picture

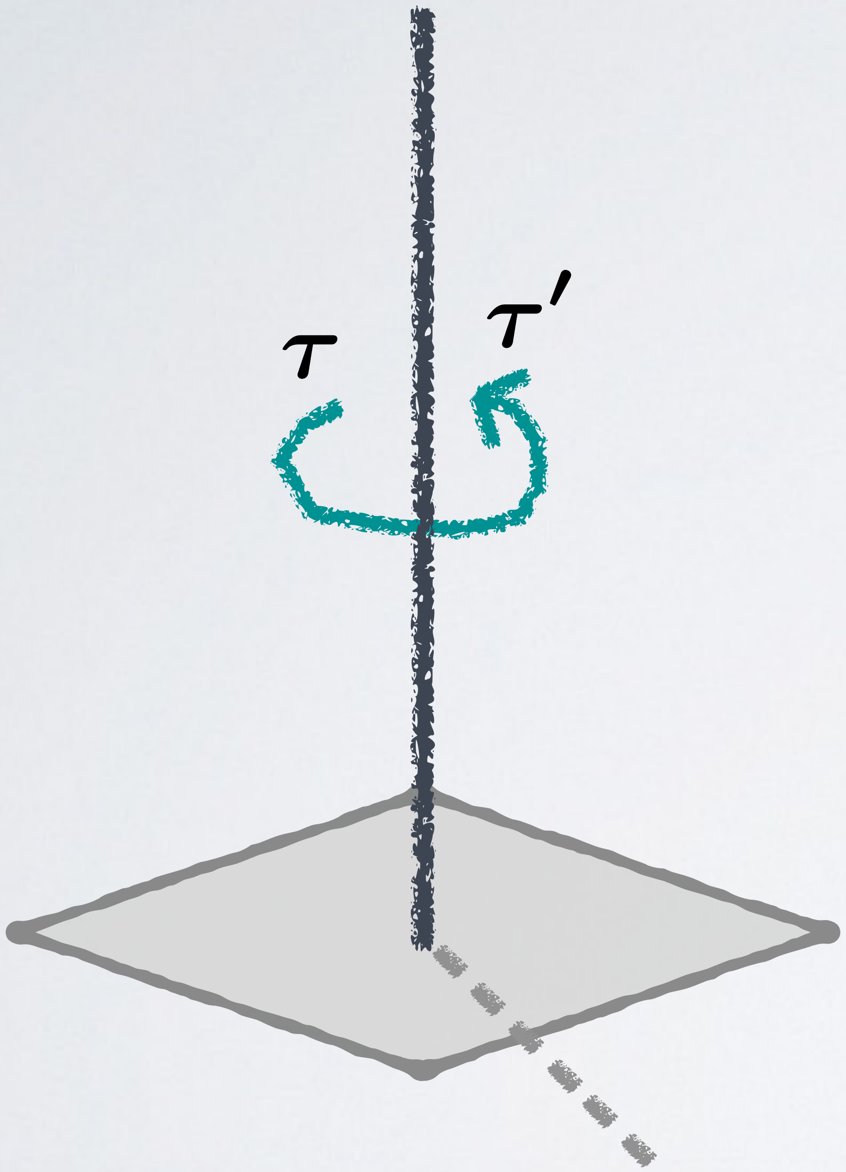


7-brane monodormy



7-brane monodormy

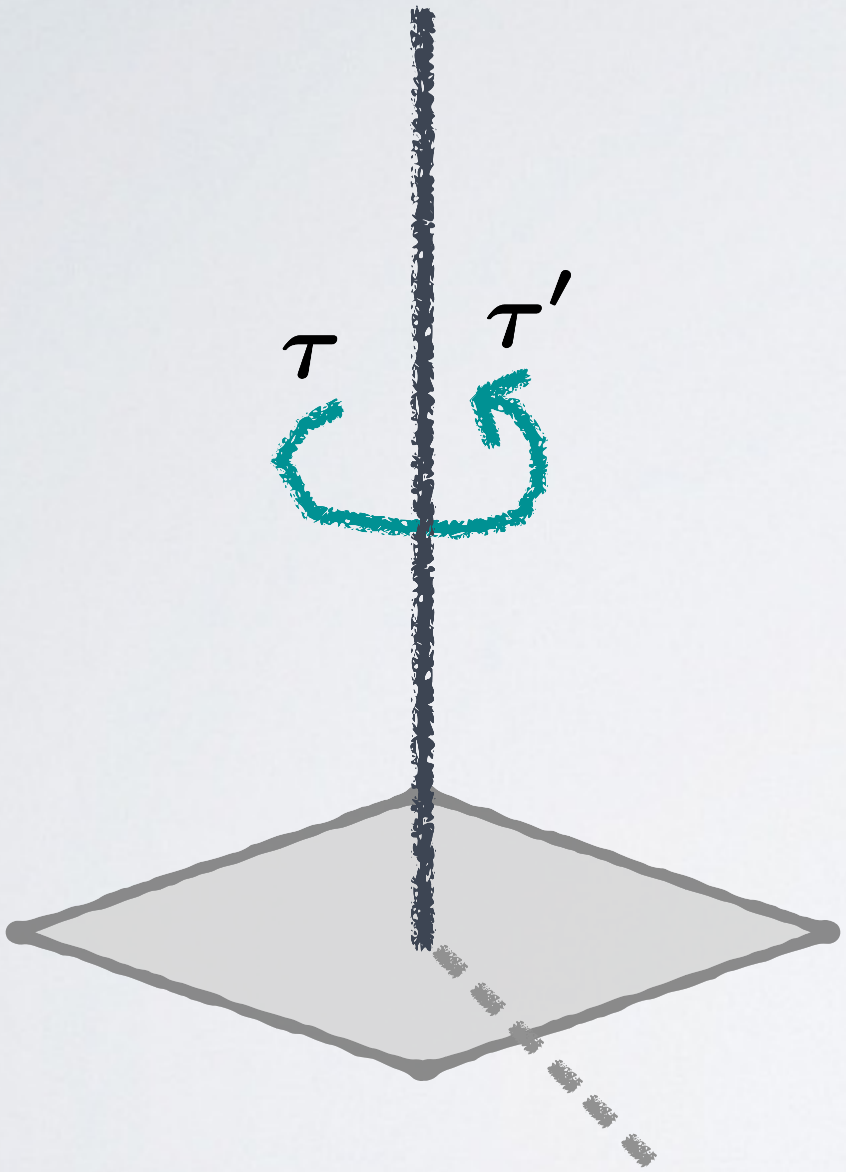
7-brane ← **axion & dilation**



$$\tau_{\text{IIB}} \longrightarrow \frac{a\,\tau_{\text{IIB}} + b}{c\,\tau_{\text{IIB}} + d}$$

7-brane monodormy

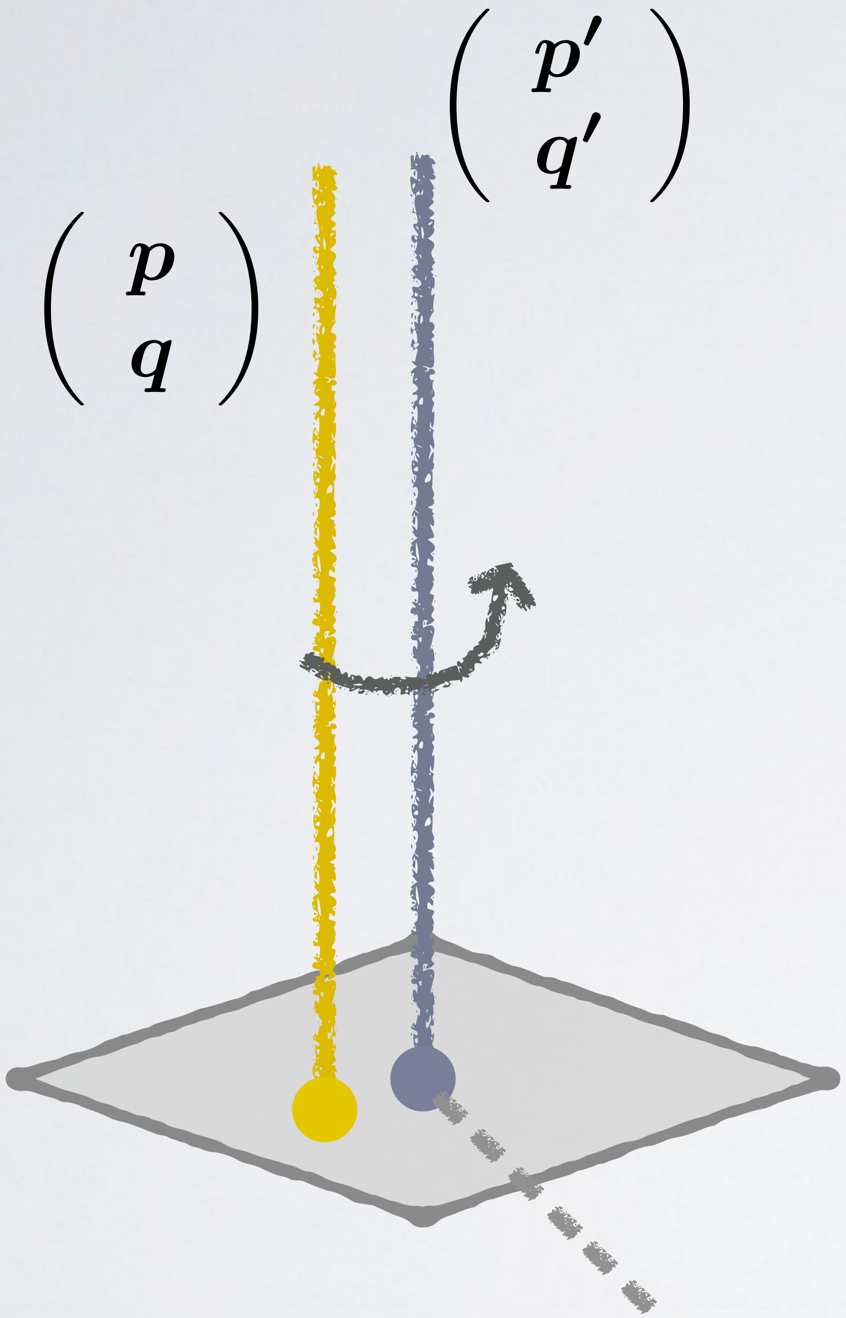
7-brane ← **axion & dilation**



$$\tau_{\mathrm{IIB}} \longrightarrow \frac{a\,\tau_{\mathrm{IIB}} + b}{c\,\tau_{\mathrm{IIB}} + d}$$

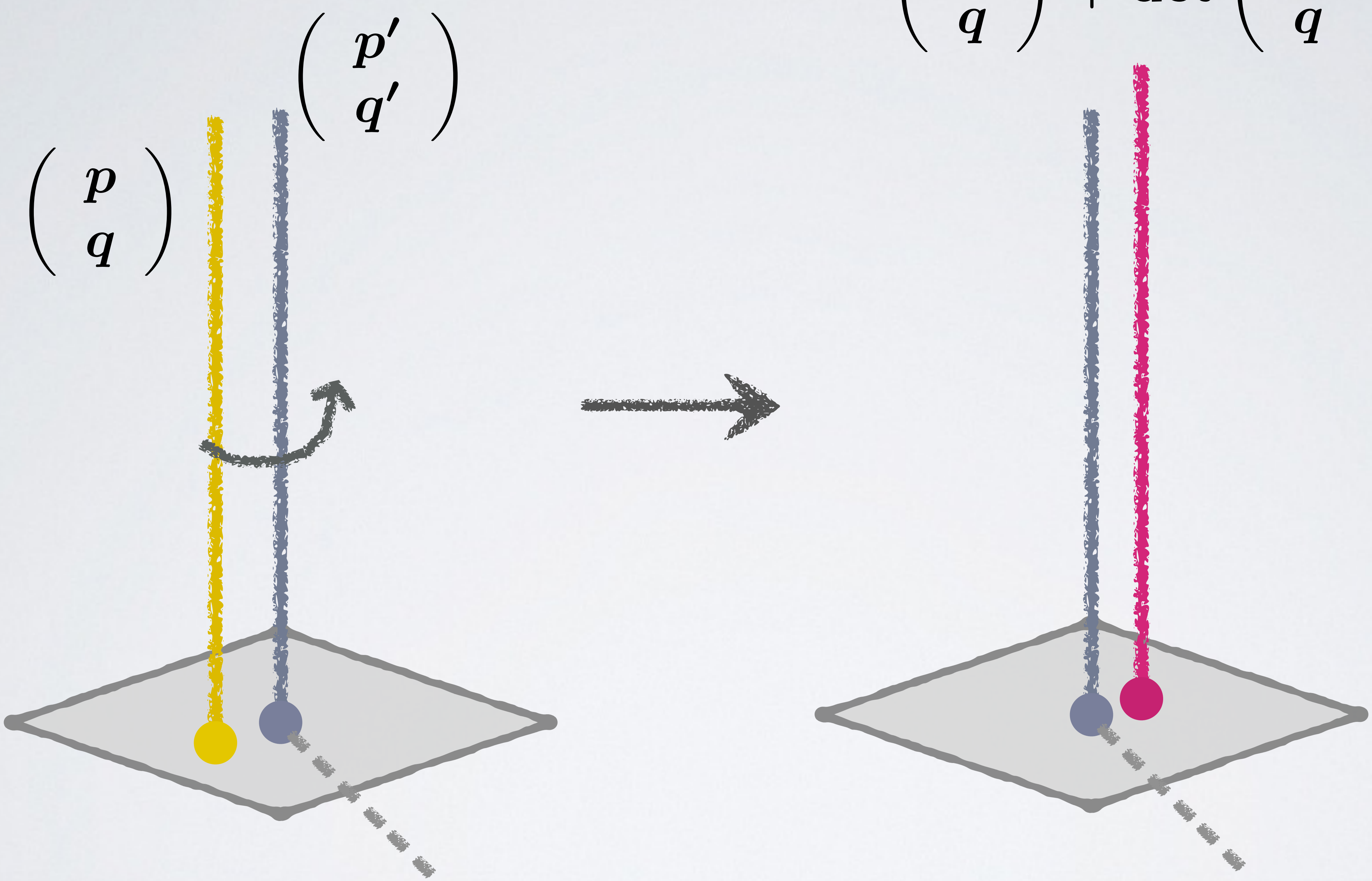
$$\begin{pmatrix} 1 + pq & -p^2 \\ q^2 & 1 - pa \end{pmatrix} \in SL(2, \mathbb{Z})$$

7-brane monodormy

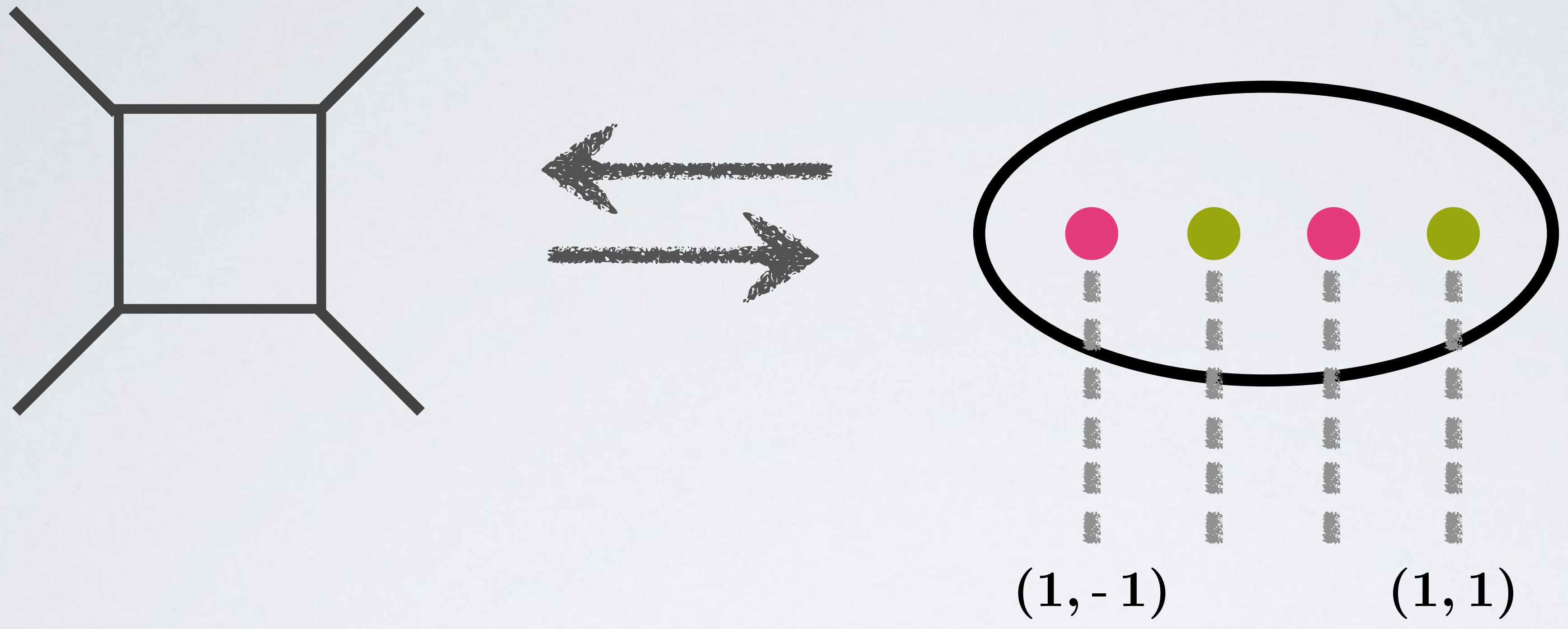


7-brane monodormy

$$\begin{pmatrix} p \\ q \end{pmatrix} + \det \begin{pmatrix} p & p' \\ q & q' \end{pmatrix} \begin{pmatrix} p' \\ q' \end{pmatrix}$$



“Seiberg duality” for $\mathbb{F}_0$



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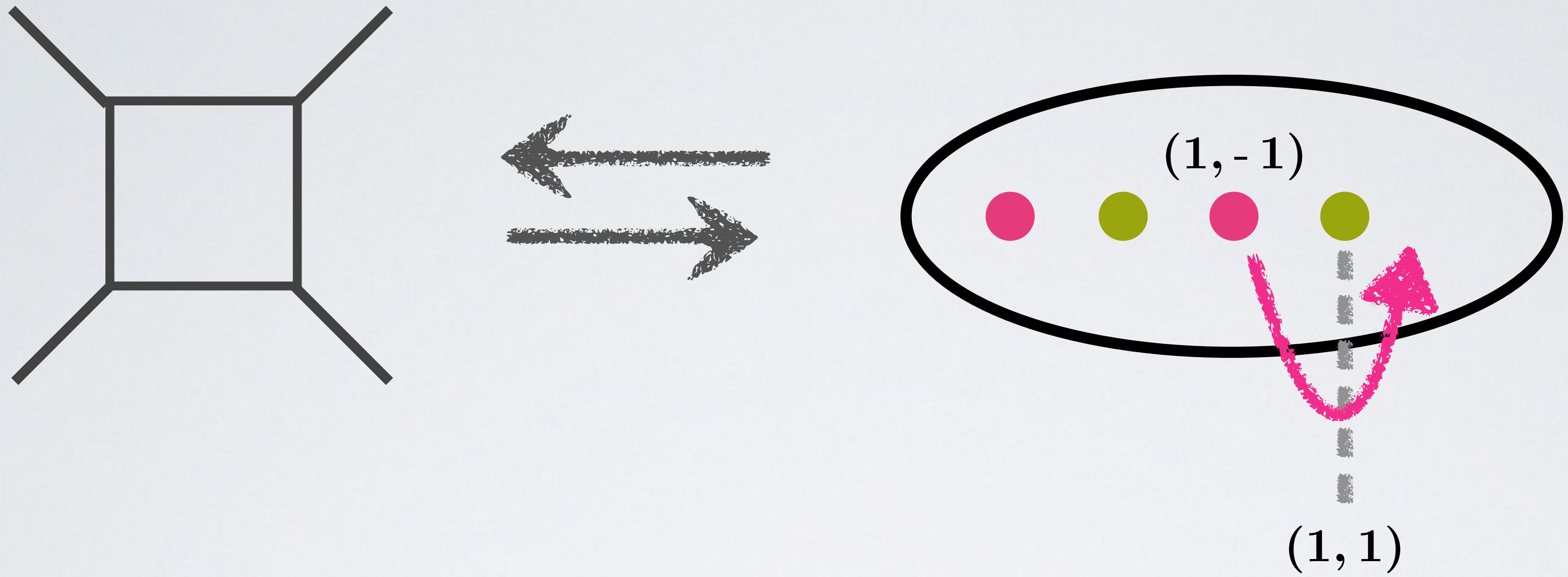

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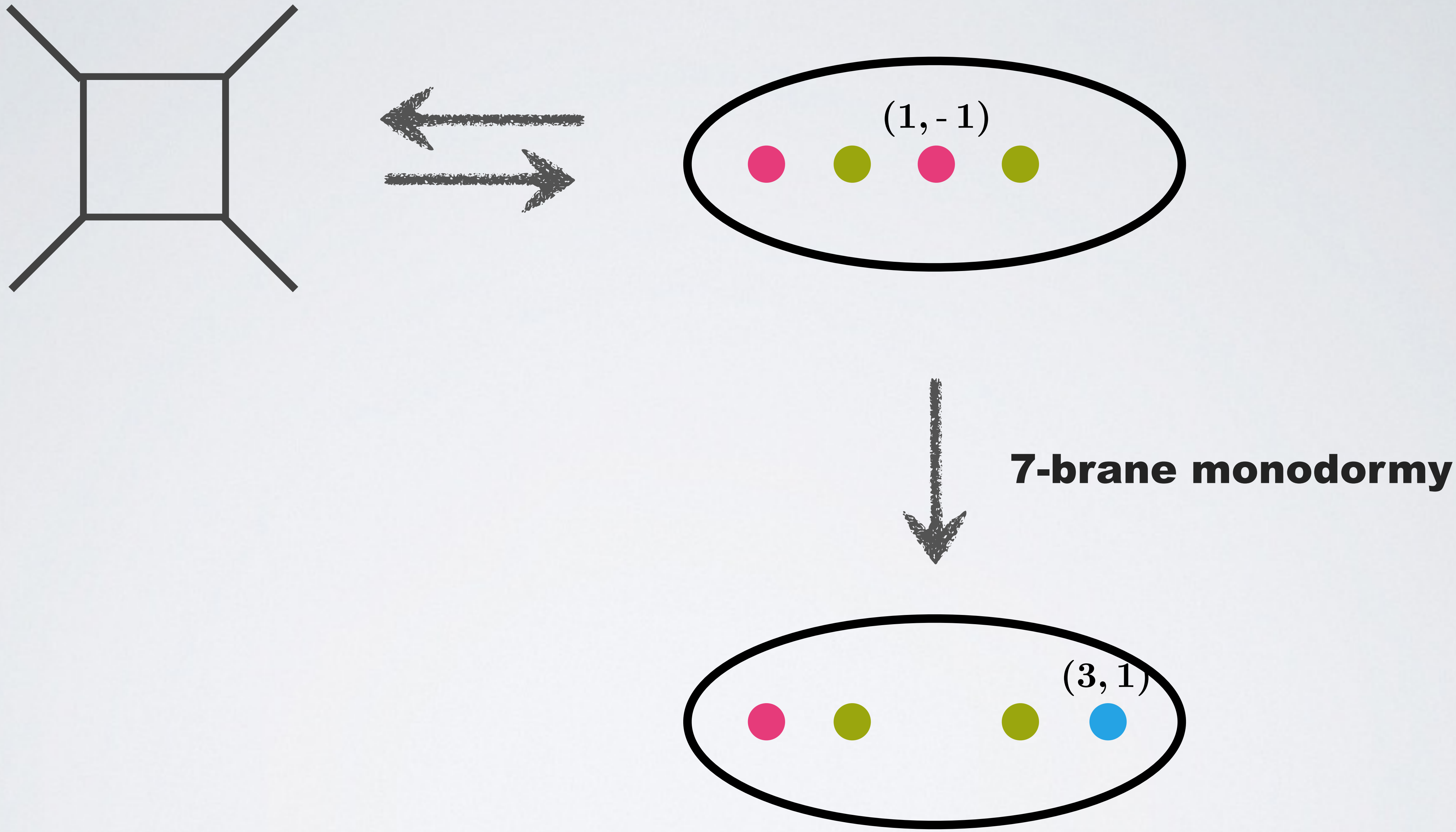

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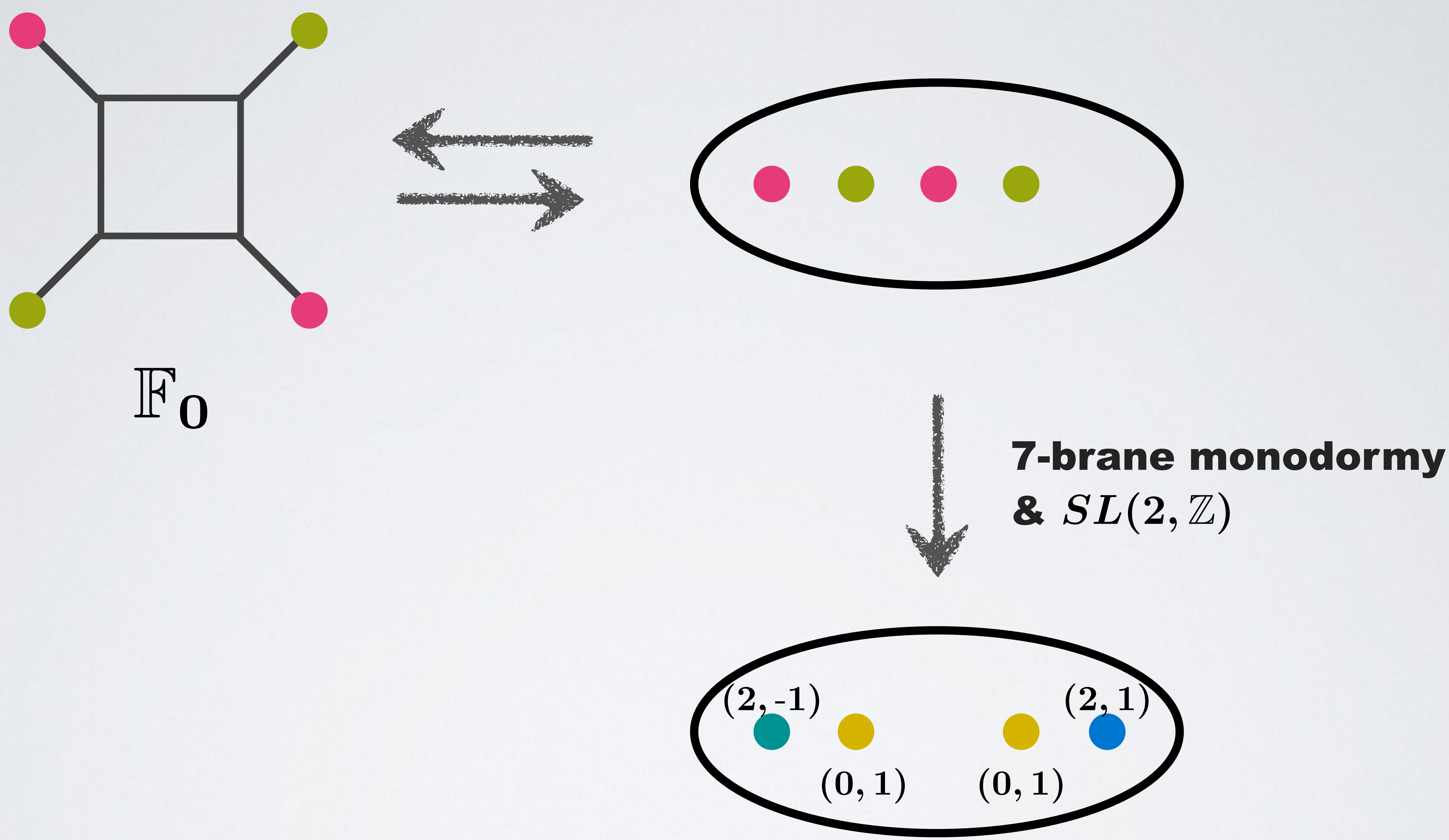

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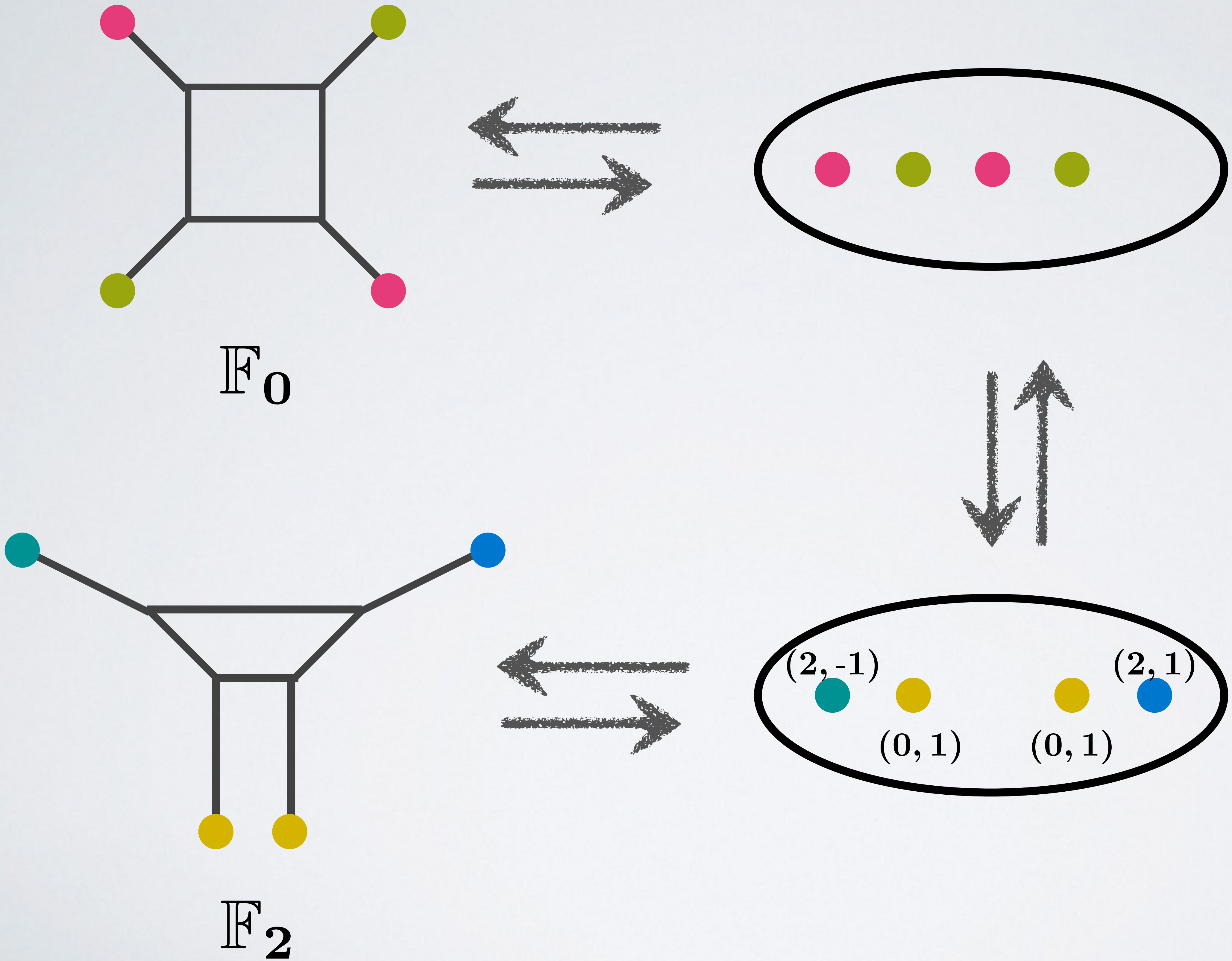
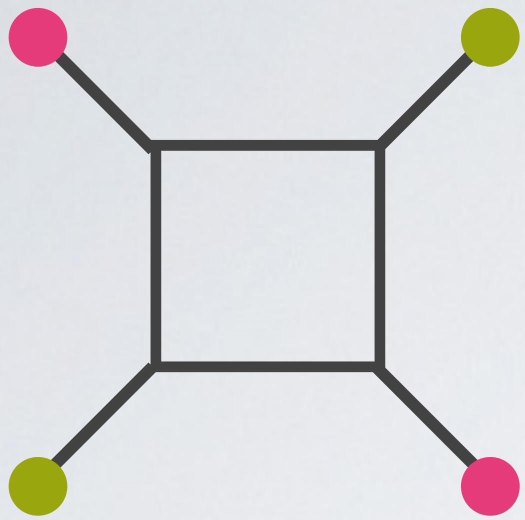
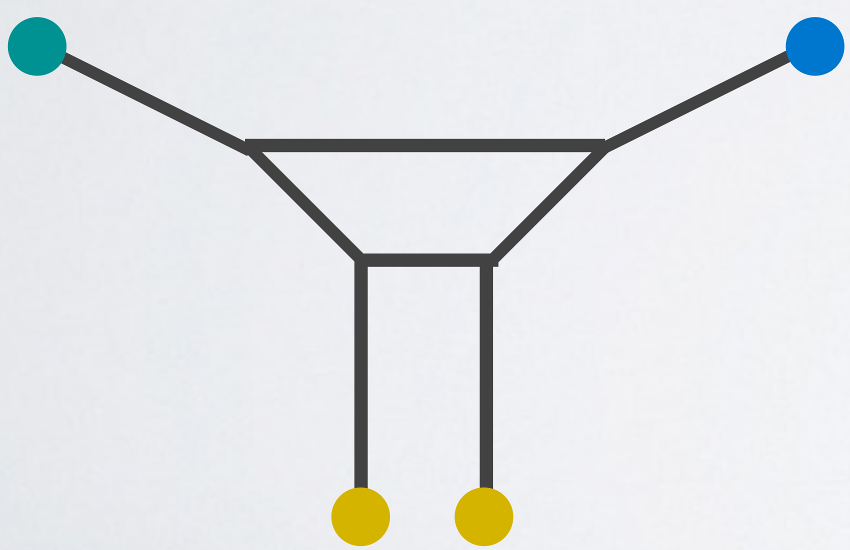


"Seiberg duality" for $\mathbb{F}_0$



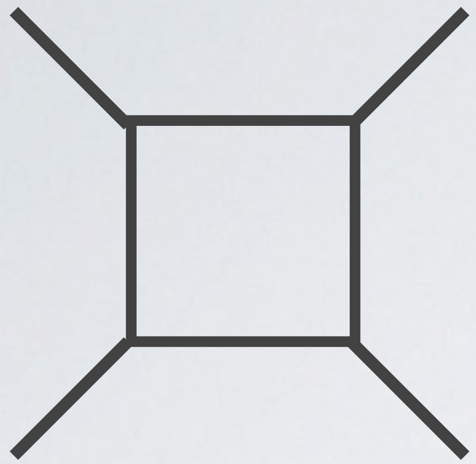
$\mathbb{F}_0$

same (5d) theory !!

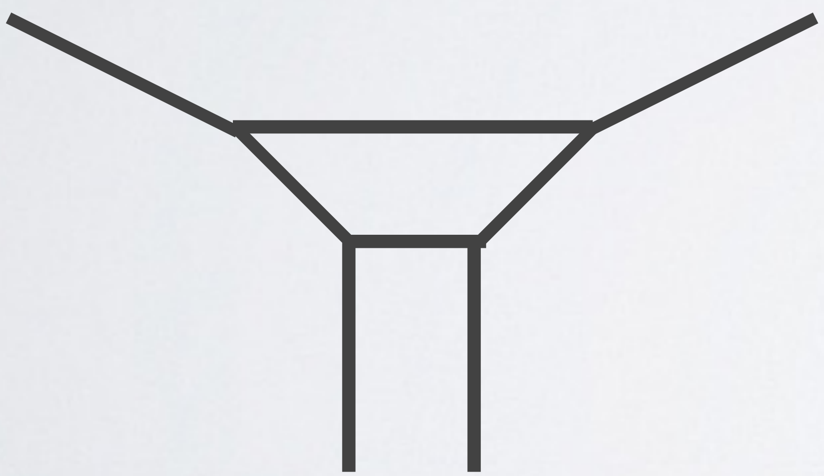


$\mathbb{F}_2$

“Seiberg duality” for $\mathbb{F}_0$



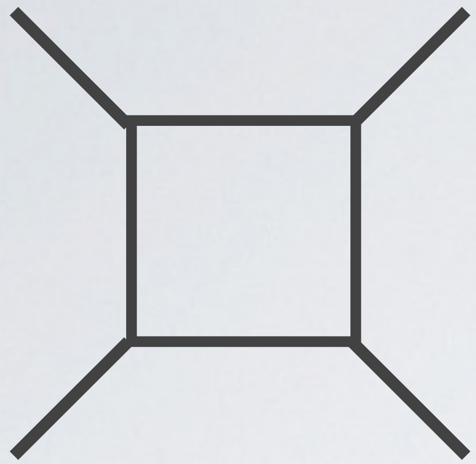
$\mathbb{F}_0$



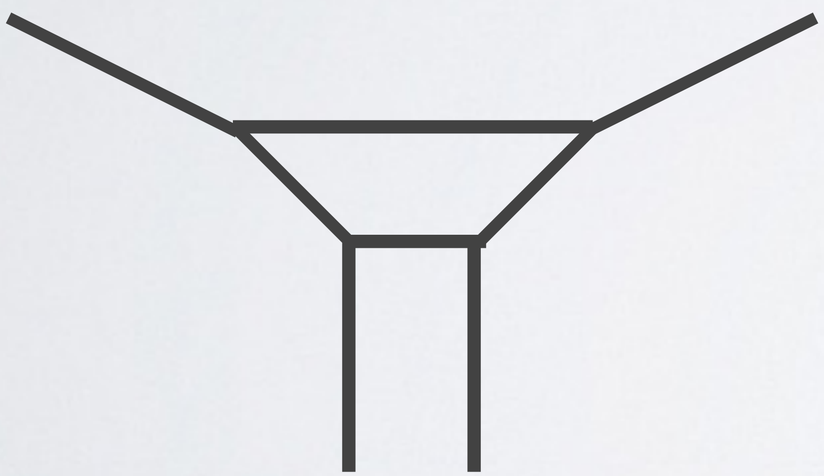
$\mathbb{F}_2$

BUT, a problem arises

“Seiberg duality” for $\mathbb{F}_0$



$\mathbb{F}_0$



$\mathbb{F}_2$

BUT, a problem arises

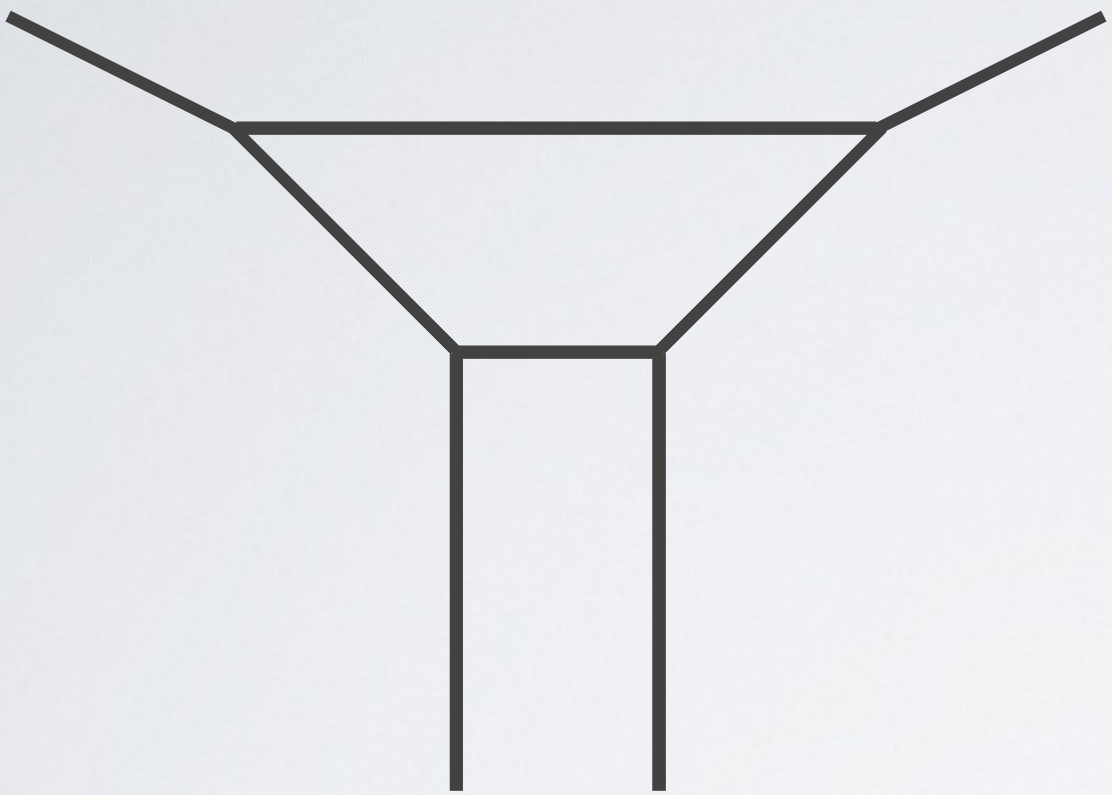
$$Z_{\mathbb{F}_0} \neq Z_{\mathbb{F}_2} \; !?$$

We will resolve this inconsistency

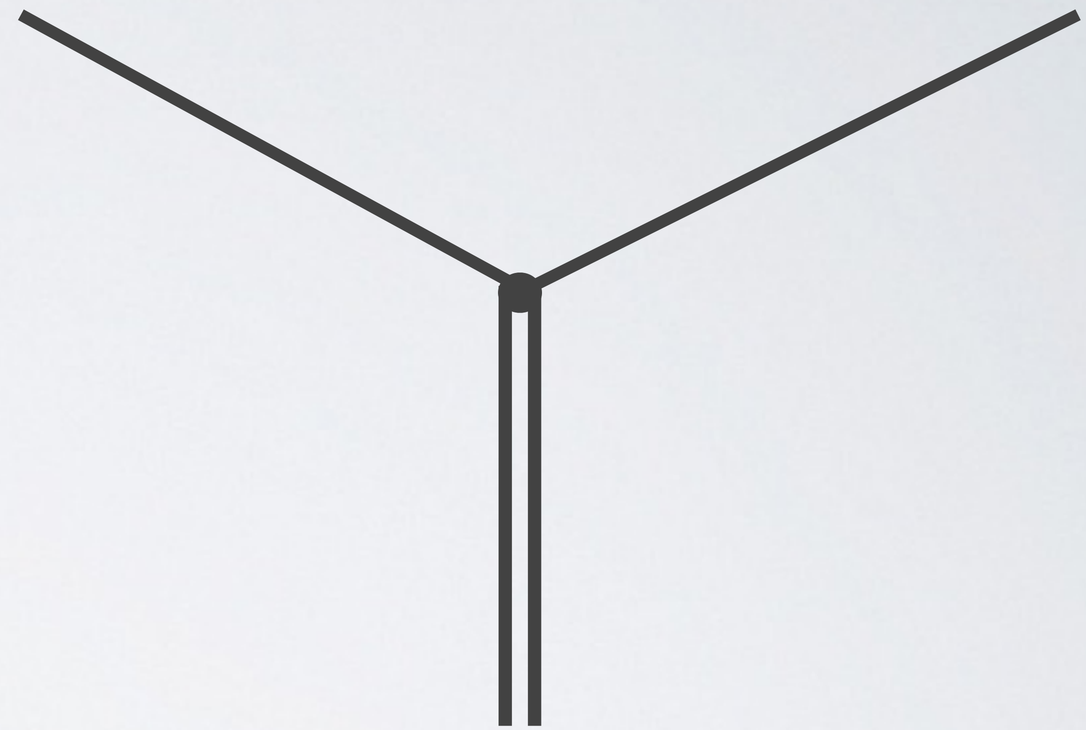
3. Resolution to discrepancy

[MT, ’13, ’14]
[Bao-Mitev-Pomoni-MT-Yagi, “13]

5d gauge theory & topological string

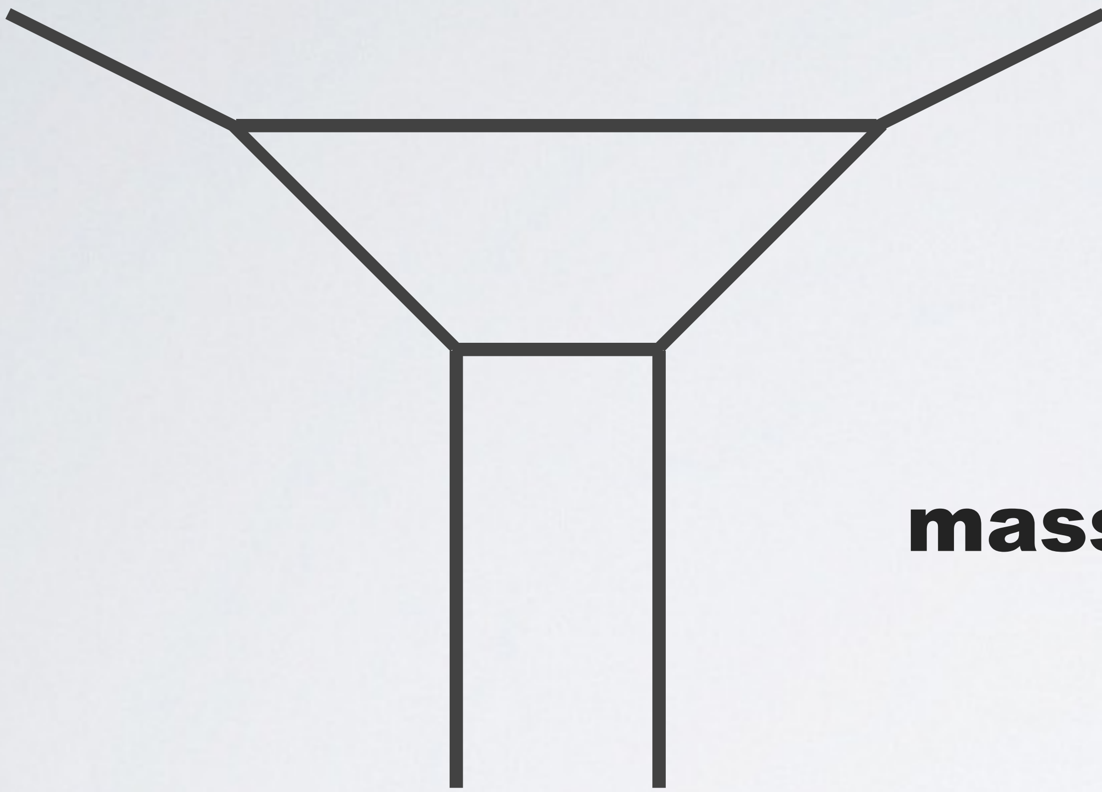


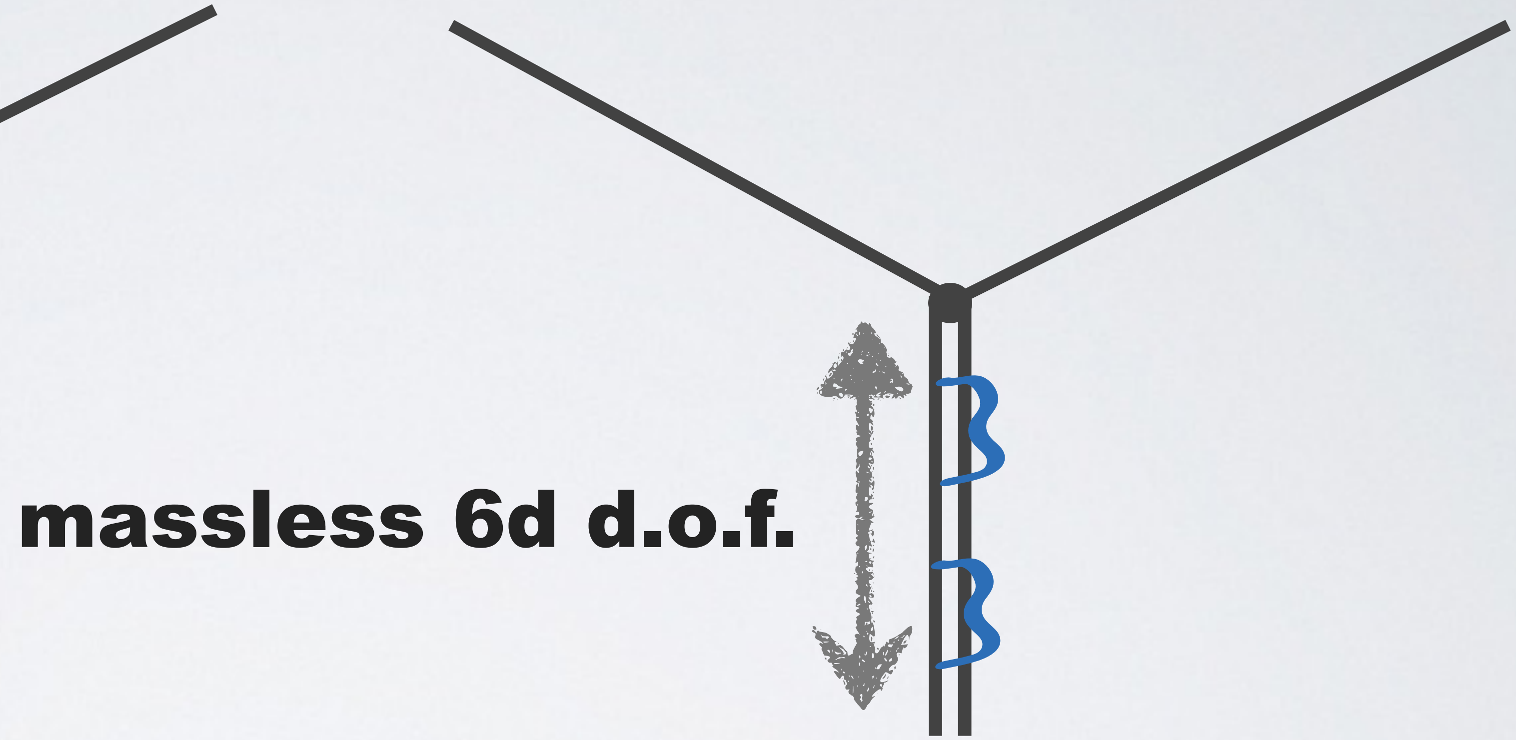
$\mathbb{F}_2$



E1 ?

5d gauge theory & topological string





$\mathbb{F}_2$

E1 + extra contribution

5d gauge theory & topological string

Claim (factorization conjecture)
[BMPTY] [HKN]

$$Z^{5\text{d}}_{\text{Nek}} = \frac{Z_{\text{top. str.}}}{Z_{\text{extra}}}$$

$$I = \frac{I[Z_{\text{top. str.}}]}{I_{\text{extra}}}$$

extra contribution is **factored out** ("decoupled")

5d gauge theory & topological string

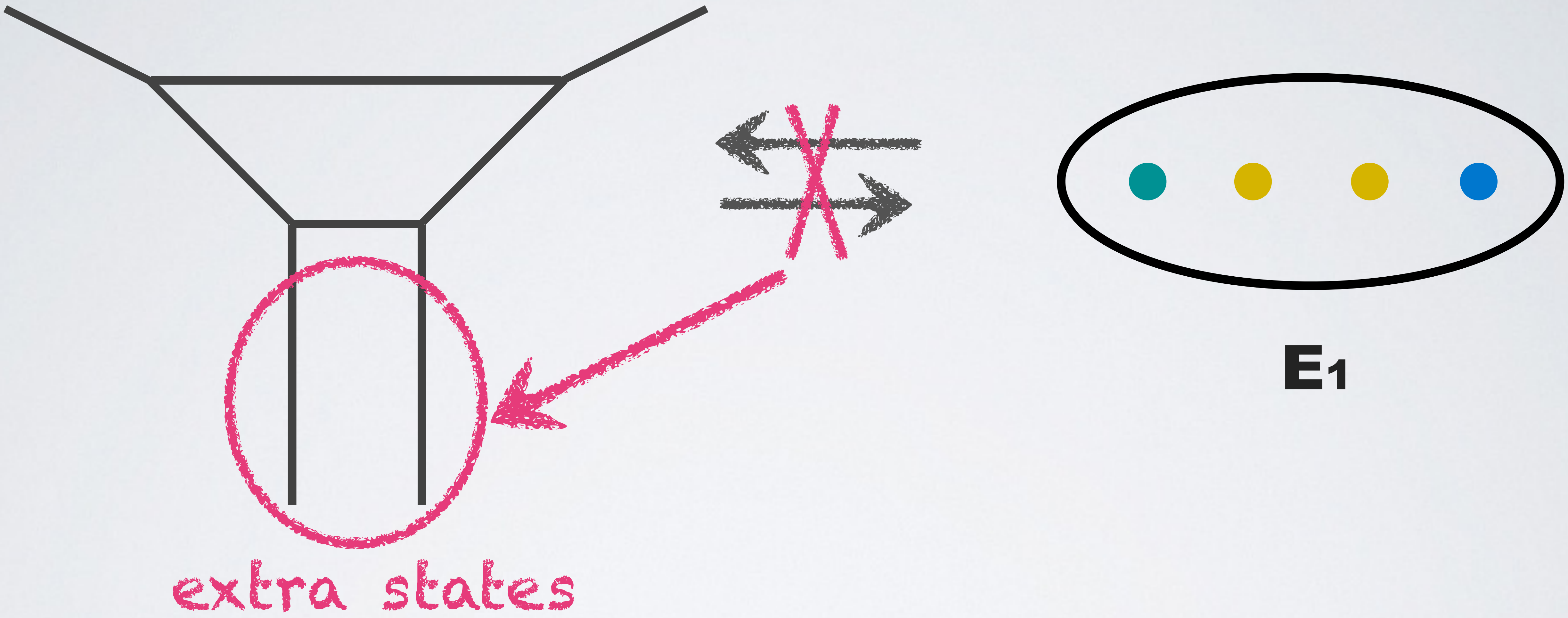
Claim (factorization conjecture)
[BMPTY] [HKN]

$$Z^{5d}_{\rm Nek} = \frac{Z_{\rm top.\,str.}}{Z_{\rm extra}}$$

$$I = \frac{I[Z_{\rm top.\,str.}]}{I_{\rm extra}}$$

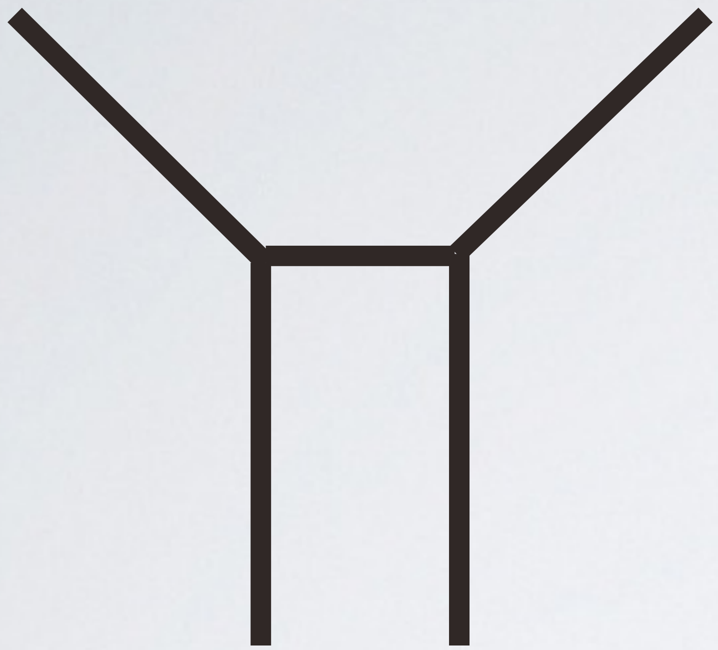
Q. How can we compute $Z_{\rm extra}$ & $I_{\rm extra}$??

Extra contribution: typical example



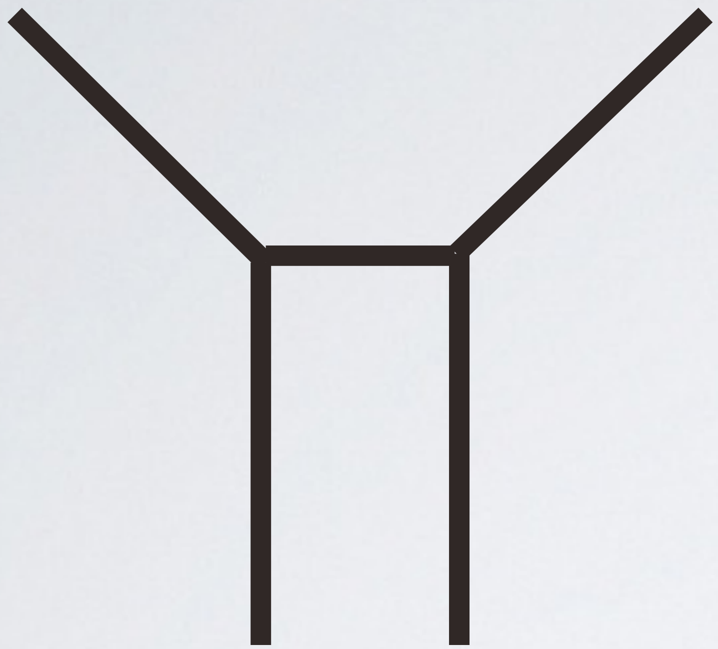
E_1 + **extra contribution**

Extra contribution: typical example



$$= Z_{\mathrm{extra}}$$

Extra contribution: typical example



$$= Z_{\text{extra}}$$

$$= \sum_{\lambda} \left(-u \sqrt{\frac{q}{t}} \right)^{|\lambda|} P_{\lambda^t}(t^{\rho}; q; t) P_{\lambda}(q^{\rho}; t; q)$$

$$= \prod_{i,j=1}^{\infty} \frac{1}{1 - u\, t^{i-1}\, q^{j}}$$

conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

$$Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}$$

conjecture [MT, ’13][Bergman-Gomez-Zafrir,’13]

$$Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1} q^{j}) \times Z_{\mathbb{F}_2}$$

$$Z_{\mathbb{F}_0} = \sum_{R_{1,2}} \left(u \frac{q}{t} \right)^{|\vec{R}|} Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

$$Z_{\mathbb{F}_2} = \sum_{R_{1,2}} \left(u \frac{q}{t} \right)^{|\vec{R}|} Z_{\vec{R}}^{\text{CS}, m=2}(Q_F; t, q)\, Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

Nekrasov formula

- vector contribution

$$Z^{\text{vect.}}_{\vec{R}}(Q_F;t,q) = \left(\frac{q}{t}\right)^{|\vec{R}|} \frac{1}{\prod_{\alpha,\beta=1,2} N_{R_\alpha R_\beta}(Q_{\beta\alpha};t,q)},$$

$$Q_{21} = Q_{12}^{-1} = Q_F \equiv e^{2Ra}, \quad Q_{11} = Q_{22} = 1$$

$$N_{R_\alpha R_\beta}(Q;t,q) = \prod_{s\in R_\alpha}\left(1 - Q\, t^{\ell_{R_\beta}(s)} q^{a_{R_\alpha}(s)+1}\right) \prod_{t\in Y_\beta}\left(1 - Q\, t^{-(\ell_{R_\alpha}(t)+1)} q^{-a_{R_\beta}(t)}\right)$$

- 5d Chern-Simons term

$$Z^{\text{CS},m}_{\vec{R}}(Q_{21};t,q) = \prod_{\alpha} Q_\alpha^{-m|R_\alpha|} t^{-m\frac{\|R_\alpha^T\|^2}{2}} q^{m\frac{\|R_\alpha\|^2}{2}}$$

conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

$$Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}$$

I don't have mathematical proof

$$Z_{\mathbb{F}_0} = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

$$Z_{\mathbb{F}_2} = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{CS}, m=2}(Q_F; t, q)\, Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q)$$

conjecture [MT, ’13][Bergman-Gomez-Zafrir,’13]

Let us test at 1-instanton level

$$Z_{\mathbb{F}_0}^{\text{1-inst}}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$Z_{\mathbb{F}_2}^{\text{1-inst}}(Q_F;t,q) = u\left(\frac{q}{t}\right)^2\frac{Q_F+1+\frac{1}{Q_F}-\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

Let us test at 1-instanton level

$$Z_{\mathbb{F}_0}^{\text{1-inst}}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$Z_{\mathbb{F}_2}^{\text{1-inst}}(Q_F;t,q) = u\left(\frac{q}{t}\right)^2\frac{Q_F+1+\frac{1}{Q_F}-\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$\longrightarrow \quad Z_{\mathbb{F}_0}(Q_F;t,q) - Z_{\mathbb{F}_2}(Q_F;t,q) = -u\frac{q}{(1-q)(1-t)}.$$

conjecture [MT, '13][Bergman-Gomez-Zafrir,'13]

Let us test at 1-instanton level

$$Z^{\text{1-inst}}_{\mathbb{F}_0}(Q_F;t,q) = u\frac{q}{t}\frac{1+\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$Z^{\text{1-inst}}_{\mathbb{F}_2}(Q_F;t,q) = u\left(\frac{q}{t}\right)^2\frac{Q_F+1+\frac{1}{Q_F}-\frac{q}{t}}{(1-q)(1-t^{-1})(1-Q_F t^{-1}q)(1-Q_F^{-1}t^{-1}q)}$$

$$\longrightarrow \quad Z_{\mathbb{F}_0}(Q_F;t,q) - Z_{\mathbb{F}_2}(Q_F;t,q) = -u\frac{q}{(1-q)(1-t)}$$

$$\longleftrightarrow \quad \prod_{i,j=1}^{\infty}(1-u\,t^{i-1}\,q^j) = 1 - u\frac{q}{(1-q)(1-t)} + \cdots$$

conjecture [MT, ’13][Bergman-Gomez-Zafrir,’13]

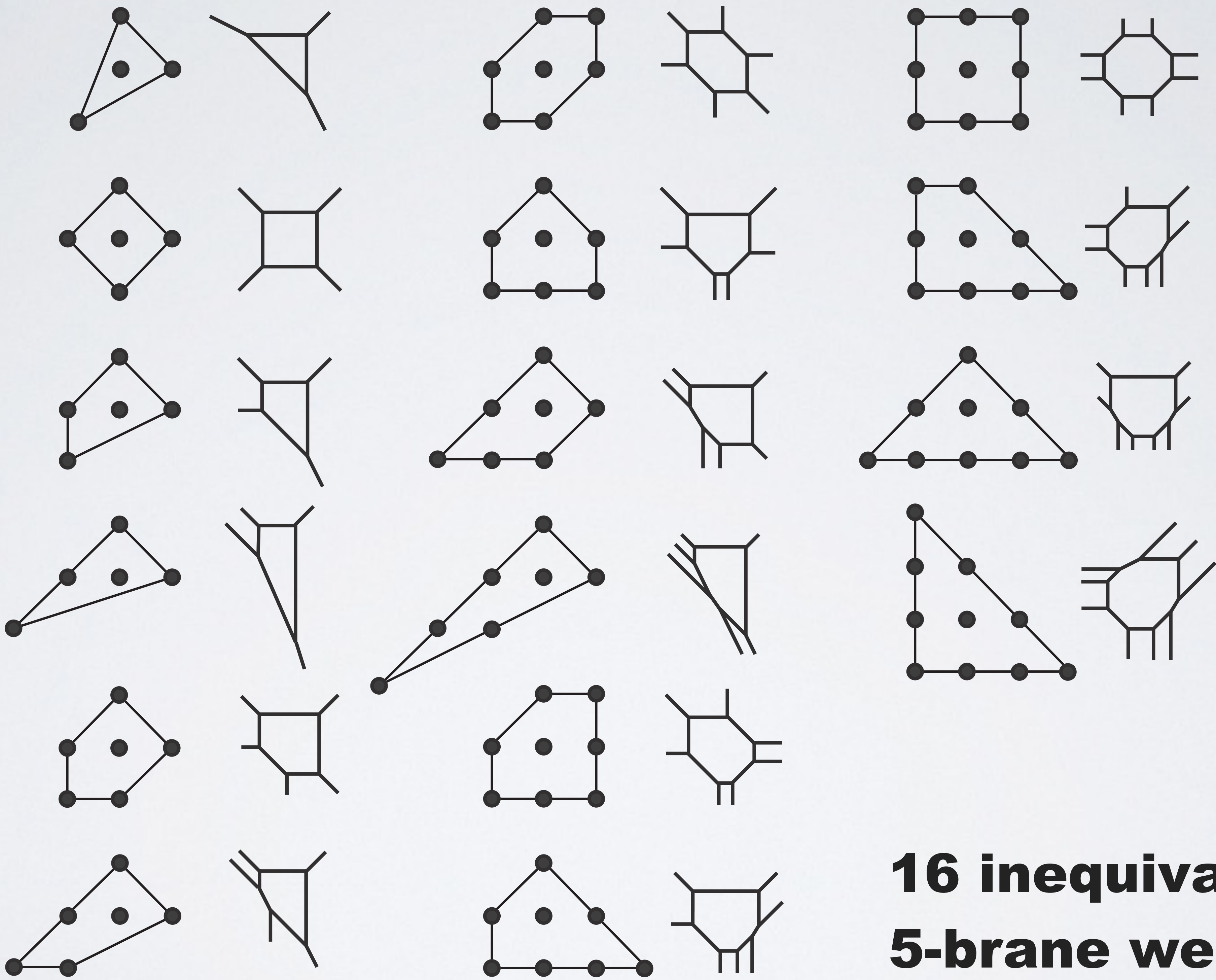
Let us test at 1-instanton level

$$Z_{\mathbb{F}_0} = \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) \times Z_{\mathbb{F}_2}$$

$$Z_{\mathbb{F}_0}(Q_F; t, q) - Z_{\mathbb{F}_2}(Q_F; t, q) = -u \frac{q}{(1-q)(1-t)}.$$

$$\Longleftrightarrow \quad \prod_{i,j=1}^{\infty} (1 - u\, t^{i-1}\, q^j) = 1 - u \frac{q}{(1-q)(1-t)} + \cdots$$

generalization: all the webs with 1d Coulomb branch



16 inequivalent 5-brane webs

All known candidate SCFTs with 1d Coulomb branch

$N_f = 0 \qquad E_1 = SU(2) \qquad \tilde{E}_1 = U(1)$

$N_f = 1 \qquad E_2 = SU(2) \times U(1)$

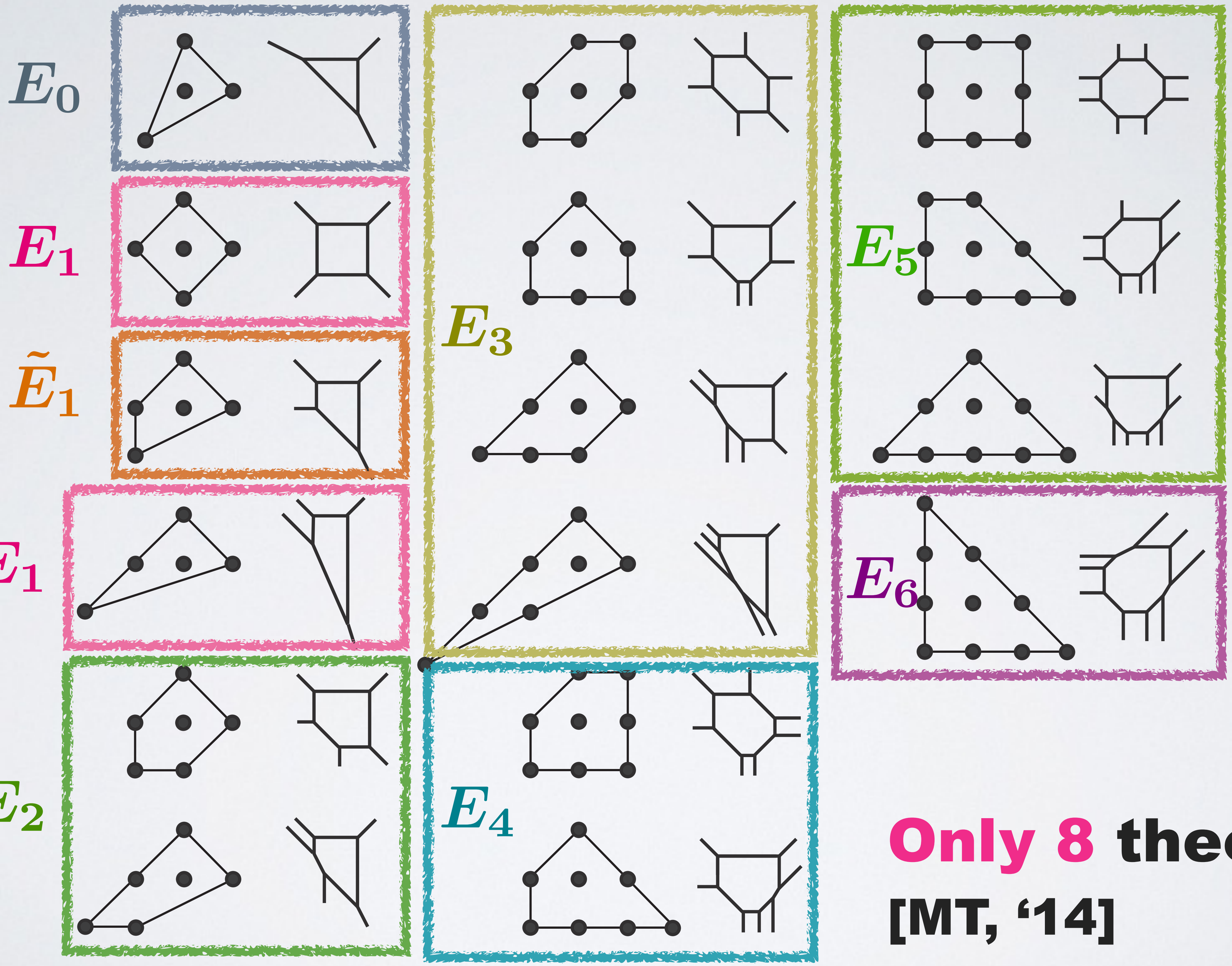
$N_f = 2 \qquad E_3 = SU(3) \times SU(2)$

$N_f = 3 \qquad E_4 = SU(5)$

$N_f = 4 \qquad E_5 = SO(10)$

$N_f = 5 \qquad E_6$

7-brane picture leads to ...



Only 8 theories !!

[MT, '14]

5D UV SCFT via CY compactification

$N_f = 0$ $E_1 = SU(2)$: **local del Pezzo 1**

$N_f = 1$ $E_2 = SU(2) \times U(1)$: **local del Pezzo 2**

$N_f = 2$ $E_3 = SU(3) \times SU(2)$: **local del Pezzo 3**

$N_f = 3$ $E_4 = SU(5)$: **local del Pezzo 4**

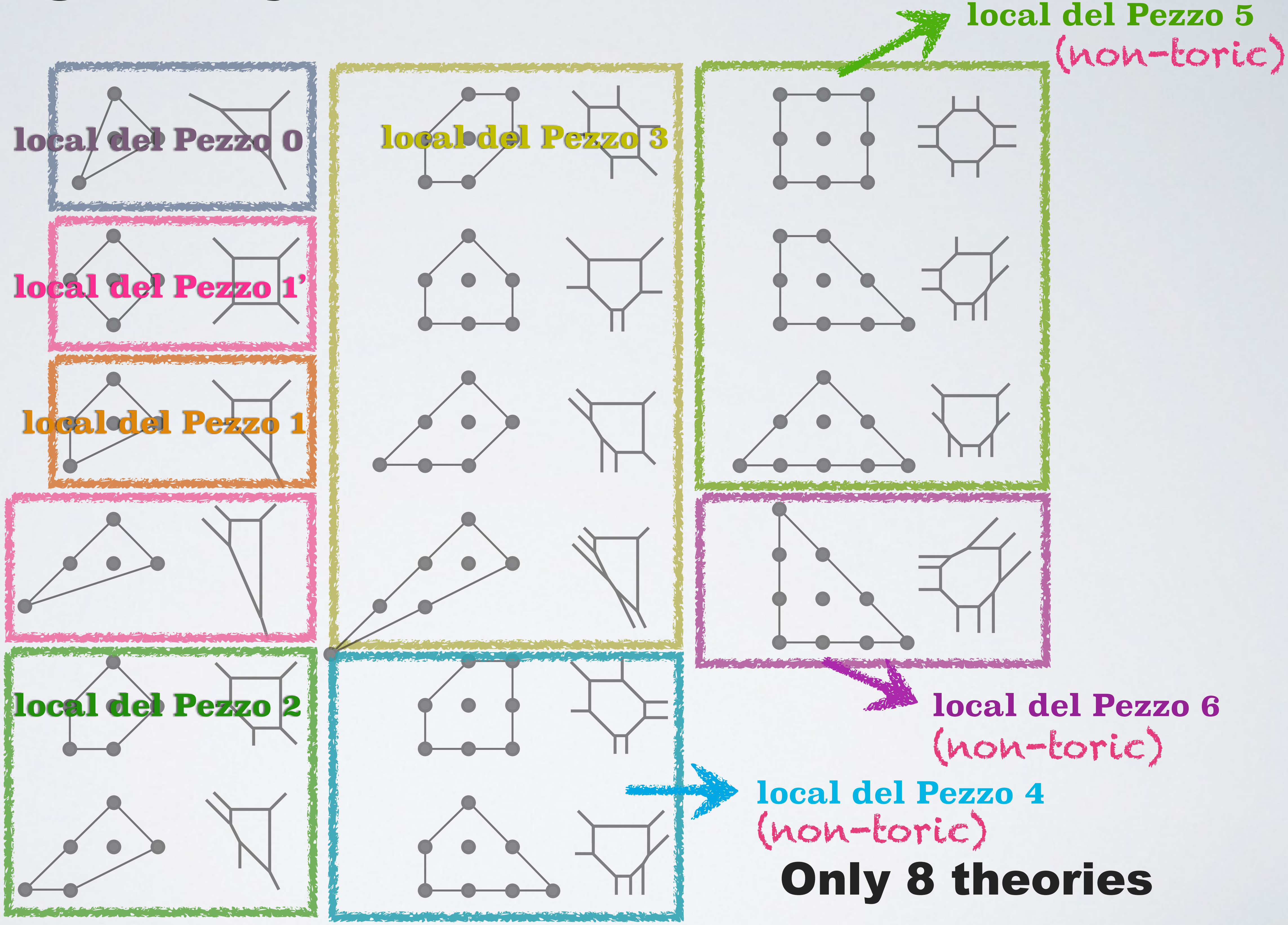
$N_f = 4$ $E_5 = SO(10)$: **local del Pezzo 5**

$N_f = 5$ E_6 : **local del Pezzo 6**

non-toric

$Z_{\text{non-toric}} = Z_{\text{toric}} \div Z_{\text{extra}}$

“Seiberg duality” [MT, ‘14]

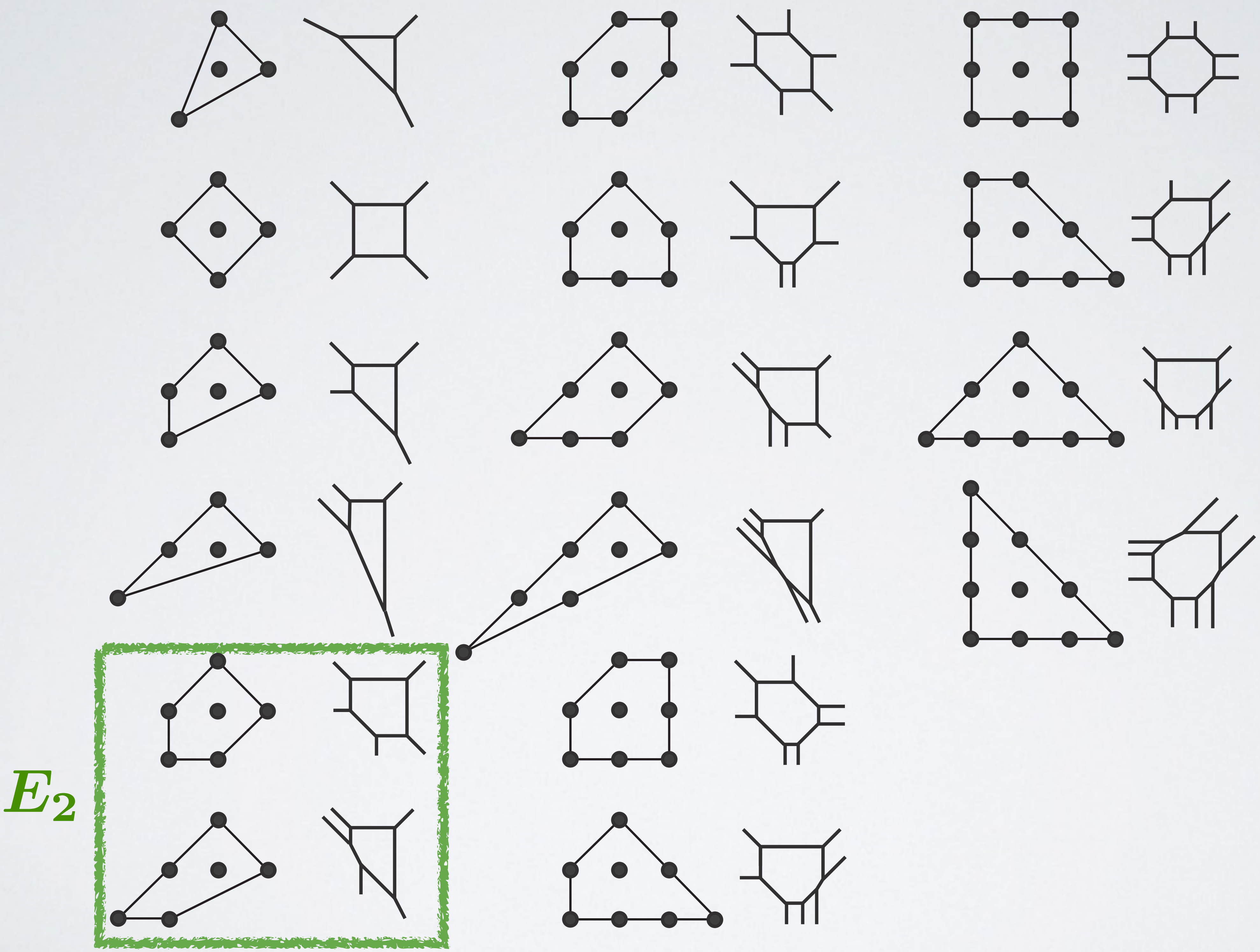


Only 8 theories

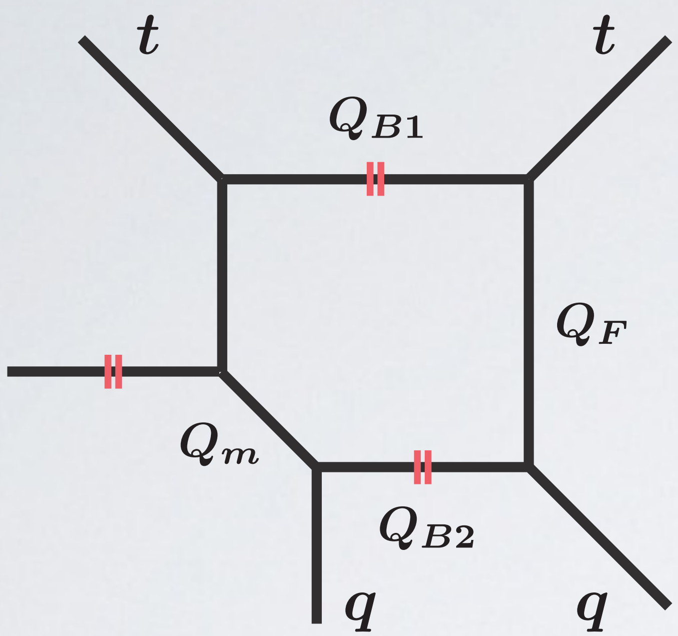
4. Various conjectures on Nekrasov functions

[MT, ‘14] [BMPTY, ‘13] [HKN, ‘13]

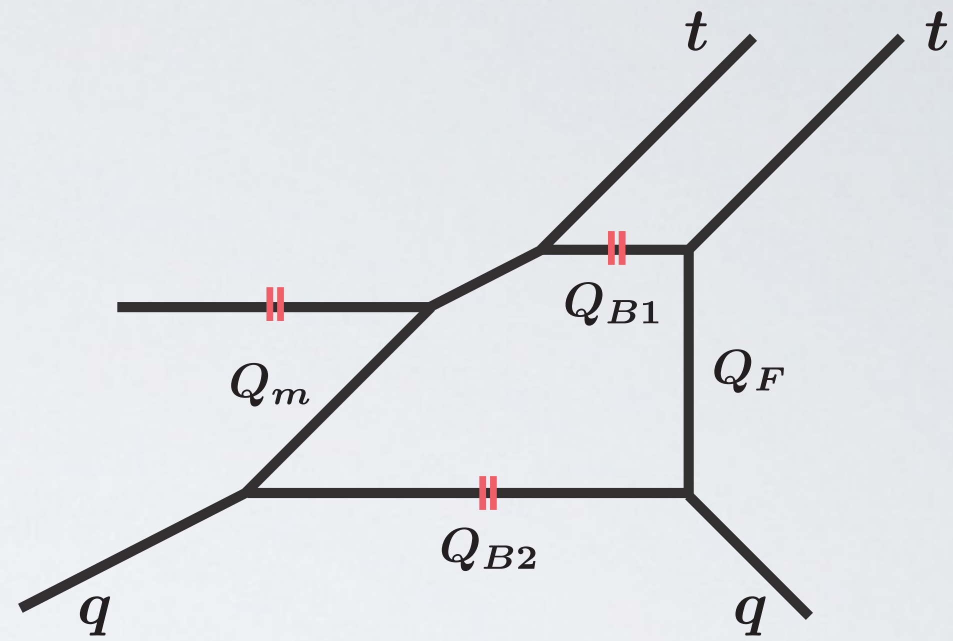
E2 case [MT, '14] [BMPTY, '13] [HKN, '13]



E2 case [MT, ‘14] [BMPTY, ‘13] [HKN, ‘13]

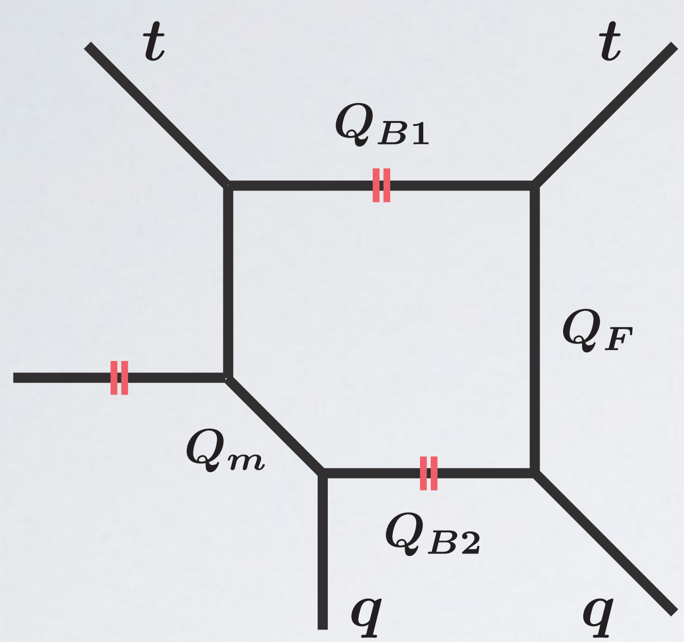


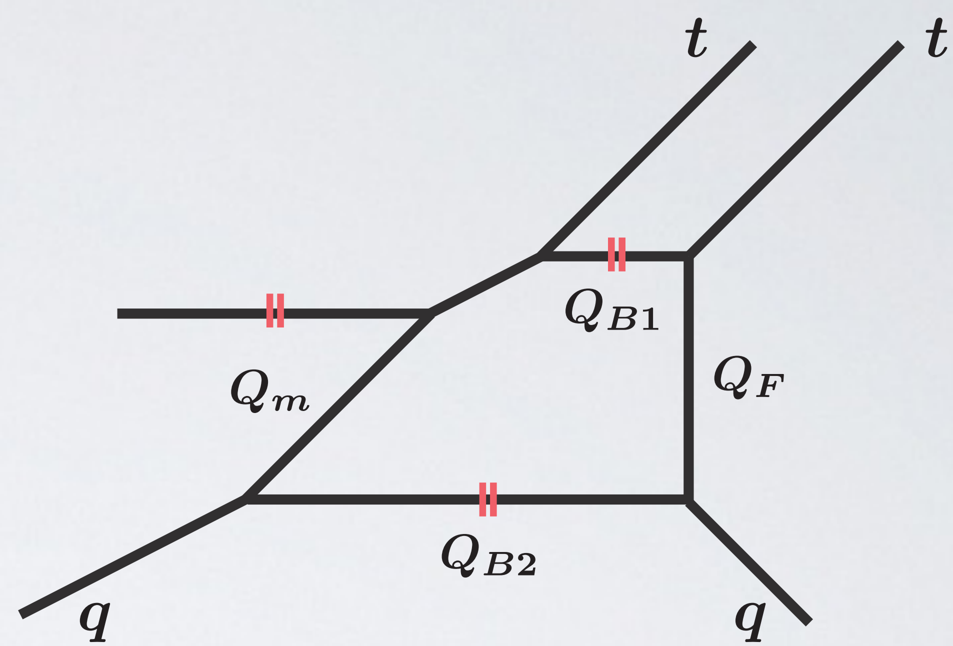
dP_2



PdP_2

E2 case [MT, '14] [BMPTY, '13] [HKN, '13]





dP_2 $\quad\quad$ PdP_2

$$Z_{dP_2}(u, Q_F, Q_m; t, q) = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q) Z_{\vec{R}}^{\text{matt.}}(Q_F, Q_m; t, q)$$

$$Z_{PdP_2}(u, Q_F, Q_m; t, q) = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z_{\vec{R}}^{\text{CS},m=-2}(Q_F; t, q)$$

$$\times Z_{\vec{R}}^{\text{vect.}}(Q_F; t, q) Z_{\vec{R}}^{\text{matt.}}(Q_F, Q_m; t, q)$$

E2 case [MT, '14] [BMPTY, '13] [HKN, '13]

- fundamental matter contribution

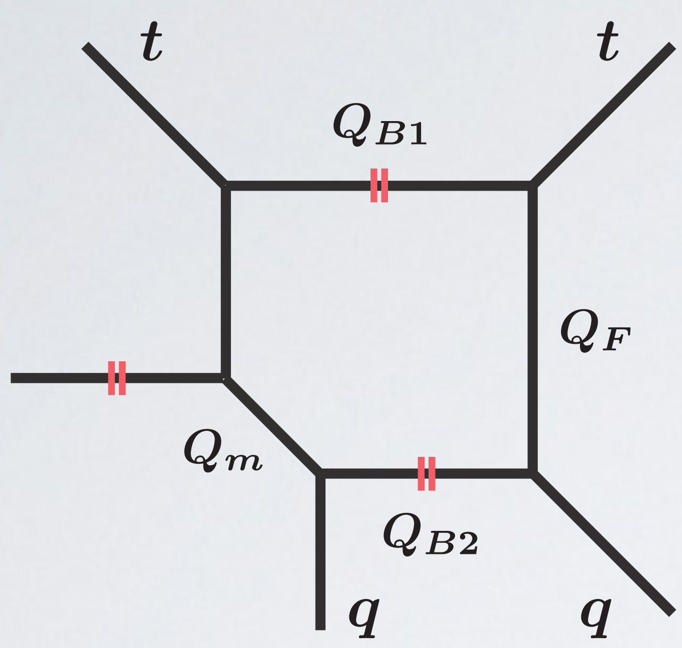
$$Z^{\text{matt.}}_{\vec{R}}(Q_{21}, Q_m; t, q) = \prod_{(i,j)\in R_1} \left(1 - \frac{Q_{21}}{Q_m} t^{-i+\frac{1}{2}} q^{j-\frac{1}{2}}\right) \prod_{(i,j)\in R_2} \left(1 - \frac{1}{Q_m} t^{-i+\frac{1}{2}} q^{j-\frac{1}{2}}\right)$$

$$Z_{dP_2}(u, Q_F, Q_m; t, q) = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z^{\text{vect.}}_{\vec{R}}(Q_F; t, q) Z^{\text{matt.}}_{\vec{R}}(Q_F, Q_m; t, q)$$

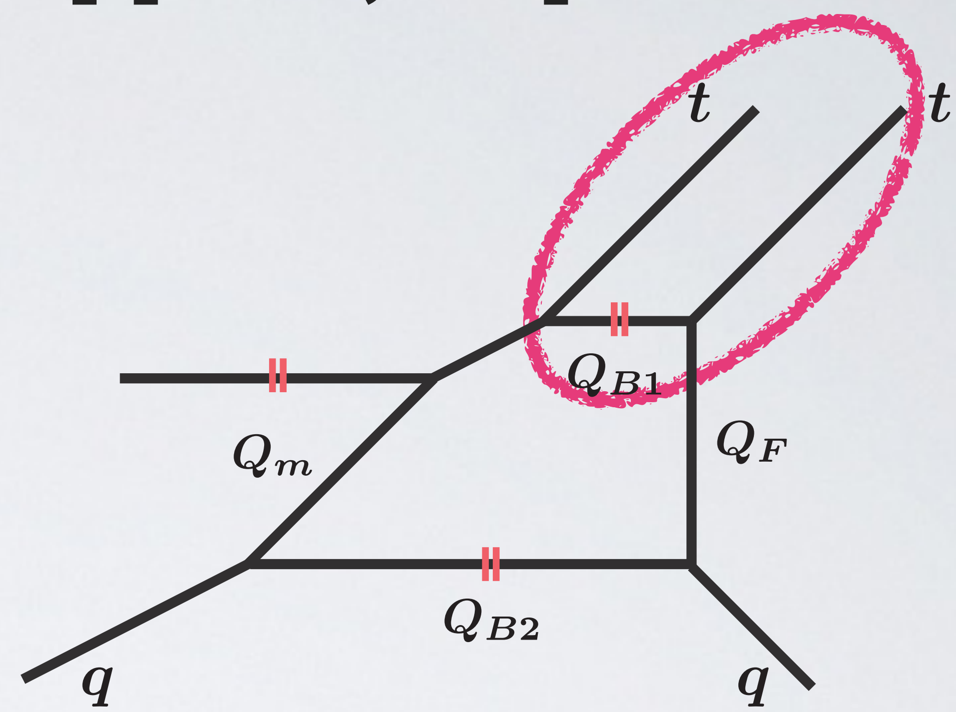
$$Z_{PdP_2}(u, Q_F, Q_m; t, q) = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z^{\text{CS},m=-2}_{\vec{R}}(Q_F; t, q)$$

$$\times Z^{\text{vect.}}_{\vec{R}}(Q_F; t, q) Z^{\text{matt.}}_{\vec{R}}(Q_F, Q_m; t, q)$$

E2 case [MT, '14] [BMPTY, '13] [HKN, '13]



dP_2

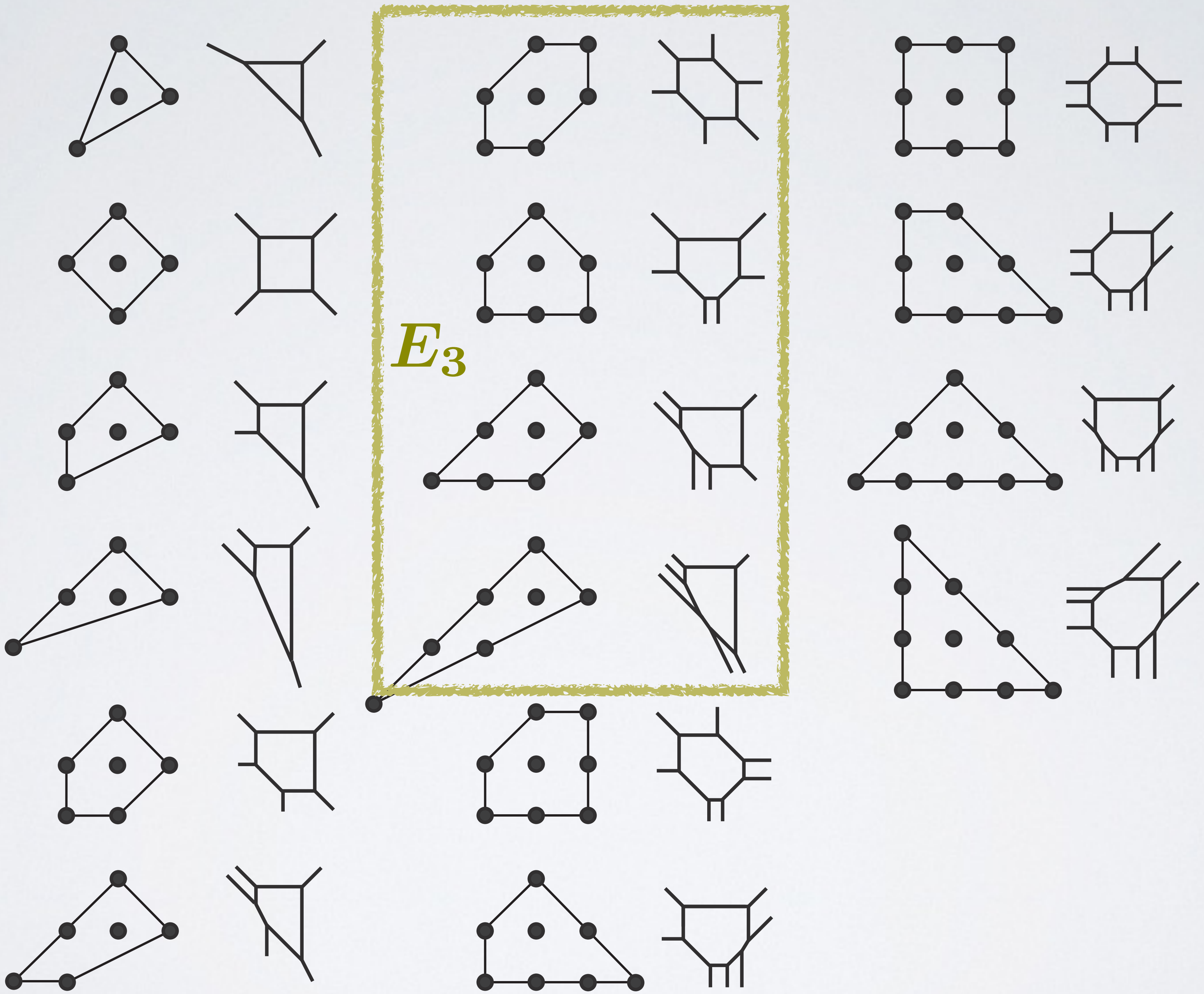


PdP_2

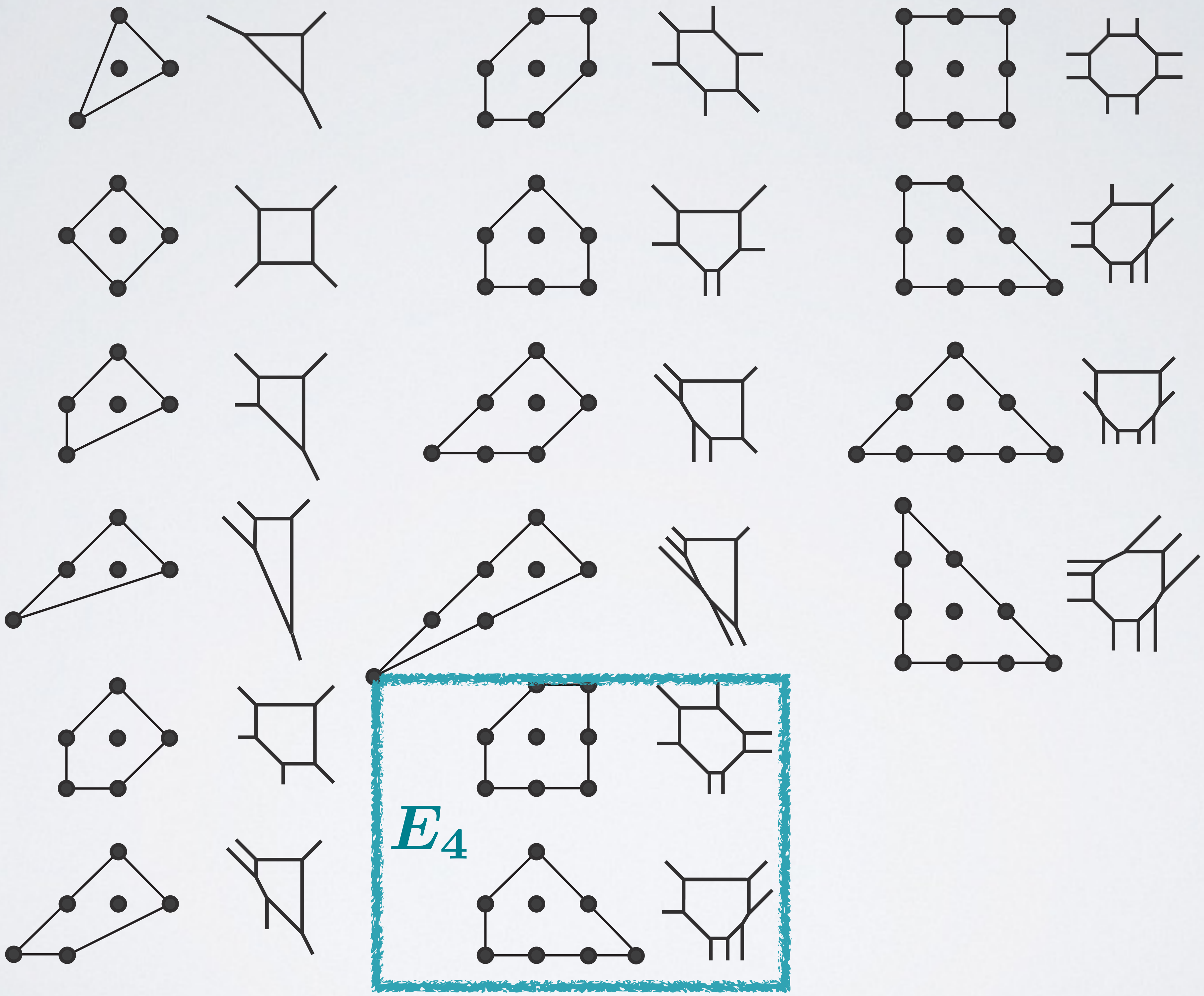
$$Z_{dP_2} = \prod_{i,j=1}^{\infty} (1 - u\, t^i\, q^{j-1}) \times Z_{PdP_2}$$

1- & 2-instanton test is straightforward.

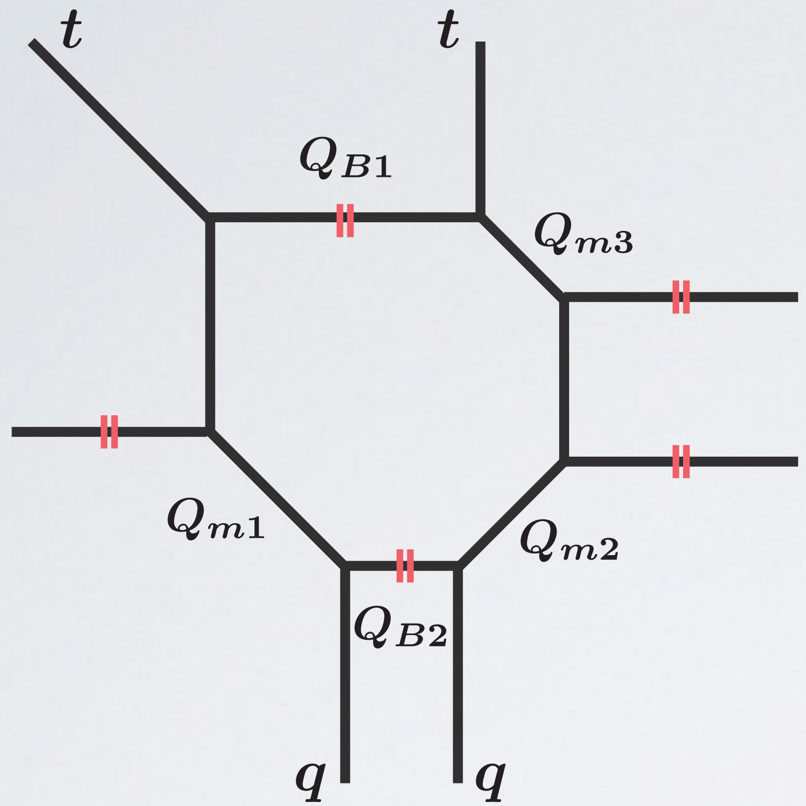
E3 case [MT, '14]

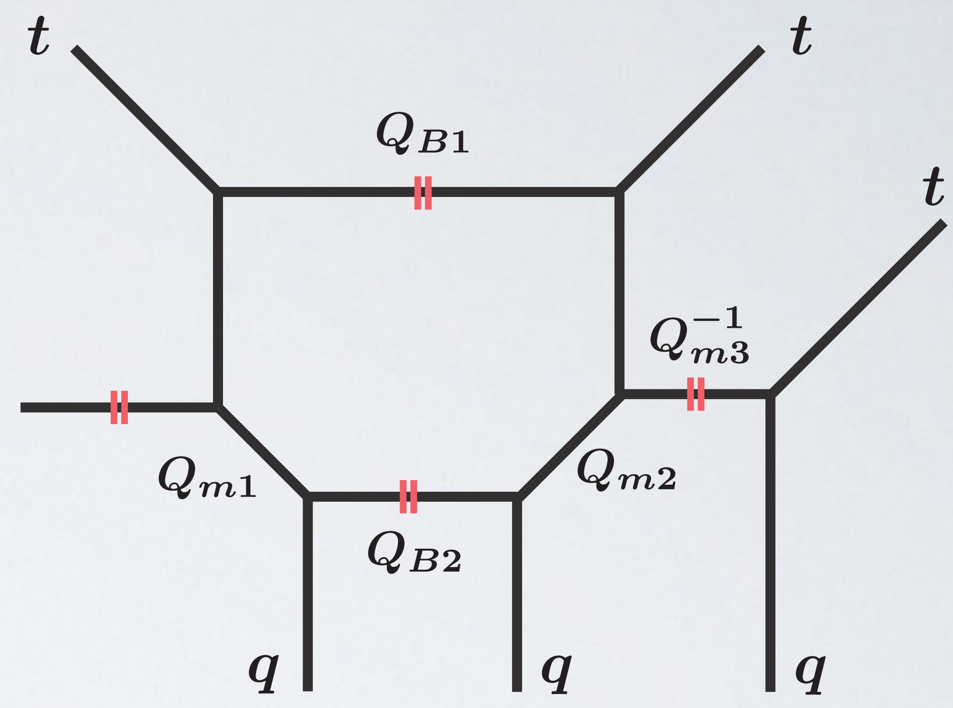


E_4 case [MT, '14]



E4 case [MT, '14]

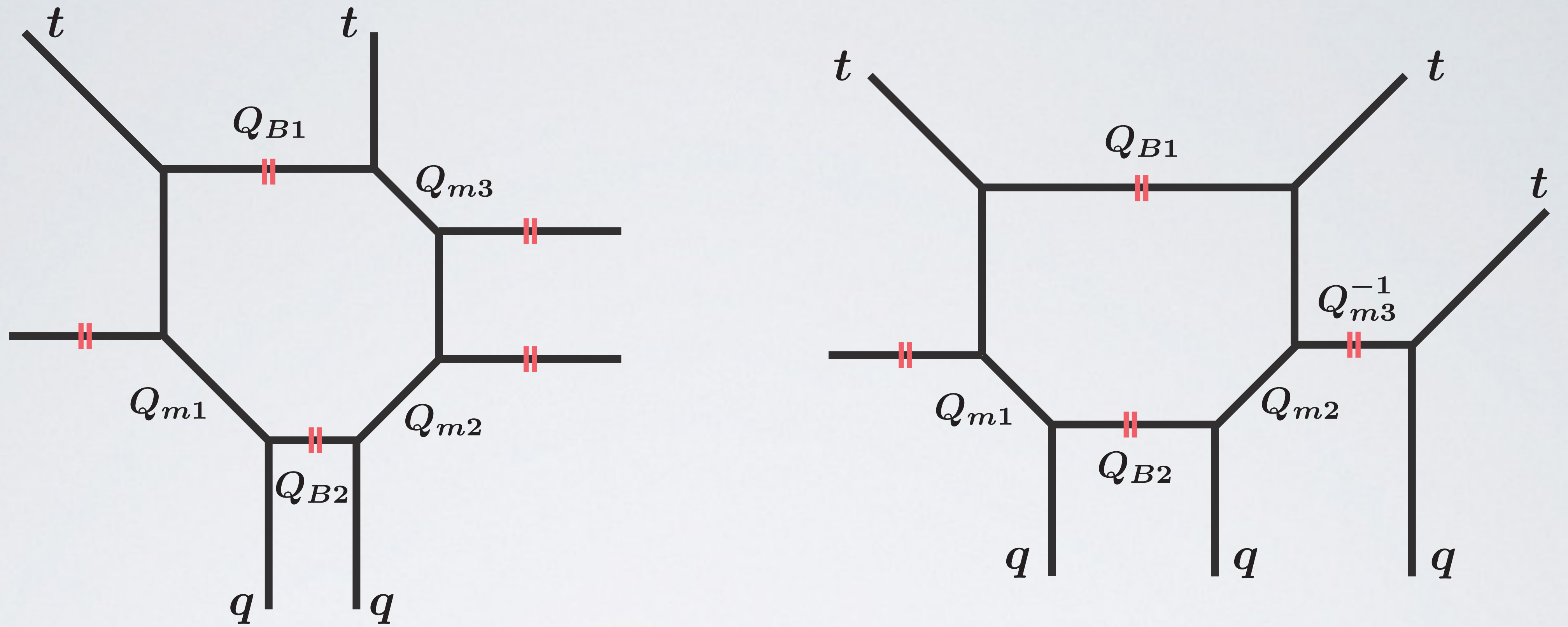




$Z_{PdP_4^I}$ $Z_{PdP_4^{II}}$

local geometries of pseudo del Pezzo surface

E4 case [MT, ‘14]



$Z_{PdP_4^I}$ $Z_{PdP_4^{II}}$

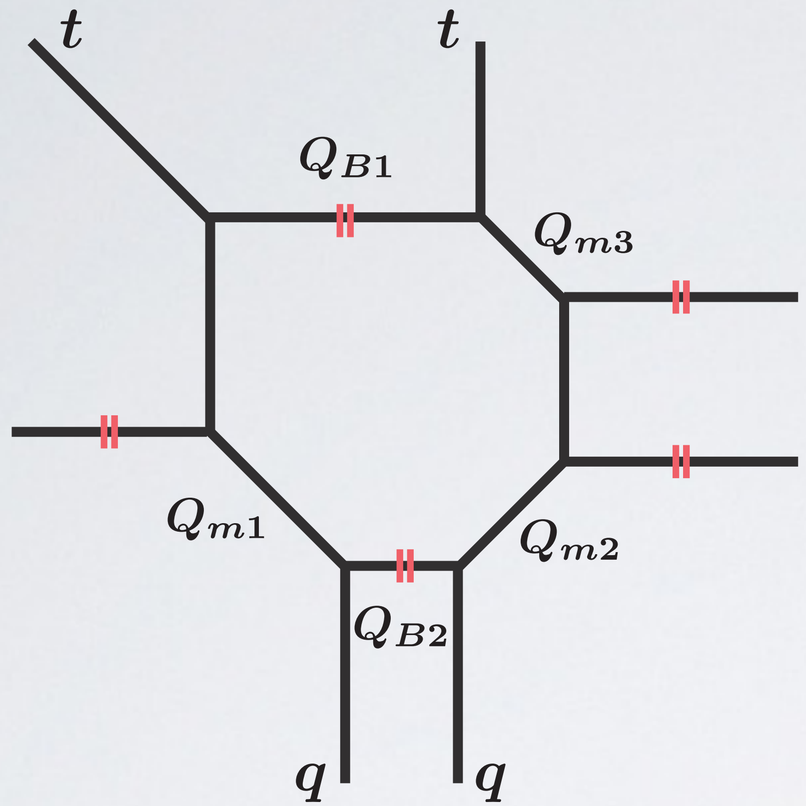
↑ ↑

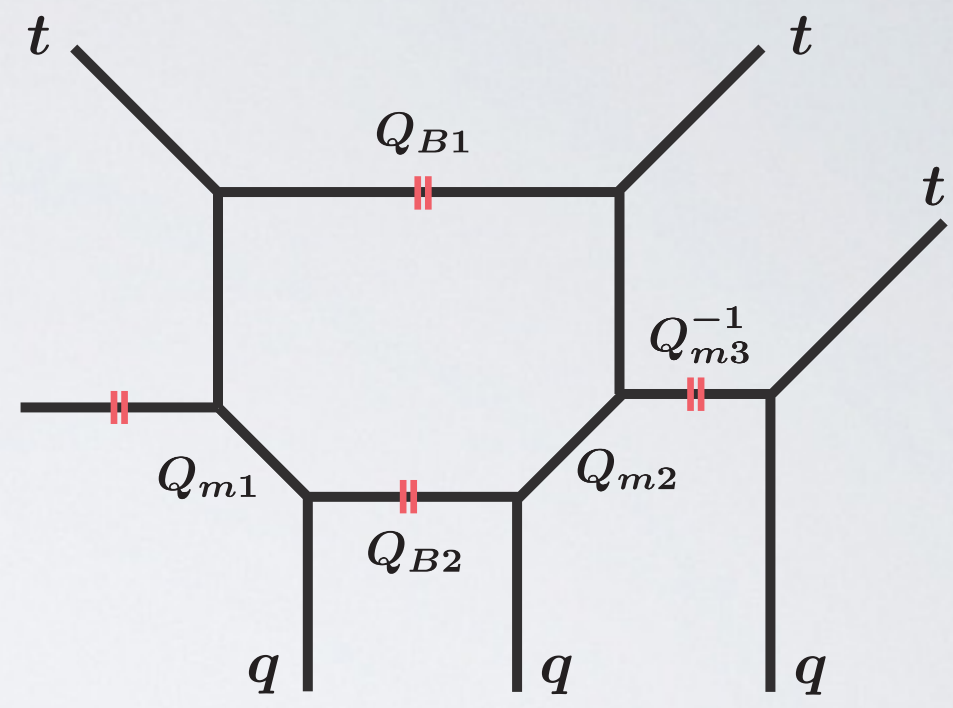
local geometries of pseudo del Pezzo surface

↑

specially-tuned complex structure

E_4 case [MT, '14]



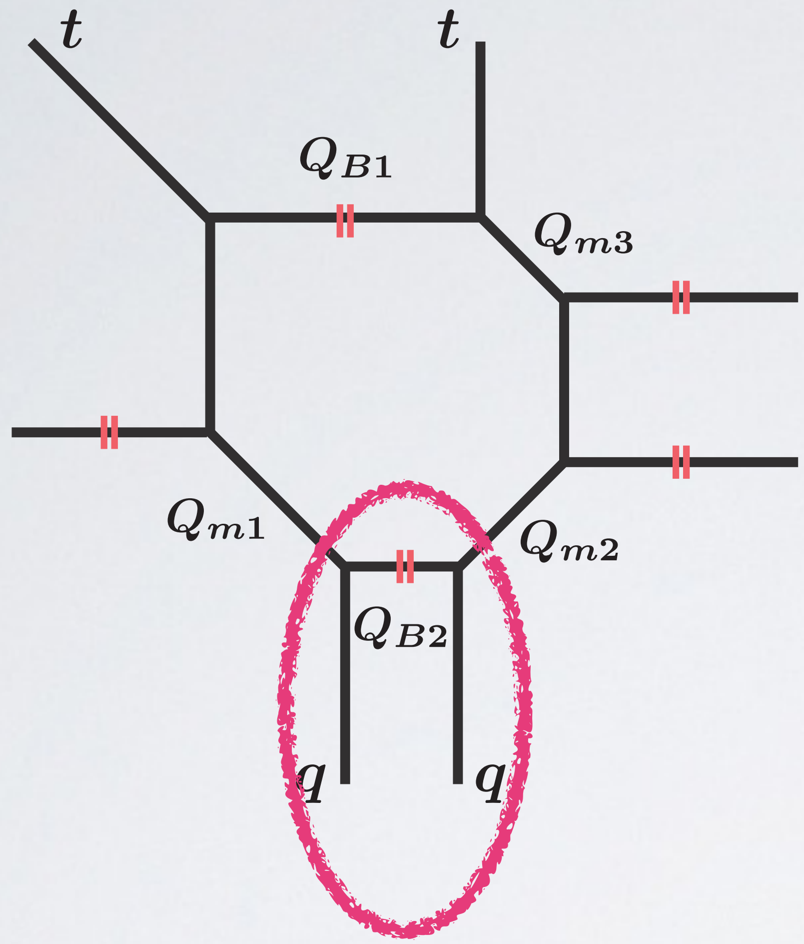


$Z_{PdP_4^I}$ $\quad$ $Z_{PdP_4^{II}}$

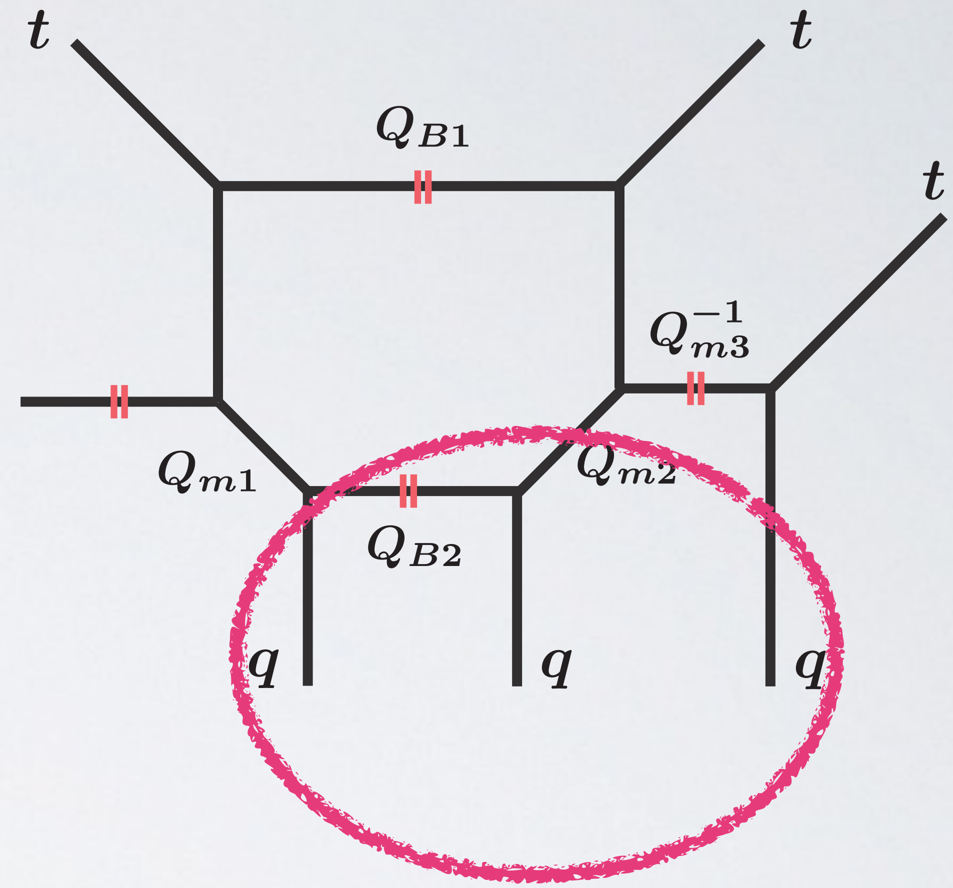
local geometries of pseudo del Pezzo surface

not generic del Pezzo

E_4 case [MT, '14]

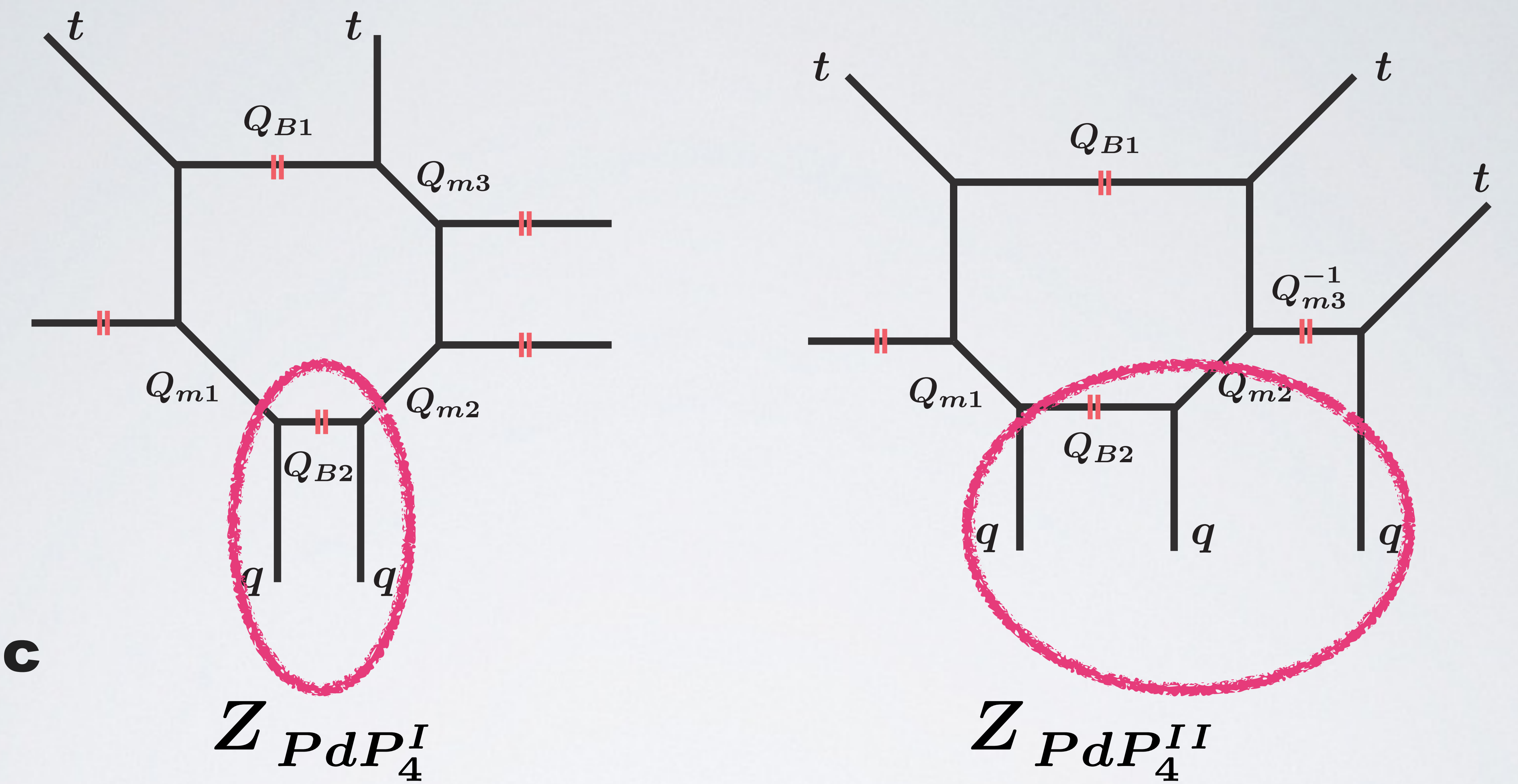


$Z_{PdP_4^I}$



$Z_{PdP_4^{II}}$

E4 case [MT, '14]



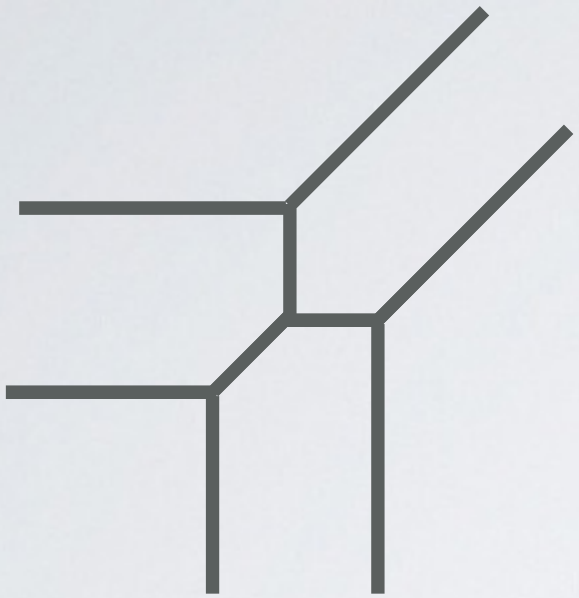
non-toric

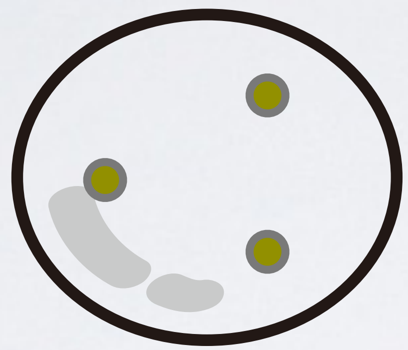
$$Z_{dP_4} = \prod_{i,j}\left(1 - \frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right) \times Z_{PdP_4^{II}}$$

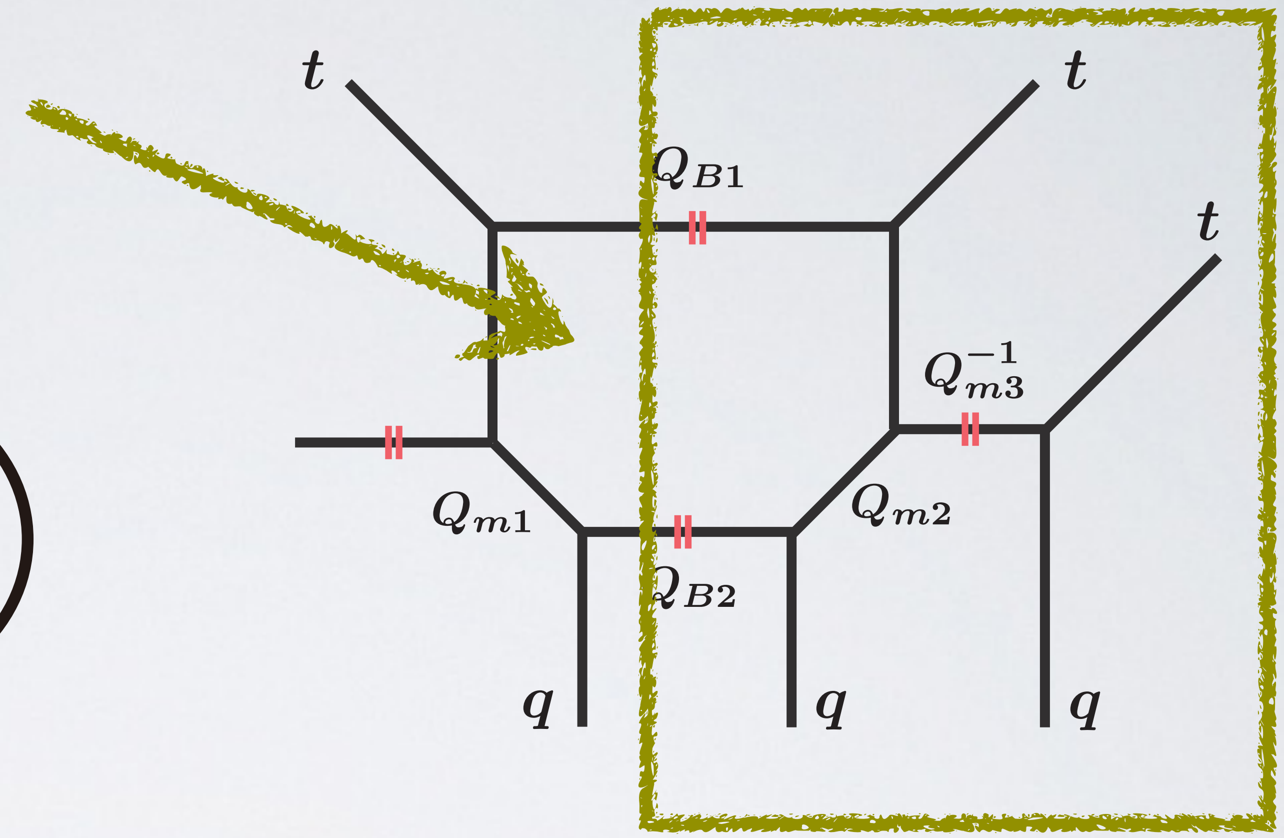
$$= \prod_{i,j}\left(1 - \frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right)\left(1 - \frac{uQ_F}{Q_{m1}Q_{m3}}q^i t^{j-1}\right) \times Z_{PdP_4^{II}}$$

E4 case [MT, '14]

(5d) Gaiotto's T2 theory







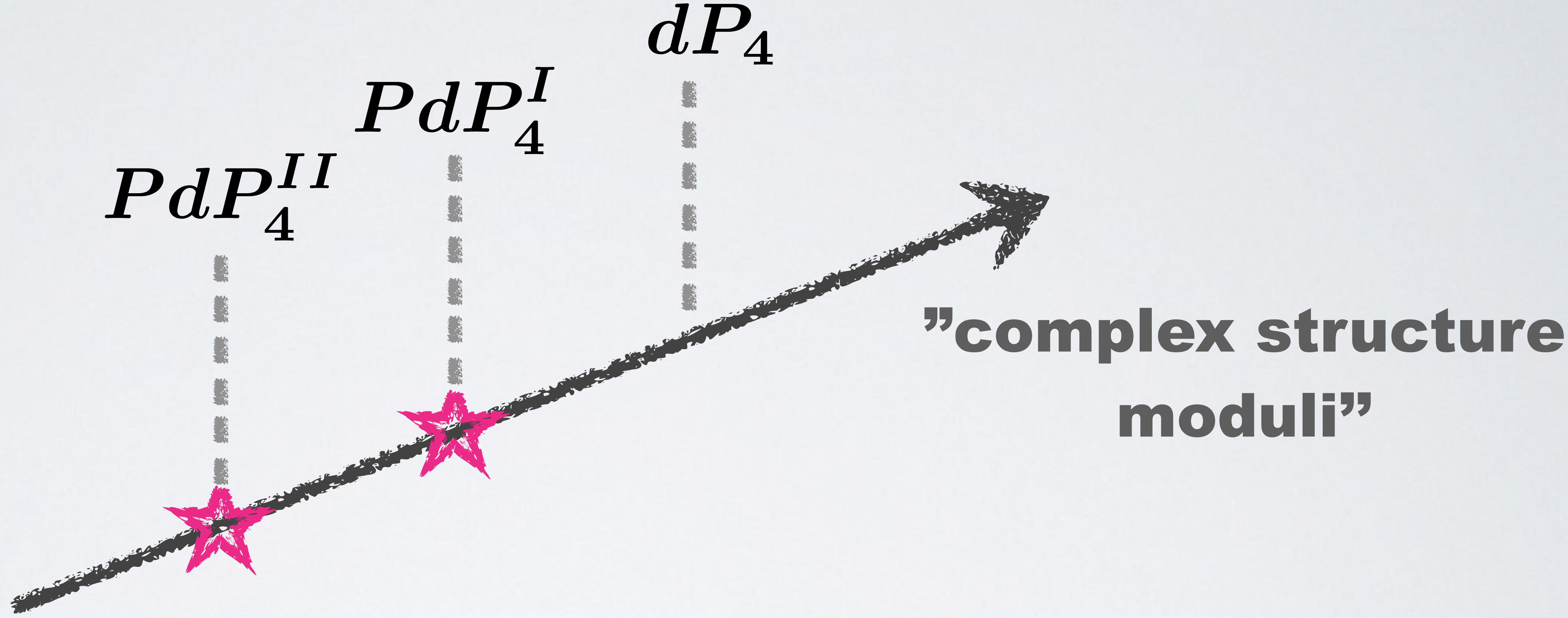
$$Z_{PdP_4^I} = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} Z^{\text{vect}}_{\vec{R}}(Q_F)\, Z^{\text{matt.}}_{\vec{R}}(Q_{m1})\, Z^{\text{matt.}}_{\vec{R}}(Q_{m2})\, Z'^{\text{matt.}}_{\vec{R}}(Q_{m3})$$

$$Z_{PdP_4^{II}} = \sum_{R_{1,2}} \left(u\frac{q}{t}\right)^{|\vec{R}|} (Q_{m2})^{|R_2|} f^{-1}_{R_2}(t,q)\, Z^{\text{vect}}_{\vec{R}}(Q_F)\, Z^{\text{matt.}}_{\vec{R}}(Q_{m1})\, Z^{T_2}_{(R_2^T, R_1^T)}(Q_{m2}, Q_F Q_{m2}^{-1}, Q_{m3}^{-1})$$

tri-fundamental like

wall-crossing!?

A-model partition function is independent of cpx str. but...

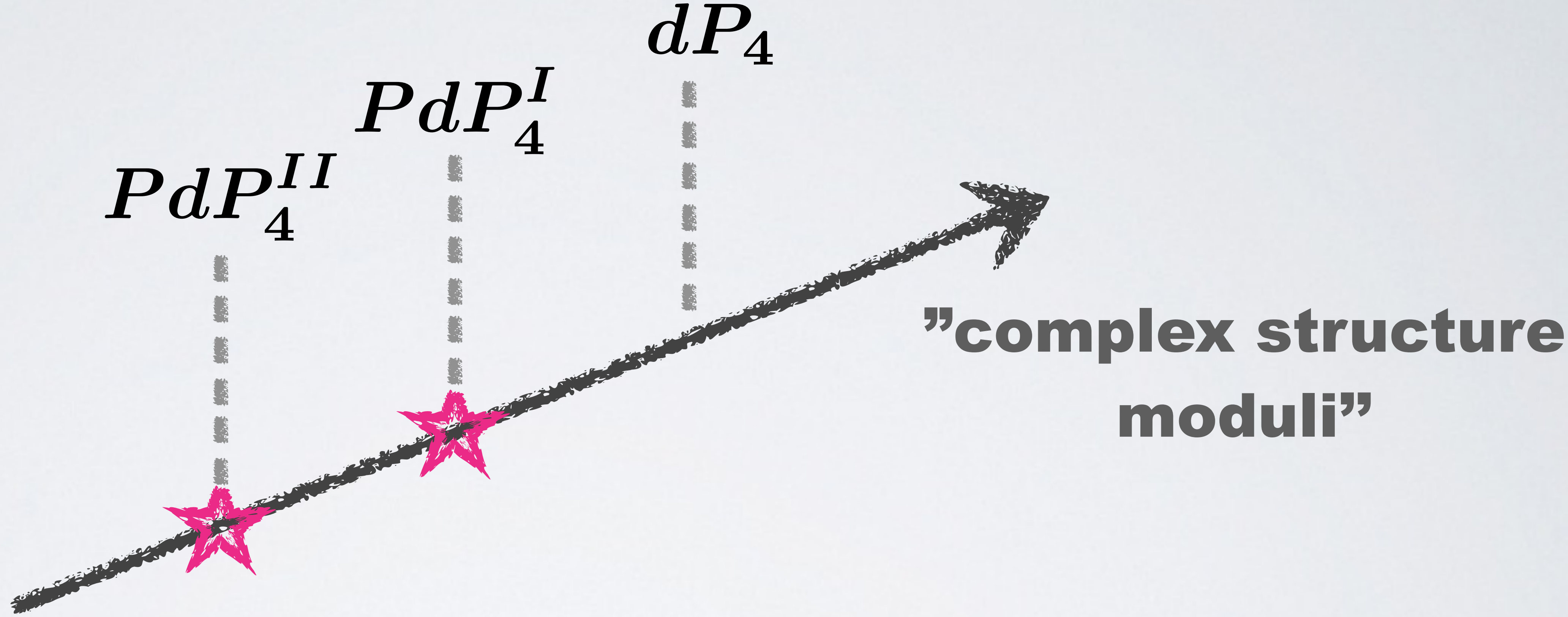


$$Z_{dP_4} = \prod_{i,j}\left(1-\frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right)\times Z_{PdP_4^{II}}$$

$$= \prod_{i,j}\left(1-\frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right)\left(1-\frac{uQ_F}{Q_{m1}Q_{m3}}q^i t^{j-1}\right)\times Z_{PdP_4^{II}}$$

wall-crossing!? or modified topological vertex!?

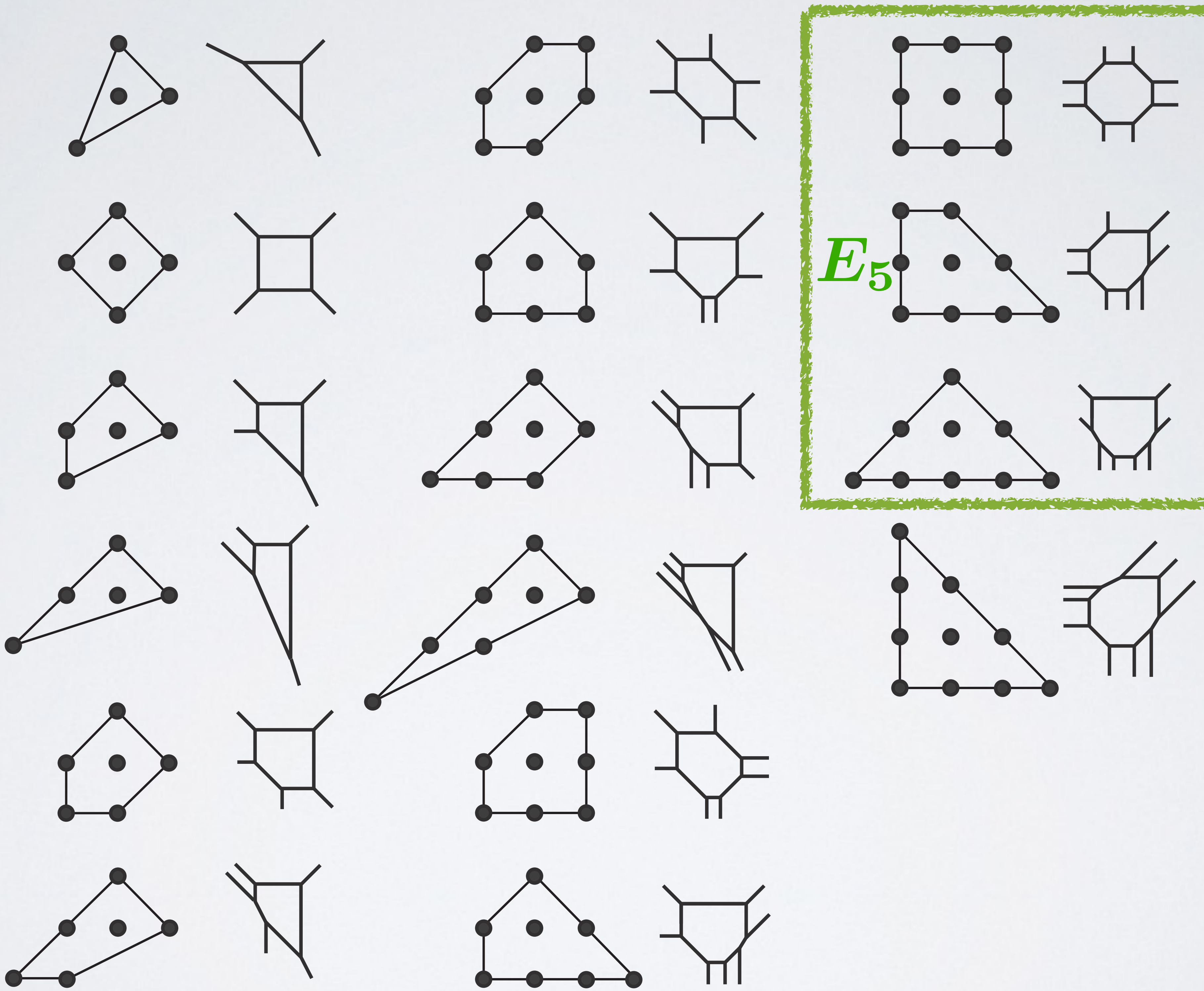
A-model partition function is independent of cpx str. but...



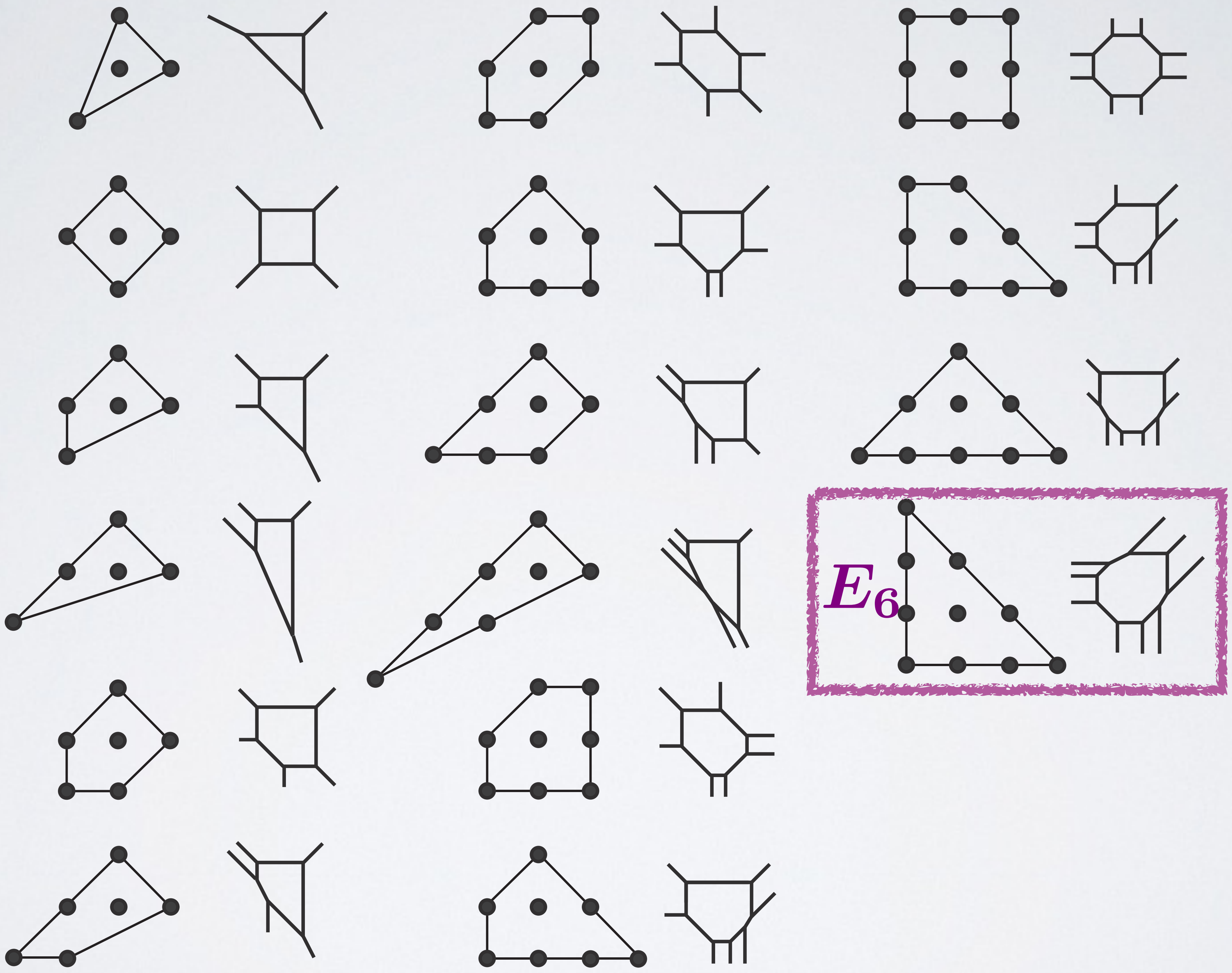
$$Z_{dP_4} = \prod_{i,j}\left(1-\frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right)\times Z_{PdP_4^{II}}$$

$$= \prod_{i,j}\left(1-\frac{uQ_F}{Q_{m1}Q_{m2}}q^i t^{j-1}\right)\left(1-\frac{uQ_F}{Q_{m1}Q_{m3}}q^i t^{j-1}\right)\times Z_{PdP_4^{II}}$$

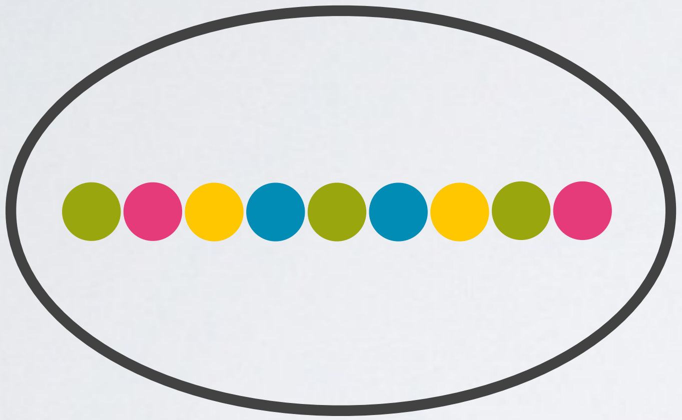
E_5 case [MT, '14]

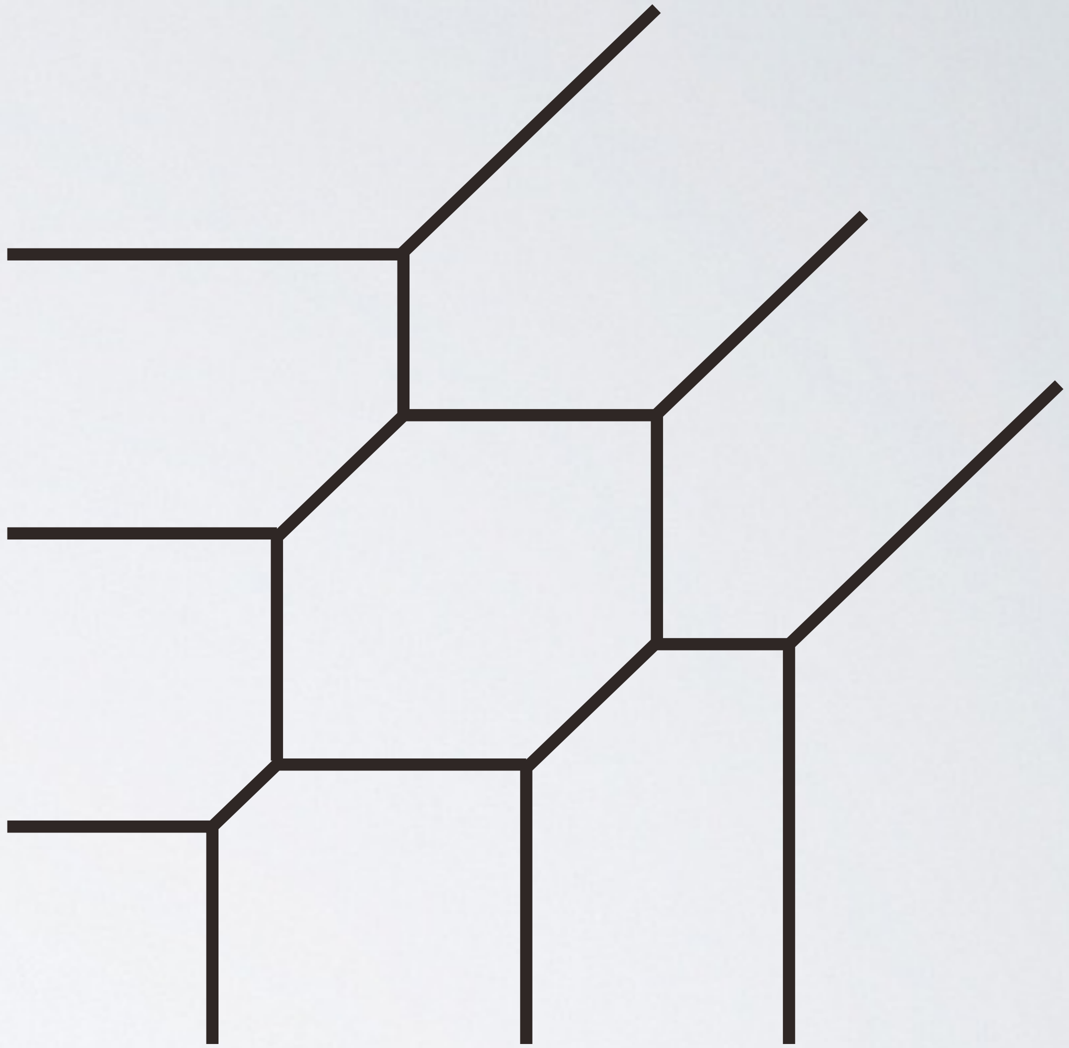


E6 case [BMPTY, '13] [HKN, '13]



E_6 case = 5d Gaiotto's T_3

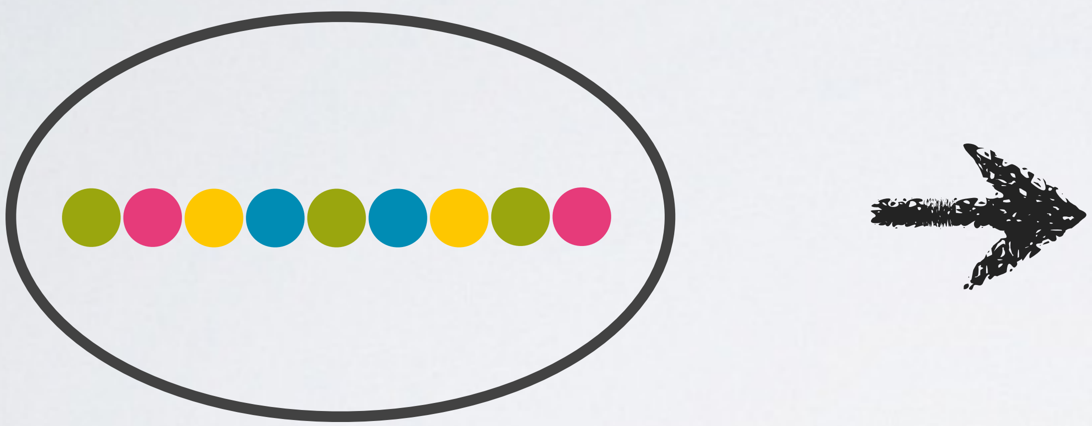


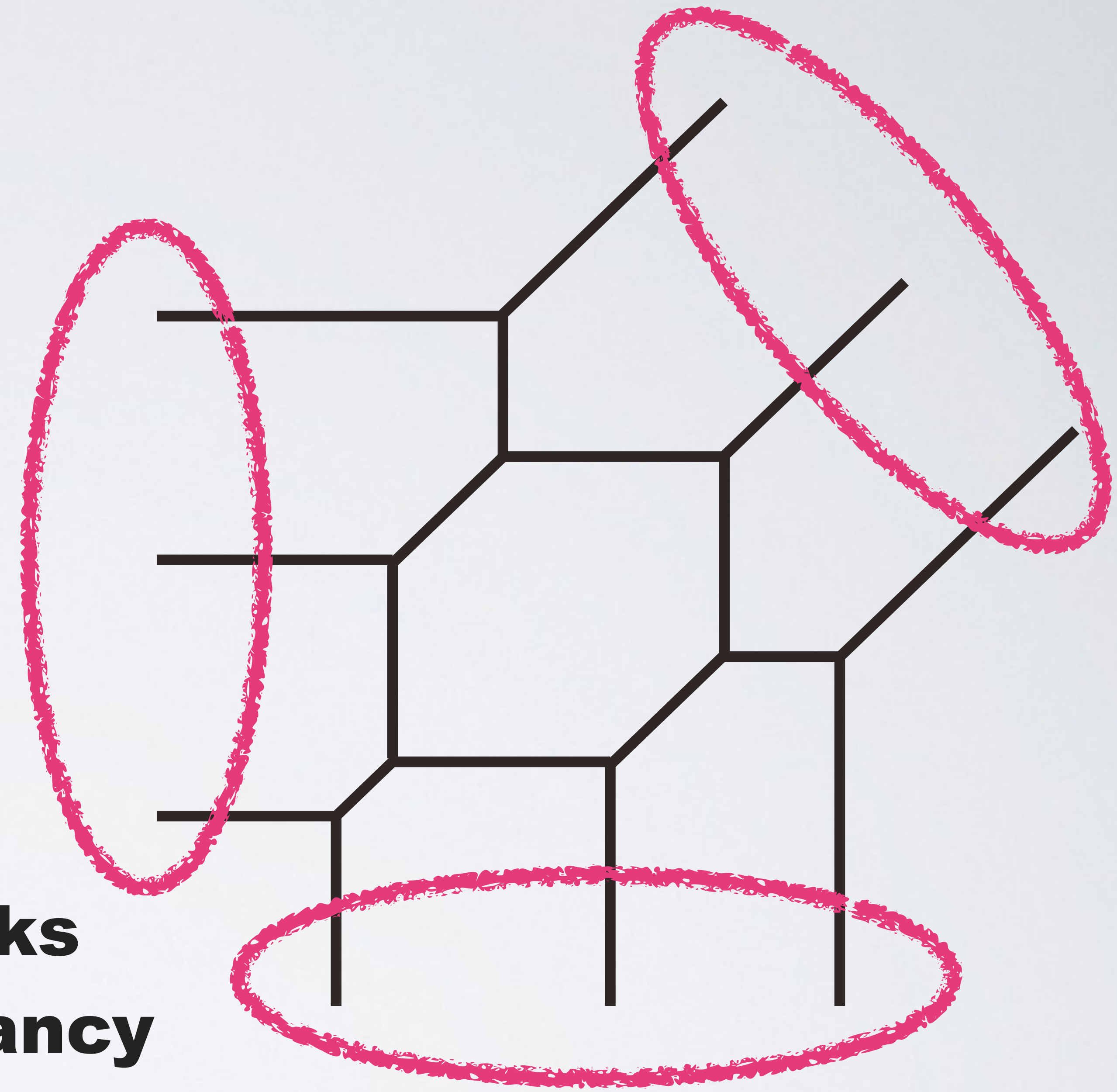


$$PdP_6 = \mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$$

[Benini-Benvenuti-Tachikawa,'09]

E_6 case = 5d Gaiotto's T_3



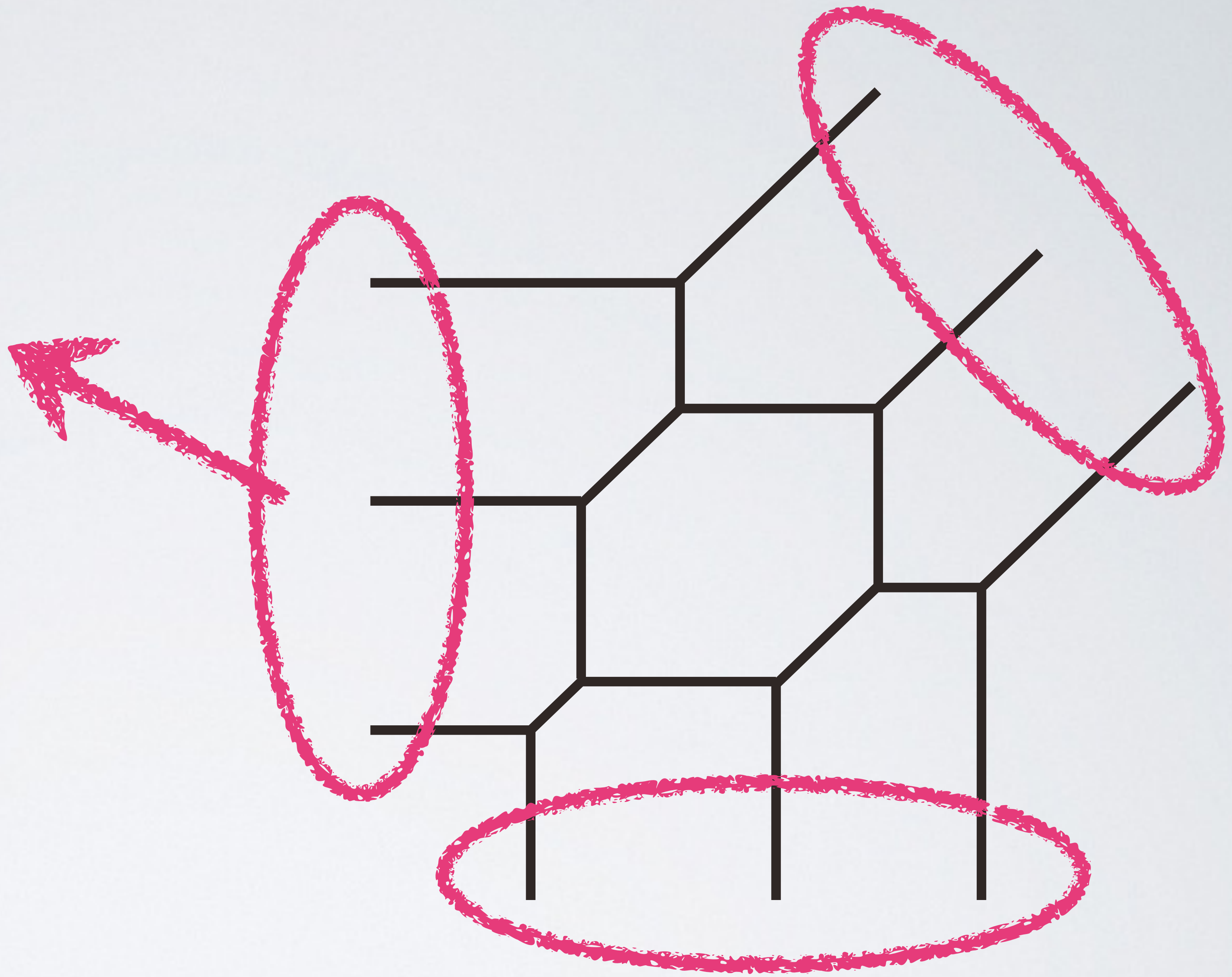


these three stacks cause the discrepancy

$$PdP_6 = \mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$$

E_6 case = 5d Gaiotto's T_3

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\text{extra}}}$$



ugly asymmetric function of masses

E_6 case

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\mathrm{extra}}}$$

$\mathbf{E_6}$ case

ugly **asymmetric** function of masses

$$Z_{dP_6} = \frac{Z_{PdP_6}}{Z_{\text{extra}}}$$

$$Z^{\text{1-inst.}}_{dP_6} = u\frac{q}{t}\frac{1}{(1-q)(1-t^{-1})(1-Q_F t^{-1} q)(1-Q_F^{-1} t^{-1} q)}$$

$$\times\Bigg[\left(1+\frac{q}{t}\right)\left(1+\sum_{f_1\neq f_2}\frac{1}{Q_{mf_1}Q_{mf_2}}+\sum_{f=1}^{5}\frac{Q_F^2 Q_{mf}}{Q_{m1}Q_{m2}Q_{m3}Q_{m4}Q_{m5}}\right)$$

$$-\sqrt{\frac{q}{t}}(1+Q_F)\left(Q_F^2+\sum_{f_1\neq f_2}\frac{Q_F Q_{mf_1}Q_{mf_2}}{Q_{m1}Q_{m2}Q_{m3}Q_{m4}Q_{m5}}+\sum_{f=1}^{5}\frac{1}{Q_{mf}}\right)\Bigg]$$

E_6 superconformal index [BMPTY] [HKN]

$$I_{dP_6} = \frac{I_{PdP_6}}{I_{\mathbf{extra}}}$$

$$= 1 + \chi^{E_6}_{\mathbf{78}} x^2 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_1(y) x^3$$

$$+ (1 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_2(y) + \chi^{E_6}_{\mathbf{2430}}) x^4 + \cdots$$

agrees with [Kim-Kim-Lee]'s E_6 index !!

5. Summary

&

Open Problems

we revisit 5d SCFT from the perspective of 7-branes & modified Nekrasov function.

→ new duality

topological vertex for non-toric CY

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Proof our conjecture on Nekrasov function ?

$$Z_{\text{delPezzo}} = Z_{\text{toric delPezzo}} \div Z_{\text{extra}}$$

Relation to 5d AGT [Awata-Yamada] ?

Relation to E-string [Sakai] ?

5d SCFT with higher dim. Coulomb branch ?
[work in progress]

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FIN