重力波観測における非一様宇宙の検証

関西相対論・宇宙論合同セミナー
@ 梅田

2011 4/19 (火)

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§ 0 Outline

§ 1 Indroduction

§ 2 LTB Model & z-drift

§ 3 GW observations

§ 4 Gravitational waveforms from compact binaries

§ 5 Constraining Inhomogeneous Universe with GW observations

§ 6 Discussions & Summary

§1
INTRODUCTION

§ 1 Introduction

1-1 Acceleration of our Universe

- Cosmological observations

⇒ **accelerating expansion** of our Universe!

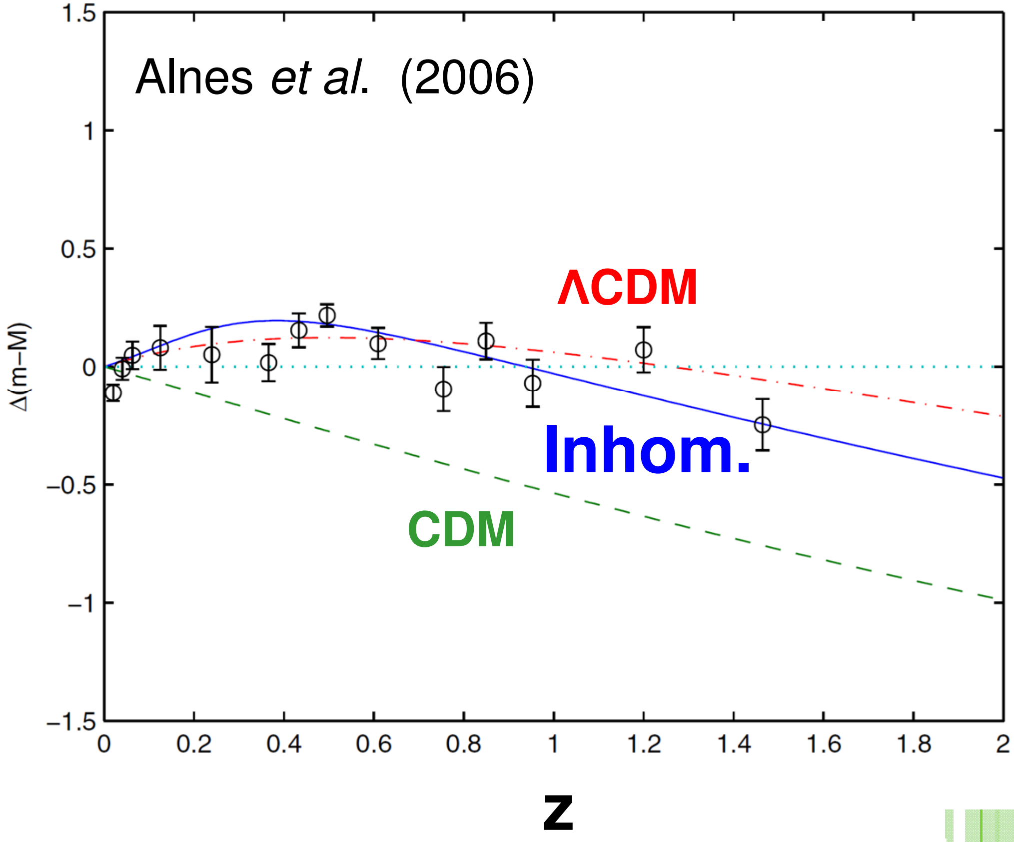
- homogeneous & isotropic

$$\underset{②}{\underline{G_{\mu\nu}}} = \frac{8\pi G}{c^4} \underset{①}{\underline{T_{\mu\nu}}}$$

① **G.R.**+ **Dark Energy**

② **Modified Gravity**

TypeIa SN observation



③ **Inhomogeneous: G.R., no Dark Energy needed.**

1-2 Inhomogeneous Universe

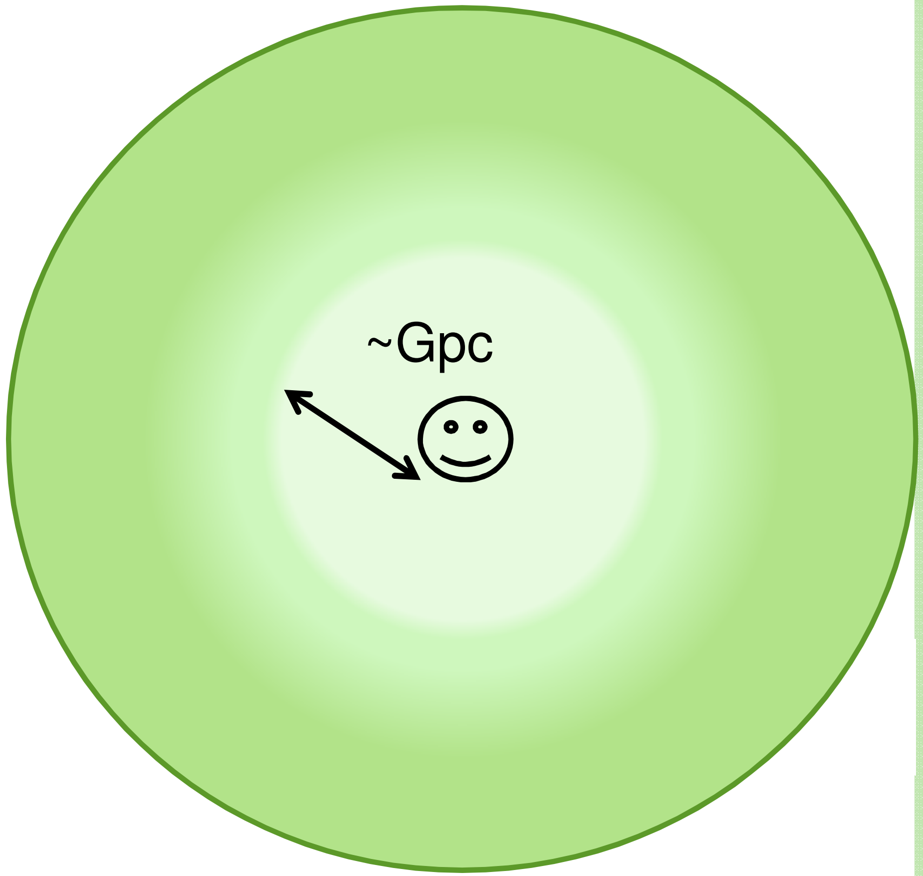
- Simplest example of the **inhomogeneous** universe model
- Spherically symmetric, dust, inhomogeneous (**LTB** model)

- We live at the center of **void** (less dense region).

Density gradient

↓

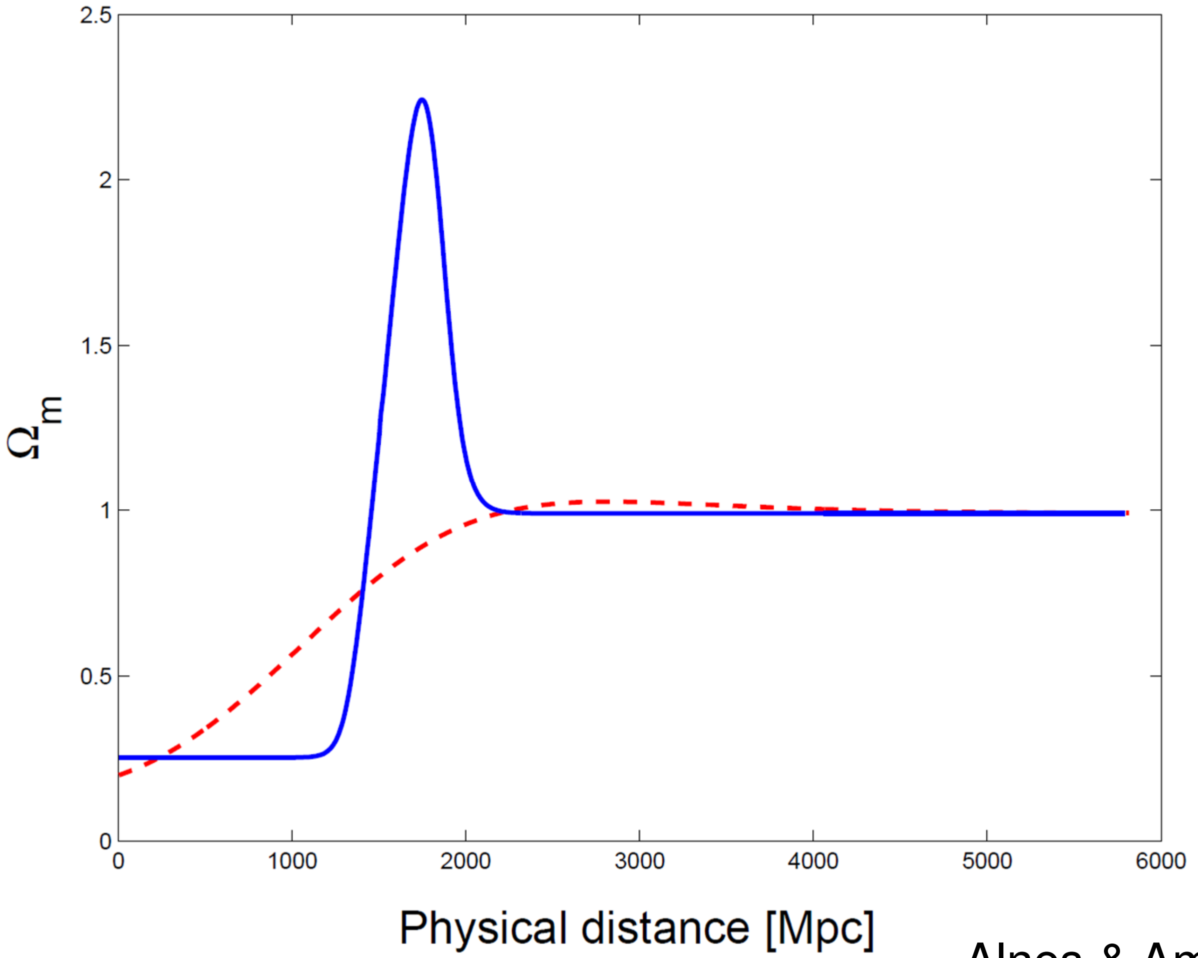
- Apparent accelerating expansion



We consider 2 types of LTB void models

- **monotonic LTB Void**・・・Density increasing monotonically
- **general LTB Void**・・・Any void density profile is OK

e.g.



Alnes & Amarzguioui (2006)

How well can LTB void model explain current observations?

- Strong constraints have already been put on some models.

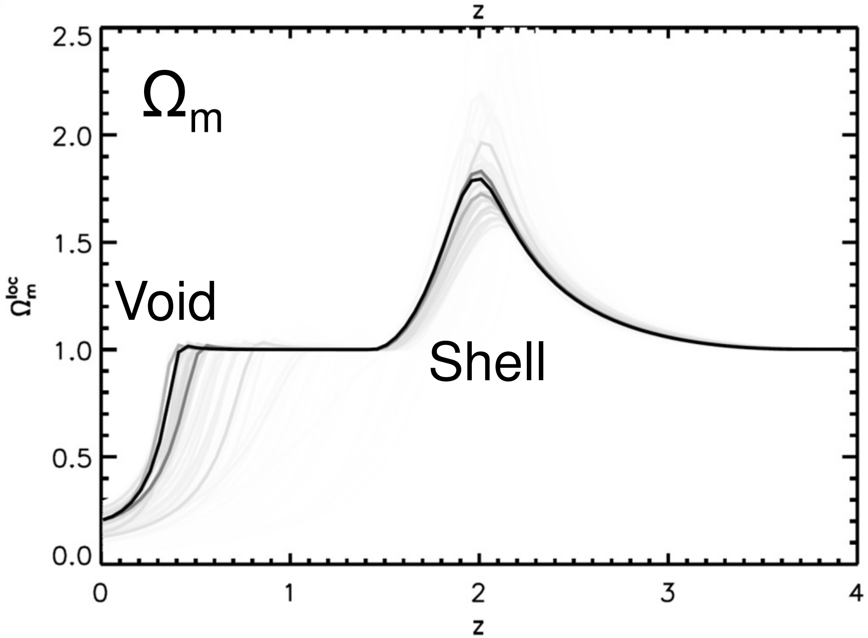
Precision cosmology defeats void models for acceleration

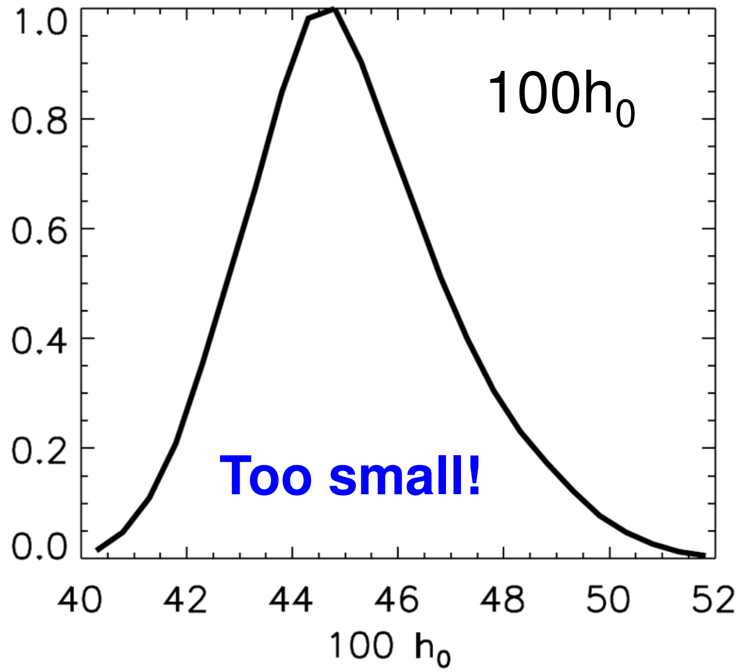
Adam Moss,* James P. Zibin,† and Douglas Scott‡
Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, V6T 1Z1 Canada
(Dated: March 28, 2011)

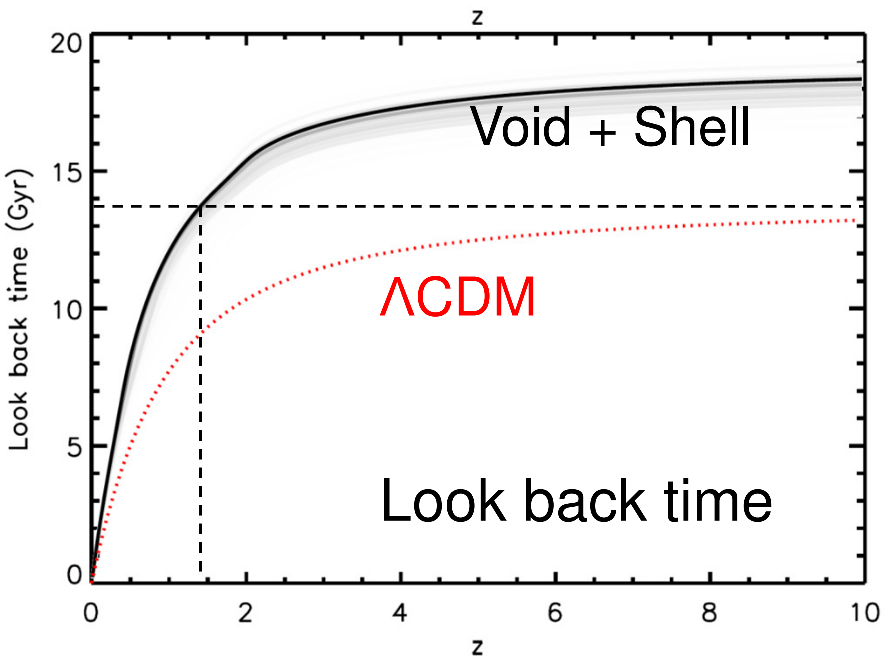
The suggestion that we occupy a privileged position near the centre of a large, nonlinear, and nearly spherical void has recently attracted much attention as an alternative to dark energy. Putting aside the philosophical problems with this scenario, we perform the most complete and up-to-date comparison with cosmological data. We use supernovae and the *full* cosmic microwave background spectrum as the basis of our analysis. We also include constraints from radial baryonic acoustic oscillations, the local Hubble rate, age, big bang nucleosynthesis, the Compton y-distortion, and for the first time include the local amplitude of matter fluctuations, σ_8. These all paint a consistent picture in which voids are in severe tension with the data. In particular, void models predict a very low local Hubble rate, suffer from an "old age problem", and predict much less local structure than is observed.

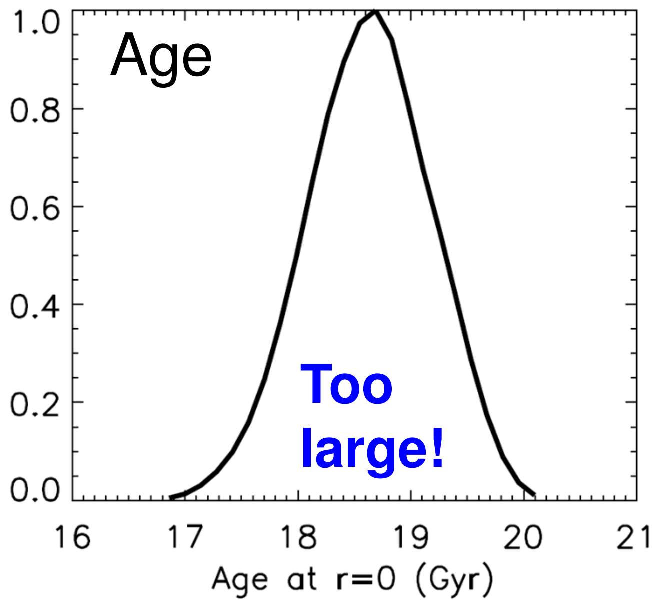
arXiv:1007.3725

▪Best fit results for CMB + SN data









- Moss *et al*. assumed the primordial power spectrum of the form

$$\ln \mathcal{P}_S(k) = \ln A_S + (n_S - 1)\ln(k/k_0) + \frac{n_{run}}{2}\left[\ln(k/k_0)\right]^2$$

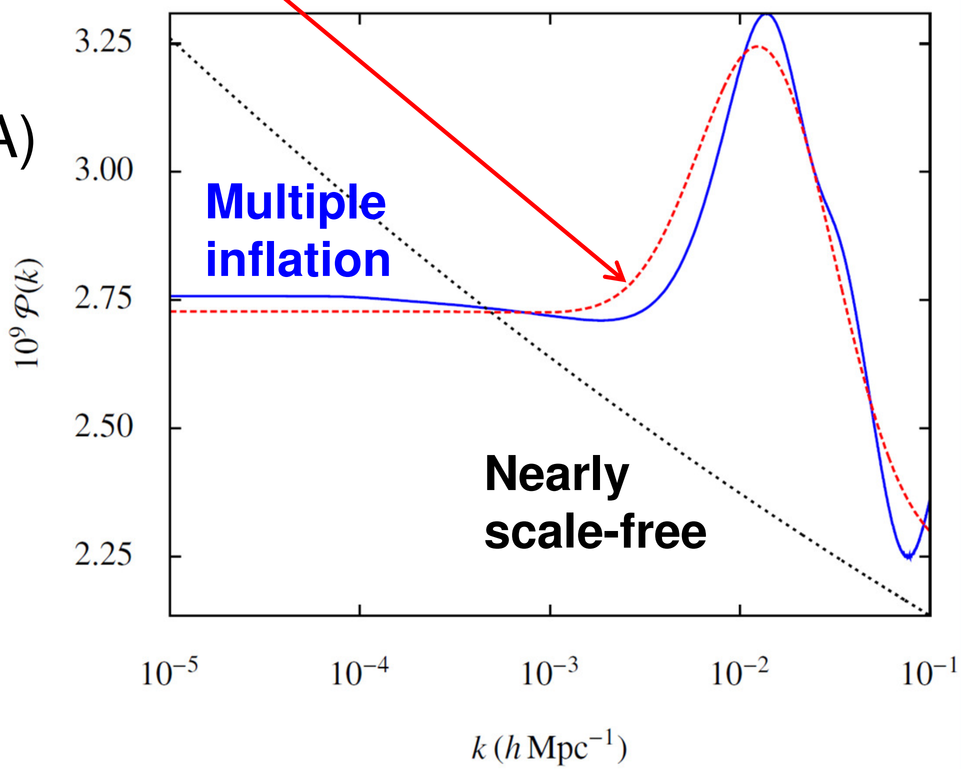
with best fit parameter n_s~0.94 & n_{run}~-0.05 (nearly scale-free)

- Can we fit the data with other ansatz? Nadathur & Sarkar (2011)

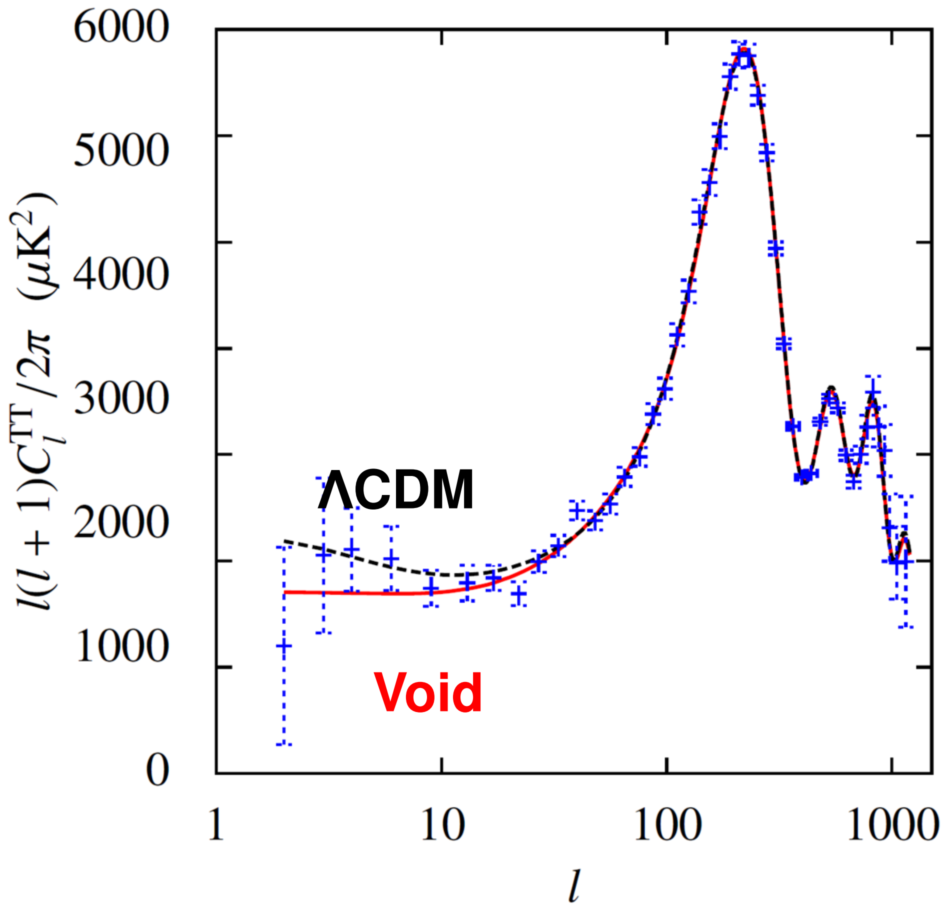
$$\mathcal{P}(k) = \mathcal{P}_0\left(1 - a\tanh(bx) + ce^{-(bx)^2}\right)$$

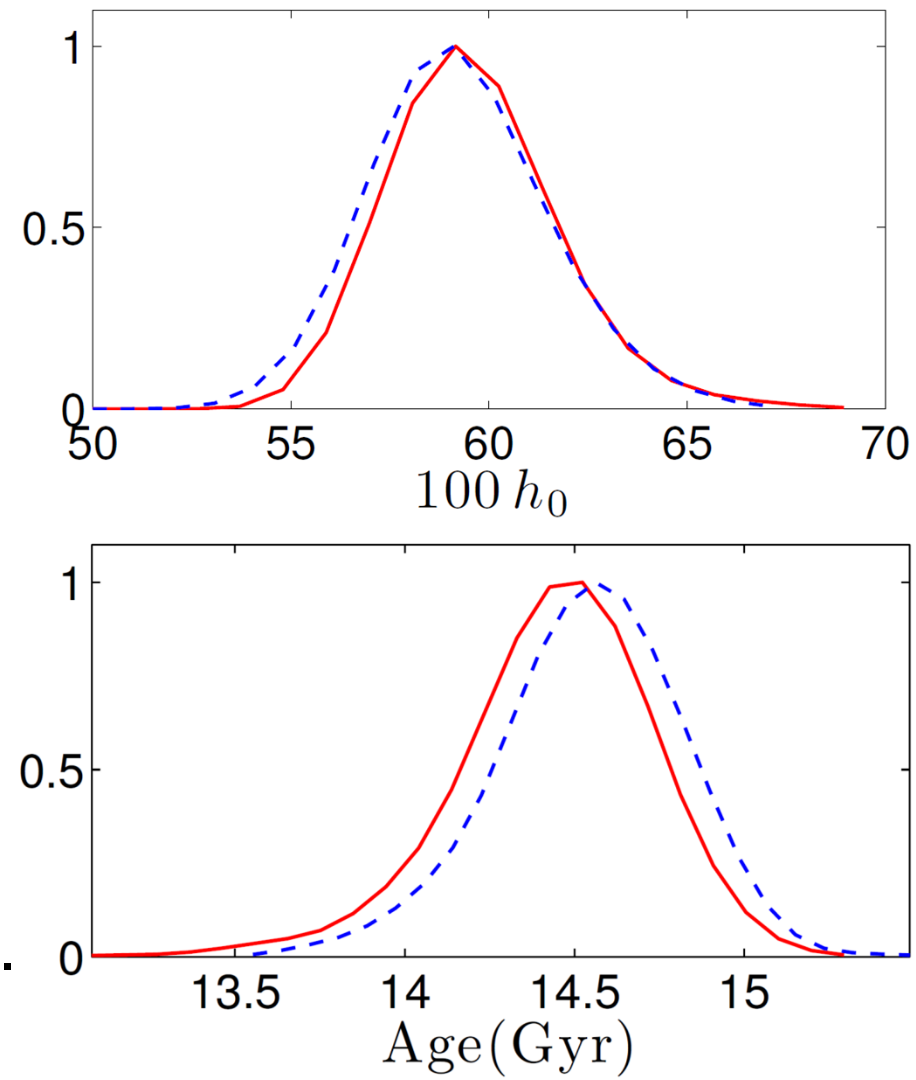
Bumpy primordial power spectrum

e.g. multiple inflation (from SUGRA)
DBI inflation



・Best fit results for CMB + SN data





・much better than Moss et al. results.

・Also consistent with BAO & BBN.

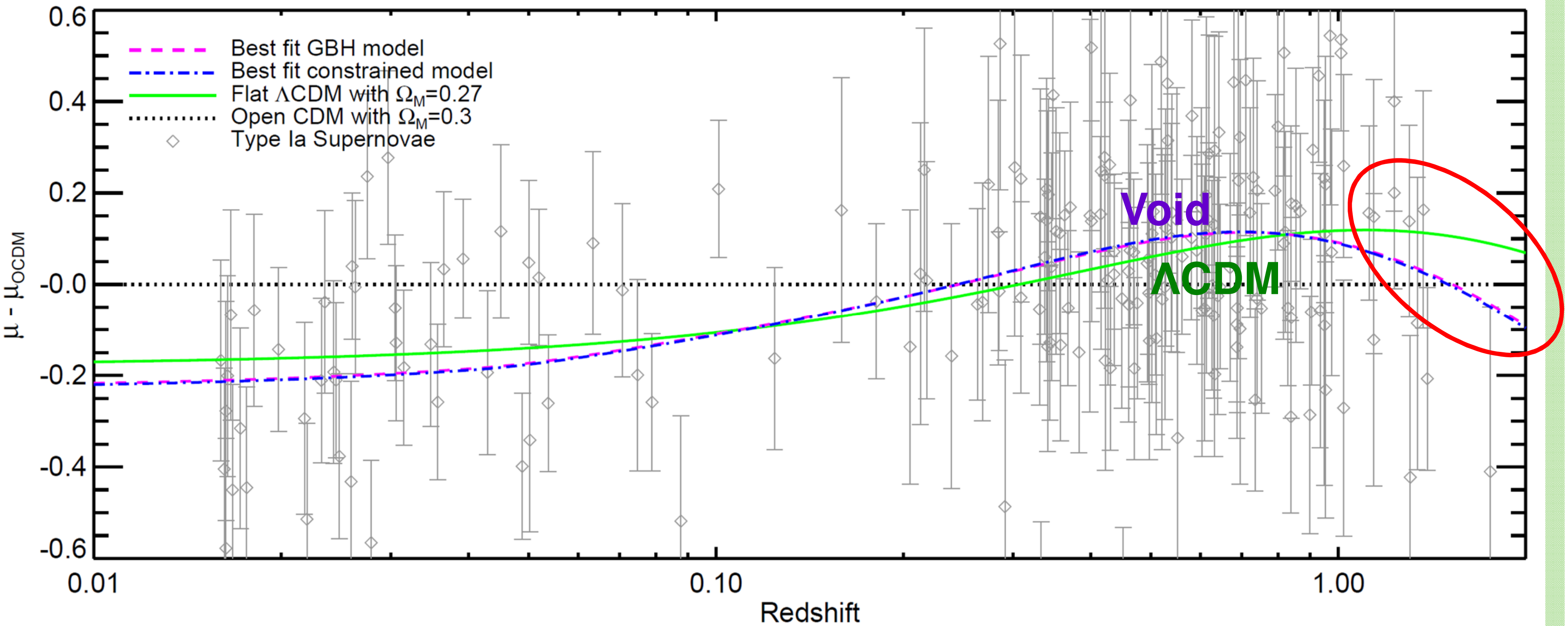
・We need **observables** that can **probe the LTB void model more clearly**!

Candidate observables that can test LTB better

- radial BAO, SZ・・・depends on primodial power spectrum

  incomplete LTB cosmological perturbations

- D_L-z relation at high z・・・SN, GRB, GW



Garcia-Bellido & Haugboelle (2008)

- **z-drift**

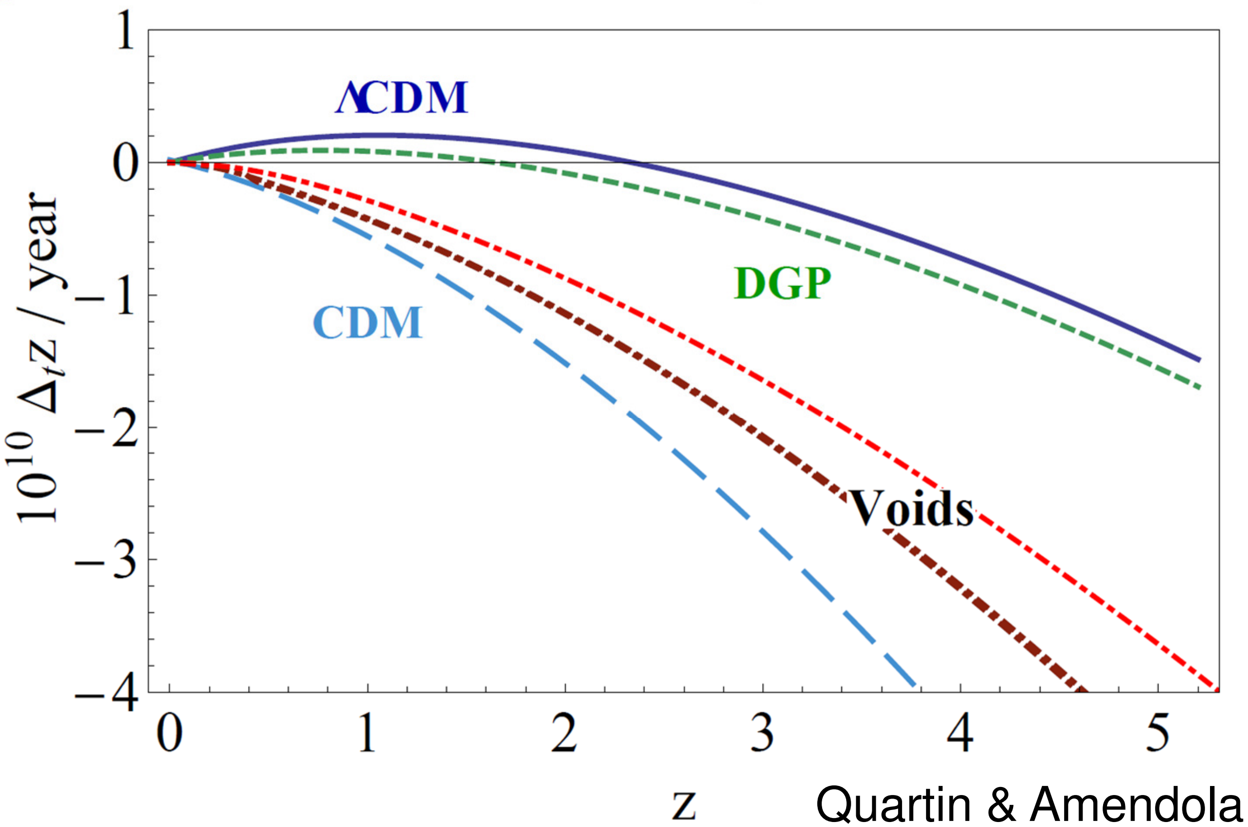
1-3 Redshift Drift $\Delta_t z$

▪Time evolution of the **redshift *z*** Loeb (1998)

z-drift reflects the accelerating expansion more directly!

$$\Delta_t z = H_0 \Delta t_0 \left(1 + z - \frac{H(z)}{H_0}\right) \quad \text{(FLRW)}$$

$$\sim \frac{\text{obs. time}}{\text{cosmic age}} \sim 10^{-10}$$



Quartin & Amendola (2010)

Monotonic LTB Void model

$\Rightarrow \Delta_t z < 0$ for $^{\forall}z>0$

General LTB Void model

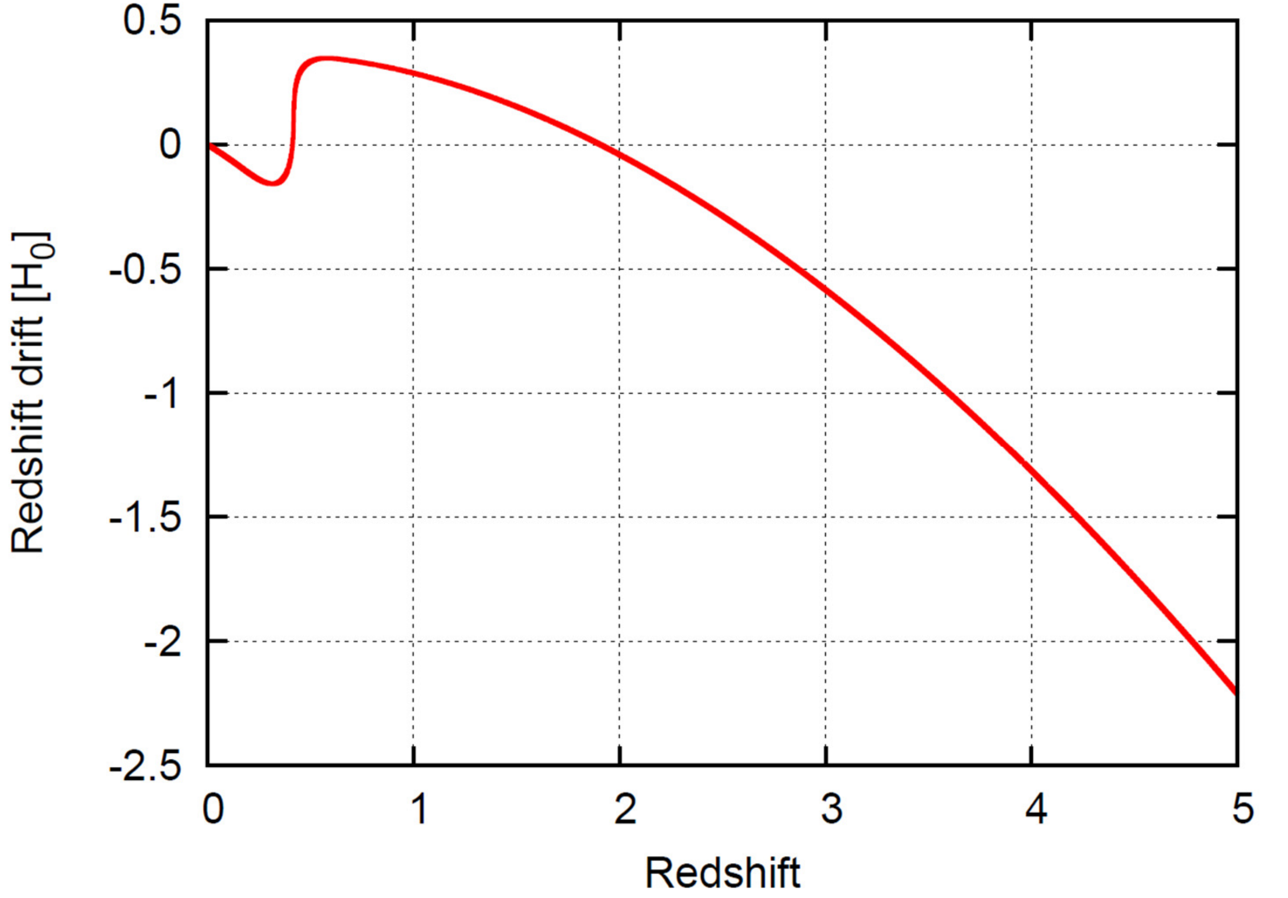
$\Rightarrow \frac{d}{dz}\Delta_t z < 0$ at z=0

Kai, Nakao & Yoo (2010)

We can make $\Delta_t z > 0$ for general LTB void model but they conflict with current observations

(too large inhomogenities).

(Kai, Nakao & Yoo 2010)



1-4 z-driftの観測

・**Quasor**スペクトルの**Lyα**の吸収線(**Lyα forest**)からz-driftを測る。

European Extremely Large Telescope (E-ELT)

・口径**42m**

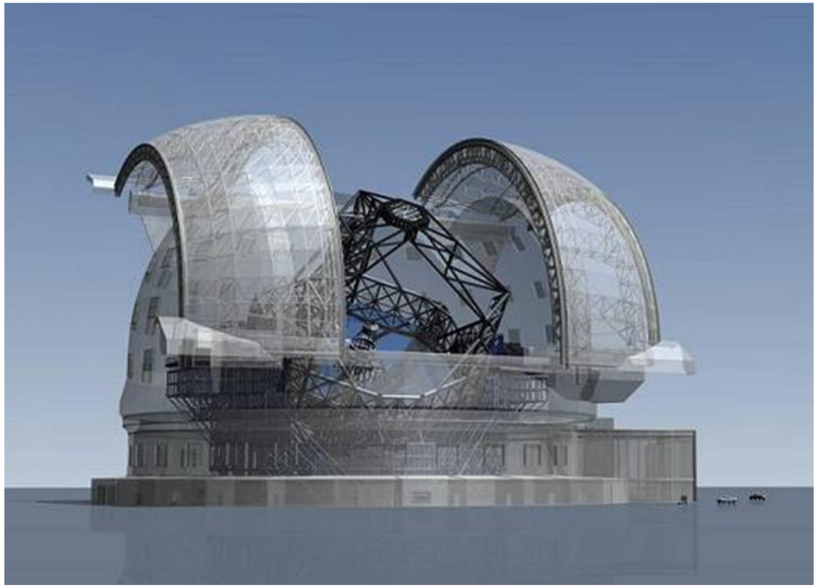
・建設予定地:アタカマ (チリ)

・最大の可視・赤外望遠鏡

・現在のものよりも100倍感度良い

・**2018年**operation開始予定

（DECIGOより9年も早い！）



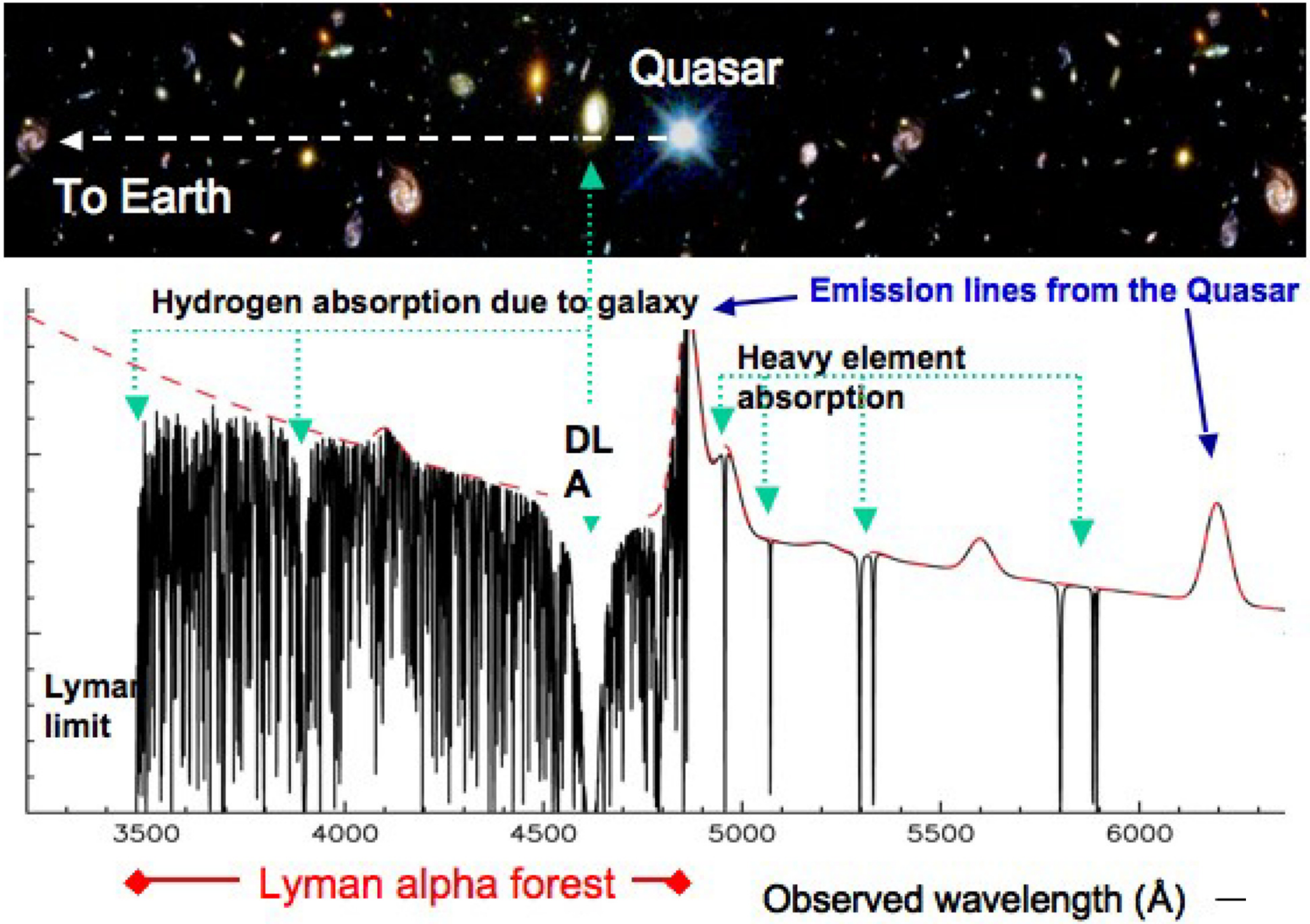
http://www.eso.org/sci/facilities/eelt/

COsmic Dynamics and EXo-earth experiment (CODEX)

・E-ELTに搭載予定の

optical, very stable, **high spectral resolution instrument**

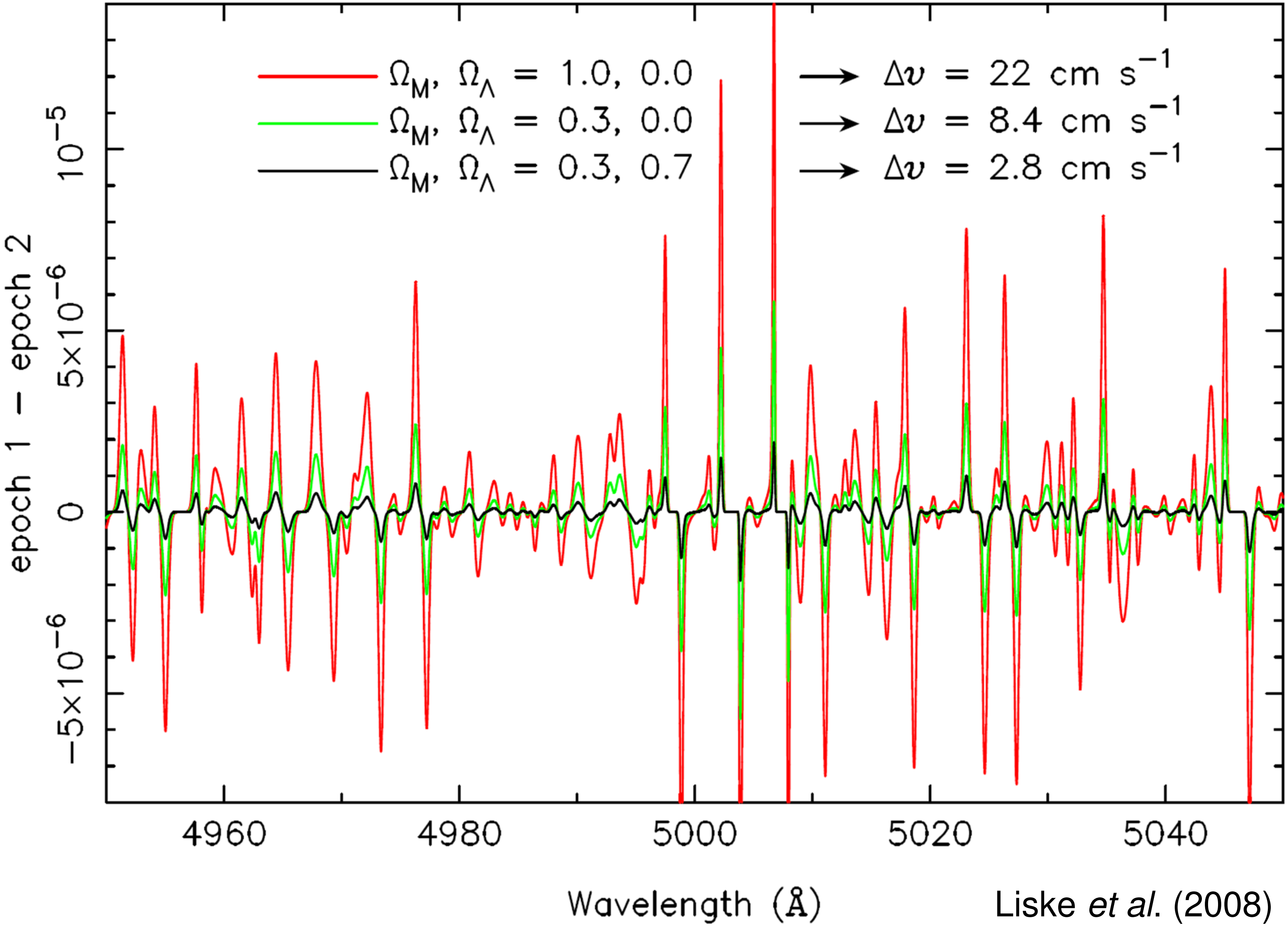
QuasarスペクトルのLyα forest



E-ELT PROGRAMME CODEX Phase A Report
Haehnelt et al. (2010)

10年観測した場合のスペクトルの変化

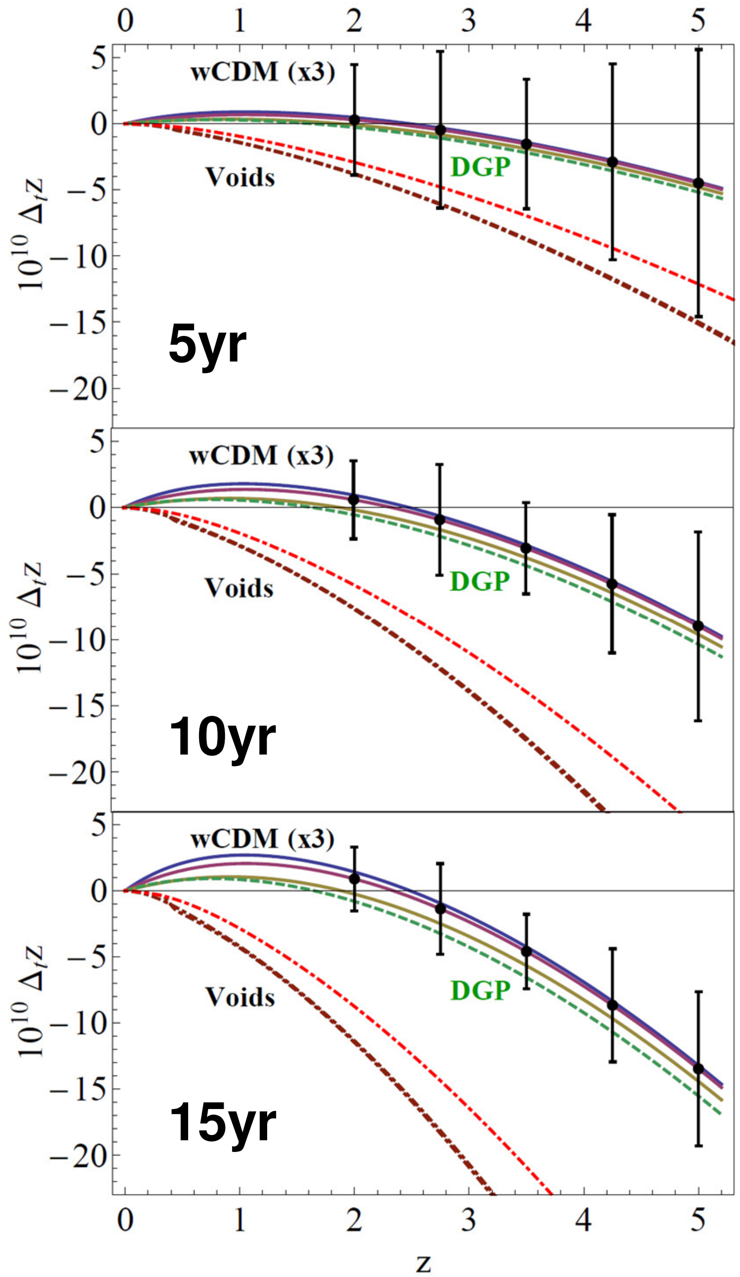
$$\Delta v = \frac{c\Delta_t z}{1+z}$$



Liske *et al.* (2008)

1-5 CODEXによるz-drift観測とLTBモデルの検証

Quartin & Amendola (2010)



・ΛCDMが正しい場合、CODEXでのz-driftの測定精度を計算

5 years	10 years	15 years
$.5\sigma$	4.3σ	9.2σ
$\chi^2 = 3.7$	$\chi^2 = 30$	$\chi^2 = 100$

・ΛCDMが正しい場合、**10年観測**すれば、**4σ以上で典型的なVoidモデルを棄却**できる!

・**low z**でもz-driftを観測したい

(low zの方がLTBを良くprobeできる)

⇒ **GW**!!

§2
LTB MODEL
& Z-DRIFT

§2-1 LTBモデル

・球対称、ダスト、非一様

$$ds^2 = -dt^2 + \frac{[R'(t,r)]^2}{1-k(r)}dr^2 + R^2(t,r)d\Omega^2$$

FRW: $R(t,r) = a(t)r \qquad k(r) = kr^2$

・Einstein Eqs.:

$$\begin{cases} \dfrac{\dot{R}^2 + k(r)}{R^2} + 2\dfrac{\dot{R}\dot{R}'}{RR'} + \dfrac{k'(r)}{RR'} = 8\pi\rho_m & \cdots ① \\ \dot{R}^2 + 2R\ddot{R} + k(r) = 0 & \cdots ② \end{cases}$$

②をtで1回積分：

$$H^2 = \frac{F(r)^3}{R^3} - \frac{k(r)^2}{R^2} \quad \cdots ③$$

matter　曲率

$$H(t,r) \equiv \frac{\dot{R}(t,r)}{R(t,r)}$$

$F(r)$は任意関数

これと①より：

$$\frac{F'(r)}{R'R^2} = 8\pi\rho_m$$

$F(r)$はmassに相当

未知関数はk(r), F(r), $R_0(r)(=R(t=t_0,r))$の3つ。

このうち1つはゲージで消せる。

$$R(t=t_0,r)=r$$

§ 2-2 Redshift Drift

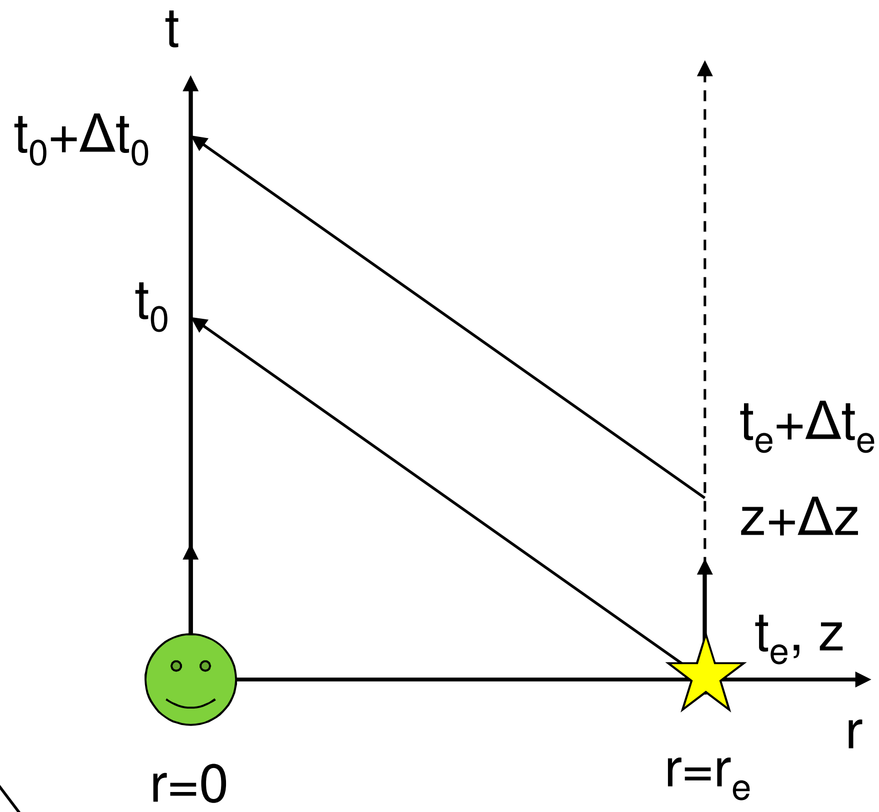
- Time evolution of the **redshift *z***

z-drift reflects the accelerating expansion more directly!

FLRW

$$\begin{cases} z(t_0) = \dfrac{a(t_0)}{a(t_e)} - 1 \\ z(t_0+\Delta t_0) = \dfrac{a(t_0+\Delta t_0)}{a(t_e+\Delta t_e)} - 1 \end{cases}$$

$$\Delta_t z = z(t_0+\Delta t_0) - z(t_0) \simeq \Delta t_0 \left(\frac{\dot{a}(t_0) - \dot{a}(t_e)}{a(t_e)} \right)$$



Rewriting the equation by using H(z)

$$\Delta_t z = H_0 \Delta t_0 \left(1 + z - \frac{H(z)}{H_0} \right) \sim \frac{\text{obs. time}}{\text{cosmic age}} \sim 10^{-10}$$

$$\begin{cases} \Delta_t z > 0 \Rightarrow \text{accelerating exp.} \\ \Delta_t z < 0 \Rightarrow \text{deccelerating exp.} \end{cases}$$

LTBの場合

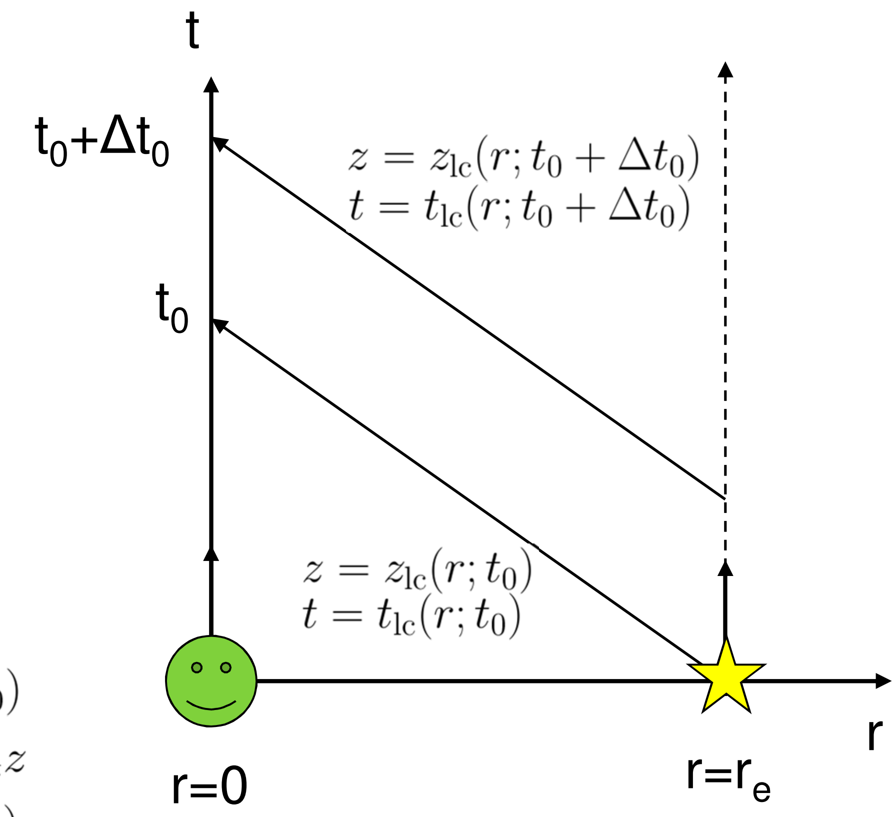
Kai, Nakao & Yoo (2010)

$$ds^2 = d\Omega^2 = 0 \quad \text{(null)}$$

$$\Rightarrow \frac{dt}{dr} = -\frac{R'(t,r)}{\sqrt{1-k(r)}}$$

$$+ \; 1+z = \frac{dt_0}{dt_e} \quad \text{(def. of z)}$$

$$\Rightarrow \frac{dz}{dr} = \frac{(1+z)\dot{R}'(t,r)}{\sqrt{1-kr^2}}$$



2 null rays:

$$\begin{cases} z = z_{\rm lc}(r; t_0) \\ t = t_{\rm lc}(r; t_0) \end{cases}$$

$$\begin{cases} z = z_{\rm lc}(r; t_0 + \Delta t_0) \\ \quad = z_{\rm lc}(r; t_0) + \Delta_t z \\ t = t_{\rm lc}(r; t_0 + \Delta t_0) \\ \quad = t_{\rm lc}(r; t_0) + \Delta t \end{cases}$$

Substitute these into the equation above and differentiate

$$\Rightarrow \frac{d}{dr}\Delta_t z = \frac{\dot{R}'}{\sqrt{1-kr^2}}\Delta_t z + \frac{(1+z)\ddot{R}'}{\sqrt{1-kr^2}}\Delta t$$

$$\frac{d}{dr} = \frac{dz}{dr}\frac{d}{dz} = \frac{(1+z)\dot{R}'}{\sqrt{1-kr^2}}\frac{d}{dz}$$

Change r der. to z der.

$$\frac{d}{dz}\left(\frac{\Delta_t z}{1+z}\right) = \frac{1}{(1+z)^2}\frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R}\Delta t_0$$

z-drift が従う微分方程式

Behaviour near the center:

$$k(r) \approx k_0$$

$$F(r) \approx \frac{4\pi}{3}\rho_0 r^3$$

$$\begin{cases} \partial_t \partial_r R|_{t=t_0, r=0} = H_0, \\ \partial_t^2 \partial_r R|_{t=t_0, r=0} = -\frac{4\pi G \rho_0}{3} = -\frac{1}{2}\Omega_{m0} H_0^2, \end{cases}$$

$$\frac{\Delta_t z}{\Delta t_0} \approx -\frac{1}{2}\Omega_{m0} H_0 z$$

for z~0

Negative!

for general LTB void

Redshift Drift $\Delta_t z$

- Time evolution of the **redshift *z***

Loeb (1998)

z-drift reflects the accelerating expansion more directly!

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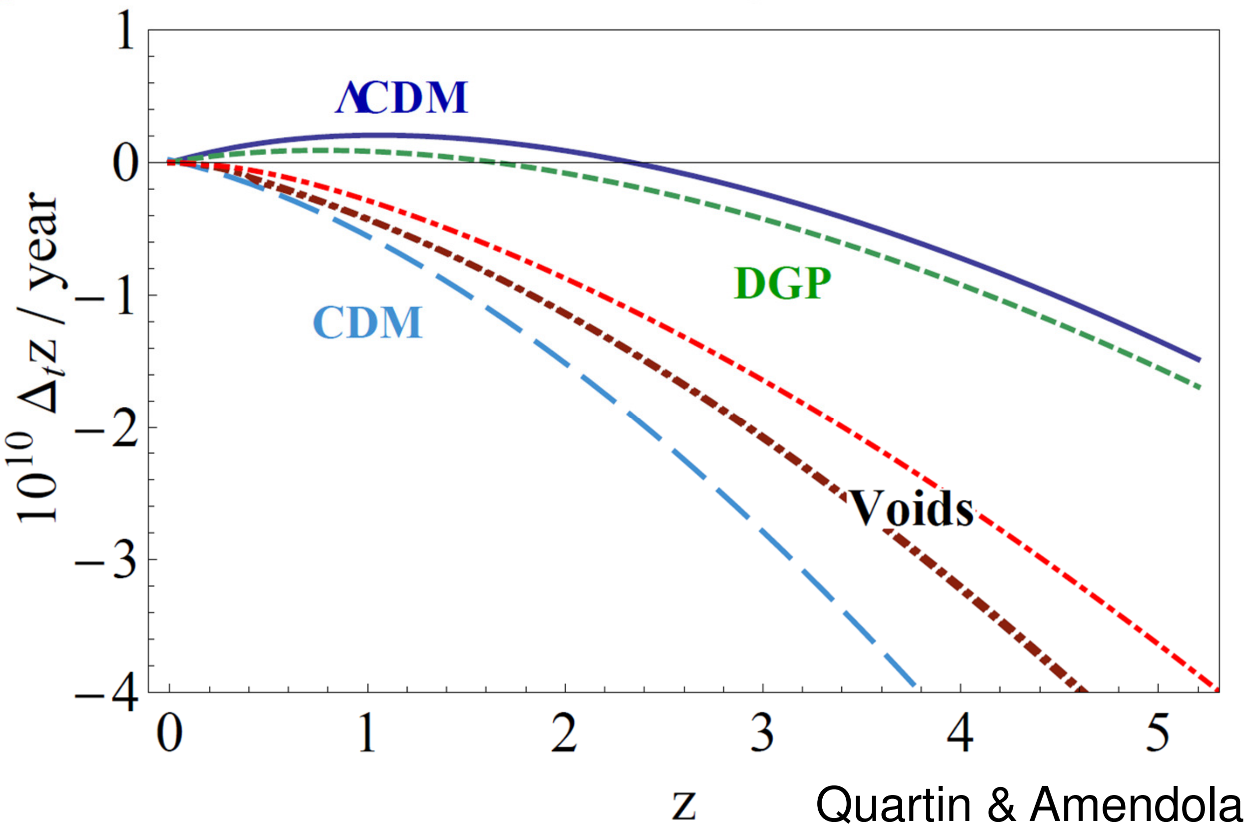

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$\Rightarrow \Delta_t z < 0$ for $^\forall z > 0$

General LTB Void model

$\Rightarrow \frac{d}{dz}\Delta_t z < 0$ at z=0

Kai, Nakao & Yoo (2010)

Monotonic LTB void :

$$\partial_r \rho > 0 \qquad \partial_r R > 0$$

$$\frac{d}{dz}\left(\frac{\Delta_t z}{1+z}\right) = \frac{1}{(1+z)^2}\frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R}\Delta t_0$$

$$\frac{dz}{dr} = \frac{(1+z)\partial_t \partial_r R}{\sqrt{1-kr^2}} > 0 \quad \Rightarrow \quad \partial_t \partial_r R > 0$$

$$H^2 = \frac{F(r)^3}{R^3} - \frac{k(r)^2}{R^2}$$

$$\Rightarrow \quad \partial_t^2 \partial_r R = -\frac{\partial_r F}{R^2} + \frac{M \partial_r R}{R^3}$$

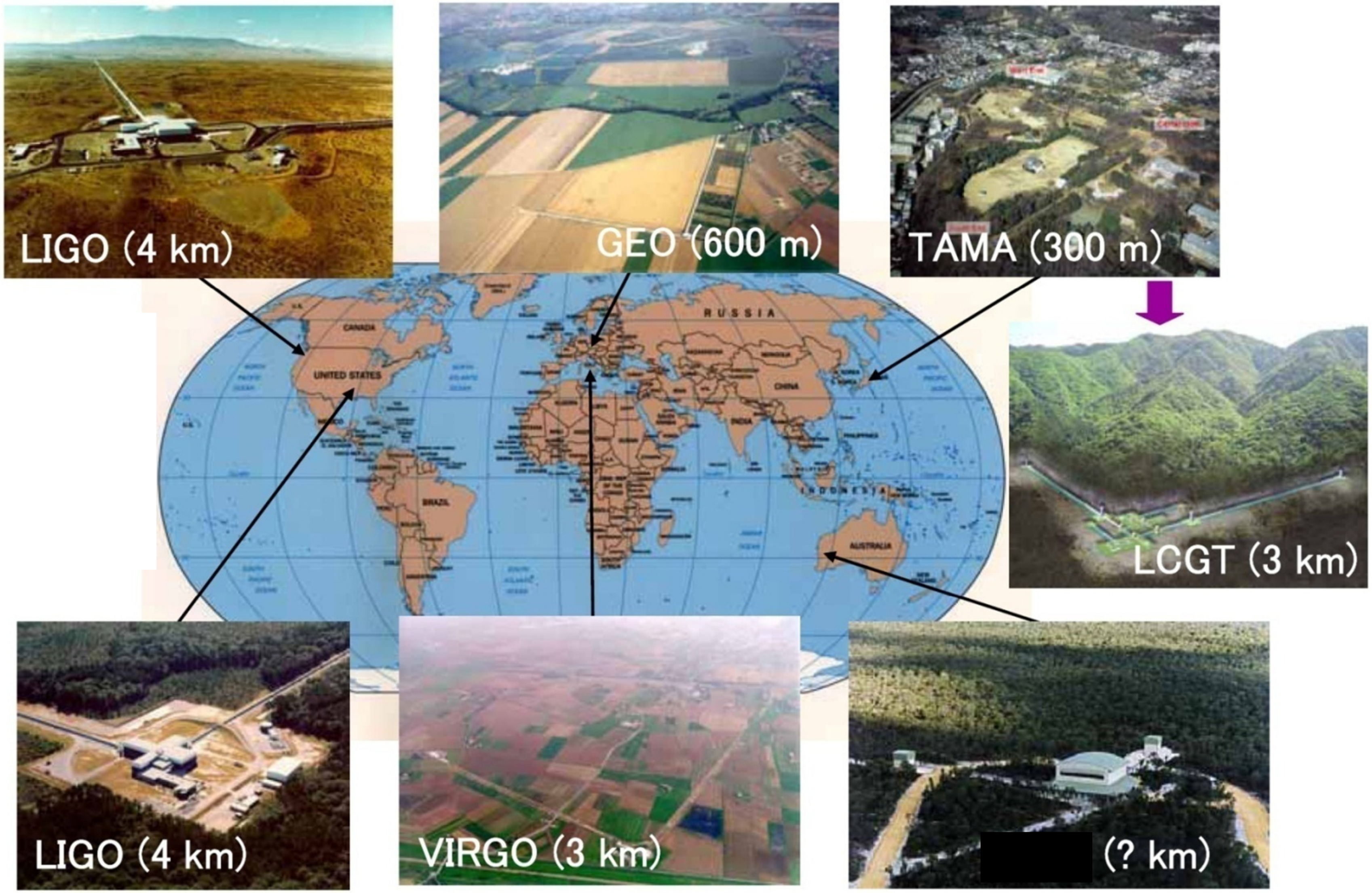
$$= -4\pi \frac{\partial_r R}{R^3}\int_0^R \left(\frac{d\rho}{dR}R^3 + \rho R^2\right) dR \quad < 0$$

$$\Rightarrow \quad \frac{d}{dz}\left(\frac{\Delta_t z}{1+z}\right) < 0 \quad + \quad \Delta_t z\big|_{z=0} = 0$$

$$\Rightarrow \quad \Delta_t z < 0$$

§3
GW
OBSERVATIONS

§ 3-1 Ground-based Detectors



(Kawamura)

Sources

Many Interesting Sources

Compact Binaries

NSs

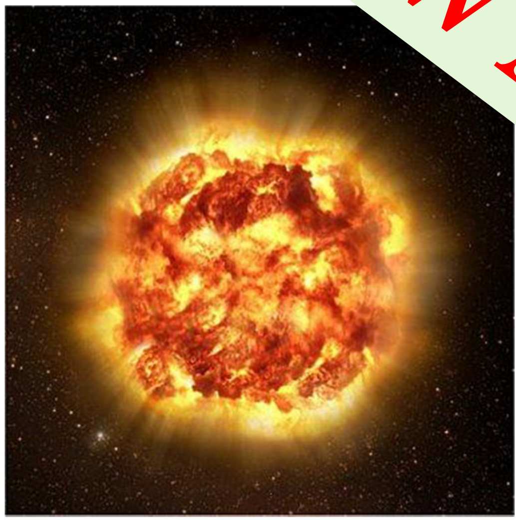
SNe

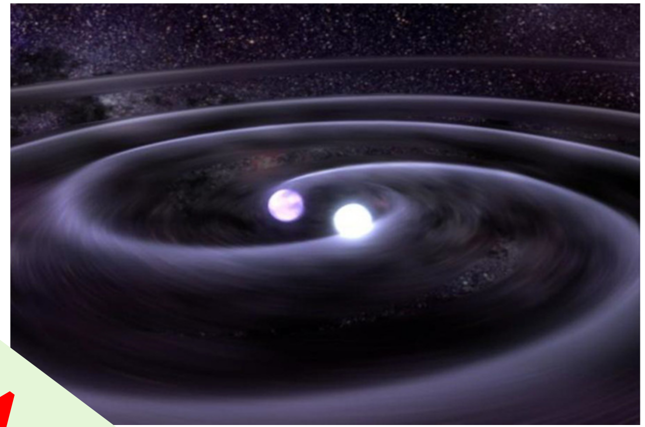
Magnetars

Cosmic Strings

etc….

Unknown Ones!





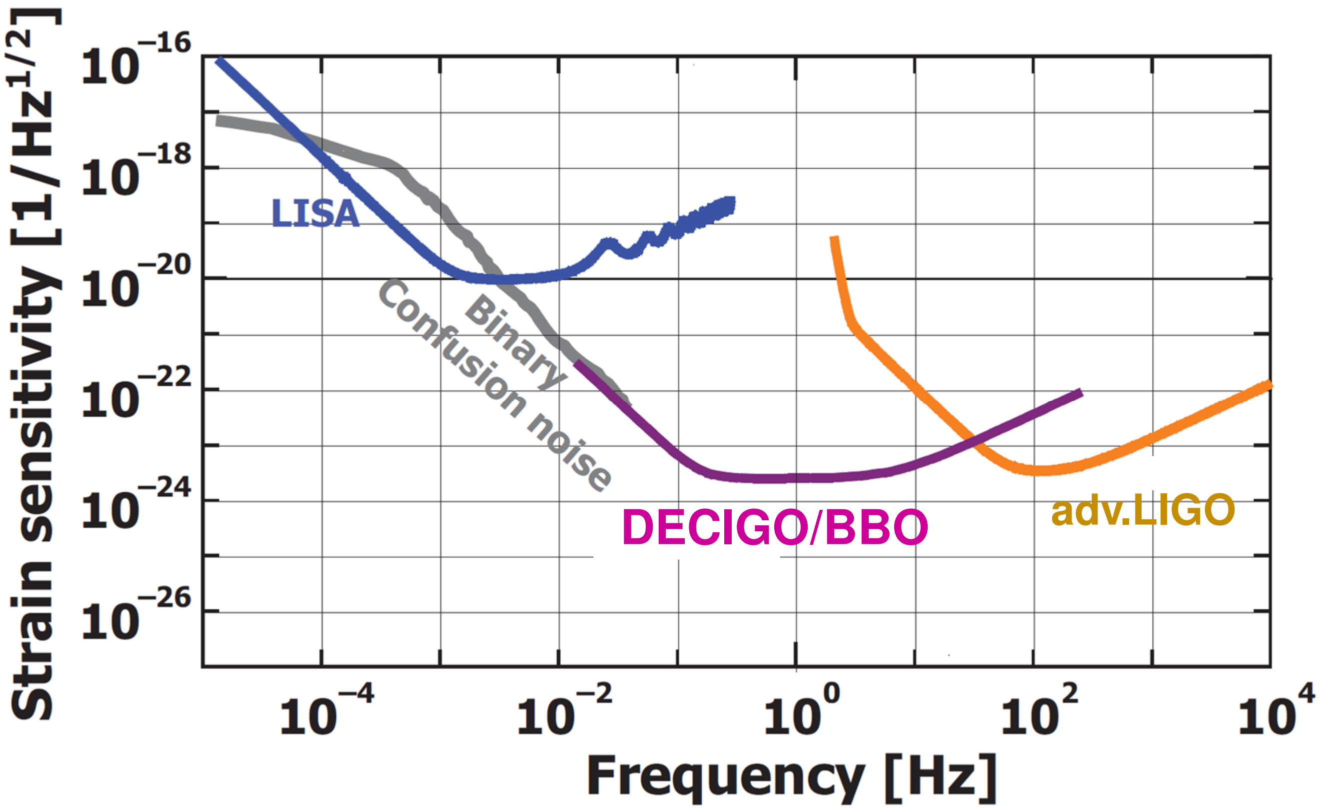
GW ASTRONOMY!!

Limitations on Ground

- Arm Length
- Seismic Noise

→ **Unable to detect lower freqency**

§ 3-2 Space GW Interferometers

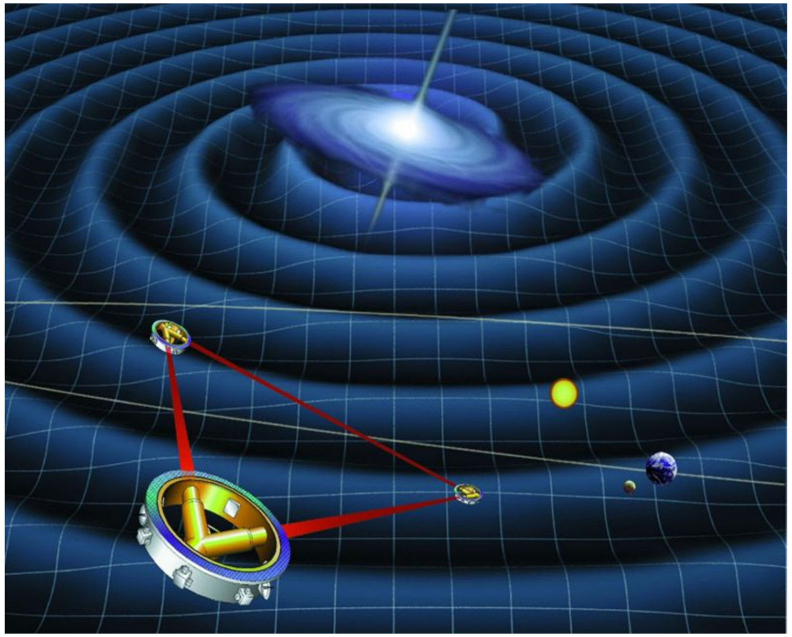


Sato *et al.* (2009)

§ 3-2-1 LISA

- ESA/(NASA)
- arm-length : 5×10^6km
- launching year: ???
- LISA Path Finder

  2013~1014



Targets & Sciences

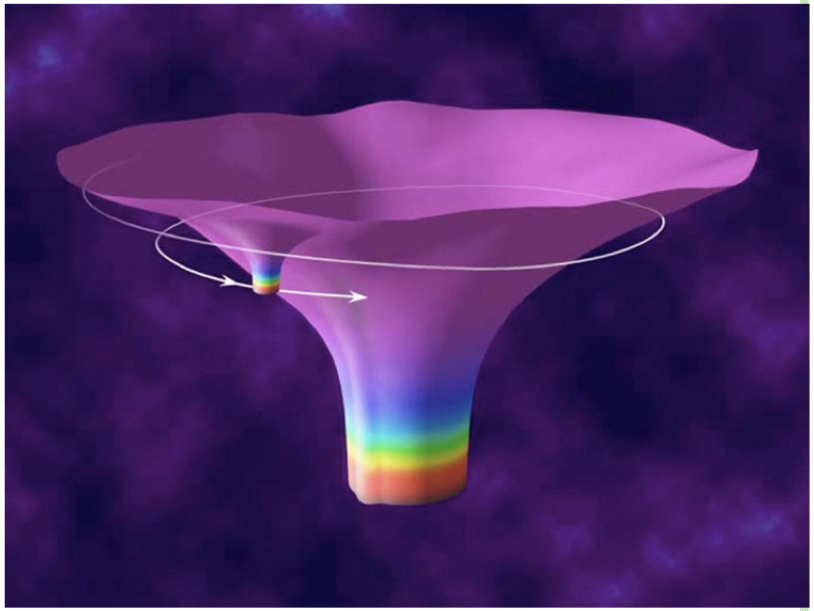
- Super-Massive BH mergers

  ⟹ **Standard Siren**

- Extreme Mass Ratio Inspiral

  ⟹ **GR test**

- WD/WD mergers



§ 3-2-2 DECIGO & BBO

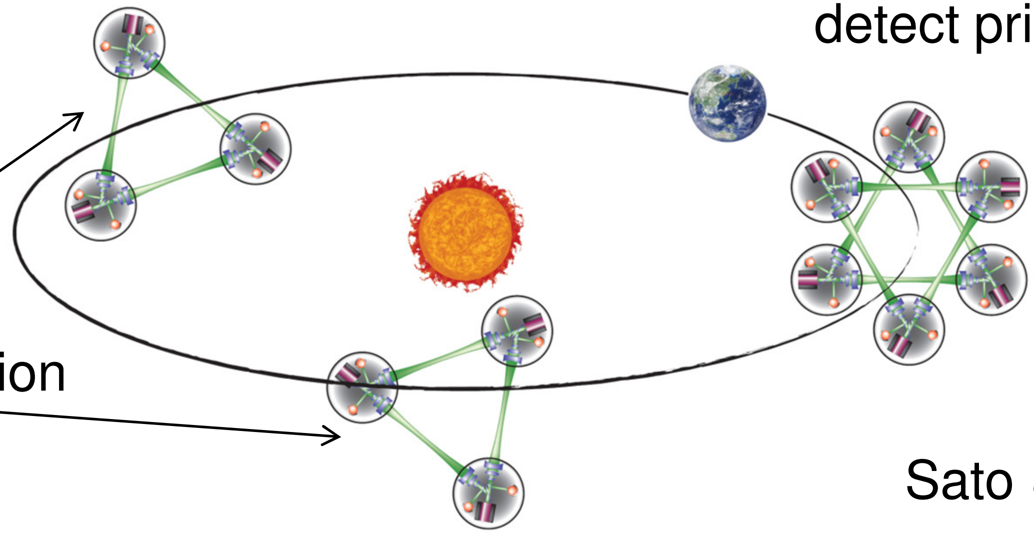
DECIGO

- Japan
- Fabry-Perrot cavity
- arm-length：1000km
- laser power： 10W
- launching year: 2027

BBO

- NASA/ESA
- arm-length：50000km
- laser power： 100W

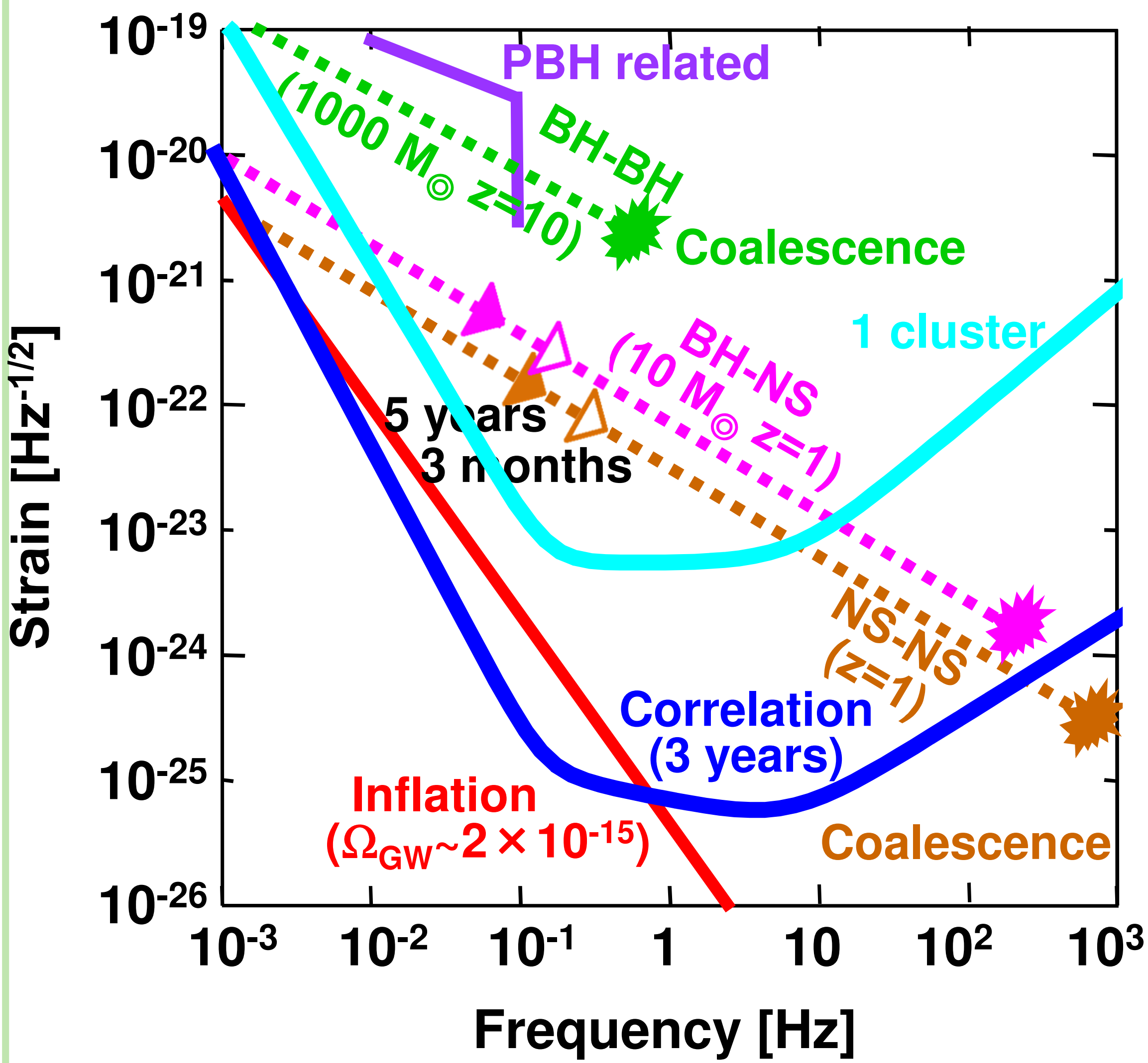
Taking correlations to detect primordial GW



To increase angular resolution

Sato *et al.* (2009)

Targets & Sciences with DECIGO & BBO

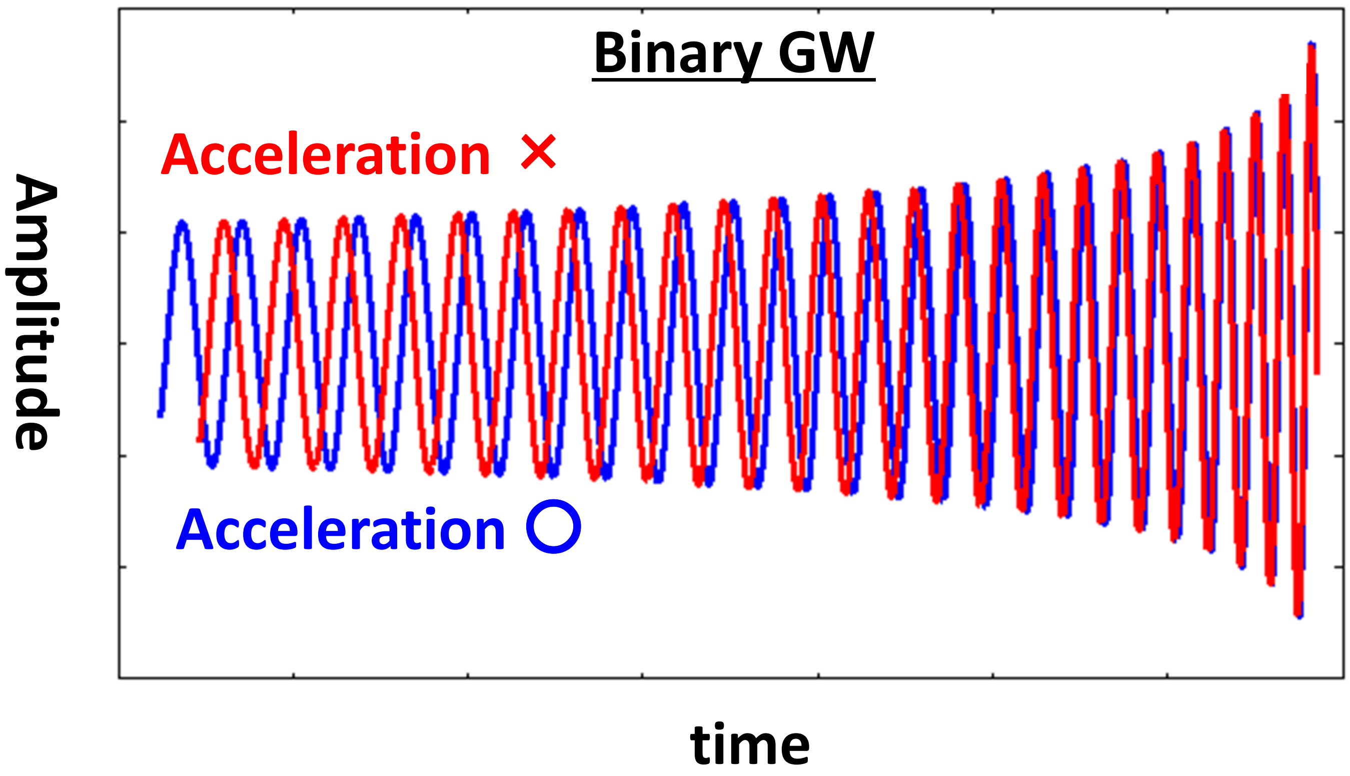


(Kawamura)

- **z-drift** (seto et al. 2001)
- **standard siren**
  (Cutler & Holz 2009)
- **SMBH formation**
- **GR test**
  (K.Y. & Tanaka 2010)
  (Nishizawa et al. 2010)
- **Dark matter (PBH)**
  (Saito & Yokoyama 2009)
- **Inflation, reheating**
  (Seto et al. 2001)
  (Nakayama et al. 2008)

§ 3-3 Data Analysis

§ 3-3-1 Correction in waveform



How accurately can we measure this difference?

§ 3-3-2 determining parameters

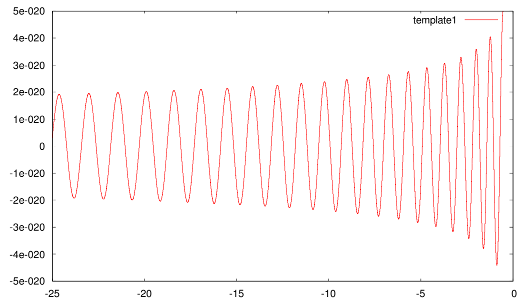
Matched Filtering

・Observed signals are buried in the detector noise.

⇒**Take the correlation with templates,**

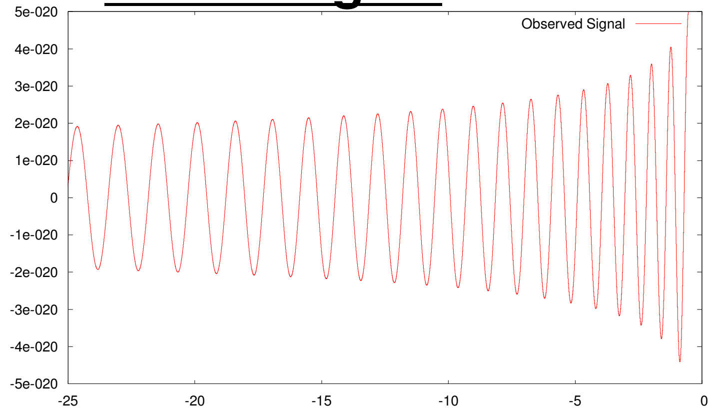
determining binary parameters.

template

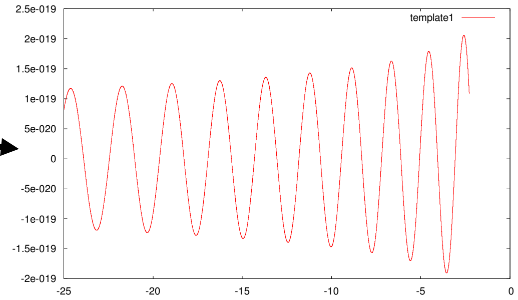


Corr.
large

Buried signal



Corr. small



Fisher Analysis

・We assume the noise is stationary & Gaussian.
⇒the probability distributions of parameters are Gaussian.

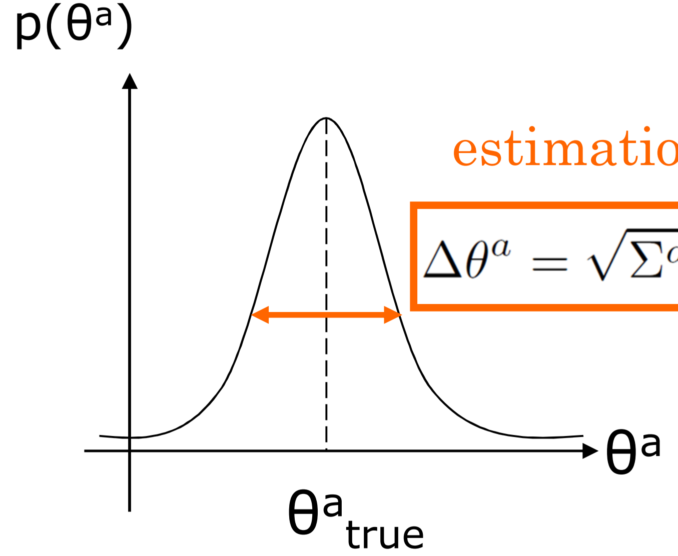
$$p(\boldsymbol{\theta}|s) = p^{(0)}(\boldsymbol{\theta}) \exp\left[-\frac{1}{2}\Gamma_{ab}\Delta\theta^a\Delta\theta^b\right]$$

Fisher Matrix:

$$\Gamma_{ij} = 4\mathrm{Re}\int_0^\infty \frac{\partial_i\tilde{h}\partial_j\tilde{h}}{S_n(f)}df$$

$S_n(f)$:noise spectrum

$$\partial_i\tilde{h} \equiv \frac{\partial\tilde{h}}{\partial\theta^i}$$



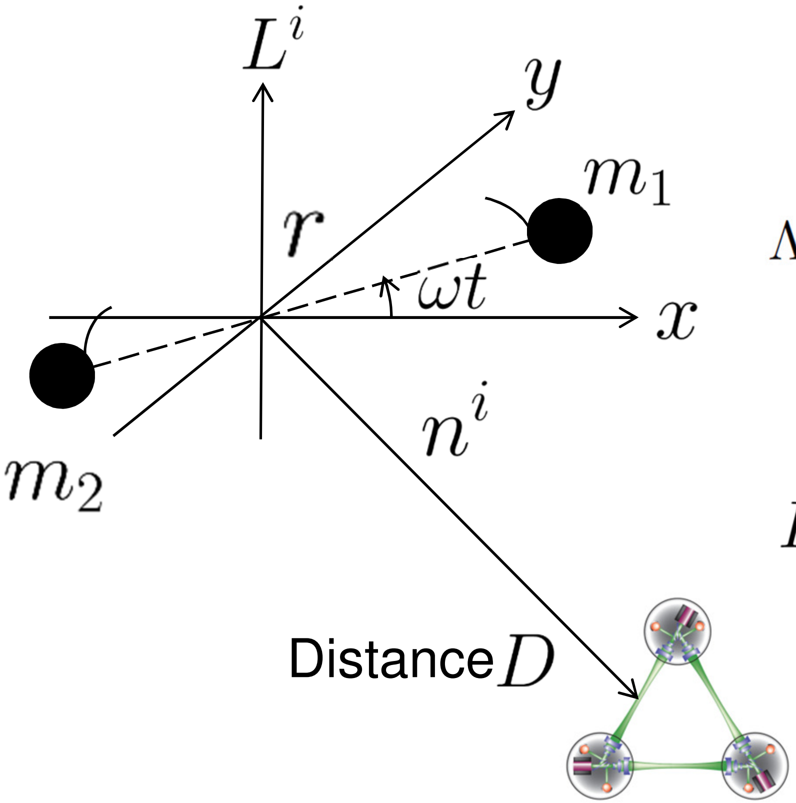
estimation error

$$\Delta\theta^a = \sqrt{\Sigma^{aa}}, \qquad \Sigma = \Gamma^{-1}$$

Step 1. Derive **correction** in the **GW phase**
Step 2. Estimate the **Fisher matrix**
Step 3, Take the inverse

§4
GRAVITATIONAL WAVES FROM COMPACT BINARIES

WAVEFORMS FROM BINARY SYSTEMS



$$h_{ij}^{\mathrm{TT}} = \frac{2}{D}\Lambda_{ij,kl}\ddot{I}_{kl}$$

$$\Lambda_{ij,kl}(\hat{\mathbf{n}}) = P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl}$$ TT operator

$$P_{ij}(\hat{\mathbf{n}}) = \delta_{ij} - n_i n_j$$

$$I_{ij} = \sum_{k=1}^{2} m_k x_i x_j$$ Quadrupole moment

Mode decomposition: $$h_{ij}^{\mathrm{TT}} = A_+ \cos\phi(t) H_{ij}^+ + A_\times \sin\phi(t) H_{ij}^\times$$

Polarization basis: $$H_{ij}^+ = p_i p_j - q_i q_j \qquad H_{ij}^\times = p_i q_j + q_i p_j$$

$$p^i = \varepsilon^{ijk} n_j \hat{L}_k \qquad q^i = -\varepsilon^{ijk} n_j p_k$$

$$h_{ij}^{\rm TT} = \boxed{A_+} \cos \boxed{\phi(t)} H_{ij}^{+} + \boxed{A_\times} \sin \boxed{\phi(t)} H_{ij}^{\times}$$

$$\boxed{A_+} = \frac{2m_1m_2}{rD}(1 + (\hat{L}^a n_a)^2) \qquad \boxed{\phi(t)} = \int 2\pi f dt$$

$$\boxed{A_\times} = -\frac{4m_1m_2}{rD}\hat{L}^a n_a \qquad f = \frac{\omega}{\pi}$$

GW signal:

Beam Pattern Functions

$$h(t) = \frac{\sqrt{3}}{2}A_+ \underline{F^+(\theta_S, \phi_S, \psi_S)} \cos\phi(t) + \frac{\sqrt{3}}{2}A_\times \underline{F^\times(\theta_S, \phi_S, \psi_S)} \sin\phi(t)$$

$$= \frac{\sqrt{3}}{2}\frac{2m_1m_2}{rD}A(t)\cos(\boxed{\phi(t)} + \phi_{\rm pol}(t))$$

$$A(t) = \sqrt{(1 + (L^a n_a)^2)^2 F^{+2} + 4(L^a n_a)^2 F^{\times 2}}$$

$$\phi_{\rm pol}(t) = \tan^{-1}\left(\frac{-A_\times F^\times}{A_+ F^+}\right)$$

Fourier transform:

$$\tilde{h}(f) = \frac{\sqrt{3}}{2}\mathcal{A} f^{-7/6} e^{i\Psi(f)} \left(\frac{5}{4}A(t(f))\right) e^{-i\phi_{\rm pol}(t(f))}$$

If we sky-average, this factor goes to 1.

$$\mathcal{A} = \frac{1}{\sqrt{30}\pi^{2/3}} \frac{\mathcal{M}^{7/6}}{D}$$

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[1 + (\text{higher PN})\right]$$

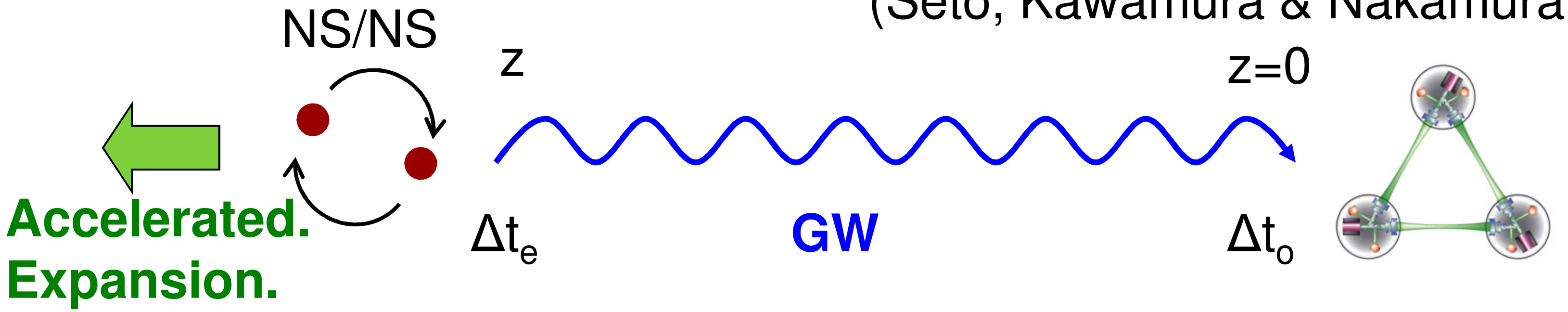
$$t(f) = t_c - \frac{5}{256}\mathcal{M}(\pi\mathcal{M}f)^{-8/3}\left[1 + (\text{higher PN})\right]$$

§5

Constraining Inhomogeneous Universe with GW observations

§ 5-1 Effect of accelerated expansion in GW waveform

(Seto, Kawamura & Nakamura 2001)



$$\Delta t_o = \Delta T(1 + X(z)\Delta T)$$

Obs. time — correction

$$\Delta T \equiv (1+z)\Delta t_e$$

Accelerating. parameter

$$X(z) = \frac{H_0}{2}\left(1 - \frac{H(z)}{(1+z)H_0}\right)$$

$$\Delta_t z = 2(1+z)\Delta t_0 X(z)$$

・correction in GW phase:

$$\delta\Psi_{\rm acc} = -2\pi f X(z)\Delta t^2$$

Δt: time to coalescence

・Advantages of using **DECIGO** over ground-based detectors:

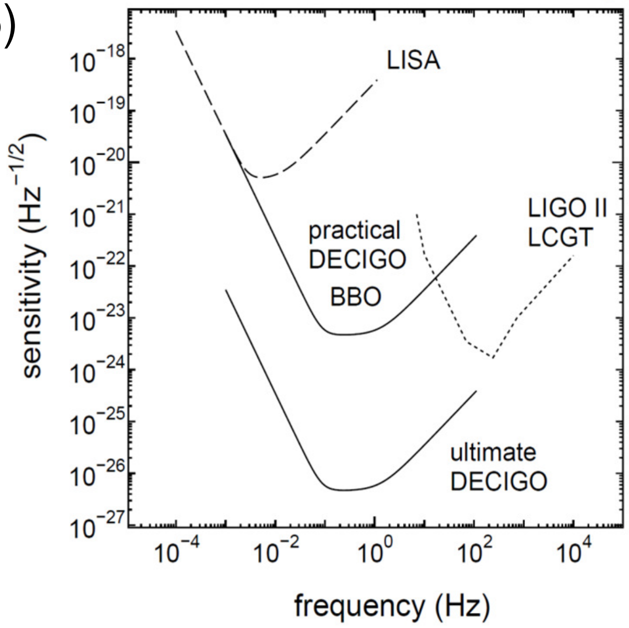
(i) Longer **observation time**

(ii) Larger **numbers of GW cycles**: 0.1Hz × 5yr~10^7

(iii) Larger NS/NS binary **detection rate**: **10^6 / yr**

§ 5-2 Determination error of X(Z) (Ultimate DECIGO)

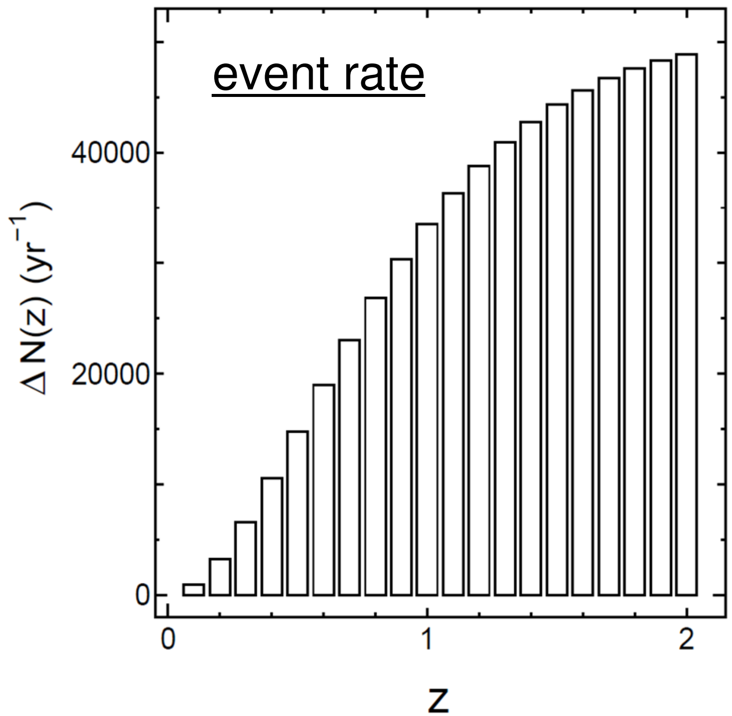
Takahashi & Nakamura (2005)

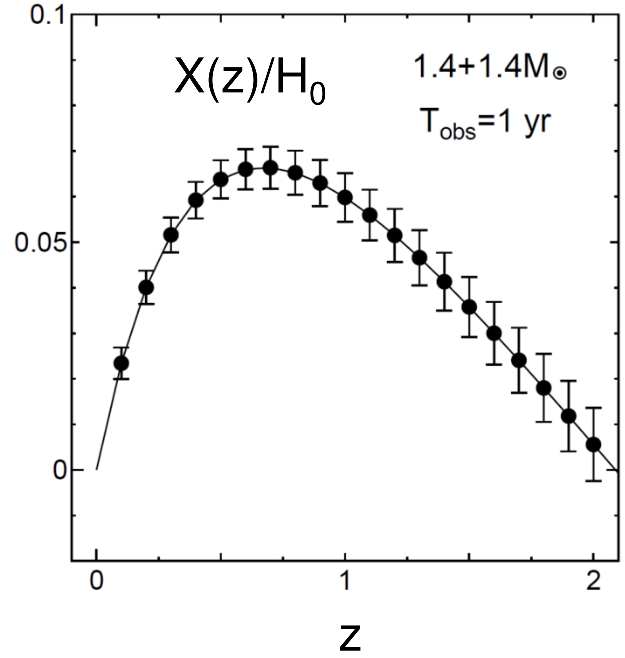


Ultimate DECIGO

NS/NS binaries at z=0~2

Merger rate=10^{-6} Mpc^{-3} yr^{-1}





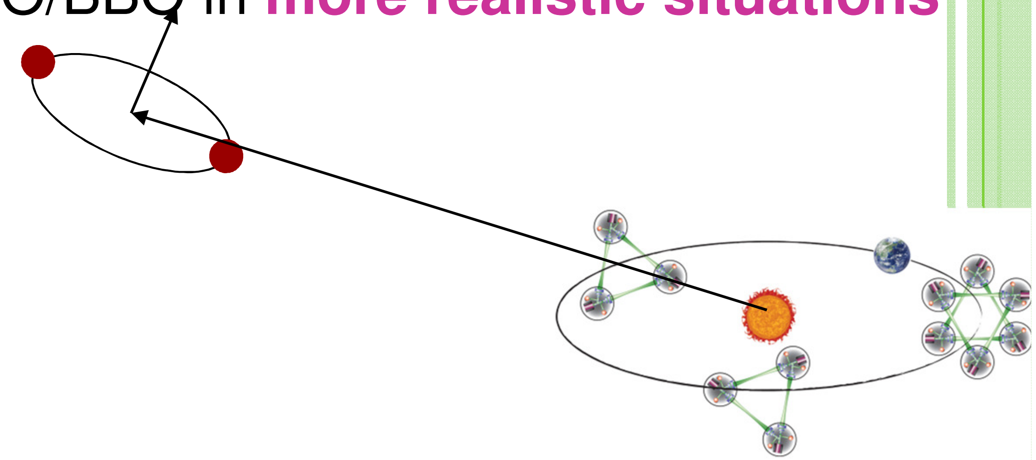
- GW obs. ⇒ **z degenerate with mass**

⇒z is determined from EM obs.

§ 5-3 Numerical Calculations

K.Y. & A. Nishizawa & C. Yoo (in prep.)

・Determining z-drift with DECIGO/BBO in **more realistic situations**



New Effects

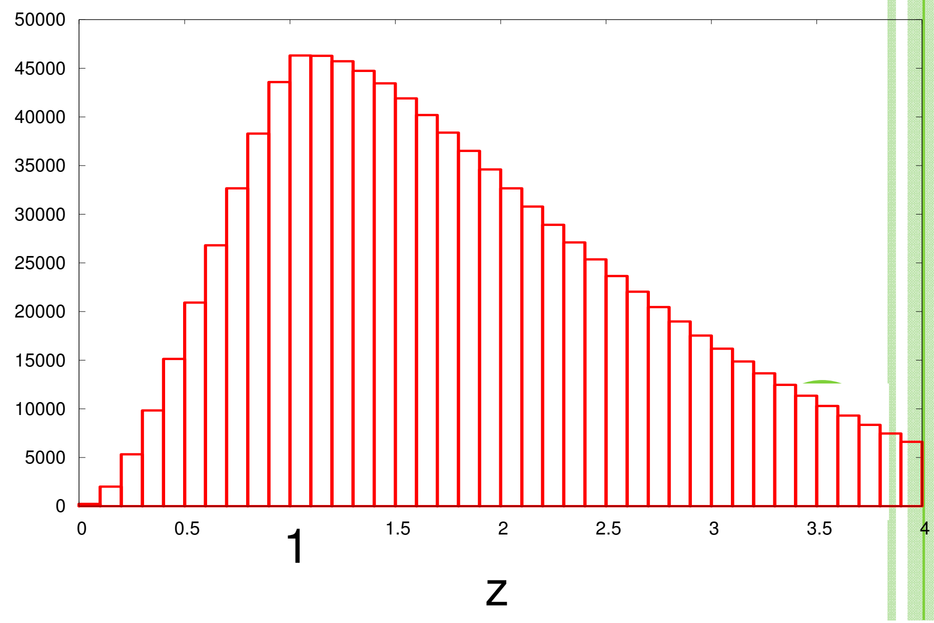
- Randomly distributing binaries at each z

  (Directions and orientations)

  Monte Carlo simulation

  Motion of Detectors
- spins (& eccentricities)
- **WD/WD confusion noise**
- **merger rate evolution** based on **star formation galaxy** obs.

event rate ΔN(z) [yr^{-1}]



- Restricted 2PN waveform (circular orbit)

$$\tilde{h}(f) = \frac{\sqrt{3}}{2}\mathcal{A}f^{-7/6}e^{i\Psi(f)}\left(\frac{5}{4}A(t(f))\right)e^{-i\phi_{\rm pol}(t(f))}$$

Accel. Exp.

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi\mathcal{M}_z f)^{-5/3}\left[1 - \frac{25}{768}X\mathcal{M}_z\eta^{-8/5}x^{-4} + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - 4(4\pi - \beta)x^{\frac{3}{2}} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right)x^2\right]$$

$$x = v^2 = (\pi(1+z)M_{\rm tot}f)^{2/3}$$

$$\mathcal{M}_z = (1+z)M_{\rm tot}\eta^{3/5} \qquad \beta: \text{ spin orbit coupling}$$

$$\eta = \mu/M_{\rm tot}$$

- Binary parameters:

$$\theta^i = (\mathcal{M}, \eta, \beta, t_c, \phi_c, D_L, \bar{\theta}_S, \bar{\phi}_S, \bar{\theta}_L, \bar{\phi}_L, X)$$

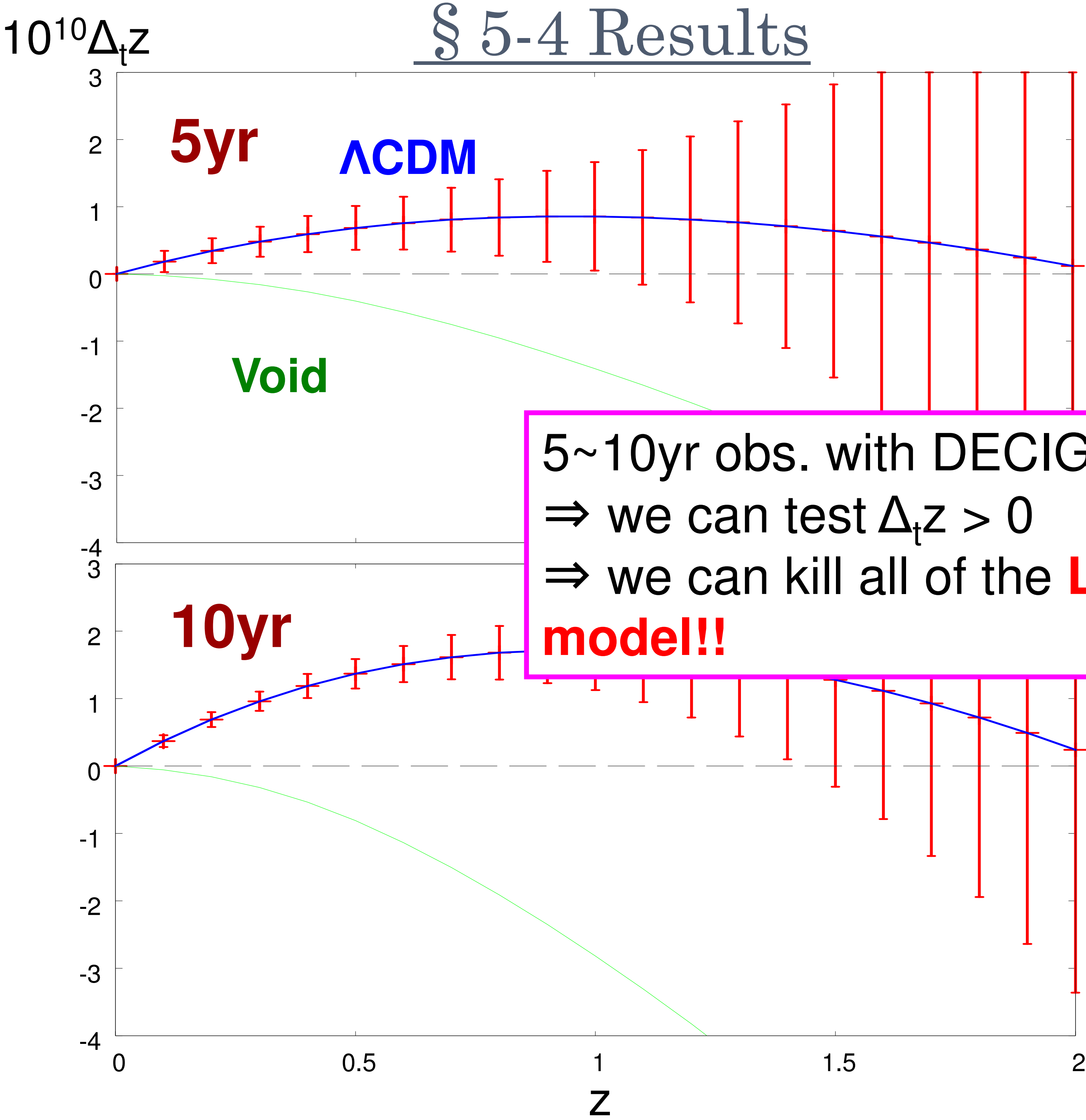
- Estimate Fisher Matrix at each z

$$\Gamma_{ij} = 4\mathrm{Re}\int_0^\infty \frac{\partial_i\tilde{h}\partial_j\tilde{h}}{S_n(f)}df$$

The error is determined from the inverse matrix: $\Delta\theta^i = \sqrt{\frac{\Gamma^{-1}_{ii}}{\Delta N}}$

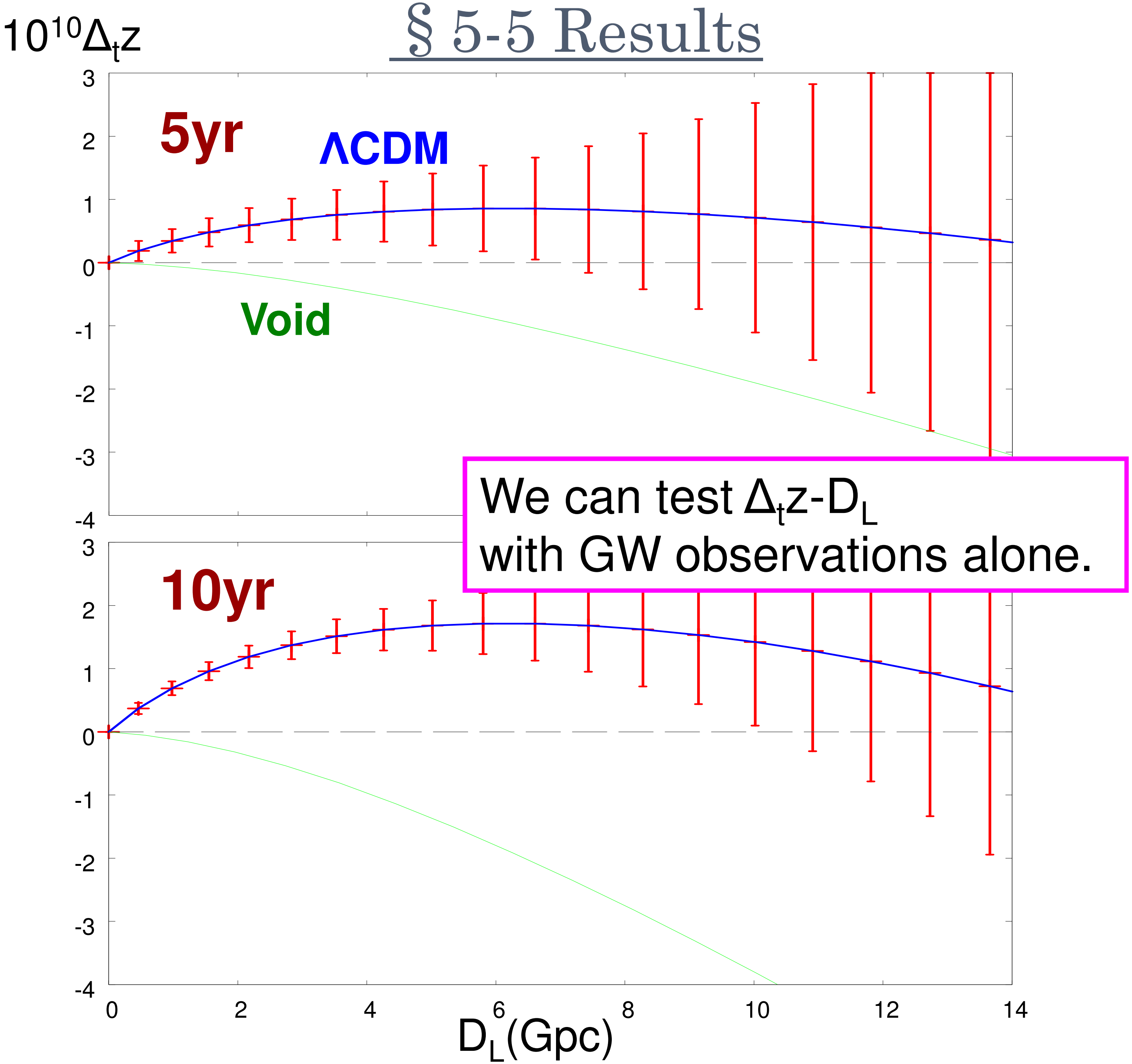
ΔX ⇨ $\Delta(\Delta_t z)$

§ 5-4 Results



5~10yr obs. with DECIGO/BBO
⇒ we can test $\Delta_t z > 0$
⇒ we can kill all of the **LTB void model!!**

§ 5-5 Results



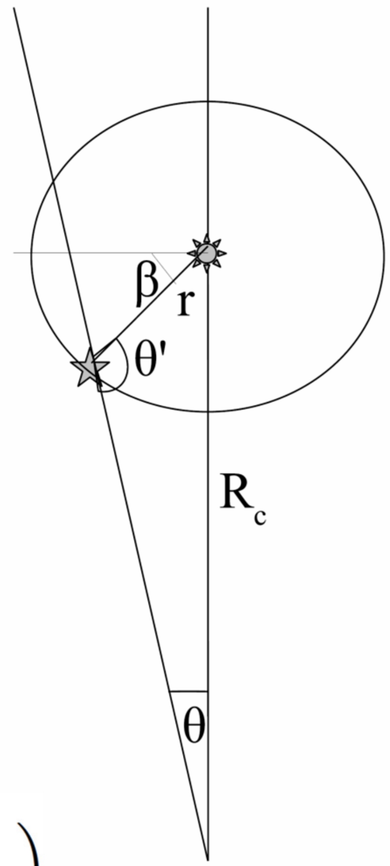
$10^{10}\Delta_t z$
5yr
ΛCDM
Void
10yr
D_L(Gpc)
We can test $\Delta_t z$-D_L
with GW observations alone.

§6 Discussions & Summary

§ 6-1 Peculiar Accelerations 1

Amendola et al. (2008)

Peculiar acceleration: $a_s = \cos\theta' \Phi_{,r}$



Navarro-Frenk-White profile (NFW)

$$\rho(r) = \frac{\delta_c \rho_{cr}}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

$$\delta_c = \frac{C\Delta_c c^3}{3}$$

$$C = [\log(1+c) - \frac{c}{1+c}]^{-1}$$

Mass within radius r:

$$M(r) = M_v C\Big(\log(1+\frac{r}{r_s}) - \frac{\frac{r}{r_s}}{1+\frac{r}{r_s}}\Big)$$

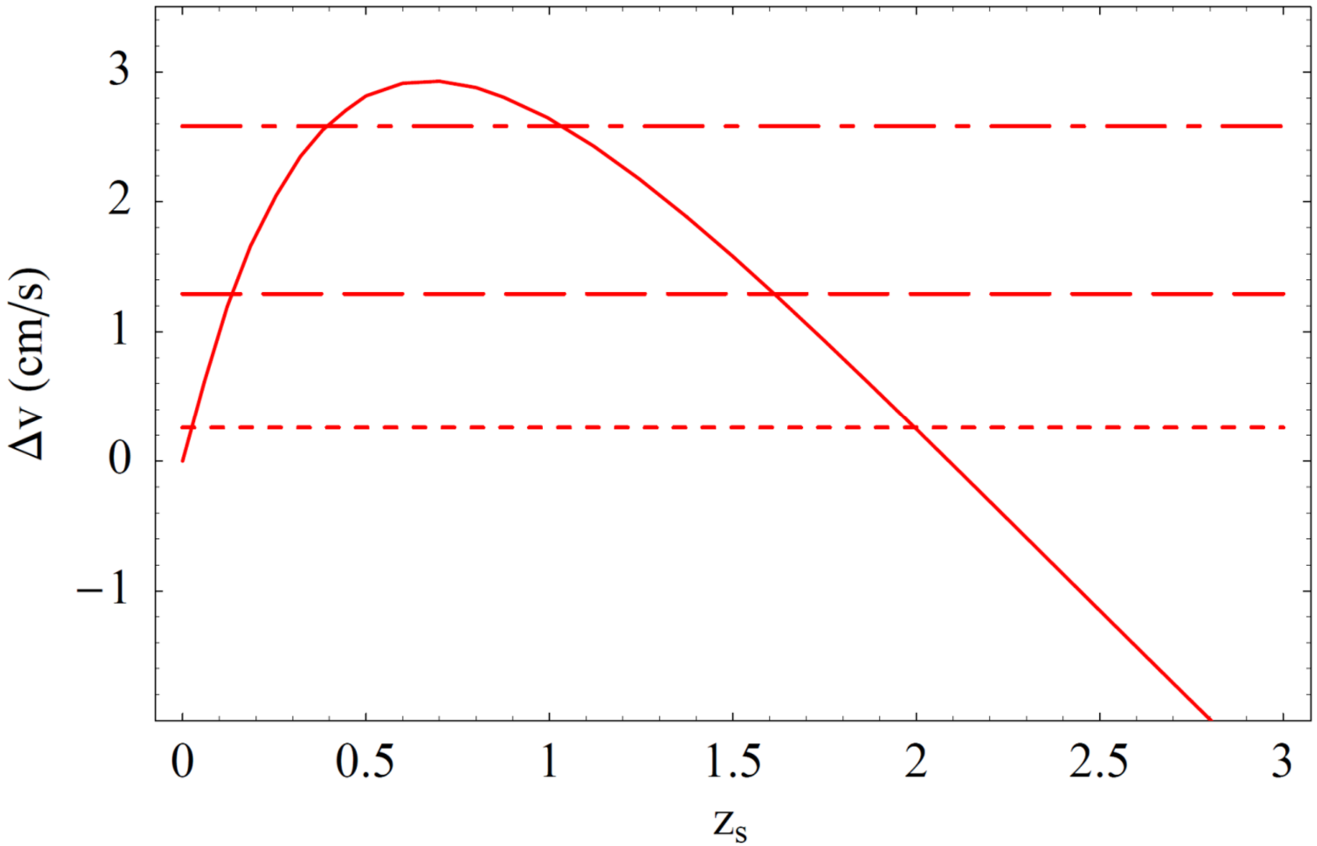
$$M_v = 4/3\pi r_v^3 \Delta_c \rho_{cr}$$

$$\Phi(r)_{,r} = \frac{GM_v}{r_s^2} C\Big(\frac{\log(1+\frac{r}{r_s})}{(\frac{r}{r_s})^2} - \frac{1}{\frac{r}{r_s}(1+\frac{r}{r_s})}\Big)$$

Velocity shift:

$$\Delta v = a_s \Delta T = \frac{GM_v}{r_s^2} C \Delta T \cos\theta' \left(\frac{\log\left(1+\frac{r}{r_s}\right)}{\left(\frac{r}{r_s}\right)^2} - \frac{1}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)}\right)$$

Max velocity shift (r→0, θ'→0) : $\Delta v_{\rm max} = \frac{C \Delta T}{2} \frac{G M_v}{r_s^2}$



Peculiar accelerations are comparable to the cosmological z-drift.
The former is reduced by a factor $\sqrt{N}$

⇒ peculiar accel. < detector noises
⇒ we can **safely neglect the effect of peculiar acceleration**.

§ 6-2 Summary

- **Space-borne GW interferometers**

  ⇒ Many interesting sciences

- **Direct detection of the acceleration of the Universe** with DECIGO/BBO

  ⇒ Maybe able to **kill monotonic LTB void models**

  or **even general ones!**

Ground-Based Detectors ⇒ **GW Astronomy**!

Space-borne Detectors ⇒ **GW Cosmology**!!!