5次元ブラックホールの形成条件と時間発展数値解析

大阪工業大学　情報科学部
山田　祐太

共同研究者：真貝寿明

関西相対論・宇宙論セミナー
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0. Motivation

Brane-World models give new viewpoints to gravity and cosmology.

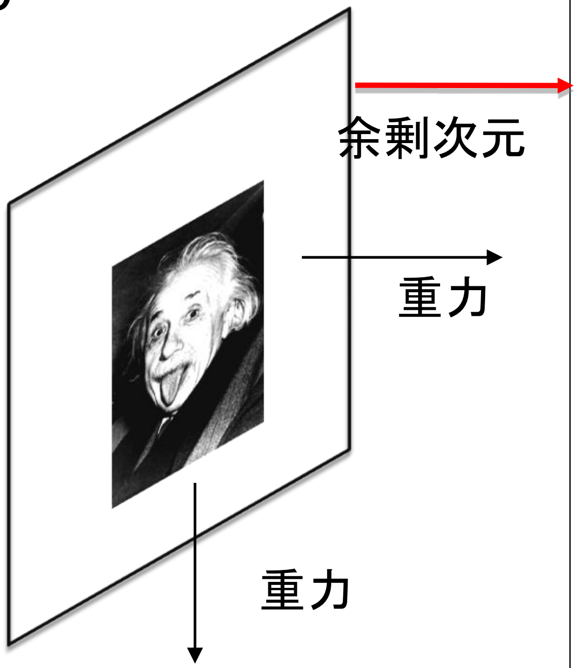
LHC experiments will (or will not) reveal Higher-Dim BHs in near future.

4-dim BH : horizon is S^{2},

Stable solutions

Schwarzschild --- Birkoff theorem (M)

Kerr --- uniqueness theorem (M, J)



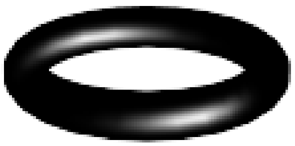
0. Motivation

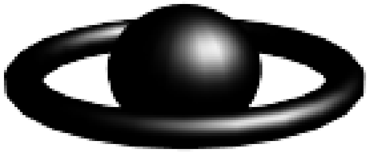
4-dim BHs — Higher-dim BHs :

Schwarzschild → Tangherlini
--- unique & stable

Kerr → Myers-Perry
--- maybe unstable in higher J

Black Objects → black ring (Emparan-Reall)
black Saturn
di-rings, orthogonal di-rings, ...

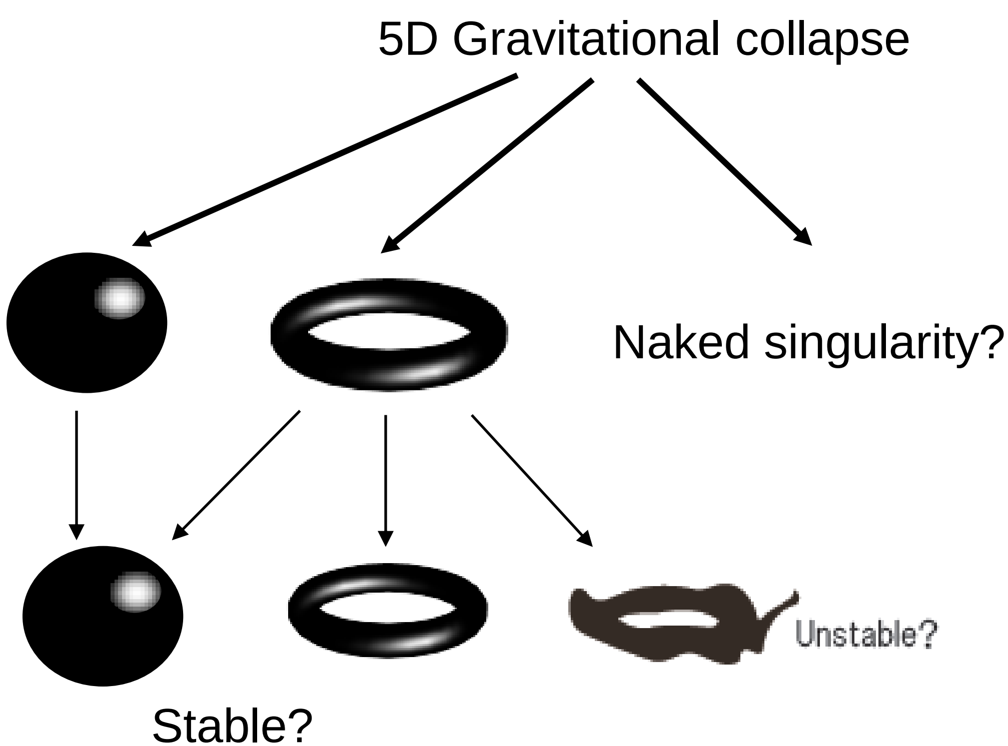




0. Motivation

Black Objects →

- black hole
- black string
- black ring (Emparan-Reall)
- black Saturn
- di-rings, orthogonal di-rings, ...



Formation Process ?

Stability ?

Dynamical Features?

Outline

1. 重力崩壊とフープ仮説

2. 5次元時空の初期値問題

・4次元楕円体とリング形状分布に対する初期解
(Yamada and Shinkai, CQG 27, 045012)
・時間発展予想
・ハイパー・フープ仮説の検証

3. 5次元時空での重力崩壊シミュレーション

・数値計算手法
・4次元楕円体分布の重力崩壊 (Yamada and Shinkai, PRD 83, 064006)
・4次元リング分布の重力崩壊

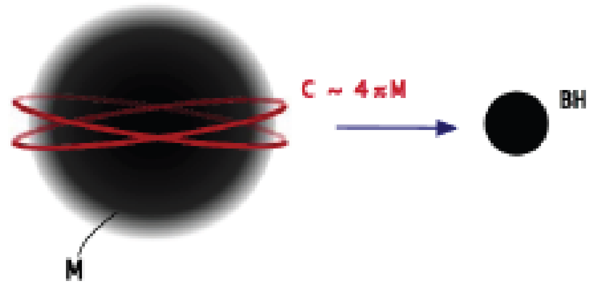
1. 重力崩壊とフープ仮説

フープ仮説

K. S. Thorne, in Magic Without Magic, edited by J. R.Klauder (Freeman, San. Francisco, 1972), p. 231

> Horizons (probably) form when and only when a mass M gets compacted into a region whose circumference in every direction is C ≦ 2π×(2GM/c^2) .

$C \leq 4\pi GM$ ⇄ BH form



もし、C≧ 2π×(2GM/c^2) なら

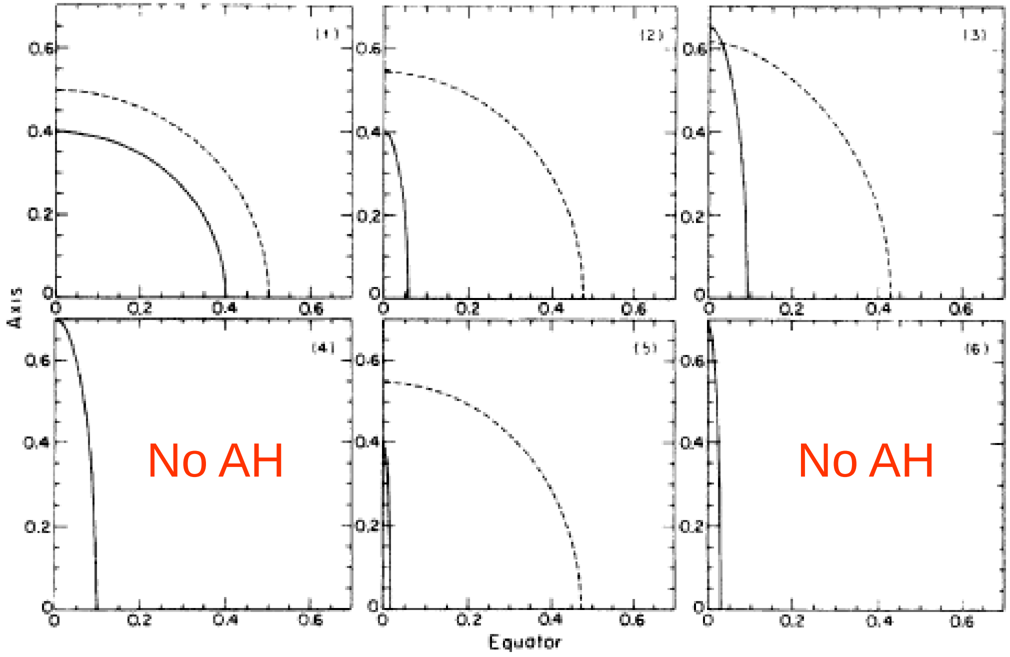
→ 裸の特異点？

Cの定義は？　Mの定義は？

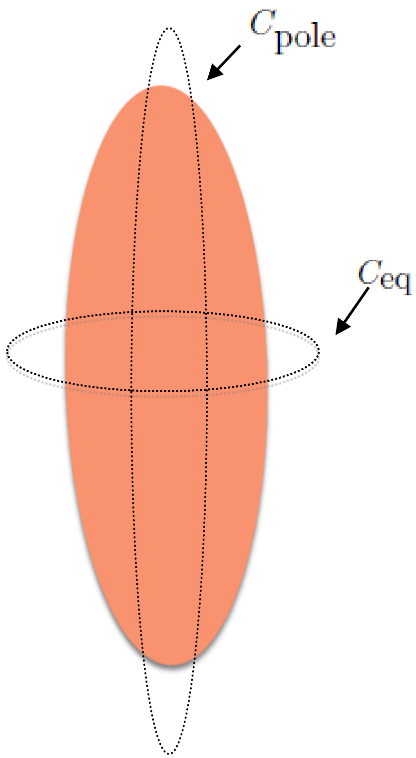
初期値解系列の解析

Nakamura, Shapiro, Teukolsky, PRD 38, 2972 (1988)

- 回転楕円体を重力源としておいた場合の、初期値解系列の解析。
- 時間反転対称。



極端に細長い物質分布の場合、Apparent horizonは形成されない。



Minimum circumference(物質周りで円周Cが最小)を計算。

$C^{\rm min}_{\rm pole}$

$$C_{pole}=\int \psi^2\sqrt{r^2+(r_{,\theta})^2}d\theta$$

$$\delta C_{pole}=0 \longrightarrow r_{,\theta\theta}-2\frac{r_{,\theta}^2}{r}-r+2\left[1+\frac{r_{,\theta}^2}{r^2}\right]\left[\frac{\psi_{,\theta}}{\psi}r_{,\theta}-\frac{\psi_{,r}}{\psi}r^2\right]=0 .$$

$C^{\rm min}_{\rm eq}$ 簡単に求められる。

TABLE I. Properties of prolate configurations in Fig. 1.

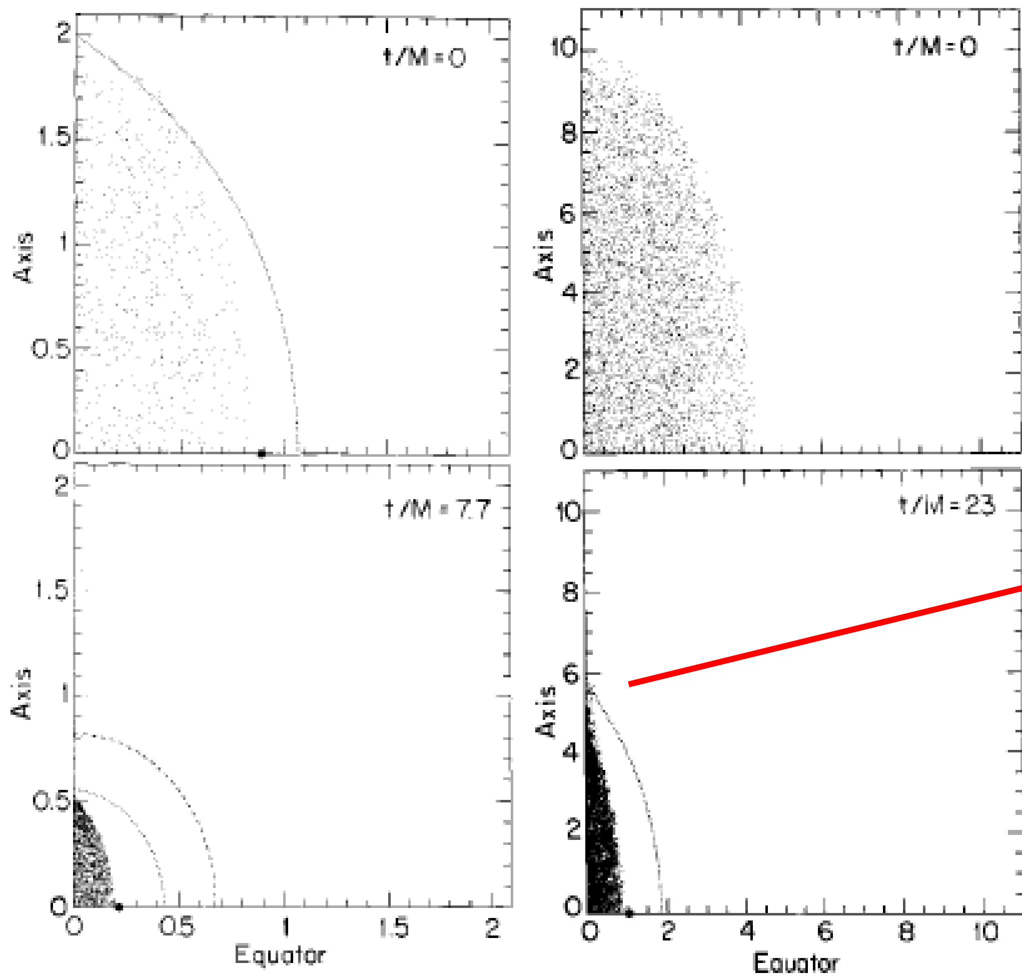
No.	e	c/M	M/M_0	Apparent horizon?	$\mathcal{C}^{\rm AH}_{\rm eq}/4\pi M$	$\mathcal{C}^{\rm Ah}_{\rm pole}/4\pi M$	$\mathcal{A}/16\pi M^2$	$\mathcal{C}^{\rm surf}_{\rm eq}/4\pi M$	$\mathcal{C}^{\rm surf}_{\rm pole}/4\pi M$	$\mathcal{C}^{\rm min}_{\rm eq}/4\pi M$	$\mathcal{C}^{\rm min}_{\rm pole}/4\pi M$
1	0.01	0.40	0.40	Yes	1.00	0.99	1.00	1.01	1.01	1.00	1.00
2	0.99	0.40	0.20	Yes	0.94	1.03	0.99	0.74	2.49	0.74	1.03
3	0.99	0.65	0.29	Yes	0.86	1.10	0.96	0.57	2.00	0.57	1.07
4	0.99	0.70	0.30	No				0.55	1.95	0.55	1.08
5	0.999	0.40	0.15	Yes	0.94	1.03	0.99	0.46	4.44	0.46	1.03
6	0.999	0.70	0.24	No				0.32	3.21	0.32	1.08

MはADM MASS

楕円体分布の重力崩壊

VOLUME 66, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1991

Formation of Naked Singularities: The Violation of Cosmic Censorship



ronomy and Physics,

neral relativity. In all cases
ompact, the singularities are
, there are no apparent hor-
nstrate that naked singulari-

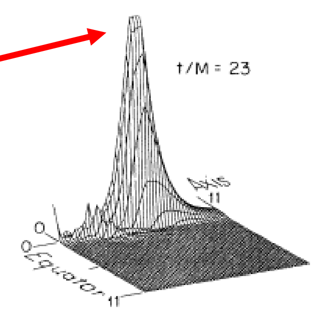


FIG. 4. Profile of I in a meridional plane for the collapse shown in Fig. 2. For the case of 32 angular zones shown here, the peak value of I is $24/M^4$ and occurs on the axis just outside the matter.

5次元時空でのフープ仮説

ブラックストリング周りの円周cは、明らかに無限大。

$C \leq 4\pi GM$ ←×— BH form

フープ仮説をみたしていない。

高次元時空での物質のコンパクトさを測る

指標は何か？

ハイパー・フープ仮説

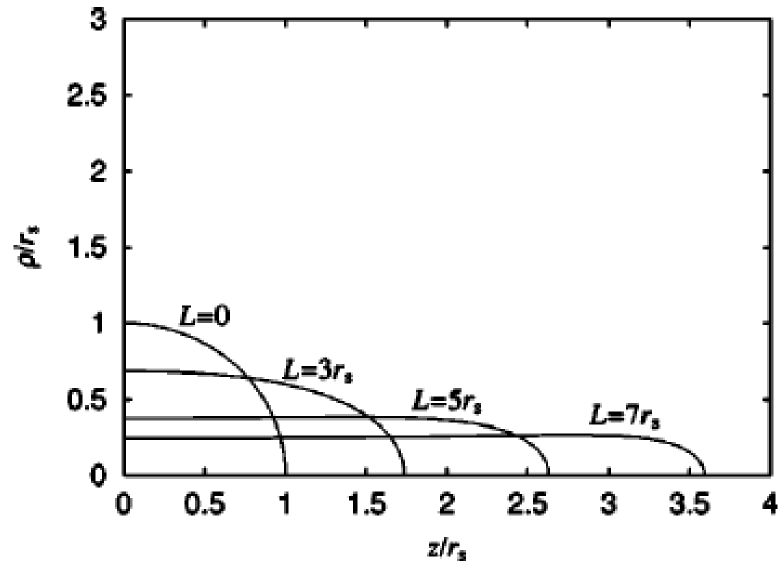
Ida, Nakao, PRD 66, 064026 (2002)

スピンドル形状の初期値解系列の解析(5D)

$$f^3 \varrho = \frac{M_{ADM}}{4\pi L \rho^2}\delta(\rho)\theta(L/2-|z|),$$

$$f := 1 + \frac{2GM_{ADM}}{3\pi L\rho}\left(\arctan\frac{z+L/2}{\rho} - \arctan\frac{z-L/2}{\rho}\right)$$

AHの3次元体積、2次元表面積、1次元長さ



$$\mathrm{Vol}(H) = 8\pi \int_0^{\pi/2} f^3 \sqrt{(r_\xi)^2 + r^2}\, r^2 \sin^2 \xi d\xi.$$

$$\mathrm{Area}(S_1) = 4\pi \int_0^{\pi/2} f^2 \sqrt{(r_\xi)^2 + r^2}\, r \sin \xi d\xi,$$

$$\mathrm{Area}(S_2) = \max\{4\pi f^2 r^2 \sin^2 \xi; \xi \in [0,\pi/2]\}.$$

$$\mathrm{Length}(C_1) = 4 \int_0^{\pi/2} f \sqrt{(r_\xi)^2 + r^2} d\xi,$$

$$\mathrm{Length}(C_2) = \max\{2\pi f r \sin \xi; \xi \in [0,\pi/2]\}.$$

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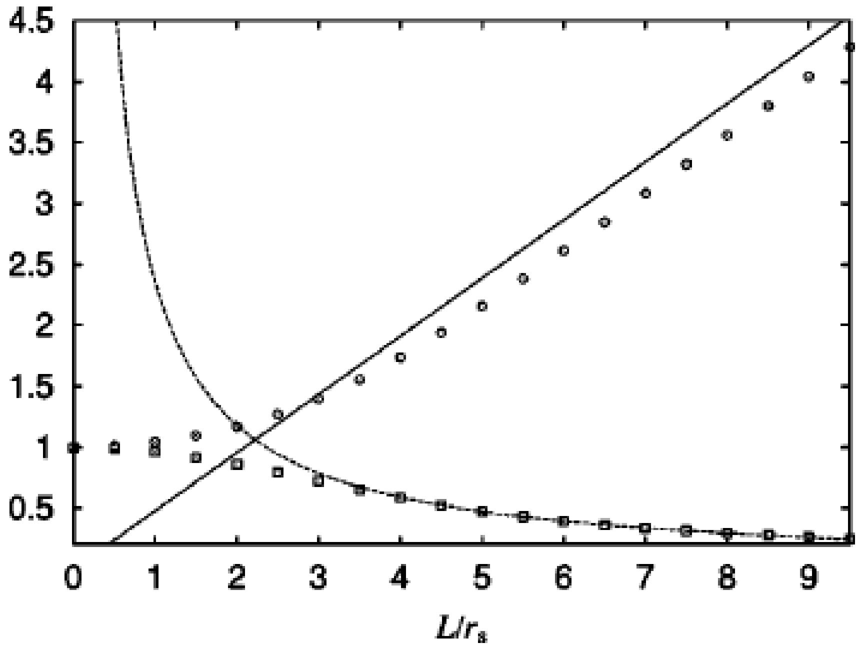

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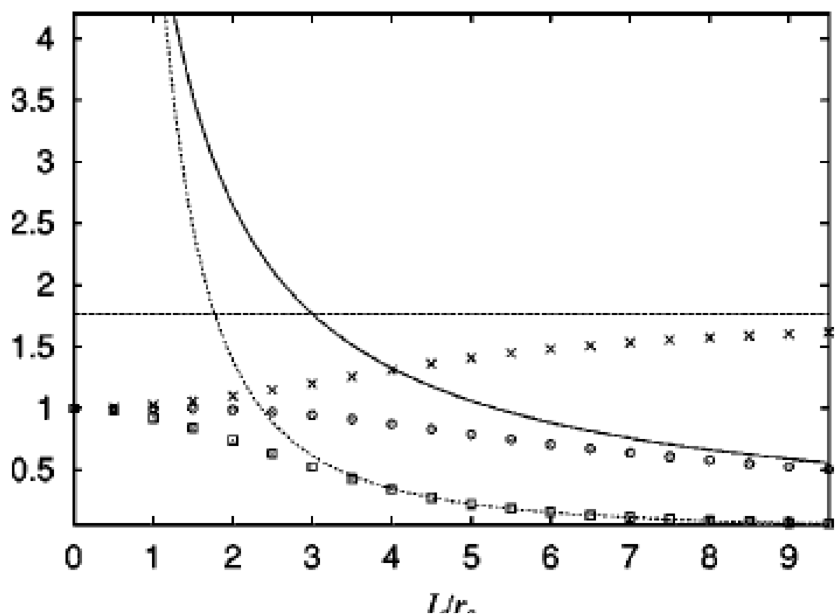
AHの3次元体積、2次元表面積、1次元長さ



$$\mathrm{Vol}(H) = 8\pi\int_0^{\pi/2} f^3\sqrt{(r_\xi)^2+r^2}\,r^2\,\sin^2\xi\, d\xi.$$

$$\mathrm{Area}(S_1) = 4\pi\int_0^{\pi/2} f^2\sqrt{(r_\xi)^2+r^2}\,r\,\sin\xi\, d\xi,$$

$$\mathrm{Area}(S_2) = \max\{4\pi f^2 r^2\,\sin^2\xi; \xi\in[0,\pi/2]\}.$$

面積S_1には、上限がある。

$$\frac{\mathrm{Area}(S)}{32GM_{ADM}/3} < O(1). \quad \Rightarrow \quad \mathrm{Area} \lesssim GM, \quad \Rightarrow \quad V_{n-2} \lesssim GM,$$

ハイパー・フープ仮説

Ida, Nakao, PRD 66, 064026 (2002)

リング形状の初期値解系列の解析(5D)

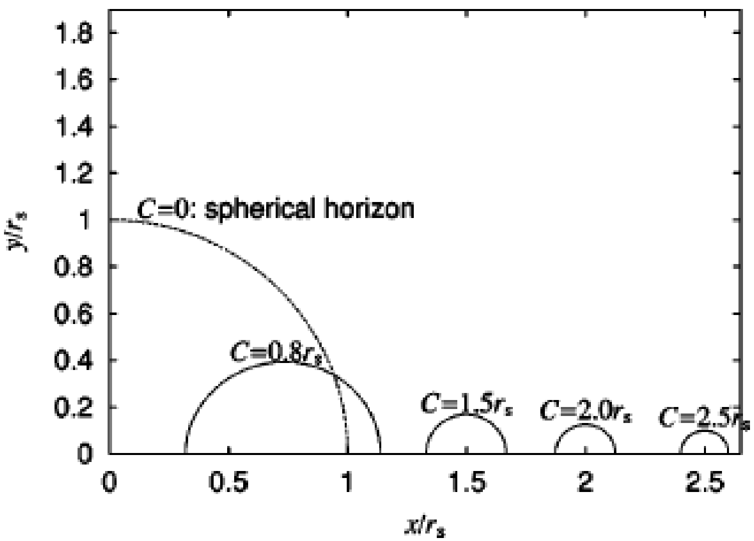
$$f^3 \varrho = \frac{M_{ADM}}{4\pi^2 C y}\delta(x-C)\delta(y).$$

$$f = 1 + \frac{2GM_{ADM}}{3\pi\sqrt{(x+C)^2+y^2}\sqrt{(x-C)^2+y^2}}.$$

AHの2次元表面積

$$\mathrm{Area}(S) = 2\pi\int_0^{\pi} f^2\sqrt{(r_\xi)^2+r^2}\, r\sin\xi\, d\xi,$$

$$\mathrm{Area}(T) = \max\{4\pi^2 f^2 r^2 \sin\xi\cos\xi;\xi\in[0,\pi]\}$$



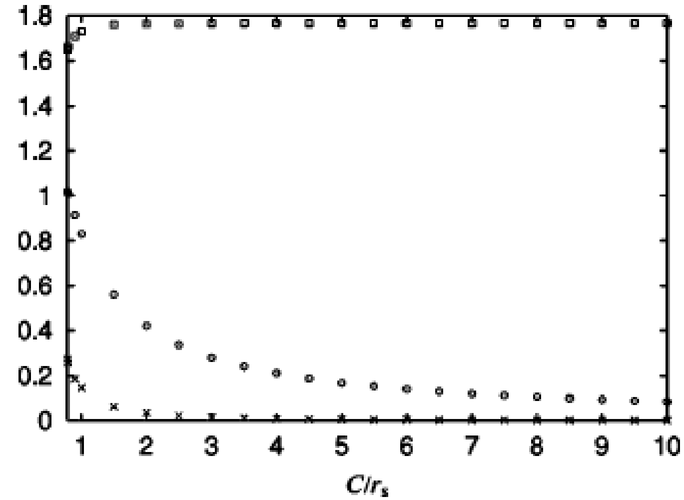
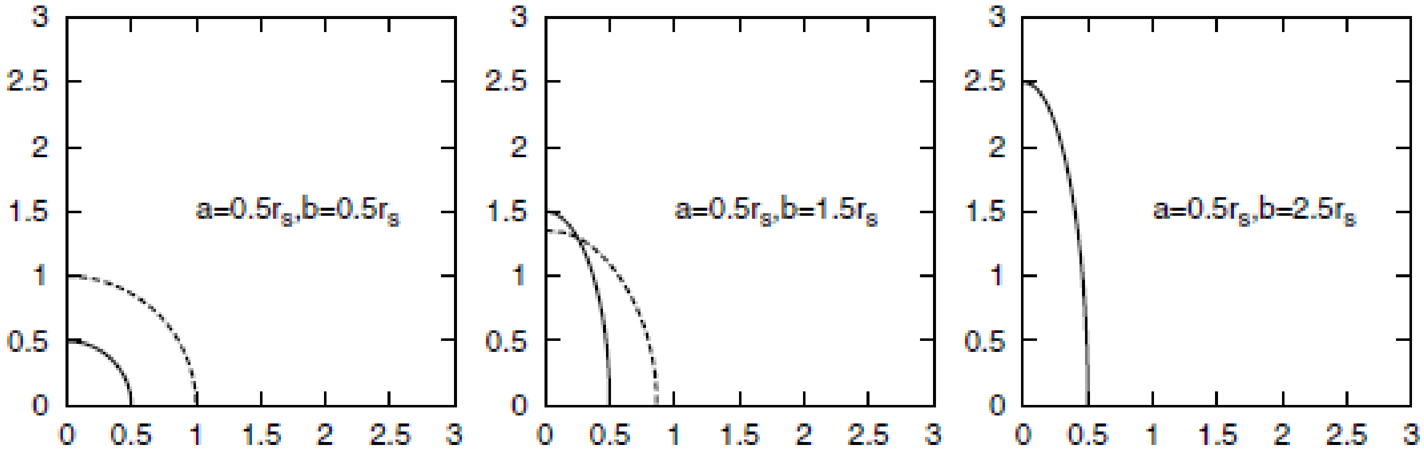


FIG. 10. The typical area scales Area(S) (crosses), Area(T) (squares) and the volume Vol(H) (circles) of a horizon are plotted as a function of the circle radius C/r_s.

Hyper hoop conjecture for spheroial matter

Yoo, Ida, Nakao, PRD 71, 104014 (2005)



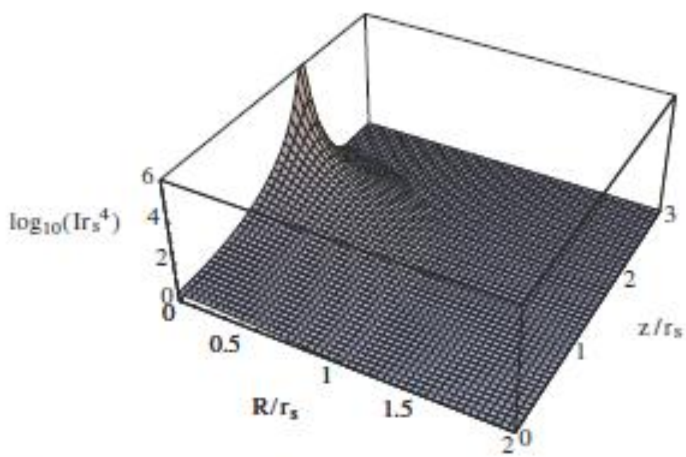


FIG. 1 (color online). The logarithm to the base 10 of the Kretschmann invariant I is plotted as function of the coordinates R and z in the case $a = 0$ and $b = 2r_s$.

$$\text{Necessary condition:} V_2 \lesssim \frac{\pi}{2} 16\pi G_5 M,$$

$$\text{Sufficient condition:} V_2 \lesssim 16\pi G_5 M.$$

2. 5次元時空の初期値問題

5次元時空での初期値設定問題

仮定

- 時間反転対称、回転なし、漸近平坦
- 共形平坦

$$\gamma_{ij} = \psi^2 \hat{\gamma}_{ij}$$

$$ds^2 = \hat{\gamma}_{ij}\, dx^i dx^j = dx^2 + dy^2 + dz^2 + dw^2$$

ハミルトニアン拘束方程式

$$\hat{\Delta}\psi = -4\pi^2 G_5 \rho$$

解く領域 $(x \geqslant 0, y \geqslant 0, z \geqslant 0, w \geqslant 0)$

境界条件 $\nabla\psi = 0$

$$\psi = 1 + \frac{M_{\mathrm{ADM}}}{r^2} \rightarrow \frac{\partial}{\partial x^i}\left[(\psi - 1) r^2\right] = 0$$

Case1. 4次元楕円体分布

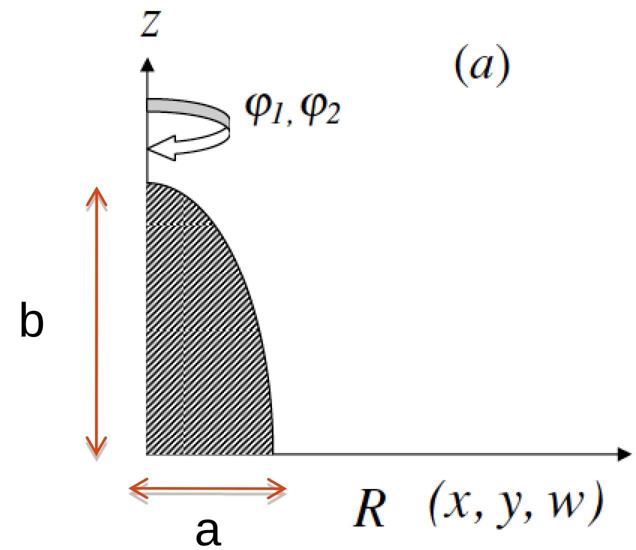
・SO(3) 対称性

$$ds^2 = \psi(R,z)^2 \left[dR^2 + R^2\left(d\varphi_1^2 + \sin^2\varphi_1\, d\varphi_2^2\right) + dz^2\right]$$

$$R = \sqrt{x^2 + y^2 + w^2} \qquad \varphi_1 = \tan^{-1}\left(\frac{w}{\sqrt{x^2+y^2}}\right) \qquad \varphi_2 = \tan^{-1}\left(\frac{y}{x}\right)$$

・ハミルトニアン拘束方程式 (Grid size 500^2)

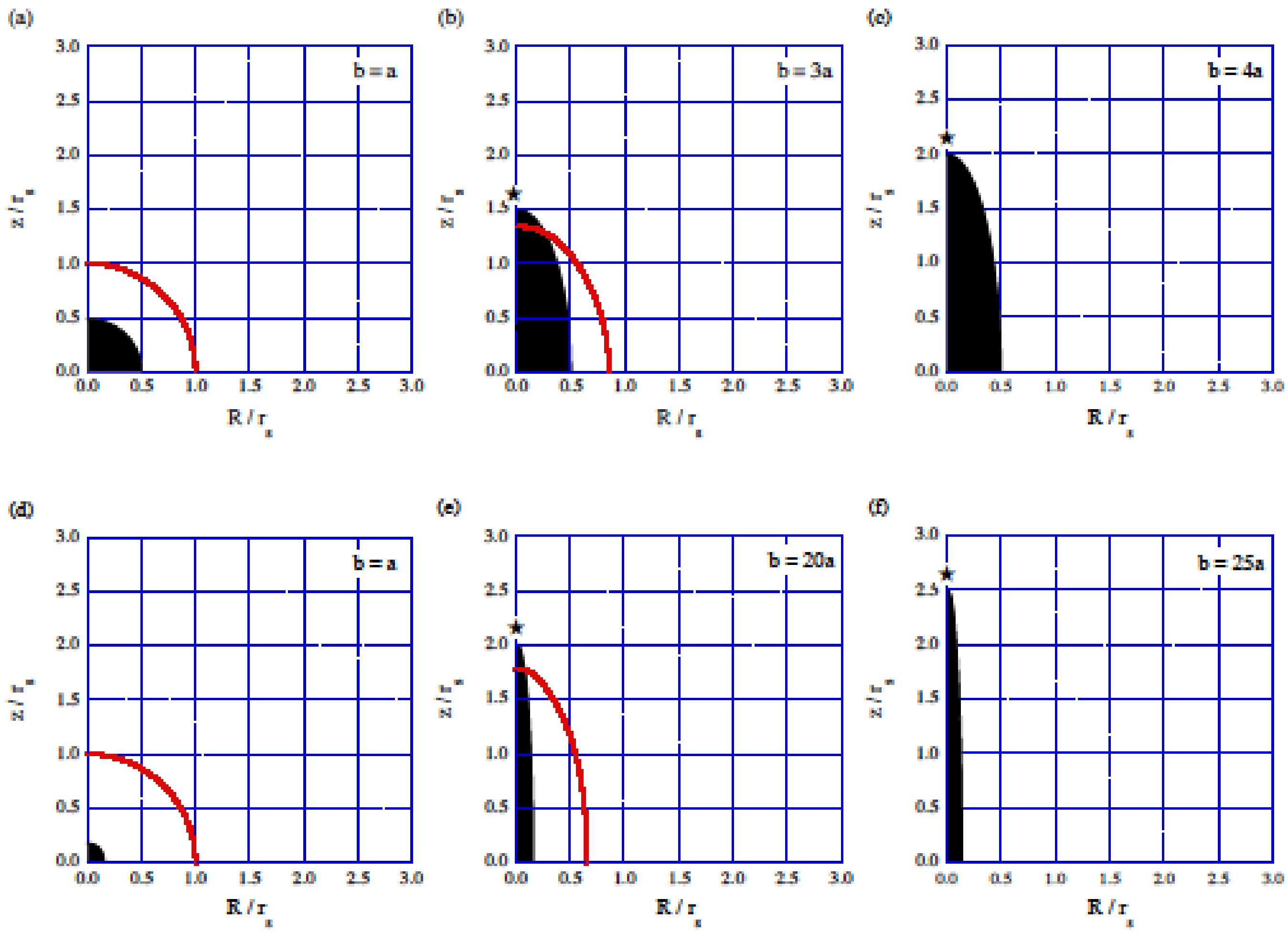
$$\frac{\partial^2\psi}{\partial R^2} + \frac{2}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial z^2} = -4\pi^2 G_5 \rho$$



・物質分布、密度

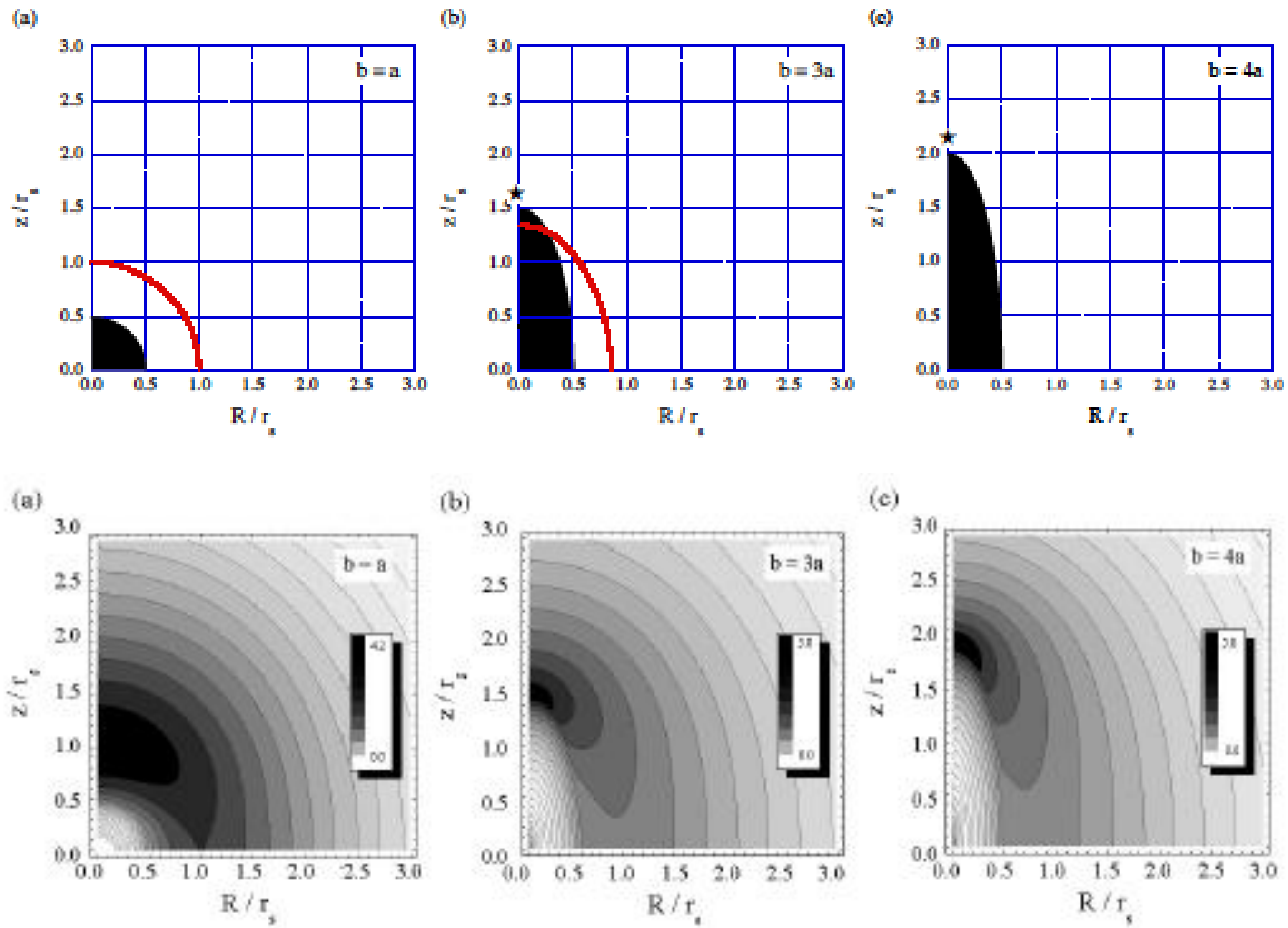
$$\frac{R^2}{a^2} + \frac{z^2}{b^2} \le 1 \qquad \rho = \frac{2M}{\pi^2 a^3 b}$$

4次元楕円体に対する初期値解系列



- 赤線はApparent horizon.
- ★はKretschmann Invariant $\mathcal{I}^{(4)} = R_{abcd}R^{abcd}$ の最大値の場所.

4次元楕円体に対する初期値解系列



- 赤線はApparent horizon.
- ★はKretschmann Invariant $\mathcal{I}^{(4)} = R_{abcd}R^{abcd}$ の最大値の場所.

Case2. 4次リング分布

- U(1)×U(1) 対称性

$$ds^2 = \psi(X,Z)^2(dX^2 + dZ^2 + X^2 d\vartheta_1^2 + Z^2 d\vartheta_2^2)$$

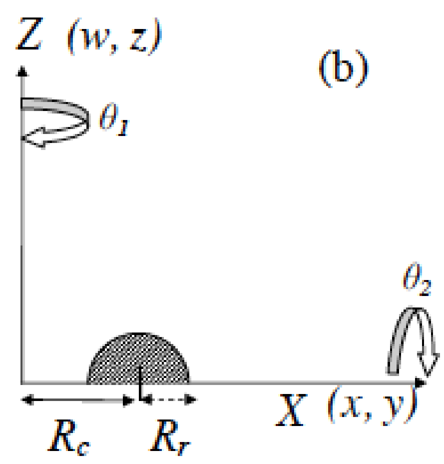
$$X = \sqrt{x^2+y^2}, \quad \vartheta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$
$$Z = \sqrt{z^2+w^2}, \quad \vartheta_2 = \tan^{-1}\left(\frac{z}{w}\right)$$

- ハミルトニアン拘束方程式 (Grid size 500^2)

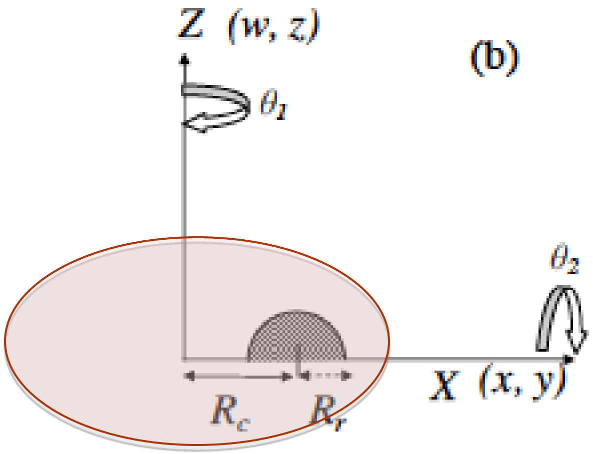
$$\frac{1}{X}\frac{\partial}{\partial X}\left(X\frac{\partial\psi}{\partial X}\right) + \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial\psi}{\partial Z}\right) = -4\pi^2 G_5 \rho,$$

- 物質分布、密度

$$(X-R_c)^2 + Z^2 \le R_r^2, \qquad \rho = \frac{3M}{8\pi^2 R_r{}^3 R_c}$$



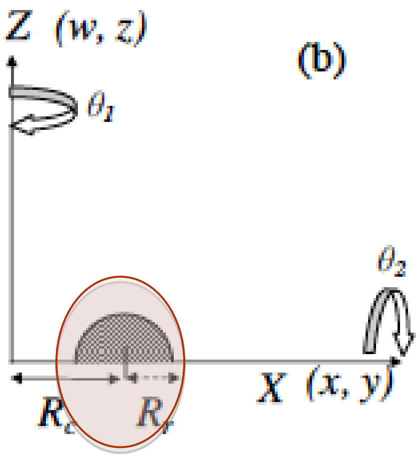
リング apparent horizon 探索



(b)

$$\ddot{r}_m - 4\frac{\dot{r}_m^{\,2}}{r_m} - 3r_m - \frac{r_m^2+\dot{r}_m^{\,2}}{r_m}\left[2\frac{\dot{r}_m}{r_m}\cot(2\phi) - \frac{3}{\psi}(\dot{r}_m\sin\phi + r\cos\phi)\frac{\partial\psi}{\partial X} + \frac{3}{\psi}(\dot{r}_m\cos\phi - r_m\sin\phi)\frac{\partial\psi}{\partial Z}\right] = 0,$$

$$A_3^{(T1)} = 4\pi^2\int_0^{\pi/2}\psi^3 r_m^2\cos\phi\sin\phi\sqrt{\dot{r}_m^{\,2}+r_m^2}\,d\phi,$$

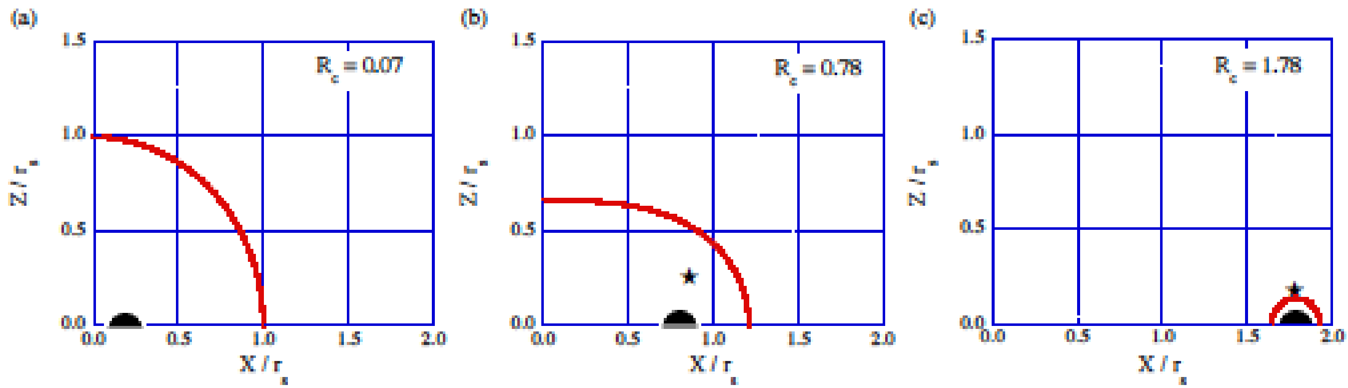


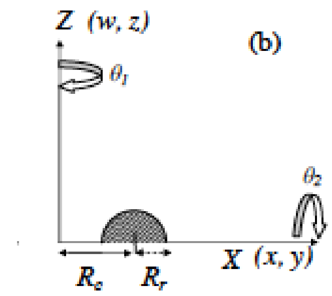
(b)

$$\ddot{r}_m - \frac{3\dot{r}_m^{\,2}}{r_m} - 2r_m - \frac{r_m^2+\dot{r}_m^{\,2}}{r_m}\times\left[\frac{\dot{r}_m\sin\xi + r_m\cos\xi}{r_m\cos\xi+R_c} - \frac{\dot{r}_m}{r_m}\cot\xi + \frac{3}{\psi}(\dot{r}_m\sin\xi + r\cos\xi)\frac{\partial\psi}{\partial x} - \frac{3}{\psi}(\dot{r}_m\cos\xi - r\sin\xi)\frac{\partial\psi}{\partial z}\right] = 0,$$

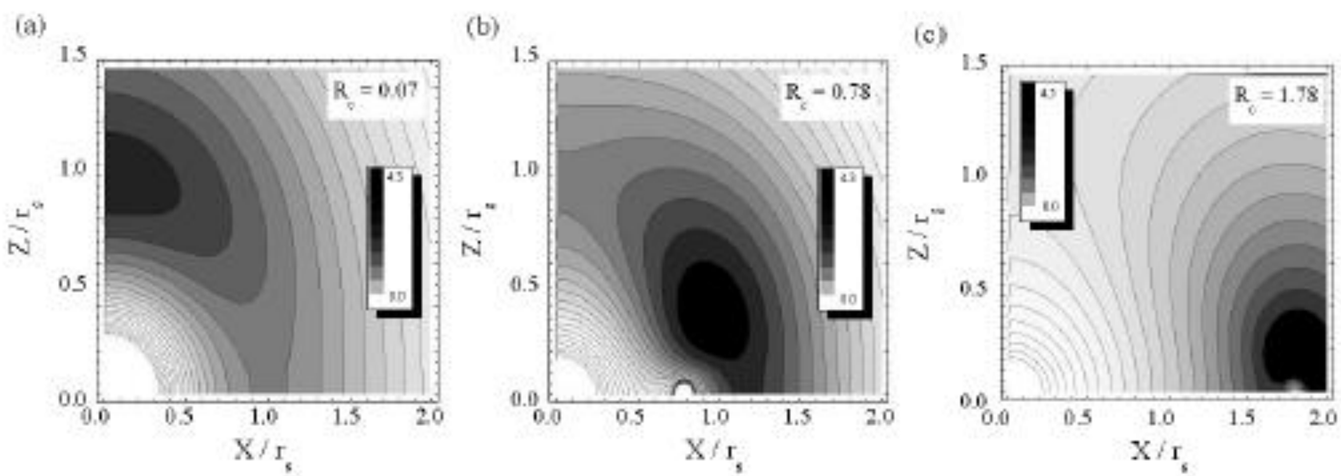
$$A_3^{(T2)} = 4\pi^2\int_0^{\pi}\psi^3(R_c + r_m\cos\xi)r_m\sin\xi\sqrt{\dot{r}_m^{\,2}+r_m^2}\,d\xi,$$

4次元リング分布に対する初期値解系列



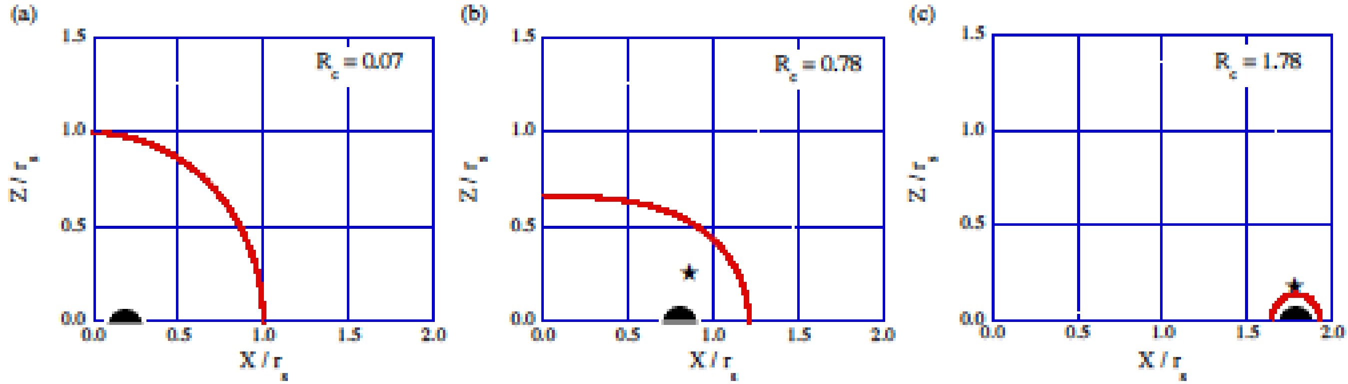


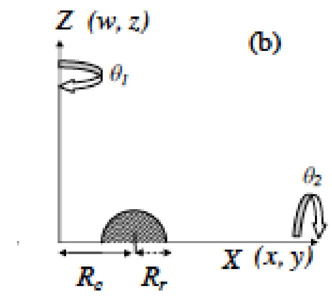
R_r/r_s=0.1で固定



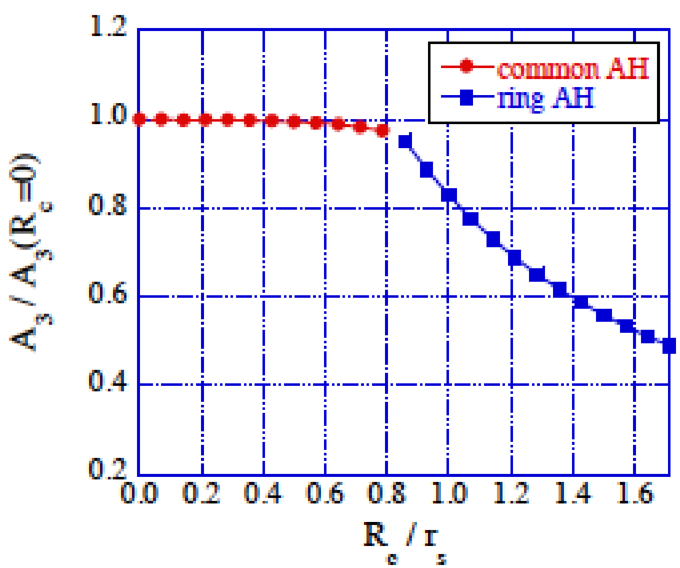
- 赤線はApparent horizon.
- ★はKretschmann Invariant $\mathcal{I}^{(4)} = R_{abcd}R^{abcd}$ の最大値の場所.

4次元リング分布に対する初期値解系列





R_r/r_s=0.1で固定

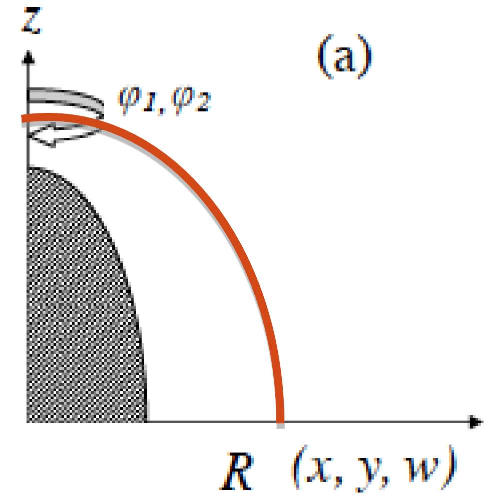


・horizonの表面積。

ハイパー・フープ仮説の検証 (楕円体分布)

・対称軸R, z に対して2次元面を定義。

・δV=0の条件を課し、面積が最小になる場所を探す。



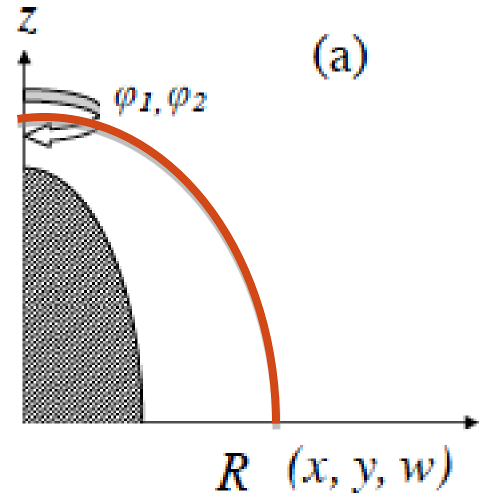
$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\theta d\theta$$ (for z-axis)

$$V_2^{(B)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\theta d\theta$$ (for R-axis)

$$r = \sqrt{R^2 + z^2}, \quad \theta = \tan^{-1}\left(\frac{R}{z}\right),$$

ハイパー・フープ仮説の検証 (楕円体分布)

$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \sin\theta d\theta$$ (for z-axis)



$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\cot\theta - \frac{2}{\psi}(\dot{r_h}\sin\theta + r_h\cos\theta)\frac{\partial\psi}{\partial z} - \frac{2}{\psi}(r_h\sin\theta - \dot{r_h}\cos\theta)\frac{\partial\psi}{\partial R}\right] = 0,$$

$$V_2^{(B)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\theta d\theta$$ (for R-axis)

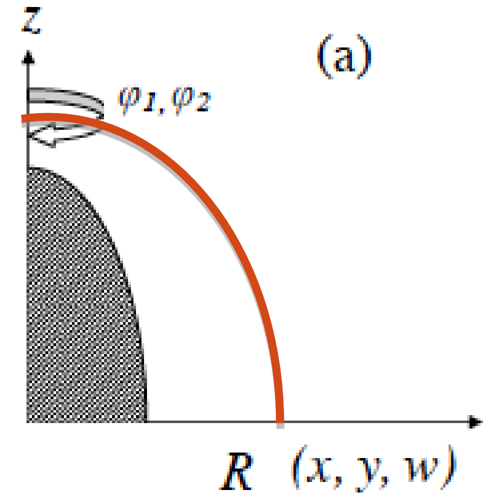
$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\tan\theta + \frac{2}{\psi}(r_h\sin\theta - \dot{r_h}\cos\theta)\frac{\partial\psi}{\partial R} + \frac{2}{\psi}(r_h\cos\theta + \dot{r_h}\sin\theta)\frac{\partial\psi}{\partial z}\right] = 0.$$

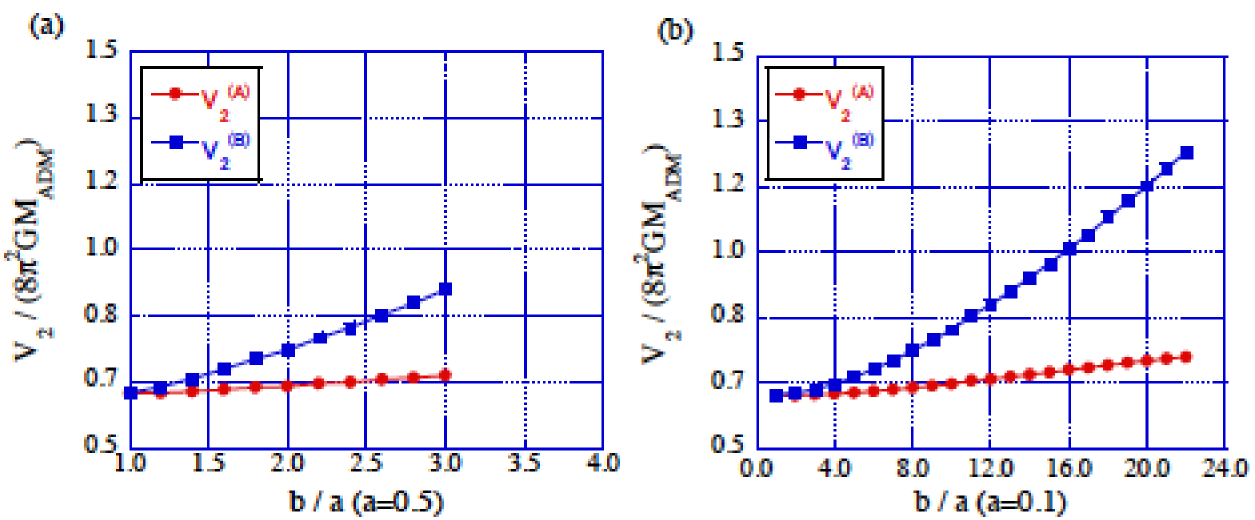
$$r = \sqrt{R^2 + z^2},$$
$$\theta = \tan^{-1}\left(\frac{R}{z}\right),$$

ハイパー・フープ仮説の検証 (楕円体分布)

$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\theta d\theta$$

$$V_2^{(B)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\theta d\theta$$

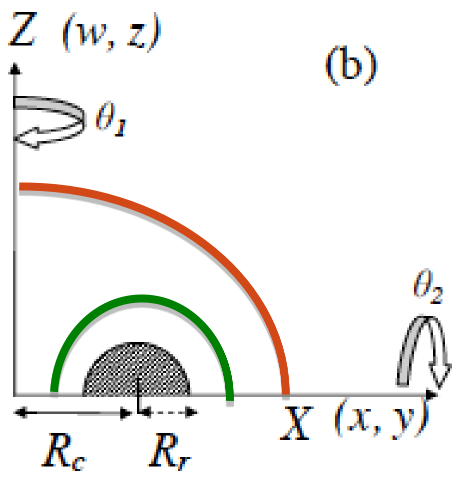




$$V_2 \leq \frac{\pi}{2} 16\pi G_5 M.$$

値は不等式の右辺項で規格化。

ハイパー・フープ仮説の検証 (リング分布)



S² hoop

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\phi d\phi,$$

$$V_2^{(D)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\phi d\phi.$$

$$r = \sqrt{X^2 + Z^2}, \quad \phi = \tan^{-1}\left(\frac{Z}{X}\right).$$

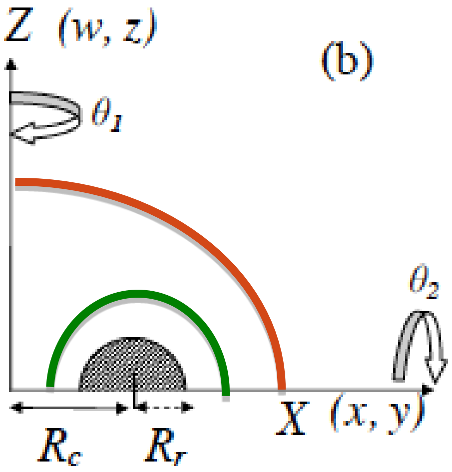
S¹ × S¹ hoop

$$V_2^{(E)} = 2\pi \int_0^{\pi} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} (r_h \cos\xi + R_c) d\xi.$$

$$r = \sqrt{(X - R_c)^2 + Z^2}, \quad \xi = \tan^{-1}\left(\frac{Z}{X - R_c}\right).$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{-R_c + \dot{r_h}\sin\xi}{R_c + r_h\cos\xi} + \frac{2}{\psi}(\dot{r_h}\sin\xi + r_h\cos\xi)\frac{\partial\psi}{\partial X} + \frac{2}{\psi}(r_h\sin\xi - \dot{r_h}\cos\xi)\frac{\partial\psi}{\partial Z}\right] = 0.$$

ハイパー・フープ仮説の検証 (リング分布)



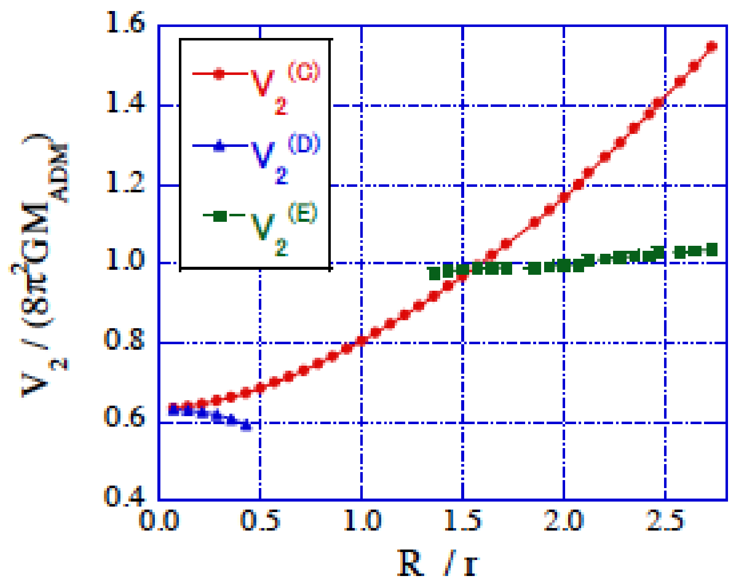
S² hoop

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\phi d\phi,$$

$$V_2^{(D)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\phi d\phi.$$

S¹ × S¹ hoop

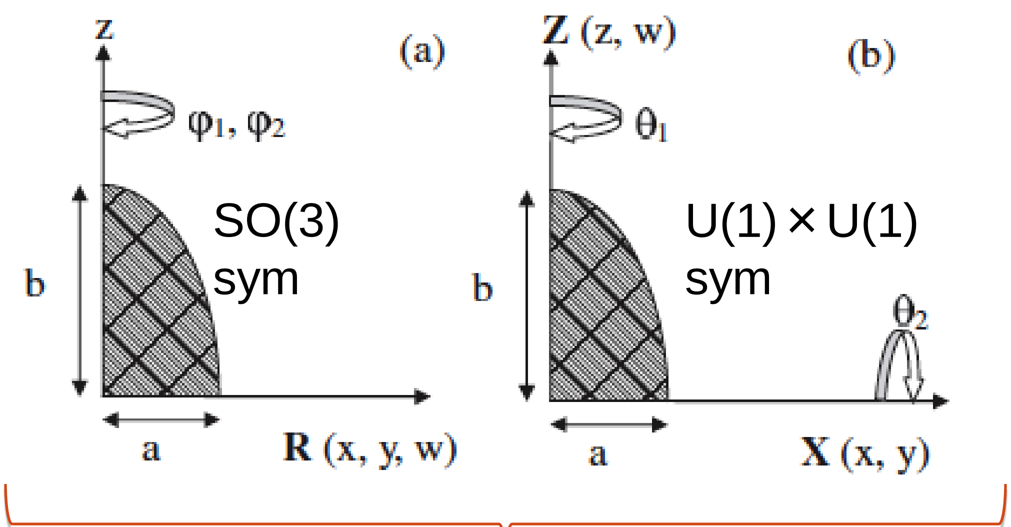
$$V_2^{(E)} = 2\pi \int_0^{\pi} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} (r_h \cos\xi + R_c) d\xi.$$



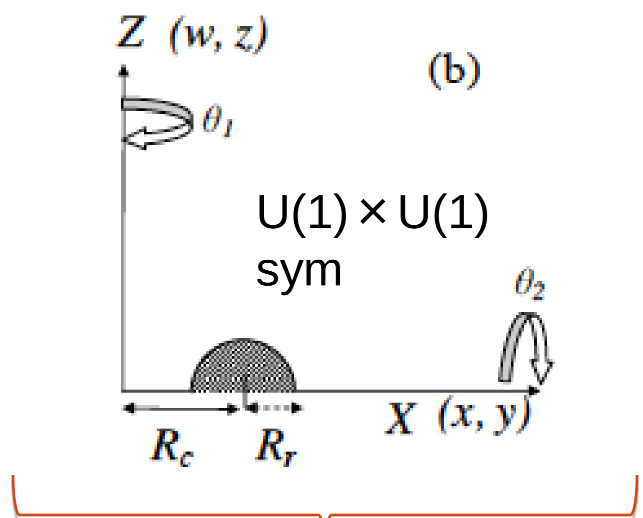
3. 5次元時空での重力崩壊シミュレーション

5次元時空での重力崩壊シミュレーション

- ADM(4+1)形式。
- 回転なし。
- 非衝突粒子で物質分布を表現し、測地線方程式を解く。
- Apparent horizon 探索と、Kretschmann invariant の計算。
- Cartoon method を用いる(Grid size $130^2 \times 2^2$)。



5次元スピンドル重力崩壊



5次元リング重力崩壊

時間発展方程式（4+1 分解）

計量： $g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_k \beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$ outer boundary $\gamma_{ij} = \delta_{ij} + \frac{\mathrm{const}}{r^2}$

$$\begin{aligned} \frac{\partial \gamma_{ij}}{\partial t} &= -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i, \\ \frac{\partial K_{ij}}{\partial t} &= \alpha({}^{(4)}R_{ij} + K K_{ij}) - 2\alpha K_{il} K^{lj} \\ &\quad -\alpha\kappa^2 (S_{ij} + \frac{1}{3}\gamma_{ij}(\rho - S)) \\ &\quad -D_i D_j \alpha + D_i \beta^m K_{mj} + D_j \beta^m K_{mi} + \beta^m D_m K_{ij}, \end{aligned}$$

2nd-order differential scheme
Iterative Crank-Nicolson method
Courant factor 0.2

スライス条件（ラプス）α

- Maximal time slicing condition

$$K=0 \iff \partial_t K \iff$$

$$\triangle\alpha = \alpha\left(K_{ij}K^{ij} + \frac{2}{3}\kappa\rho + \frac{1}{3}\kappa S\right)$$

境界条件： $(\alpha-1)r^2 = \mathrm{const} \iff \frac{\partial}{\partial x^i}\left[(\alpha-1)r^2\right] = 0$

*z軸方向の時間一定面

"Singularity avoidance"

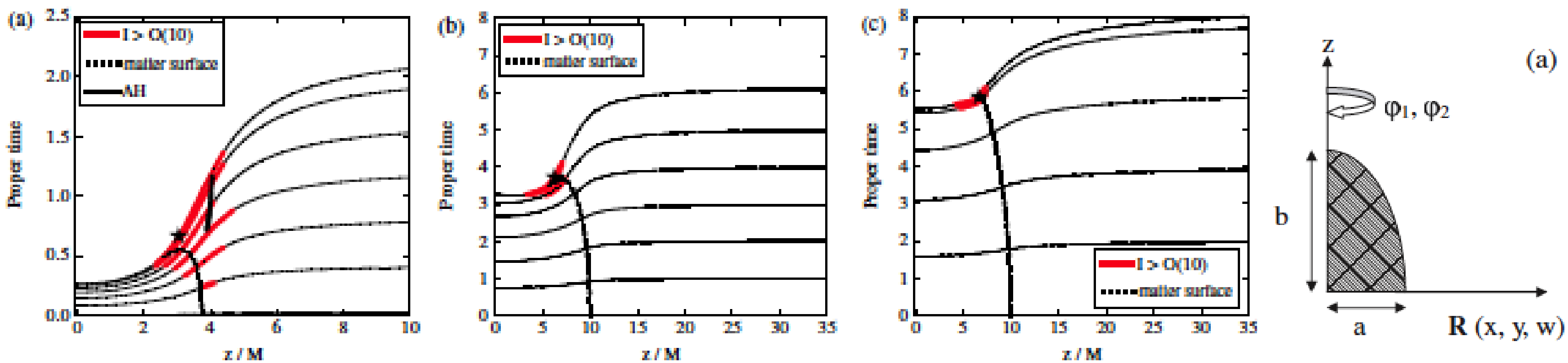


FIG. 4 (color online). The snapshots of the hypersurfaces on the z axis in the propertime versus coordinate diagram; (a) model 5DSβ, (b) model 5DSδ, and (c) model 4Dδ. The uppermost hypersurface is the final data in numerical evolution. We also mark the matter surface and the location of AH if exist. The ranges with $\mathcal{I} \gtrsim 10$ are marked with bold lines and peak value of $\mathcal{I}$ expressed by asterisks.

スライス条件（シフト）β

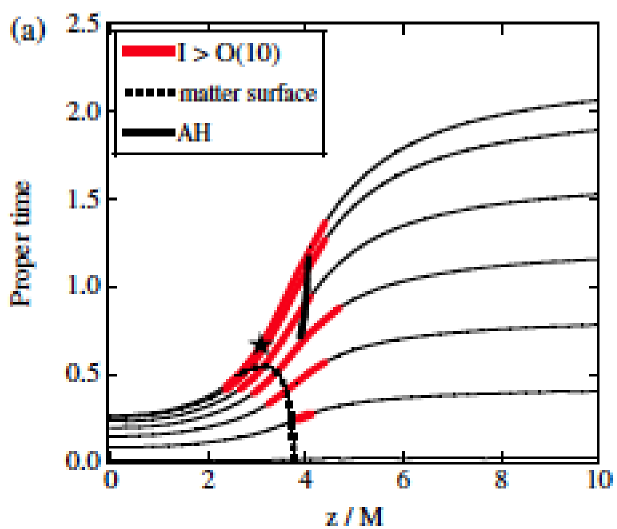
- Minimal strain condition

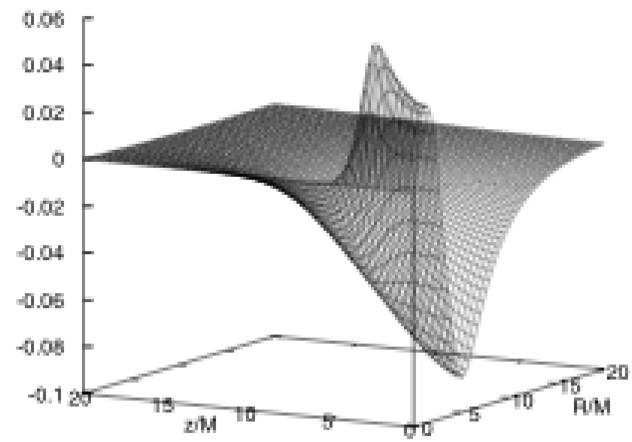
$$\Theta_{\mu\nu} = \perp\nabla_{(\nu}t_{\mu)} = -\alpha K_{\mu\nu} + \frac{1}{2}D_{(\mu}\beta_{\nu)}$$ congruence : $t_\mu = \alpha n_\mu + \beta_\mu$

$$D_j\Theta_{ij} = 0 \iff \Delta\beta^i + D^iD_j\beta^j + R_{ij}\beta^j = 2D^j(\alpha K_{ij})$$

境界条件： $\beta^i r^2 = \mathrm{const} \iff \frac{\partial}{\partial x^i}[\beta r^2] = 0$

“anti grid-stretching”





βx

Matter = particles

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

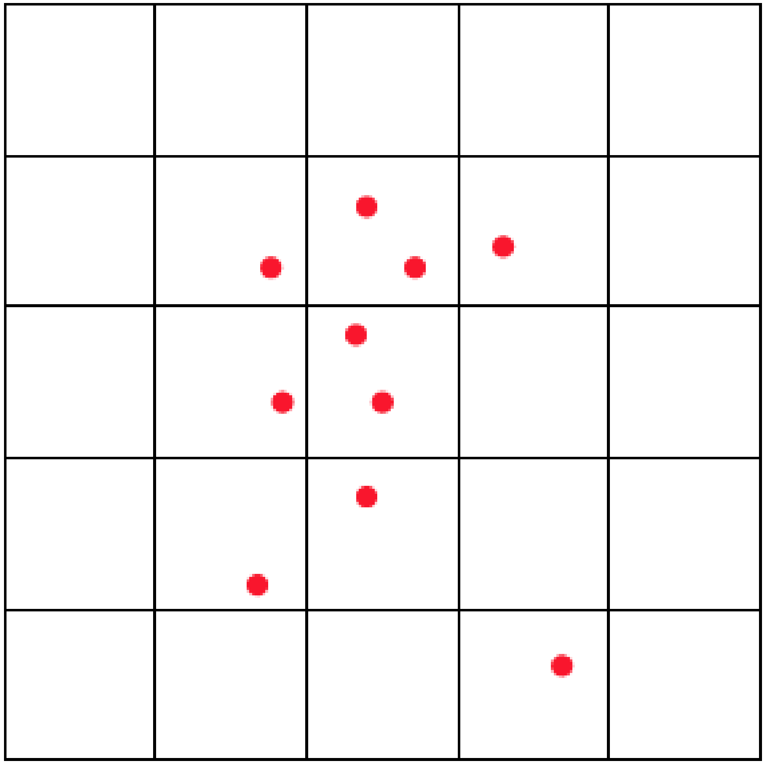
5000 particles
Runge-Kutta 法

$$T^{ij} = \sum_A mn_{(A)} u^i_{(A)} u^j_{(A)}$$

n : 数密度

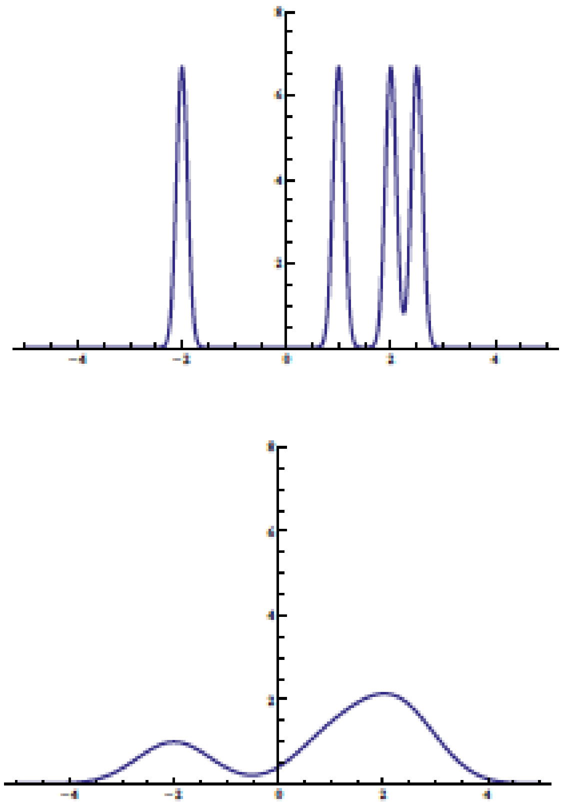
$$\rho = \sum_A mn_{(A)} (\alpha u^0_{(A)})^2$$

$$S_{ij} = \sum_A mn_{(A)} u_{i(A)} u_{j(A)}$$



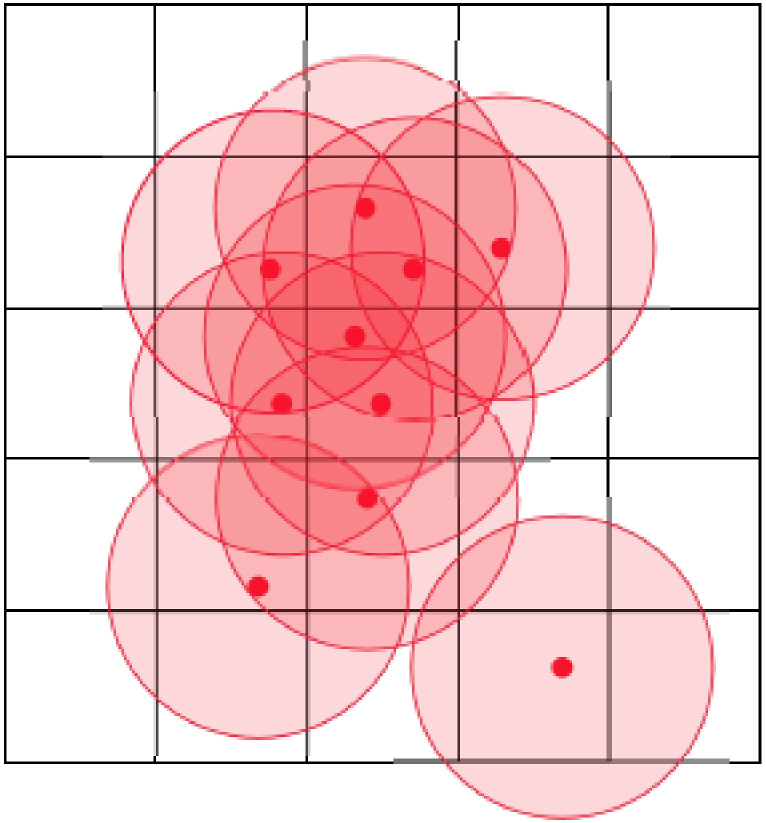
Matter = particles

cf. SPH



Smoothing kernel

$$W(x_i, y_i) = \frac{\exp\left[\{-(x-x_i)^2 - (y-y_i)^2\}/h^2\right]}{\pi h^2}$$



Matter = particles

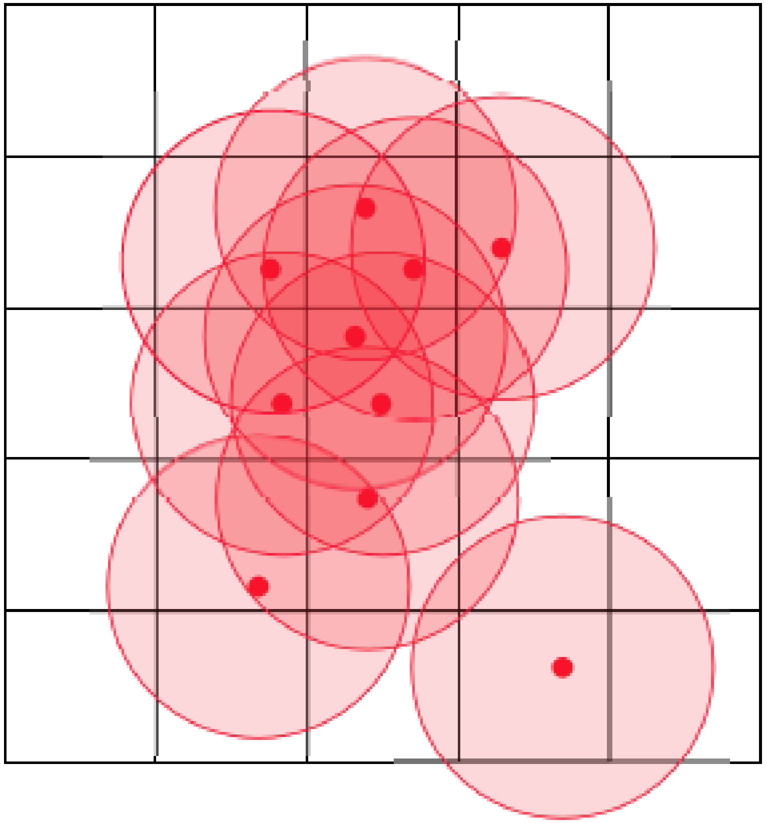
cf. SPH

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

$$W(x_i, y_i) = \frac{\exp\left[\{-(x-x_i)^2 - (y-y_i)^2\}/h^2\right]}{\pi h^2}$$

$$T^{ij} = \sum_A mn_{(A)} u^i_{(A)} u^j_{(A)}$$

n : 数密度



$$\rho = \sum_A mn_{(A)} (\alpha u^0_{(A)})^2 W$$

$$S_{ij} = \sum_A mn_{(A)} u_{i(A)} u_{j(A)} W$$

Cartoon method

Alcubierre, Brandt, Bruegmann, Holz, Seidel, Takahashi, Thornburg,
gr-qc/9908012

円筒座標

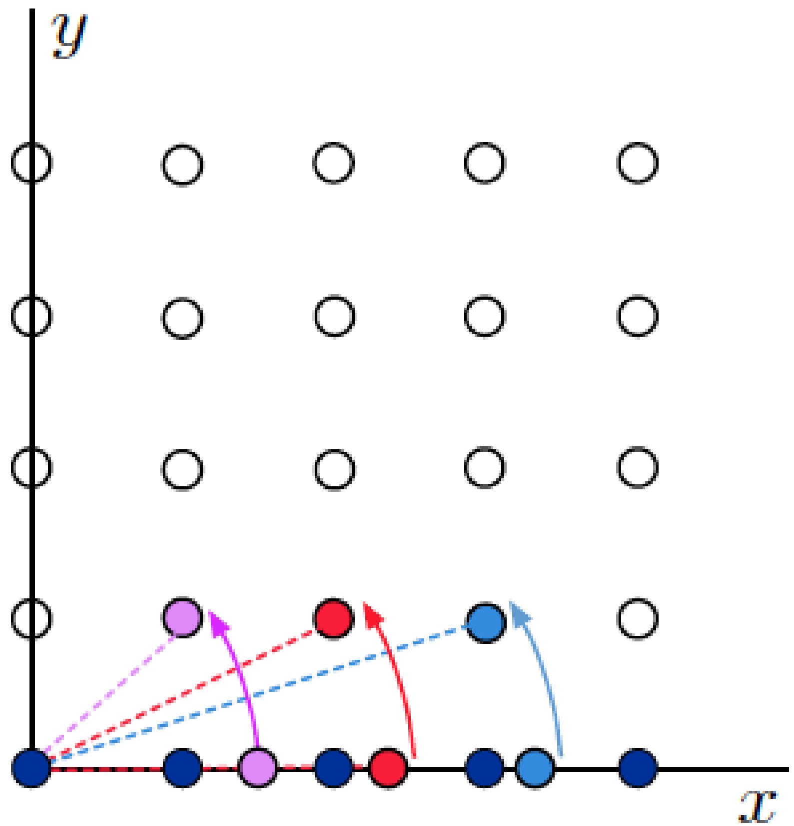
$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

Cartesian 座標

$$(x, y, z)$$



(x, 0, z)面上の値を、(x, y, z)へ補間。

Cartoon method (4-dim)

Alcubierre, Brandt, Bruegmann, Holz, Seidel, Takahashi, Thornburg, gr-qc/9908012

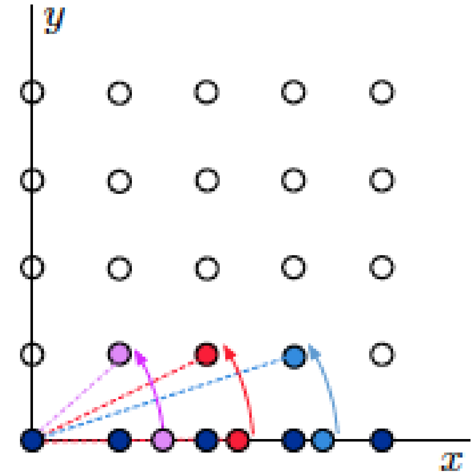
scala

$$\Psi(x,y,z) = \Psi(\rho,0,z)$$

vector

$$T^z(x,y,z) = T^z(\rho,0,z)$$

$$T^x(x,y,z) = (x/\rho)T^x(\rho,0,z) - (y/\rho)T^y(\rho,0,z)$$
$$T^y(x,y,z) = (y/\rho)T^x(\rho,0,z) + (x/\rho)T^y(\rho,0,z)$$



tensor

$$S^{zz}(x,y,z) = S^{zz}(\rho,0,z)$$

$$S^{zx}(x,y,z) = (x/\rho)S^{zx}(\rho,0,z) - (y/\rho)S^{zy}(\rho,0,z)$$
$$S^{zy}(x,y,z) = (y/\rho)S^{zx}(\rho,0,z) + (x/\rho)S^{zy}(\rho,0,z)$$

$$S^{xx}(x,y,z) = (x/\rho)^2 S^{xx}(\rho,0,z) + (y/\rho)^2 S^{yy}(\rho,0,z) - (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{yy}(x,y,z) = (y/\rho)^2 S^{xx}(\rho,0,z) + (x/\rho)^2 S^{yy}(\rho,0,z) + (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{xy}(x,y,z) = (xy/\rho)[S^{xx}(\rho,0,z) - S^{yy}(\rho,0,z)] + [(x^2-y^2)/\rho^2]S^{xy}(\rho,0,z)$$

Cartoon method (5-dim)

Shibata, Yoshino, PRD 80 (2009) 084025

SO(3)の場合（x=y=z, w）

scalar

$$\Psi(x,y,z,w) = \Psi(r,0,0,w)$$

vector

$$T^x(x,y,z,w) = (x/r)T^x(r,0,0,w)$$

$$T^y(x,y,z,w) = (y/r)T^x(r,0,0,w)$$

$$T^z(x,y,z,w) = (z/r)T^x(r,0,0,w)$$

$$T^w(x,y,z,w) = T^w(r,0,0,w)$$

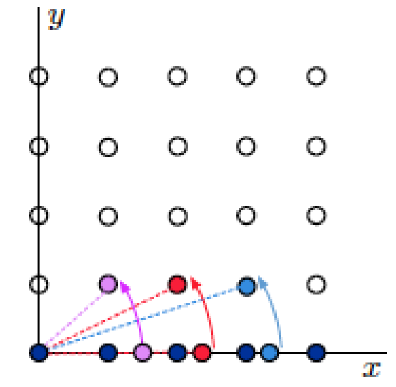
tensor

$$S^{ww}(x,y,z,w) = S^{ww}(r,0,0,w)$$

$$S^{xw}(x,y,z,w) = (x/r)S^{xw}(r,0,0,w)$$
$$S^{yw}(x,y,z,w) = (y/r)S^{xw}(r,0,0,w)$$
$$S^{zw}(x,y,z,w) = (z/r)S^{xw}(r,0,0,w)$$

$$S^{xx}(x,y,z,w) = (x^2/r^2)S^{xx}(r,0,0,w) + \left(1 - x^2/r^2\right) S^{yy}(r,0,0,w)$$
$$S^{yy}(x,y,z,w) = (y^2/r^2)S^{xx}(r,0,0,w) + \left(1 - y^2/r^2\right) S^{yy}(r,0,0,w)$$
$$S^{zz}(x,y,z,w) = (z^2/r^2)S^{xx}(r,0,0,w) + \left(1 - z^2/r^2\right) S^{yy}(r,0,0,w)$$

$$S^{yz}(x,y,z,w) = (yz/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$
$$S^{zx}(x,y,z,w) = (zx/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$
$$S^{xy}(x,y,z,w) = (xy/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$



4次元時空でのSpheroidal Collapse

ST result

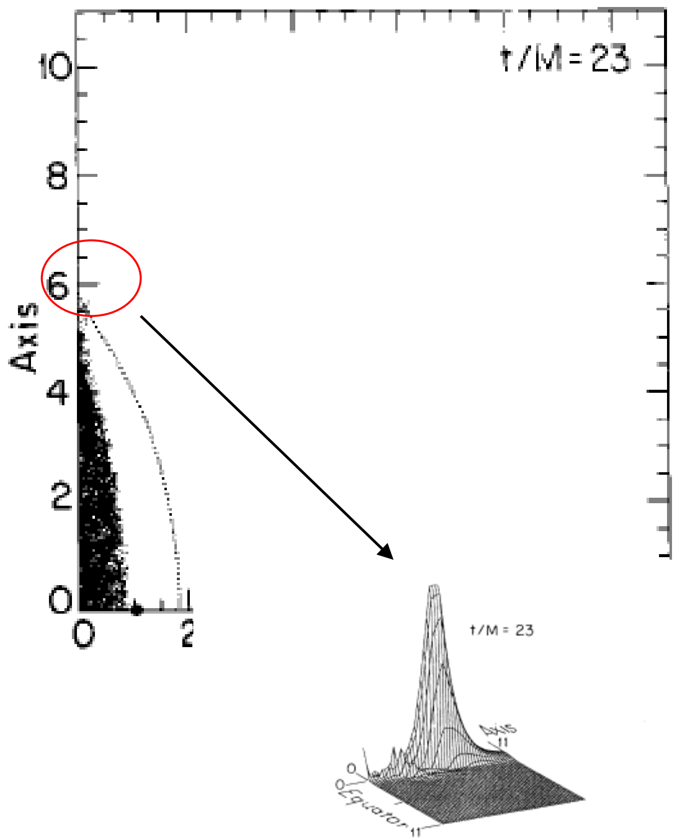
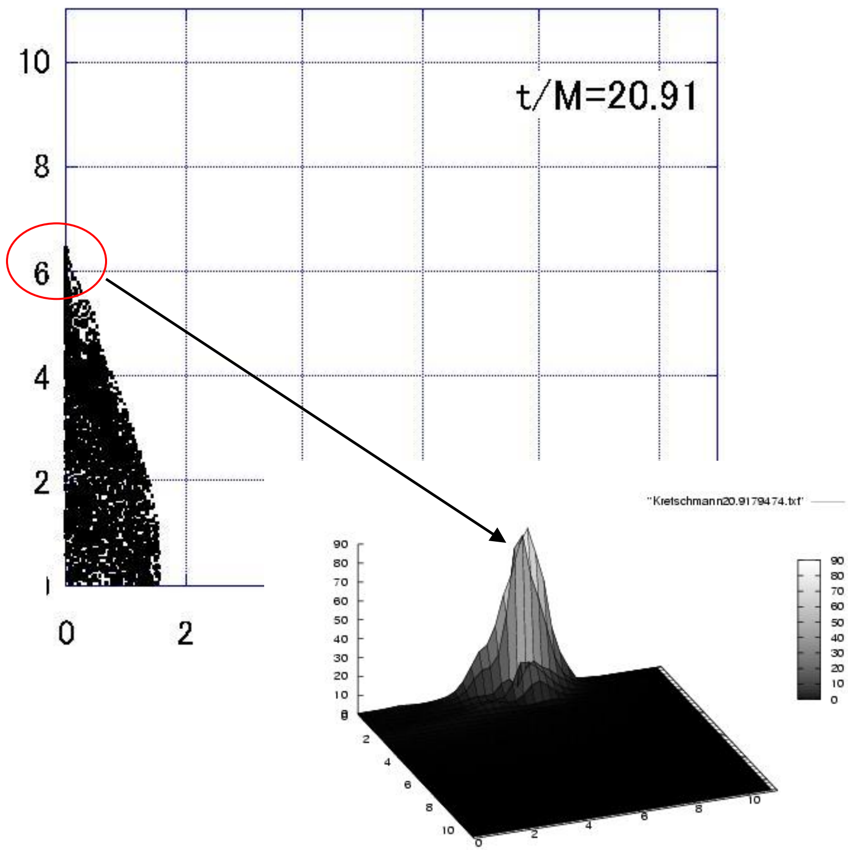


FIG. 4. Profile of I in a meridional plane for the collapse shown in Fig. 2. For the case of 32 angular zones shown here, the peak value of I is $24/M^4$ and occurs on the axis just outside the matter.

Our result



5次元時空でのSpheroidal Collapse

Yamada, Shinkai, PRD 83, 064006

SO(3) sym (初期の離心率 0.9)

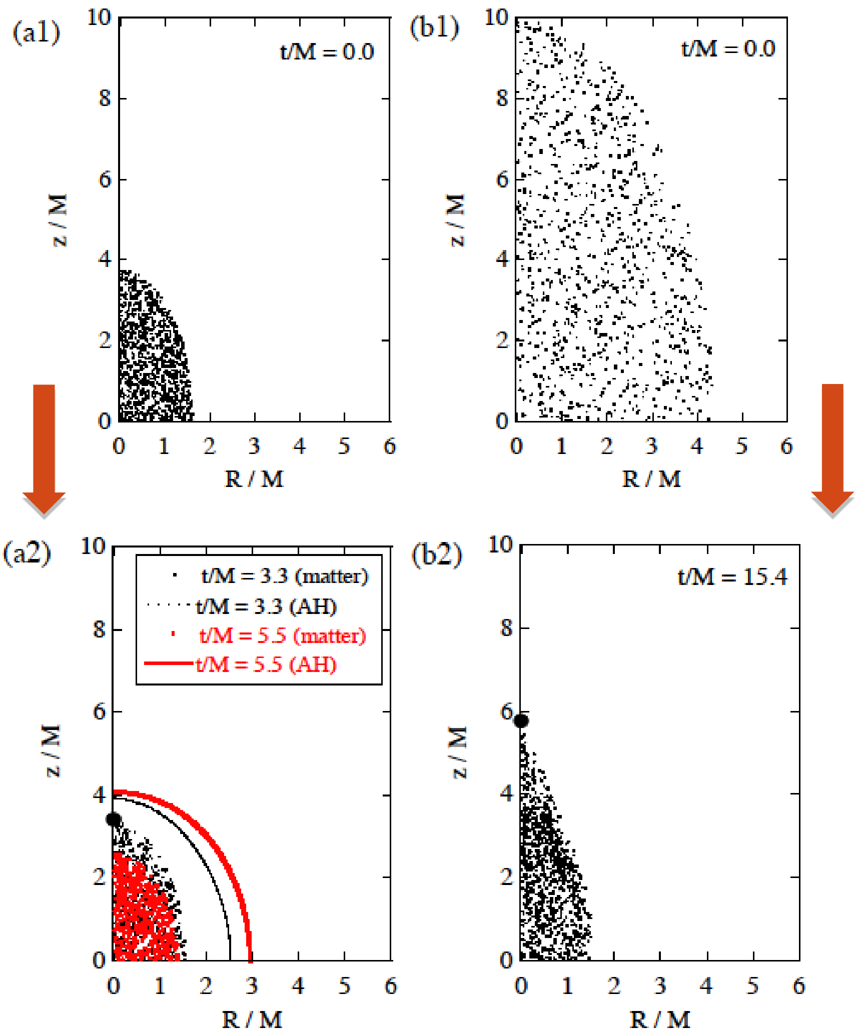


FIG. 2: Snapshots of 5D axisymmetric evolution with the initial matter distribution of $b/M = 4$ [Fig.(a1) and (a2); model 5DSβ in Table I] and 10 [Fig.(b1) and (b2); model 5DSδ]. We see the apparent horizon (AH) is formed at the coordinate time $t/M = 3.3$ for the former model and the area of AH increases, while AH is not observed for the latter model up to the time $t/M = 15.4$ when our code stops due to the large curvature. The big circle indicates the location of the maximum Kretschmann invariant $\mathcal{I}_{\max}$ at the final time at each evolution. Number of particles are reduced to 1/10 for figures.

$$\mathcal{I} = R_{abcd}R^{abcd}$$

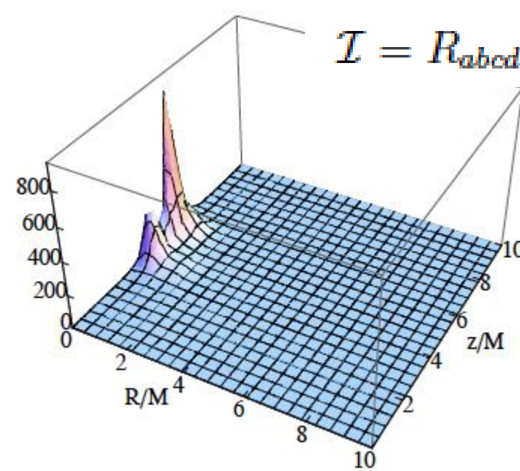
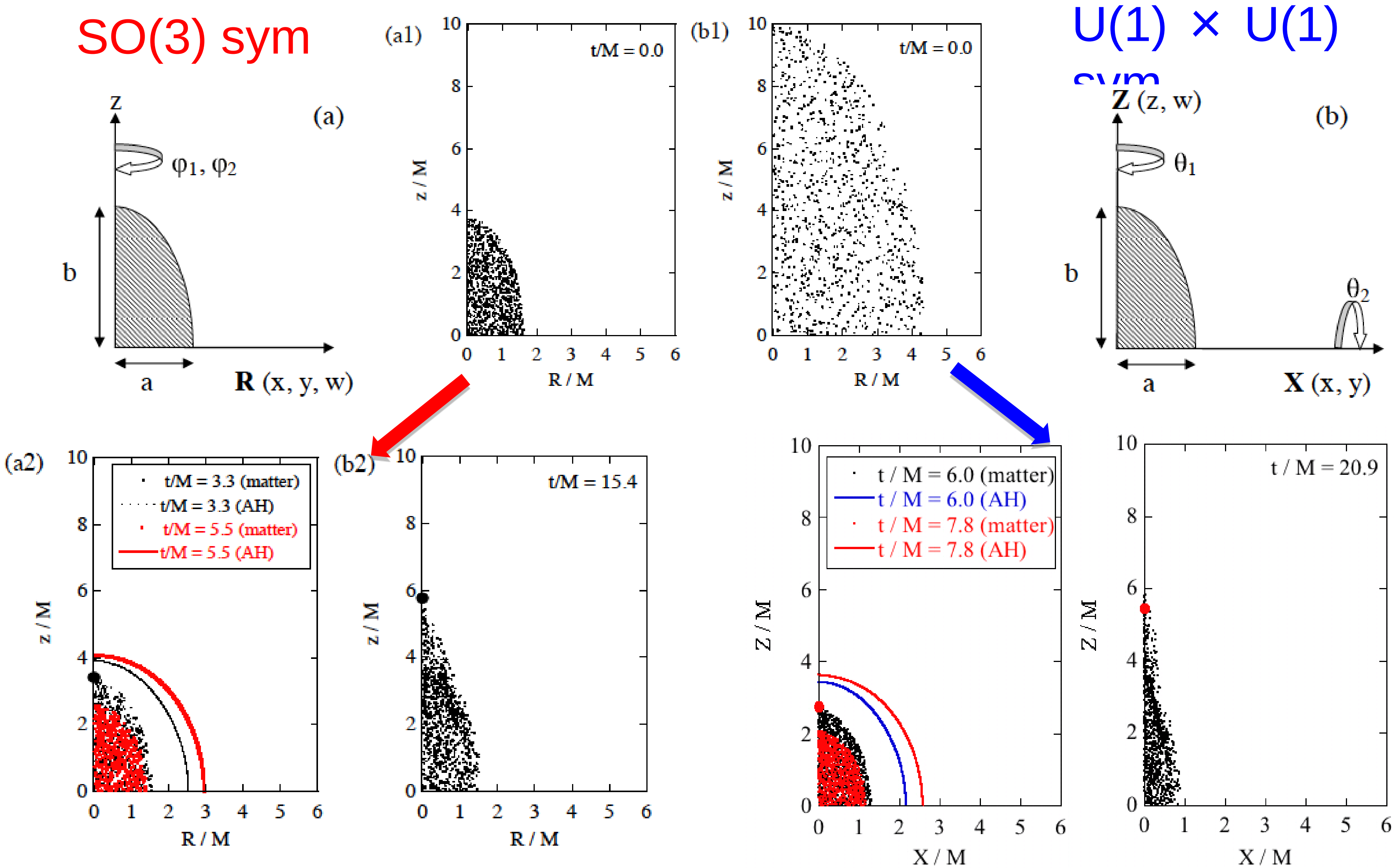


FIG. 3: Kretschmann invariant $\mathcal{I}$ for model 5DSδ at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

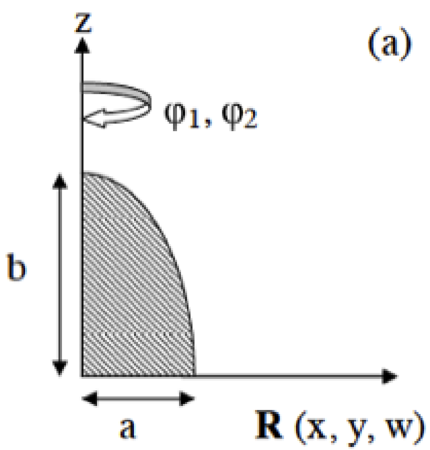
5次元時空でのSpheroidal Collapse

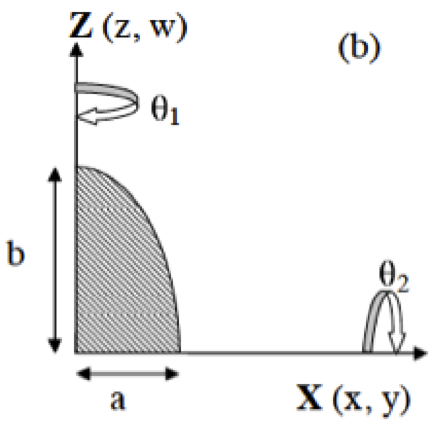
Yamada, Shinkai, PRD 83, 064006



5次元時空でのSpheroidal Collapse

4次元、5次元重力崩壊の比較





$b/M\ (t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα	4Dβ	4Dγ	4Dδ
	AH-formed	no	no	no
	$e_{\rm AH}=0.90$			
	$e_{\rm f}=0.92$	$e_{\rm f}=0.89$	$e_{\rm f}=0.92$	$e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα	5DSβ	5DSγ	5DSδ
	AH-formed	AH-formed	no	no
	$e_{\rm AH}=0.88$	$e_{\rm AH}=0.88$		
	$e_{\rm f}=0.82$	$e_{\rm f}=0.84$	$e_{\rm f}=0.88$	$e_{\rm f}=0.96$

towards spindle

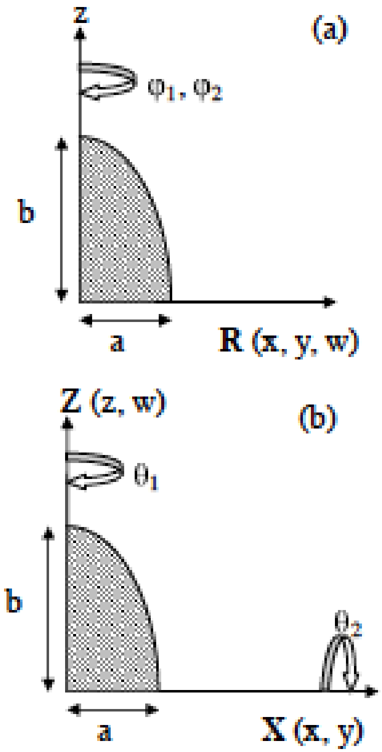
towards spherical

・5Dでは、重力波が4Dより多く放出される。

M. Shibata and H. Yoshino, Phys. Rev. D **81**, 021501 (2010); **81**, 104035 (2010).

H. Witek *et al.*, Phys. Rev. D **82**, 104014 (2010).

・重力波放出が、球形状への重力崩壊に影響。



$b/M\ (t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH}=0.90$ $e_{\rm f}=0.92$	4Dβ no $e_{\rm f}=0.89$	4Dγ no $e_{\rm f}=0.92$	4Dδ no $e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.82$	5DSβ AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.84$	5DSγ no $e_{\rm f}=0.88$	5DSδ no $e_{\rm f}=0.96$
5D double axisym. U(1)×U(1)	5DUα AH-formed $e_{\rm AH}=0.86$ $e_{\rm f}=0.79$	5DUβ AH-formed $e_{\rm AH}=0.87$ $e_{\rm f}=0.81$	5DUγ AH-formed $e_{\rm AH}=0.92$ $e_{\rm f}=0.90$	5DUδ no $e_{\rm f}=0.98$

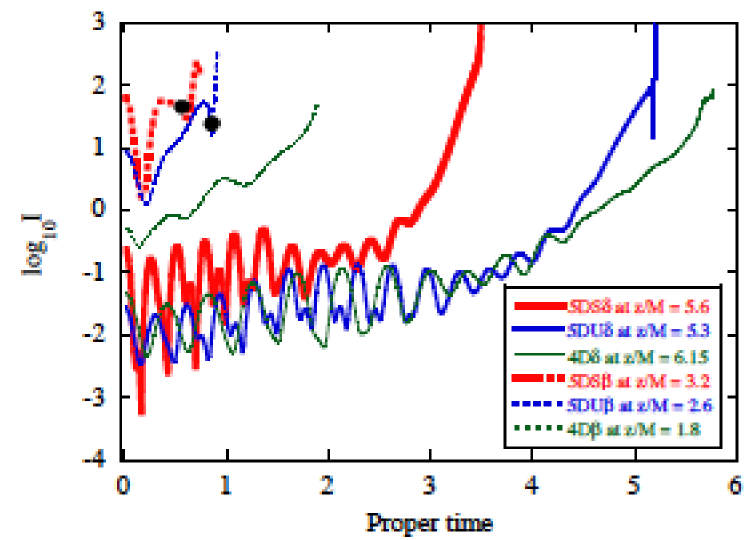


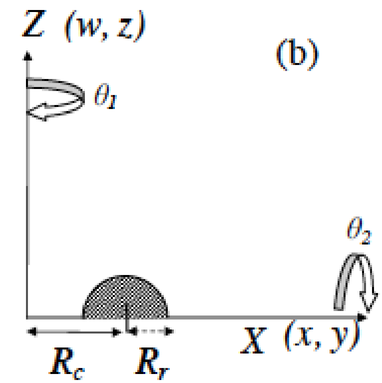
FIG. 5: The Kretschmann invariant $\mathcal{I}$ at the location of $\mathcal{I}_{\rm max}$ on the final hypersurface is plotted as a function of proper time at its location. Labels indicate model-names in Table I. The time of AH formation (t=0.6 for model 5DSβ, t=0.9 for 5DUβ) is shown by a dot.

5D collapses

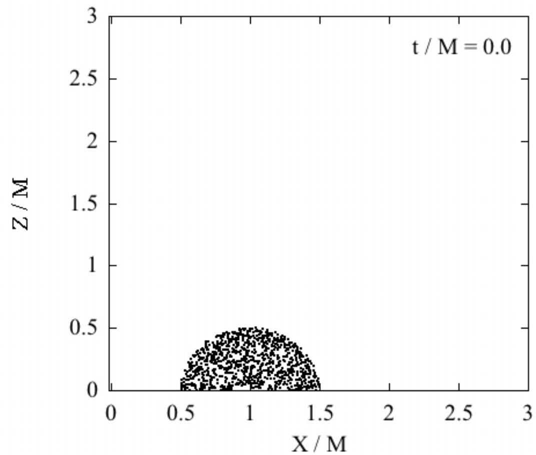
- proceed rapidly
- towards spherically
- AH forms in wider ranges

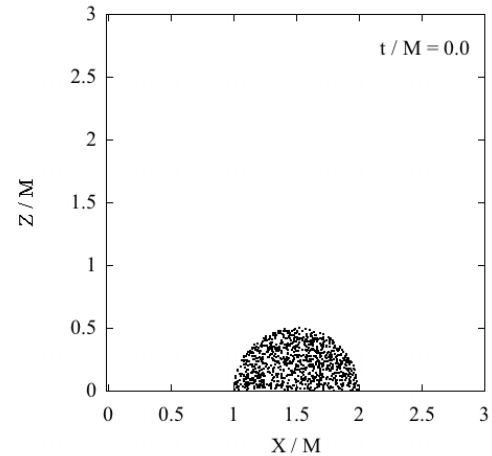
5次元時空での Ring Collapse

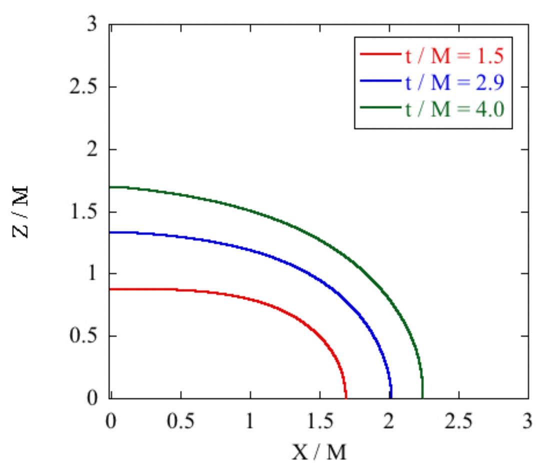
- ADM(4+1)形式。
- 非衝突粒子5000個。
- S^3, $S^2 \times S^1$トポロジーのApparent horizon 探索。
- Cartoon method (Grid size $130^2 \times 2^2$)。
- スライス条件
  ラプス：Maximal time slicing condition
  シフト：β=0

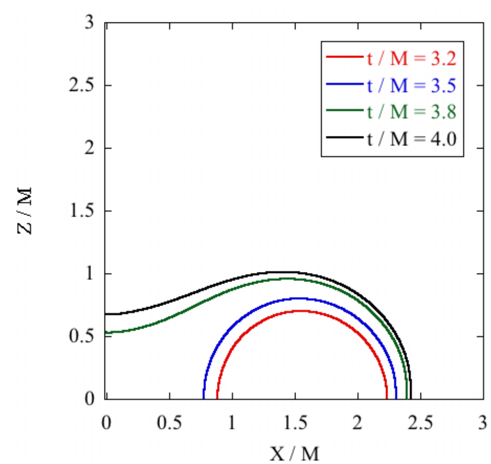


5次元時空での Ring Collapse









t/M = 1.5　共通horizon

t/M = 3.2　リング horizon

t/M = 3.8　共通 horizon

まとめと今後の課題

ハイパー・フープ仮説

・S^3トポロジーを持つhorizon形成条件としては、妥当。

・ブラック・オブジェクトに対して必ずしも有効であるとは限らない。

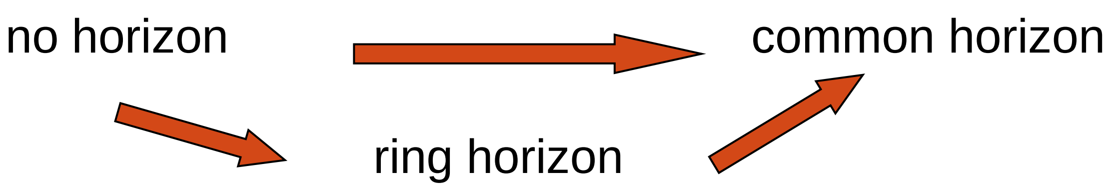
重力崩壊Sim

・5D vs. 4D　楕円体形状の重力崩壊（回転なし）

-- 重力崩壊が迅速に進む、 物質形状が球形状に近づく。

-- 極端に大きな物質分布の崩壊で、裸の特異点形成。

・5D リング重力崩壊（回転なし）



今後の課題

・回転を入れる。

・時間発展でのフープ仮説検証

・スライス条件の変更

・Event horizon探索

・安定性解析