Cosmology from the dynamical solutions in higher dimensions

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Refs) 1003.5967, 1006.2306 ,1007.1762, 1011.2376, 1103.5325/26

Introduction

Recent interests in higher dimensions

Kaluza-Klein theory (old)

↓

Superstring / M-theory (new)

quantum gravity ?

10D or 11D theory

Suggestions for cosmology

Inflation ← New degrees of freedom from compactification

Dark energy

hierarchy, and CC problem ← Brane worlds

stabilization of extra dimensions ← Fluxes of form fields

What will be investigated

Warped and dynamical solutions in higher-dim gravity

coupled to scalar and form fields

↓ string theory, brane world

↓ cosmology

string-inspired models

1) Dynamical branes

- p-branes in the presence of a scalar potential

  colliding branes

  cosmology

- cosmological solutions in 6D Nishino-Salam-Sezgin model

- intersecting branes :

  cosmology after compactifications

2) Warped de Sitter compactifications

→ de Sitter brane world models

Dynamical branes

Ref) 1003.5967
1006.2306
1007.1762
1011.2376

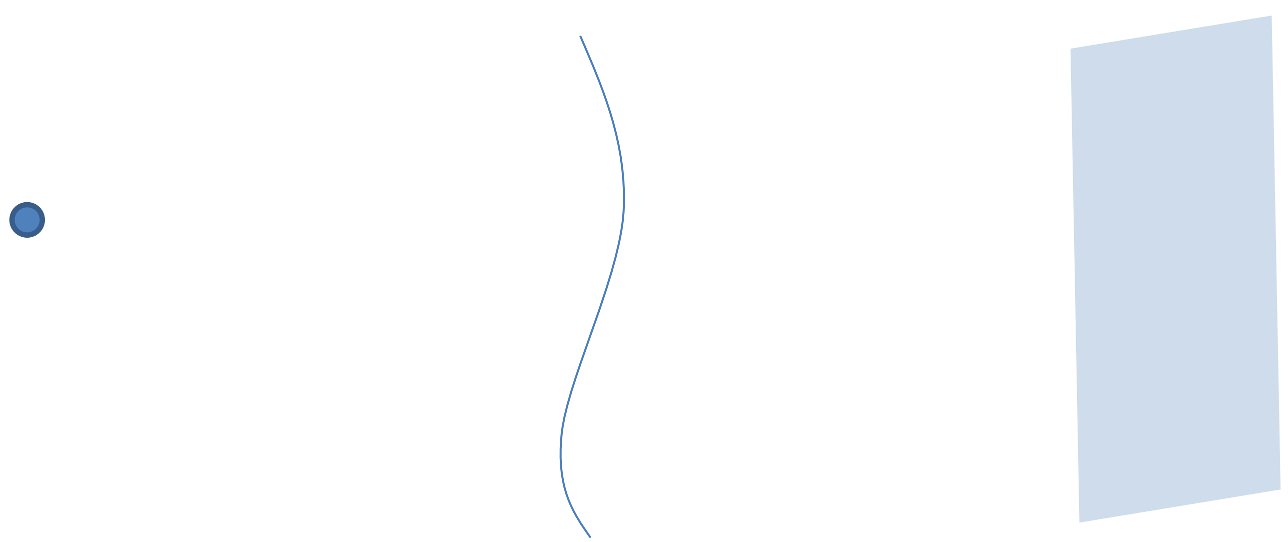
p-brane

In supergravity /string theory, there are (p+2)-form field strengths F_{p+2}

$\longrightarrow$ A charged object couples to (p+1)-form gauge field A_{p+1}
and sweeps over (p+1)-dim worldvolume .

c.f. Charge (0-brane) in 4D $\longrightarrow$ $F_{\mu\nu}$

0-brane 1-brane 2-brane



Dynamical solutions in 4D and higher dimensions

There is an analogy between 4D and higher-dim gravity.

- 4D gravity

Charged RN BH $\longrightarrow$ Dynamical BH

(Kastor-Traschen)

- Higher-dimensional gravity

p-brane solution $\longrightarrow$ Dynamical p-brane

(Gibbons-Lu-Pope)

- Dynamical BH in 4D Kastor & Traschen 93

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$\Lambda > 0$$

solution

$$ds^2 = -h^{-2}(t,y)dt^2 + h^2(t,y)\delta_{ij}dy^i dy^j$$

$$F_2 = d\left(h^{-1}\right)\wedge dt$$

$$h(t,y) = \pm\sqrt{\frac{\Lambda}{3}}t + b + \sum_\ell \frac{M_\ell}{\left|\vec{y} - \vec{y}_\ell\right|}$$

$$ds^2 = -\left(\sqrt{\frac{\Lambda}{3}}t + b + \frac{M}{r}\right)^{-2} dt^2 + \left(\sqrt{\frac{\Lambda}{3}}t + b + \frac{M}{r}\right)^2 \left(dr^2 + r^2 d\Omega^2\right)$$

$r \to \infty$ de Sitter space

$$ds^2 \approx -d\tau^2 + e^{2a\tau}\delta_{ij}dy^i dy^j$$

$$\tau := a^{-1}\ln(at+b)$$

$r \to 0$ (extremal) RN BH

$$ds^2 \approx -\left(1 - \frac{M}{R}\right)^2 dt^2 + \left(1 - \frac{M}{R}\right)^{-2} dR^2 + R^2 d\Omega^2$$

$$R := br + M$$

horizon $R = M$ or $r = 0$

$h = 0$ curvature singularities

- Dynamical p-branes

Gibbons, Lu and Pope (05), Kodama and Uzawa (05)
Binetruy, Uzawa & Sasaki (09)

$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left[R(X)-\frac{1}{2}(\partial\phi)^2-\frac{1}{2(p+2)!}e^{c\phi}F_{p+2}^2\right]$$

$$c^2=4-\frac{2(p+1)(D-p-3)}{D-2}$$

solution

$$ds^2=h^{-\left(\frac{D-p-3}{D-2}\right)}(x,y)q_{\mu\nu}(X)dx^\mu dx^\nu+h^{\frac{p+1}{D-2}}(x,y)u_{ij}(Y)dy^i dy^j$$

(p+1)-dim
world-volume

(D-p-1)-dim
transverse space

$$e^\phi=h^{-\frac{c}{2}}$$

$$F_{(p+2)}=\sqrt{-q}\,d\left(h^{-1}\right)\wedge dx^0\wedge\cdots\wedge dx^p$$

$$h(x,y)=h_0(x)+h_1(y)$$

$$R_{\mu\nu}(X)=0 \qquad R_{ij}(Y)=0$$

$$D_\mu D_\nu h_0=0 \qquad \Delta_Y h_1(y)=0$$

X and Y are Ricci-flat.

For $q_{\mu\nu}=\eta_{\mu\nu} \qquad u_{ij}=\delta_{ij}$

$$h(x,y)=A_\mu x^\mu+B+\sum_\ell \frac{M_\ell}{\left|\vec{y}-\vec{y}_\ell\right|^{D-p-3}}$$

$A_\mu \quad B$: integration constants

Single p-brane solution

$$ds^2 = \left(At + B + \frac{M}{r^{D-p-3}} \right)^{-\left(\frac{D-p-3}{D-2}\right)} \left(-dt^2 + \delta_{ij} dx^i dx^j \right) + \left(At + B + \frac{M}{r^{D-p-3}} \right)^{\frac{p+1}{D-2}} \left(dr^2 + r^2 d\Omega_{D-p-2}^2 \right)$$

$r \to \infty$ Kasner solution

$$ds^2 \approx -d\tau^2 + \tau^{-\frac{2(D-p-3)}{D+p-1}} \delta_{pq} dx^p dx^q + \tau^{\frac{2(p+1)}{D+p-1}} \delta_{ij} dy^i dy^j$$

$r \to 0$ static

$$ds^2 \approx \left(B + \frac{M}{r^{D-p-3}} \right)^{-\left(\frac{D-p-3}{D-2}\right)} \left(-dt^2 + \delta_{ij} dx^i dx^j \right) + \left(B + \frac{M}{r^{D-p-3}} \right)^{\frac{p+1}{D-2}} \left(dr^2 + r^2 d\Omega_{D-5}^2 \right)$$

$D = 10 \quad p = 3 \qquad AdS_5 \times S^5$

$$ds^2 \approx \frac{r^2}{M^{1/2}} \left(-dt^2 + \delta_{ij} dx^i dx^j \right) + \frac{M^{1/2}}{r^2} dr^2 + d\Omega_5^2$$

dilaton is not dynamical

D=10, p=3 $\quad A_0 < 0 \quad A_i = 0$

$$h(x,y) = -|A_0|t + B + \sum_{\ell} \frac{M_\ell}{|\vec{y} - \vec{y}_\ell|^4}$$

$h = 0$ curvature singularities

Gibbons, Lu and Pope (05)

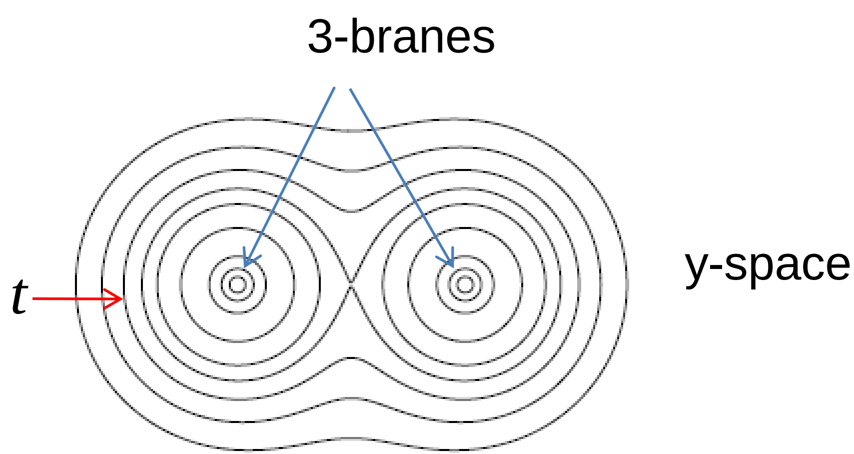
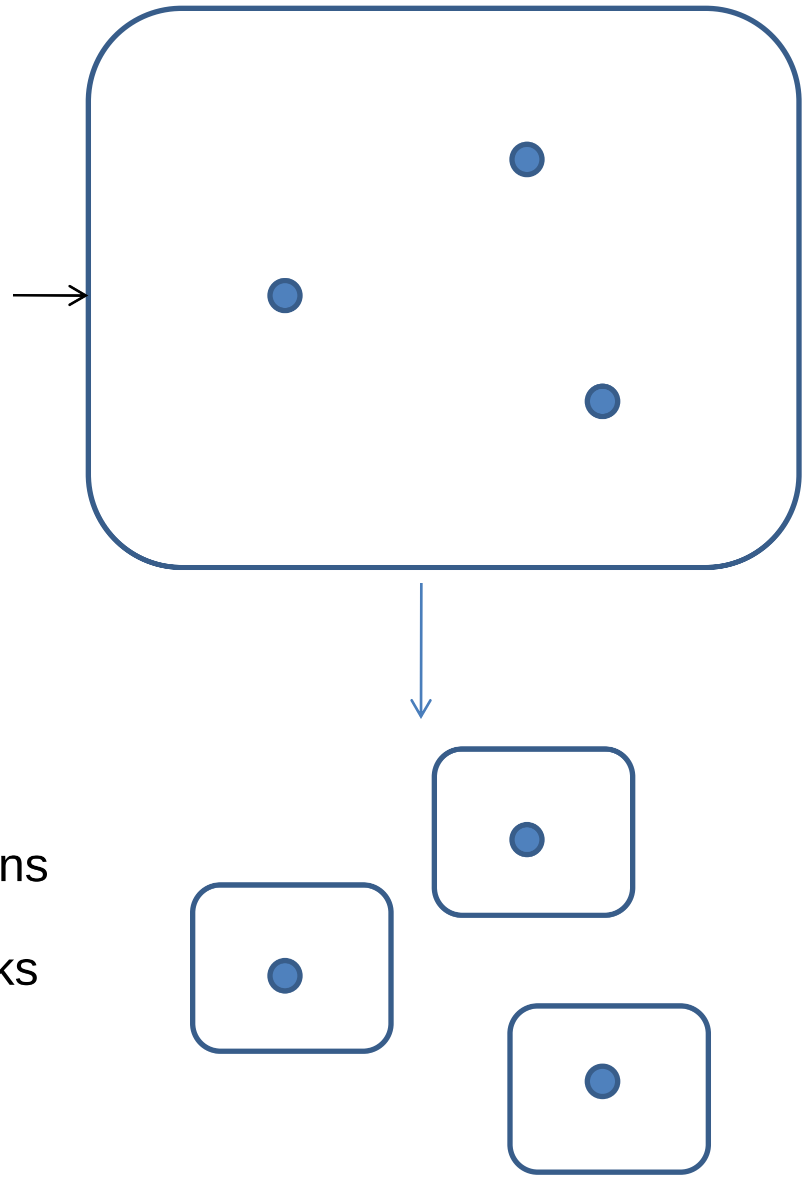


FIG. 1: The level sets of $\Phi(\mathbf{y})$ for two mass points: The universe after the collision is confined inside a level set $\Phi(\mathbf{y}) > -ht$. As time progresses, the level set ∂D_t shrinks and splits into two components, which then shrink around the two D3-branes.

Singularity
moves in from
infinity



Splitting into domains

Each domain shrinks
around each brane

Higher-dimensional theory with a potential **1006.2306**

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[R(X) - \underline{2e^{\alpha\phi}\Lambda} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(p+2)!}e^{c\phi}F_{p+2}^2\right]$$

$$c^2 = 4 - \frac{2(p+1)(D-p-3)}{D-2} \qquad \alpha = -2\left(\frac{p+1}{D-2}\right)(c)^{-1}$$

$$D = 6 \quad p = 1 \quad \Lambda > 0$$ Nishino-Salam-Sezgin model

ansatz $a = -\frac{D-p-3}{D-2} \qquad b = \frac{p+1}{D-2}$

$$ds^2 = h^a(x,y)q_{\mu\nu}(X)dx^\mu dx^\nu + h^b(x,y)u_{ij}(Y)dy^i dy^j$$

(p+1)-dim world-volume

(D-p-1)-dim transverse space

$$e^\phi = h^{\varepsilon c/2}$$

$$F_{p+2} = \sqrt{-q}\,d\left(h^{-1}\right)\wedge\left(dx^0\wedge\cdots\wedge dx^p\right)$$

Resultant equations

$$R_{\mu\nu}(X) = 0$$

$$h(x,y) = h_0(x) + h_1(y) \qquad \Delta_Y h_1(y) = 0$$

$$D_\mu D_\nu h_0 + \frac{8}{4(D-2)-2(p+1)(D-p-3)} \Lambda q_{\mu\nu} = 0$$

$$R_{ij}(Y) + \frac{4(p-1)}{4(D-2)-2(p+1)(D-p-3)} \Lambda u_{ij} = 0$$

$$\Lambda \neq 0$$

- 'X' is Ricci-flat
- 'Y' cannot be Ricci flat except for $p = 1$

$$\Lambda = 0 \qquad R_{\mu\nu}(X) = 0 \qquad R_{ij}(Y) = 0$$

- 0-brane $p=0$ **Cosmological**

$$R_{ij}(Y)=\frac{2}{D-1}\Lambda u_{ij}$$

$\Lambda>0$ $\tau=(D-2)t^{\frac{1}{D-2}}$

$$ds^2=\left[1+\left(\frac{\tau}{\tau_0}\right)^{-2(D-2)}h_1\right]^{-\left(\frac{D-3}{D-2}\right)}\left[-d\tau^2+\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2(D-2)}h_1\right\}\left(\frac{\tau}{\tau_0}\right)^2 u_{ij}dy^i dy^j\right]$$

- Milne-type universe
- Y is a closed space

$\Lambda<0$ collapsing universe

- 1-brane $p=1$

$$R_{\mu\nu}(X)=0 \qquad R_{ij}(Y)=0$$

For $q_{\mu\nu}(X)=\eta_{\mu\nu} \qquad u_{ij}(Y)=\delta_{ij}$

$$ds^2=\left[\frac{\Lambda}{2}\left(t^2-x^2\right)+h_1(\vec{y})\right]^{-\left(\frac{D-4}{D-2}\right)}\left(-dt^2+dx^2\right)+\left[\frac{\Lambda}{2}\left(t^2-x^2\right)+h_1(\vec{y})\right]^{\left(\frac{2}{D-2}\right)}d\vec{y}^2$$

$$h_1(y)=\sum_{\ell=1}^{N}\frac{M_\ell}{\left|\vec{y}-\vec{y}_\ell\right|^{D-4}} \qquad D>4$$

$$h_1(y)=\sum_{\ell=1}^{N}M_\ell\ln\left|\vec{y}-\vec{y}_\ell\right| \qquad D=4$$

$$h_1(y)=\sum_{\ell=1}^{N}M_\ell\left|\vec{y}-\vec{y}_\ell\right| \qquad D=3$$

Spacetime is inhomogeneous along x-direction.

far brane limit $h_1(\vec{y}) \to 0$

- $\Lambda > 0$

$$t = \sqrt{\frac{2}{\Lambda}} T \cosh X \quad x = \sqrt{\frac{2}{\Lambda}} T \sinh X$$

$$ds^2 = \frac{2}{\Lambda}\left[-d\tau^2 + \left(\frac{2}{D-2}\right)^2 \tau^2 \left(dX^2 + d\vec{Y}^2\right)\right] \qquad \tau = \left(\frac{D-2}{2}\right) T^{\frac{2}{D-2}}$$

cosmological

- $\Lambda < 0$

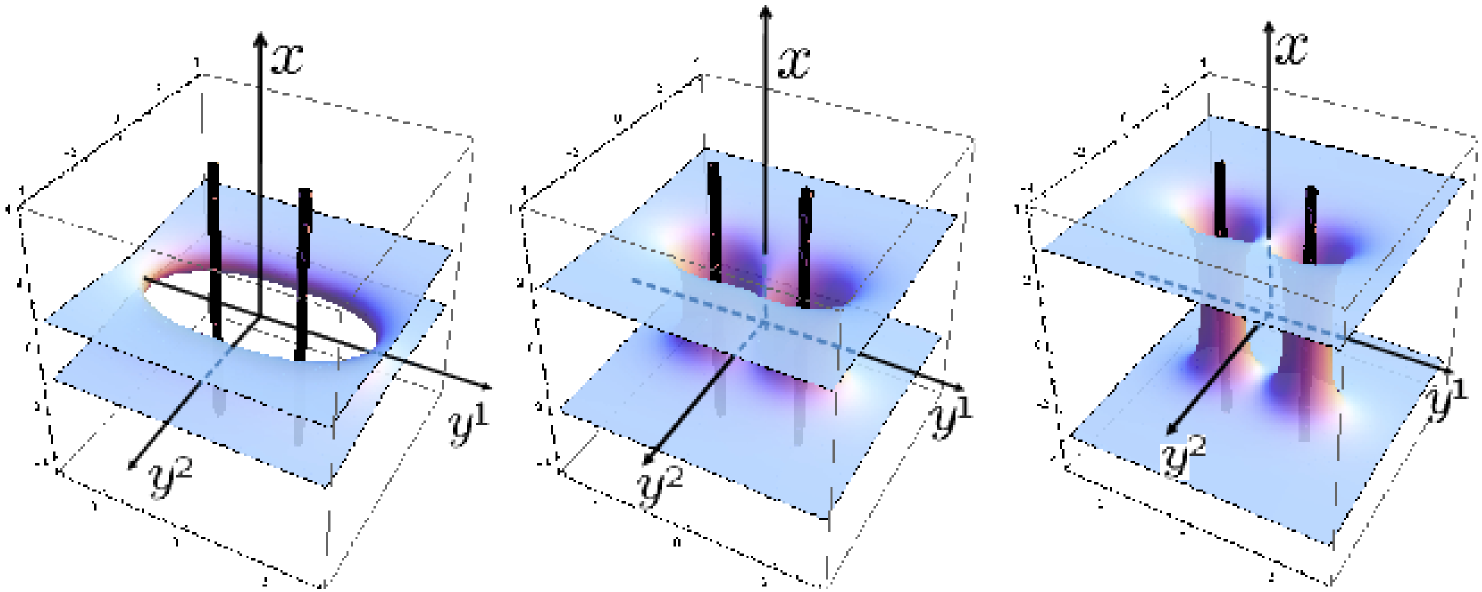
$$t = \sqrt{-\frac{2}{\Lambda}} X \sinh T \quad x = \sqrt{-\frac{2}{\Lambda}} X \cosh T$$

$$ds^2 = \frac{2}{|\Lambda|}\left[d\xi^2 + \left(\frac{2}{D-2}\right)^2 \xi^2 \left(-dT^2 + d\vec{Y}^2\right)\right] \qquad \xi = \left(\frac{D-2}{2}\right) X^{\frac{2}{D-2}}$$

conformally flat and inhomogeneous spacetime

- Brane dynamics $D > 4 \quad \Lambda < 0$

regular region $h(t, x, \vec{y}) = h_1(\vec{y}) - \frac{1}{2}|\Lambda|(t^2 - x^2) > 0$

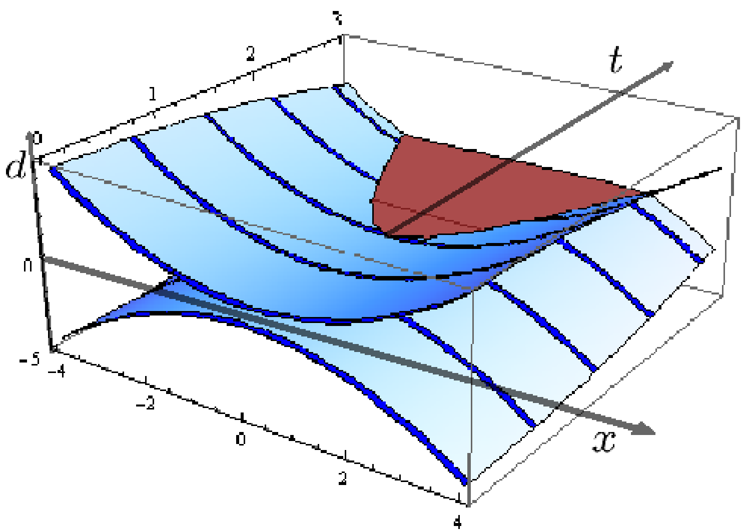


- Spatial region shrinks
  splits into small domains surrounding each brane.

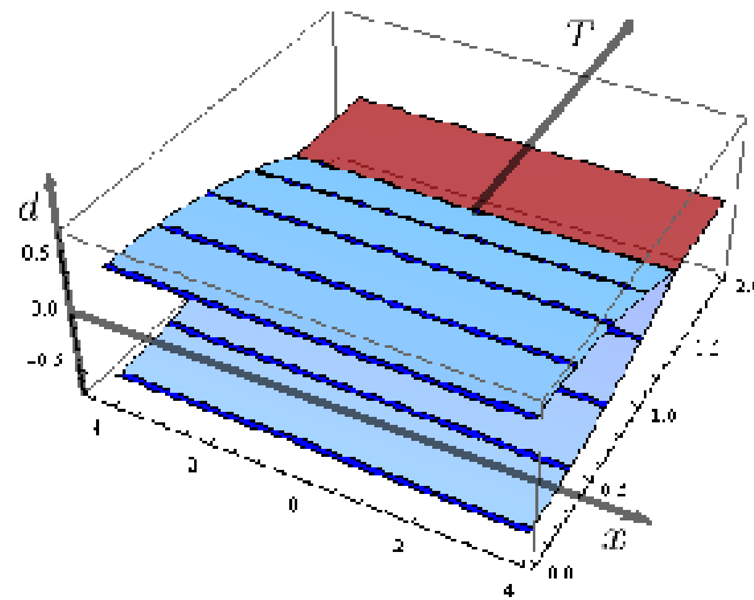
• Brane dynamics $D=3 \quad \Lambda<0$

$$ds^2=\left[\frac{\Lambda}{2}\left(t^2-x^2\right)+\sum_\ell M_\ell\left|y-y_\ell\right|\right]^{-1}\left(-dt^2+dx^2\right)+\left[\frac{\Lambda}{2}\left(t^2-x^2\right)+\sum_\ell M_\ell\left|y-y_\ell\right|\right]^2 dy^2$$

- branes can collide
- observer-dependent collision



$T=\sqrt{t^2-x^2}$



Cosmology from Nishino-Salam-Sezgin model

1003.5967

theory of $D=6 \quad p=1 \quad \Lambda>0$

$\longrightarrow$ 6D Nishino-Salam-Sezgin model with $F_{\mu\nu}=0$

Nishino & Sezgin (84), Salam & Sezgin (84)

$$S=\int d^6x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R-(\partial\phi)^2\right)-\frac{1}{4}e^{-\phi}F_{\mu\nu}^2-\frac{1}{12}e^{-2\phi}H_{\mu\nu\alpha}^2-\frac{2g^2}{\kappa^4}e^{\phi}\right]$$

- positive potential

- Minkowski vacuum after compactifying on S^2

  Salam & Sezgin (84)

- warped generalization of the Minkowski solution

  Gibbons, et. al (03)

$\longrightarrow$ brane world models

Cosmological solution with $F \neq 0 \quad H = 0$

$$ds^2 = -d\tau^2 + a^2(\tau)\delta_{ij}dx^i dx^j + b^2(\tau)ds_2^2$$

- Scaling solution $\qquad a(\tau) \propto \tau \quad b(\tau) \propto \tau$

  de Rham,et .al (06)

- Maeda-Nishino solution $\qquad a(\tau) \propto \tau^{p_\pm} \quad b(\tau) \propto \tau^{q_\pm}$

  Maeda and Nishino (85)

$$p_\pm = \frac{9 \pm 4\sqrt{3}}{33} \approx 0.483, \quad 0.063$$

$$q_\pm = \frac{1 \mp 2\sqrt{3}}{11} \approx -0.224, \quad 0.406$$

- and warped generalizations of them…

$H \neq 0 \quad F = 0$ Cosmological solution

Wick rotations from the warped static solution Afshar and Parvizi (09)

$$\downarrow$$

$$ds^2 = \frac{e^{-2G_\phi t}\cosh^{1/2}\left[f_2(t-t_2)\right]}{\left|\sinh\left[f_1(t-t_1)\right]\right|^{5/2}}\left(-\frac{\kappa^6 c^2 f_1^4}{8 f_2^2 g^4}dt^2 + e^{\frac{8}{3}G_\phi t}\left|\sinh\left[f_1(t-t_1)\right]\right|^2 \sum_i \left(dx^i\right)^2\right)$$

$$+\frac{e^{-2(2G_\phi - G_w)t}}{\left|\sinh\left[f_1(t-t_1)\right]\right|^{1/2}\cosh^{1/2}\left[f_2(t-t_2)\right]}\left(dy^2 + e^{4(G_\phi - G_w)t}d\theta^2\right)$$

$$e^\phi = \frac{2g^2 f_2^2}{c^2\kappa^4 f_1^2}e^{2G_\phi t}\left|\frac{\sinh\left[f_1(t-t_1)\right]}{\cosh\left[f_2(t-t_2)\right]}\right|^{1/2}$$

$$H = \frac{\sqrt{2}g^2 f_2^3}{\kappa^5 c^2 f_1^2}\frac{1}{\cosh^2\left[f_2(t-t_2)\right]}dt \wedge dy \wedge d\theta$$

$$\boxed{f_2^2 := 2 f_1^2 - 4\left(G_\phi - G_w\right)^2 - \frac{32}{3}G_\phi^2 > 0}$$

$t = t_1$ singularity

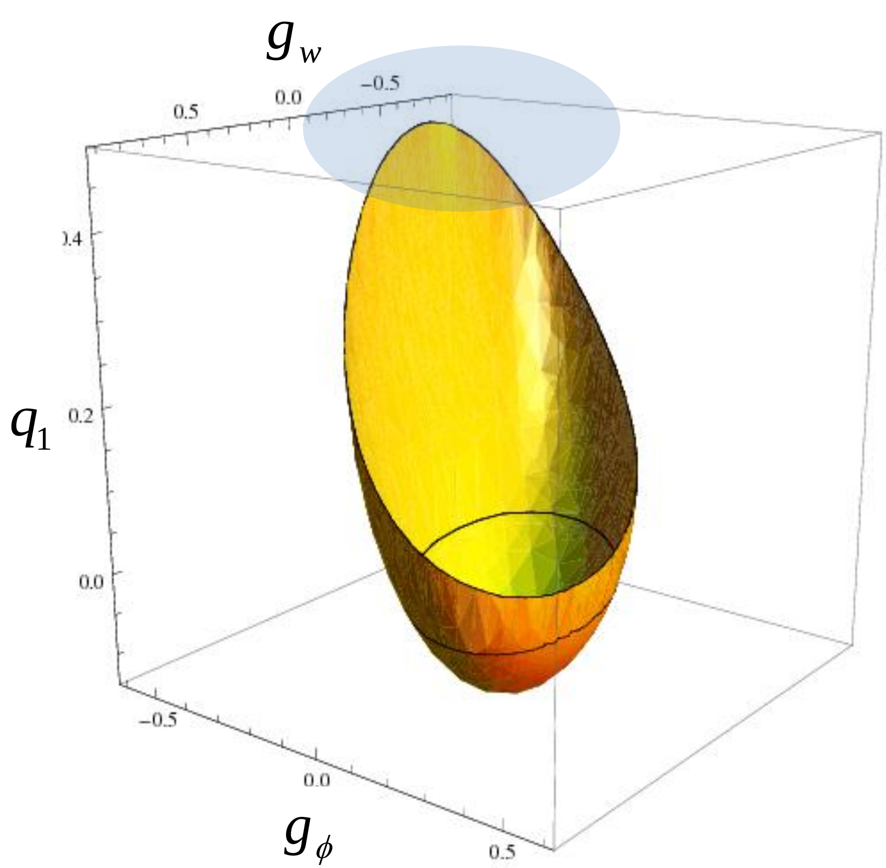
Late time behavior

$$ds^2 \approx -dT^2 + T^{2q_1}\sum_{i=1}^{3}\left(dx^i\right)^2 + T^{2q_v}dy^2 + T^{2q_w}d\theta^2$$

$$q_1 = \frac{f_2 - f_1 + 4G_\phi/3}{f_2 - 5f_1 - 4G_\phi} \quad q_v = \frac{-f_2 - f_1 + 4\left(2G_\phi - G_w\right)}{f_2 - 5f_1 - 4G_\phi} \quad q_v = \frac{-f_2 - f_1 - 4G_\phi}{f_2 - 5f_1 - 4G_\phi}$$

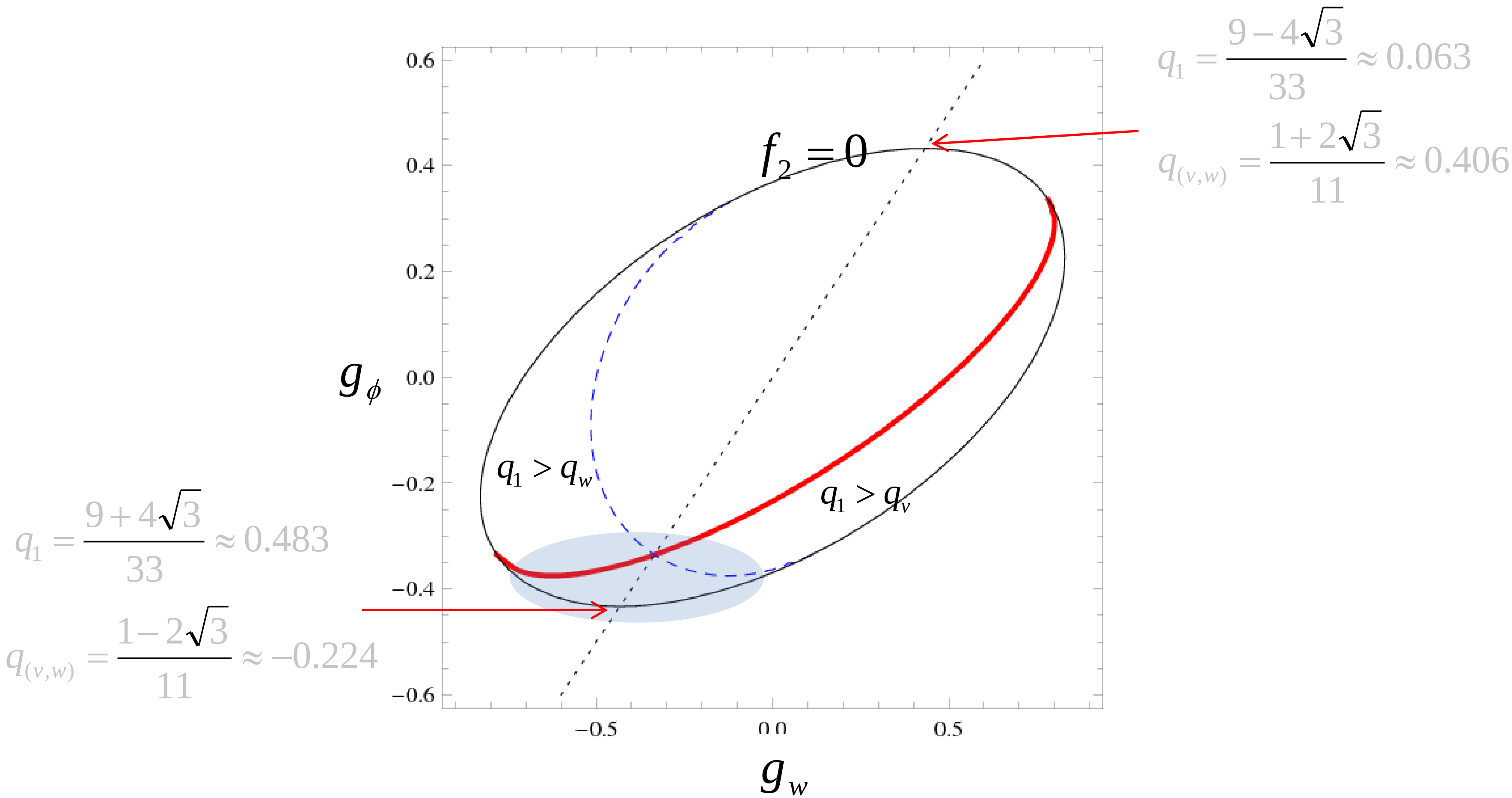
$$g_w := \frac{G_w}{f_1}$$

$$g_\phi := \frac{G_\phi}{f_1}$$



expanding universe

no acceleration



- slower expansion than in the solution with 2-form   Maeda and Nishino (85)
- stabilized extra dimensions $q_1 = \frac{1}{3} \quad q_v = q_w = 0$

Generalization **1011.2376**

$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left[R(X)-2e^{\alpha\phi}\Lambda-\frac{1}{2}(\partial\phi)^2-\frac{1}{2(p+2)!}e^{c\phi}F_{p+2}^2\right]$$

$$c^2=N-\frac{2(p+1)(D-p-3)}{D-2} \qquad \alpha=-2\left(\frac{p+1}{D-2}\right)(c)^{-1}$$

$$N\neq 4$$

Analytic solutions are obtained for limited cases.

(1) $c \neq 0$ Scalar field is dynamical

- only for N=2 and p=0
- no solution for $\Lambda < 0$
- approaching Minle-type universe

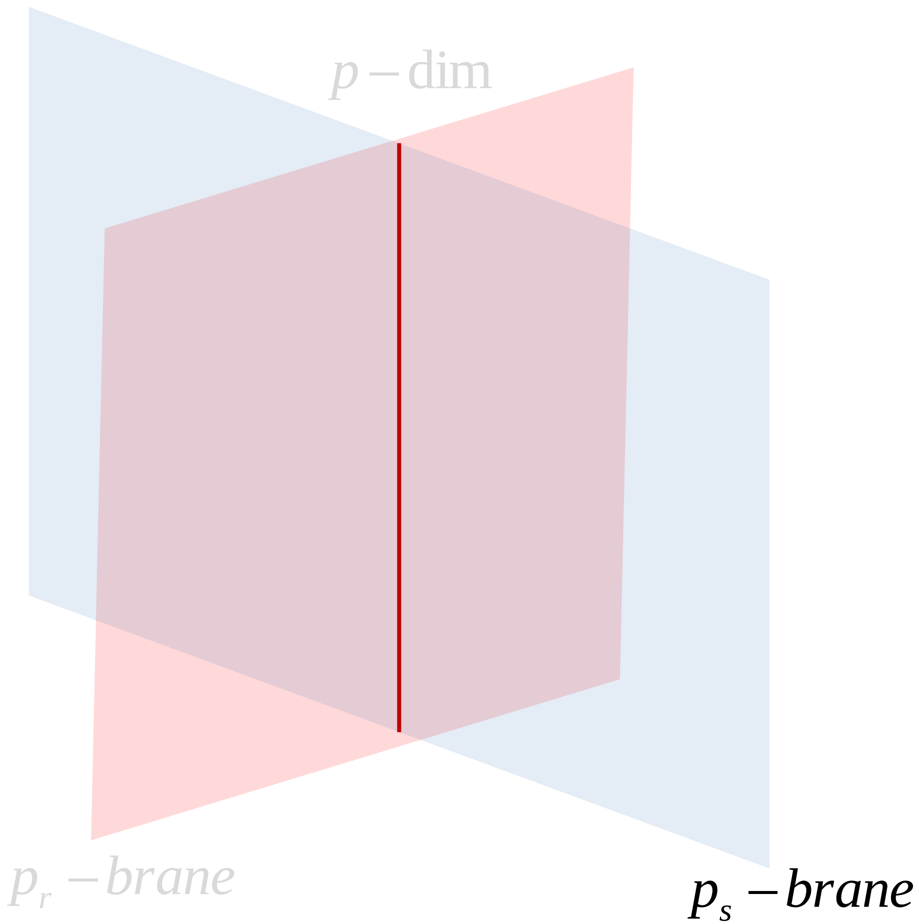
(2) $c = 0$ $N = \dfrac{2(D-p-3)(p+1)}{D-2}$ Scalar field is nondynamical

- only for p=0
- no solution for $\Lambda < 0$
- approaching de Sitter spacetime.

Maki and Shiraishi (93), case (I)

- Kastor-Traschen solution for $D = 4$

Intersecting branes **1007.1762**



F_{p_r+2} F_{p_s+2}

Two intersecting branes

$$ds^2 = h_r^{\alpha} h_s^{\beta}\Big[\overbrace{\left(h_r h_s\right)^{-1} q_{\mu\nu}\left(X\right) dx^{\mu} dx^{\nu}}^{\text{(p+1)-dim world volume}} + \overbrace{h_s^{-1}\gamma_{ij}\left(Y_1\right) dy^i dy^j}^{(p_s\text{-p})\text{-dim relative transverse}}$$

$$+ \underbrace{h_r^{-1} w_{mn}\left(Y_2\right) dv^m dv^n}_{(p_r\text{-p})\text{-dim relative transverse}} + \underbrace{u_{ab}\left(Z\right) dz^a dz^b}_{(\text{D-p-}p_r\text{-}p_s\text{-1})\text{-dim transverse}}\Big]$$

$$\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}$$

Two intersecting branes

$$
\begin{aligned}
ds^2 = h_r^{\alpha} h_s^{\beta} \Big[& \overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X)}^{\text{(p+1)-dim world volume}} dx^{\mu} dx^{\nu} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^{i} dy^{j}}^{\text{(}p_s\text{-p)-dim relative transverse}} \qquad p_s - brane \\
& + \underbrace{h_r^{-1} w_{mn}(Y_2)}_{\text{(}p_r\text{-p)-dim relative transverse}} dv^{m} dv^{n} + \underbrace{u_{ab}(Z) dz^{a} dz^{b}}_{\text{(D-p-}p_r\text{-}p_s\text{-1)-dim transverse}} \Big]
\end{aligned}
$$

$$
\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}
$$

Two intersecting branes

$$p_r - brane$$

$$ds^2 = h_r^{\alpha} h_s^{\beta} \Big[\overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X) dx^{\mu} dx^{\nu}}^{\text{(p+1)-dim world volume}} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}^{\text{(p}_s\text{-p)-dim relative transverse}} \qquad p_s - brane$$

$$+ \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{\text{(p}_r\text{-p)-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{\text{(D-p-p}_r\text{-p}_s\text{-1)-dim transverse}} \Big]$$

$$\alpha = \frac{p_r + 1}{D-2} \qquad \beta = \frac{p_s + 1}{D-2}$$

3 cases Behrndt, Bergshoeff and Janssen (96)

Case I : Both h_r and h_s depend on the overall transverse coordinates

Maeda, Ohta and Uzawa (09)

$$h_r = h_r(x, z) \qquad h_s = h_s(z) \qquad \text{or} \qquad h_r = h_r(z) \quad h_s = h_s(x, z)$$

- Intersecting rule is the same as the static case

Argurio, et. al (97), Ohta (97)

Case II : h_s depends on the overall transverse coordinates

h_r depends on the relative transverse coordinate of the other brane

$$h_r = h_r(x, y) \qquad h_s = h_s(z)$$

$\longrightarrow$ no solution as $h_r = h_r(y) \qquad h_s = h_s(x, z)$

- Intersecting rule is the same as the case I

Case III : Each depends on the corresponding relative transverse coordinate

$$h_r = h_r(x, y) \qquad h_s = h_s(v) \qquad \text{or} \qquad h_r = h_r(y) \qquad h_s = h_s(x, v)$$

- Intersecting rule is different from the previous two cases

- x dependence is linear

- Transverse space dependences is given in terms of the solutions of Laplace eq in each space.

- All spaces are Ricci-flat.

Cosmic expansion

scale factor in the far region $a(\tau) \propto \tau^{\lambda}$

(1) M-brane: M5-M5 for the case I

$$\lambda = \frac{6}{13}$$

(2) D-brane: D2-D6 and D4-D6 (case I, II, III)

$$\lambda = \frac{7}{15}$$

(3) F1-string: F1-D6 (case I, II)

$$\lambda = \frac{7}{15}$$

(4) NS5-string: F1-D6 (case I, II)

$$\lambda = \frac{7}{15}$$

slower expansion law than in radiation-dominated era $a(\tau) \propto \tau^{1/2}$

Warped de Sitter solution

Ref) 1103.5325
1103.5326

de Sitter spacetime

$$ds^2 = -dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q$$

$H:$ Hubble expansion rate

de Sitter spacetime plays the role to study

- inflation in the early universe
- cosmic acceleration in the present time (dark energy)

Realization of solutions involving 4D de Sitter (or accelerating) universe in higher-dimensional gravitational theory is required.

No-Go theorem for de Sitter compactifications

no solution realizing 4D de Sitter spacetime after compactifications under the following assumptions. Maldacena and Nunez (02)

(1) no higher–derivative / higher curvature corrections

(2) scalar potential takes a nonpositive value

(3) fields with positive kinetic terms

(4) 4-dimensional effective gravitational constant is finite.
The internal space is compact.

→ We relax the condition (4) and
assume the existence of a noncompact direction.

Warped de Sitter compactification solutions

We look for a D-dimensional warped compactificaton

$$ds_D^2 = e^{2A(y)}\left[\underline{q_{\mu\nu}(X)dx^{\mu}dx^{\nu}} + u_{ij}(Y)dy^i dy^j\right]$$

n-dimensional de Sitter space

(D-n-1) dimensional internal manifold

- Y space has a non-compact direction.

Known solutions

- 11D supergravity Gibbons and Hull (01)

  Y is the 7D hyperbolic space

- 10D vacuum and with form fields Neupane (09,10)

$$u_{ij}(Y)dy^i dy^j = dy^2 + f(y)ds^2_{S^1\times(S^2\times S^2)}$$

- D-dim pure gravity with Neupane (10)

$$u_{ij}(Y)dy^i dy^j = dy^2 + d\Omega^2_{D-5}$$

→ We extend them including both scalar and form fields.

Warped de Sitter solutions

1) Vacuum Neupane (10)

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\,R \qquad D > 6$$

$$ds_D^2 = e^{\pm 2\sqrt{\frac{n-1}{D-2}}H(y-y_0)}\left(-dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + dy^2 + \frac{D-n-2}{(n-1)H^2}\gamma_{ab}(Z)dz^a dz^b\right)$$

n-dim de Sitter

$$R_{ab}(Z) = (D-n-2)\gamma_{ab}(Z)$$

Asymptotic geometry $y \to \mp\infty$

$$\Delta\Omega_p^{(D)} = \Omega_p^{(D)}\left[1 - \left(\frac{D-n-2}{D-2}\right)^{\frac{p}{2}}\right]$$

deficit solid angle

2) With a scalar field

$$S=\frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-2e^{\beta\phi}\Lambda\right)$$

$$c=\beta^2-\frac{2}{D-2} \qquad\qquad D>6$$

Solution

$$ds_D^2=e^{2A(y)}\left(-dt^2+e^{2Ht}\delta_{pq}dx^p dx^q+dy^2+\frac{D-n-2}{(n-1)H^2}\gamma_{ab}(Z)dz^a dz^b\right)$$

$$A(y)=\pm\beta\sqrt{-\frac{\Lambda}{D-2}}(y-y_0)=\pm\sqrt{\frac{(n-1)\left(c+\frac{2}{D-2}\right)}{c(D-2)}}H(y-y_0)$$

$$\phi=-\frac{2}{\beta}A(y)=\mp 2\sqrt{-\frac{\Lambda}{(D-2)}}(y-y_0)$$

Solution exists for $\Lambda < 0$

Hubble expansion rate

$$H^2 = -\frac{c\Lambda}{n-1}$$

de Sitter solution exists if $c > 0$

$c \to \infty$ Vacuum solution

Asymptotic geometry $y \to \mp\infty$

$$\Delta\Omega_p^{(D)} = \Omega_p^{(D)}\left[1 - \left(\frac{D-n-2}{D-2}\left(1 + \frac{2}{c(D-2)}\right)\right)^{\frac{p}{2}}\right]$$

deficit solid angle disappears for

$$c_* = \frac{2(D-n-2)}{n(D-2)}$$

3) With scalar and p(=D-n-1)-form fields

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left(R - 2e^{-\frac{\alpha\phi}{p-1}}\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2p!}e^{\alpha\phi}F_p^2\right)$$

Solution

$$ds_D^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + dy^2 + \frac{1}{H^2 + f^2/2(n-1)}\left(\frac{D-n-2}{n-1}\right)\gamma_{ab}(Z)dz^a dz^b\right)$$

$$F = f\sqrt{\gamma}dz^1\wedge\cdots\wedge dz^p \longrightarrow$$ constant

$$A(y) = \pm\alpha\sqrt{\frac{1}{(p-1)(D-2)}\left(-\frac{\Lambda}{p-1} + \frac{f^2}{4}\right)}(y - y_0)$$

$$\phi(y) = -\frac{2(p-1)}{\alpha}A(y) = \mp 2\sqrt{\left(\frac{p-1}{D-2}\right)\left(-\frac{\Lambda}{p-1} + \frac{f^2}{4}\right)}(y - y_0)$$

Reality condition

$$\Lambda < \frac{(p-1)f^2}{4}$$

Hubble expansion rate

$$H^2 = \frac{1}{n-1}\left(\frac{\alpha^2}{p-1} - \frac{2(p-1)}{D-2}\right)\left(-\frac{\Lambda}{p-1} + \frac{f^2}{4}\right)$$

$$\longrightarrow \qquad \alpha^2 > \frac{2(p-1)^2}{D-2}$$

$$\beta := \frac{\alpha^2}{p-1} - \frac{2(p-1)}{D-2}$$

Asymptotic geometry $y \to \mp\infty$

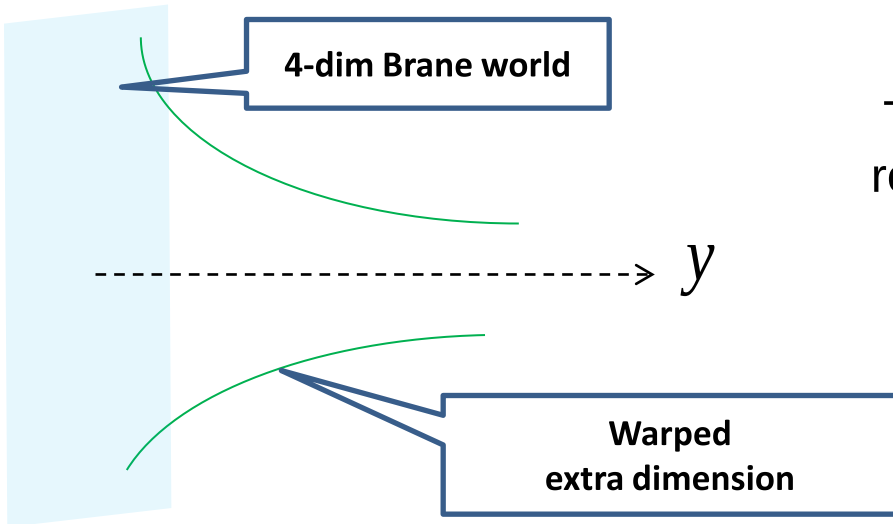
$$\Delta\Omega_p^{(D)} = \Omega_p^{(D)}\left[1 - \left(\frac{D-n-2}{D-2}\left(\frac{1+2(p-1)/(D-2)\beta}{1+f^2/2(n-1)H^2}\right)\right)^{\frac{p}{2}}\right]$$

deficit solid angle disappears for

$$\beta_*^2 = 2H^2\left(\frac{D-n-2}{D-2}\right)\left(\frac{p-1}{nH^2 + 2(D-2)f^2/(n-1)}\right)$$

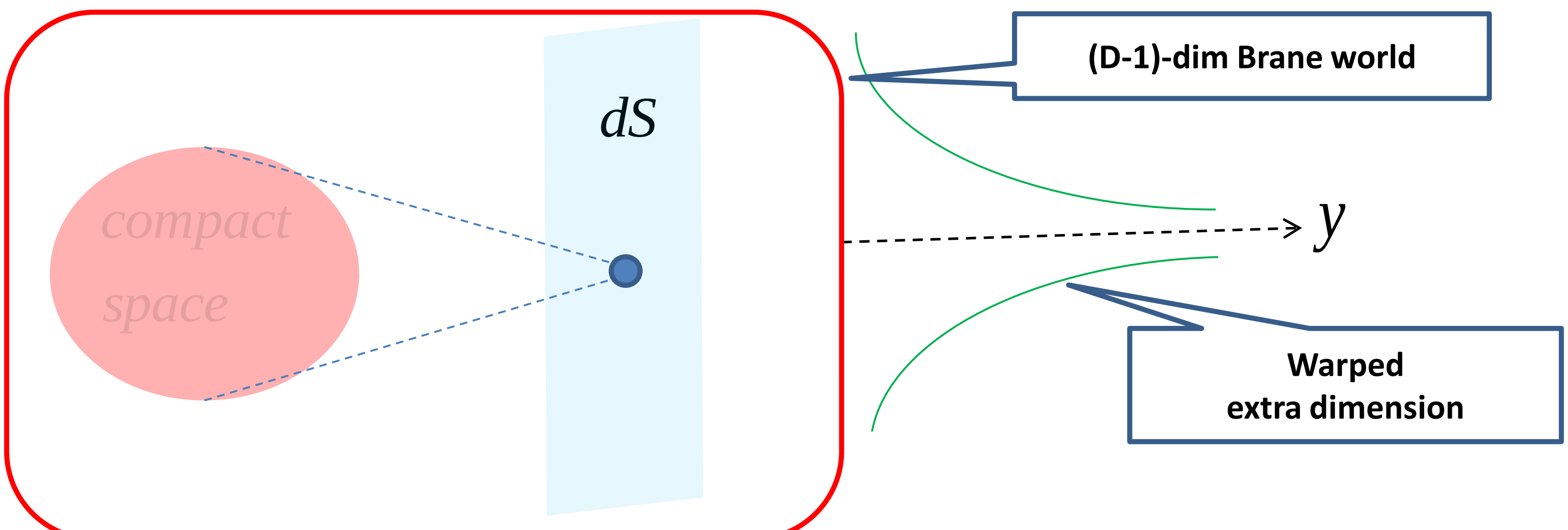
RS II warped compactification

Randall and Sundrum (99)



The warped structure helps the recovery of 4D gravity on the brane.

Warped compactification in D-dim



Brane world model

<u>1) vacuum</u> $D > 6$

Neupane (10)

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}R - \int d^{D-1}x\sqrt{-q}\sigma$$

$$ds_D^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + dy^2 + \frac{D-6}{3H^2}\gamma_{ab}(Z)dz^a dz^b\right)$$

$$A(y) = -H\sqrt{\frac{3}{D-2}}\,|\,y - y_0\,|$$

We put a brane at $y = y_0$ and impose Z_2 symmetry

$$ds_{ind}^2 = -dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + \frac{D-6}{3H^2}\gamma_{ab}(Z)dz^a dz^b$$

The brane tension

$$\kappa^2\sigma = 2\sqrt{3(D-2)}H$$

2) with scalar and form fields

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left(R - 2e^{-\frac{\alpha\phi}{(D-6)}}\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{e^{\alpha\phi}}{2p!}F_p^2 \right) - \int d^{D-1}x\sqrt{-q}e^{\gamma\phi}\sigma$$

$$ds_D^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + dy^2 + \frac{D-6}{3H^2 + f^2/2}\gamma_{ab}(Z)dz^a dz^b \right)$$

$$A(y) = -H\sqrt{\frac{3}{(D-2)}\left(1 + \frac{2(D-6)}{(D-2)\beta} \right)}|y - y_0|$$

$$H^2 = \frac{\beta}{6}\left(-\frac{2\Lambda}{D-6} + \frac{f^2}{2} \right) \qquad \beta := \frac{\alpha^2}{D-6} - \frac{2(D-6)}{D-2}$$

The brane tension $\qquad \kappa^2\sigma = 2\sqrt{3\left(D - 2 + \frac{2(D-6)}{\beta} \right)}H$

brane coupling parameter $\qquad \gamma = -\frac{D-6}{(D-2)\alpha}$

Spectrum on the brane

Behavior of each KK mode with mass m

$$\varphi_{m,k} \propto e^{-\frac{3}{2}Ht} Z_{i\nu_m}\left(ke^{-Ht}\right)$$

$$\nu_m = \frac{m^2}{H^2} - \frac{9}{4}$$

homogeneous limit $k \to 0$

$0 < \frac{m}{H} << \frac{3}{2}$ $\quad \left|\varphi_{m,0}\right|^2 \propto a^{-\frac{2m^2}{3H^2}}$ decays slowly

$\frac{m}{H} > \frac{3}{2}$ $\quad \left|\varphi_{m,0}\right|^2 \propto \frac{1}{a^3}$ damped oscillation

$m_c := \frac{3}{2}H$ critical mass of de Sitter

1) vacuum

$$ds_D^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\left(\delta_{pq} + h_{pq}\right)dx^p dx^q + dy^2 + \frac{D-6}{3H^2}\gamma_{ab}(Z)dz^a dz^b\right)$$

$$h^{pq}{}_{,q} = h^q{}_q = 0$$

$$h_{pq} = \int dmd^3k \sum_{\{L\}} f_m(y)\varphi_{m,k}(t)e^{ik_jx^j}Y_{\{L\}}\hat{e}_{pq}$$

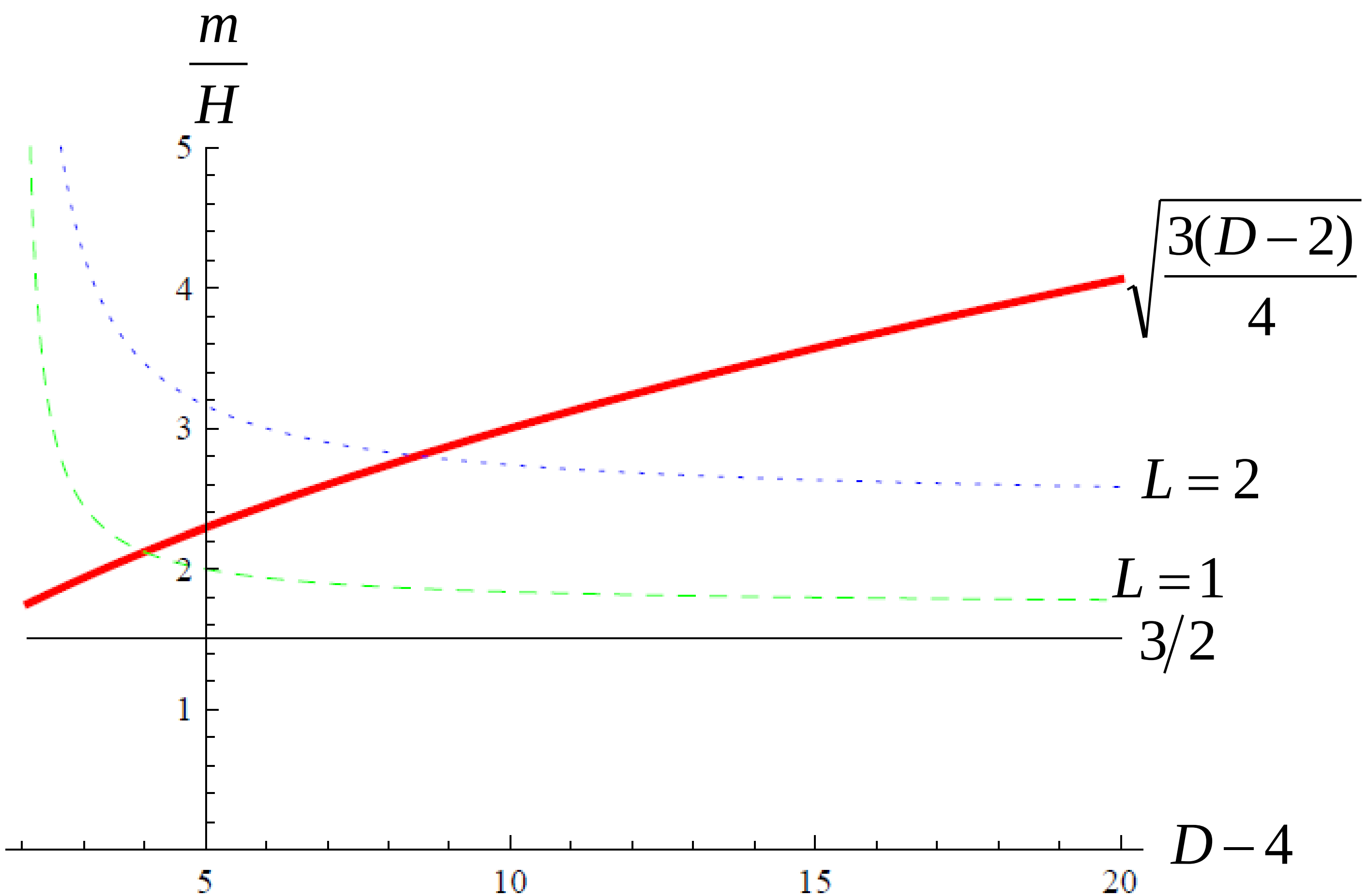
$$\Delta_{D-5}Y_{\{L\}} = -L(L+D-6)Y_{\{L\}}$$ harmonics on (D-5)-sphere

$$m^2 = 0$$ zero mode

$$\longrightarrow \quad M_4^2 = \frac{2\Omega_{D-5}}{\sqrt{3(D-2)}\kappa^2 H^{D-4}}\left(\frac{D-6}{3}\right)^{\frac{D-5}{2}}$$

$$m^2 = \frac{3L(L+D-6)}{D-6}H^2 > \left(\frac{3}{2}H\right)^2$$ massive bound states

$$\Delta m^2 = \frac{3(D-2)}{4}H^2 > \left(\frac{3}{2}H\right)^2$$ no light KK modes



No light mode of $m < \frac{3}{2}H$ on the brane

2) with scalar and form field strengths

$$ds_D^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\left(\delta_{pq} + h_{pq}\right)dx^p dx^q + dy^2 + \frac{D-6}{3H^2 + f^2/2}\gamma_{ab}(Z)dz^a dz^b\right)$$

Spectrum

$$m^2 = 0$$

zero mode

$$m^2 = \frac{3L(L+D-6)}{D-6}\left(1+\frac{f^2}{6H^2}\right)H^2$$

massive bound states

$$\Delta m^2 > \frac{3(D-2)}{4}\left(1+\frac{2(D-6)}{(D-2)\beta}\right)H^2$$

no light KK modes

$$M_4^2 = \frac{2\Omega_{D-5}}{\kappa^2}\left(\frac{D-6}{\frac{\beta}{2}\left(-\frac{2\Lambda}{D-6}+\frac{f^2}{2}\right)+\frac{f^2}{2}}\right)^{\frac{D-5}{2}} \frac{1}{\sqrt{3\left(-\frac{2\Lambda}{D-6}+\frac{f^2}{2}\right)\left(D-6+\frac{D-2}{2}\beta\right)}}$$

$$H^2 = \frac{\beta}{6}\left(-\frac{2\Lambda}{D-6}+\frac{f^2}{2}\right)$$

two ways of $H \to 0$ limit

(1) $\beta \to 0$ $$M_4^2 \to \frac{2\Omega_{D-5}}{\kappa^2}\left(\frac{2(D-6)}{f^2}\right)^{\frac{D-5}{2}}\left(3\left(-2\Lambda+\frac{f^2}{2}(D-6)\right)\right)^{-1/2}$$

well-defined

(2) $f^2 \to \frac{4\Lambda}{D-6}$ $$M_4^2 \to \infty$$

ill-defined

(n+1)-dimensional effective theory

$$ds_D^2 = e^{2A(v)}\left[w_{ij}(M)dv^i dv^j + \underline{e^{2\psi(v)}\gamma_{ab}dz^a dz^b}\right]$$

$(n+1)-\mathrm{dim}$ $\quad\quad$ $p = (D-n-1)-\mathrm{dim}$

compactified

$$w_{ij}(M)dv^i dv^j = q_{\mu\nu}(X)dx^\mu dx^\nu + dy^2$$

n-dim de Sitter

$$\phi = \frac{2}{\alpha}(p-1)A(v)$$

$$F = f\Omega(z)$$

fixed

Einstein frame

$$w_{ij}(\overline{M}) = \exp\left[\frac{(D-2)A + p\psi}{n-1}\right]w_{ij}(M)$$

(n+1)-dimensional effective theory

$$S = \frac{1}{2\tilde{\kappa}^2}\int d^{n+1}x\sqrt{-w(\overline{M})}\left[R(\overline{M}) - V(\overline{A},\overline{\psi}) - \frac{1}{2}(\partial\overline{A})^2 - \frac{c_2}{2\sqrt{c_1c_3}}\partial\overline{A}\partial\overline{\psi} - \frac{1}{2}(\partial\overline{\psi})^2\right]$$

effective potential

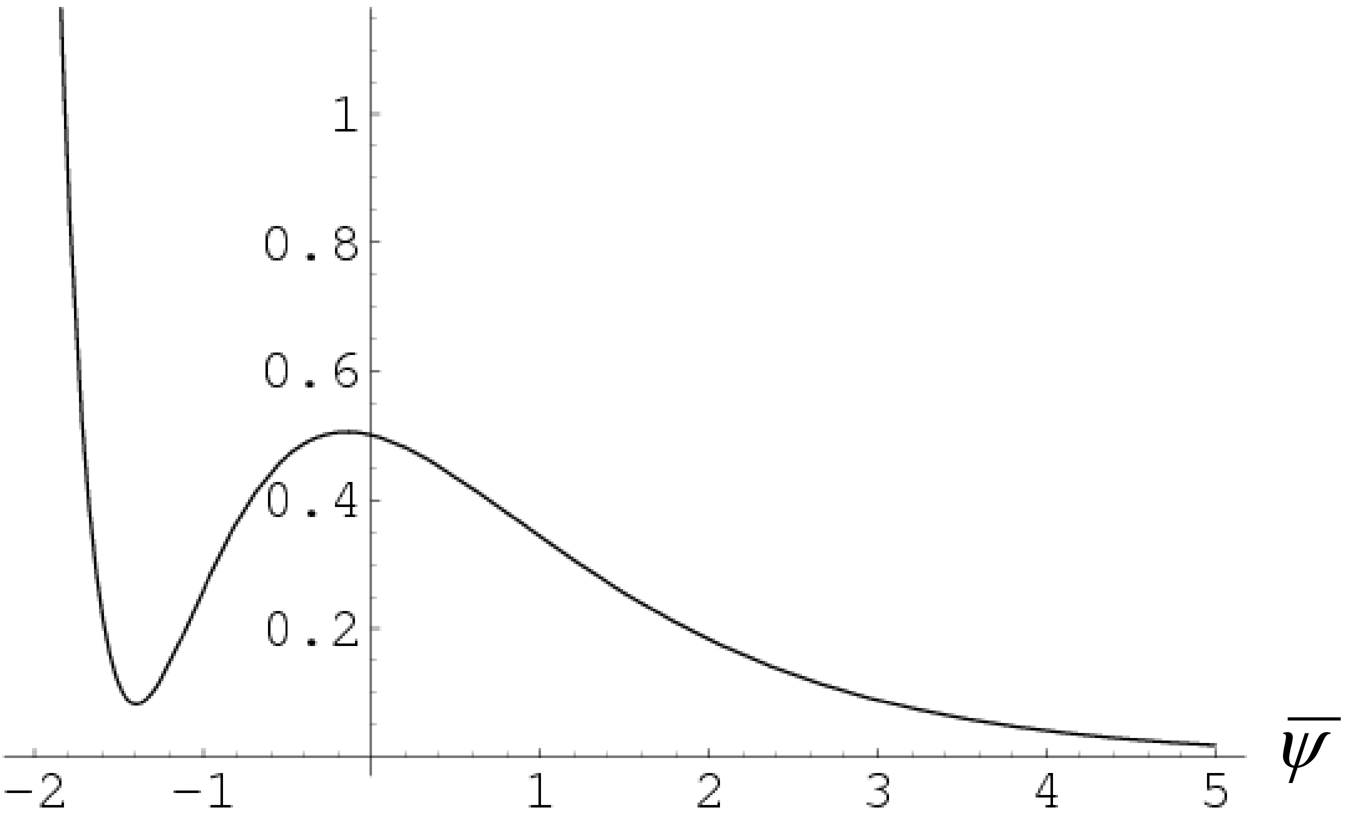
$$\boxed{\begin{aligned} V\left(\bar{A},\bar{\psi}\right) = \exp\left[-\frac{2(D-2)\bar{A}}{(n-1)\sqrt{c_1}}\right]\Bigg[& 2\Lambda\exp\left(-\frac{2p\bar{\psi}}{(n-1)\sqrt{c_3}}\right) \\ & +\frac{f^2}{2}\exp\left(-\frac{2np\bar{\psi}}{(n-1)\sqrt{c_3}}\right) - p\lambda\exp\left(-\frac{2(D-2)\bar{\psi}}{(n-1)\sqrt{c_3}}\right)\Bigg] \end{aligned}}$$

$$\bar{A} = \sqrt{c_1}A \qquad \bar{\psi} = \sqrt{c_3}\psi$$

$$c_1 = 2(D-2)\left[\frac{n(D-2)}{n-1} - 2(D-1)\right] + 2(p-1)\left[n-1+\frac{2}{\alpha^2}(p-1)\right] + 2p(D-1)$$

$$c_2 = \frac{4(D-2)p}{n-1} \qquad\qquad c_3 = 2p\left(\frac{n-1}{p}+1\right)$$

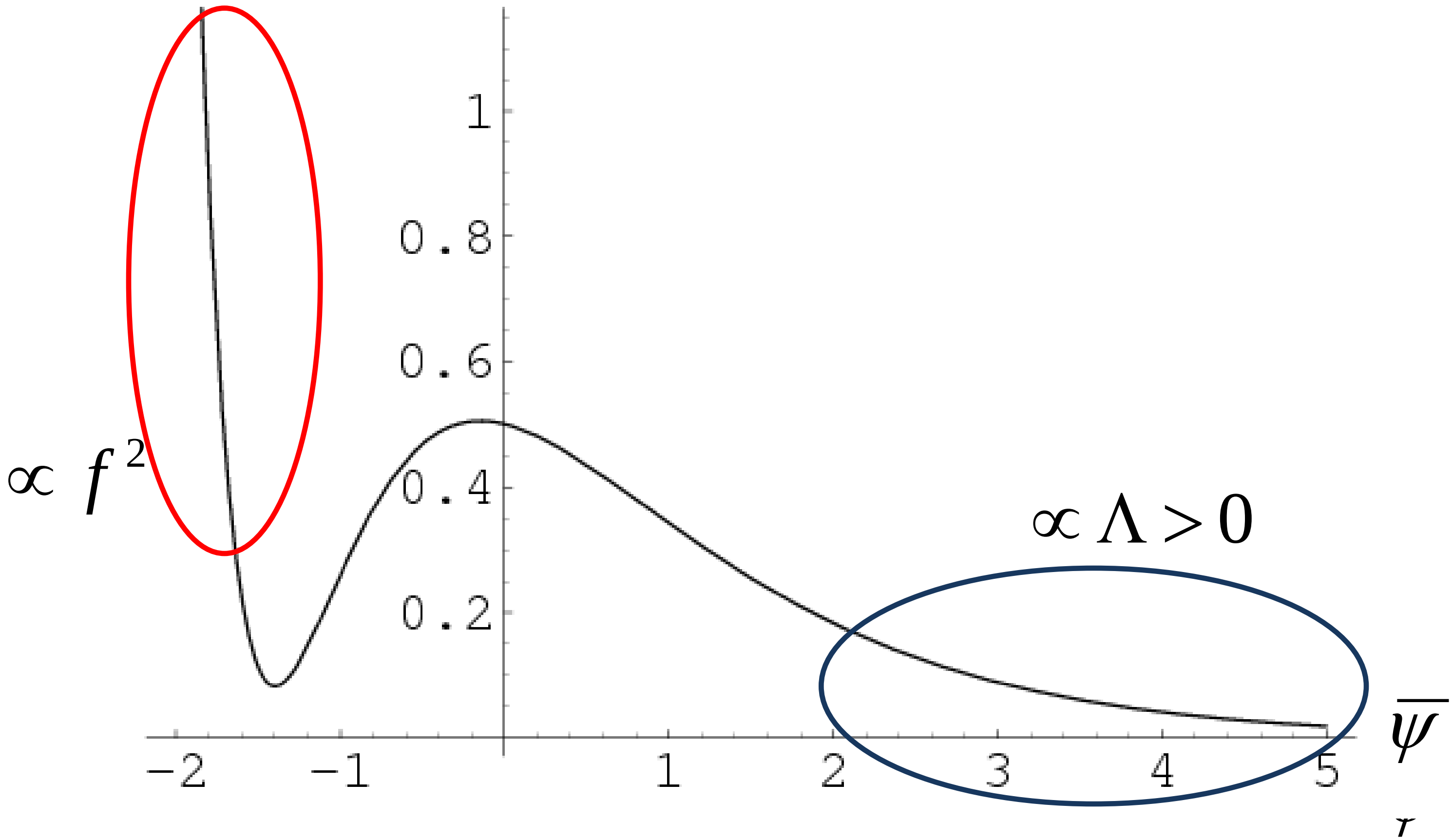
- $\bar{A}$ is not fixed
- flux supports the local minimum of $\bar{\psi}$



$\bar{A}$ cannot be fixed by the background

$\bar{\psi}$ can be fixed

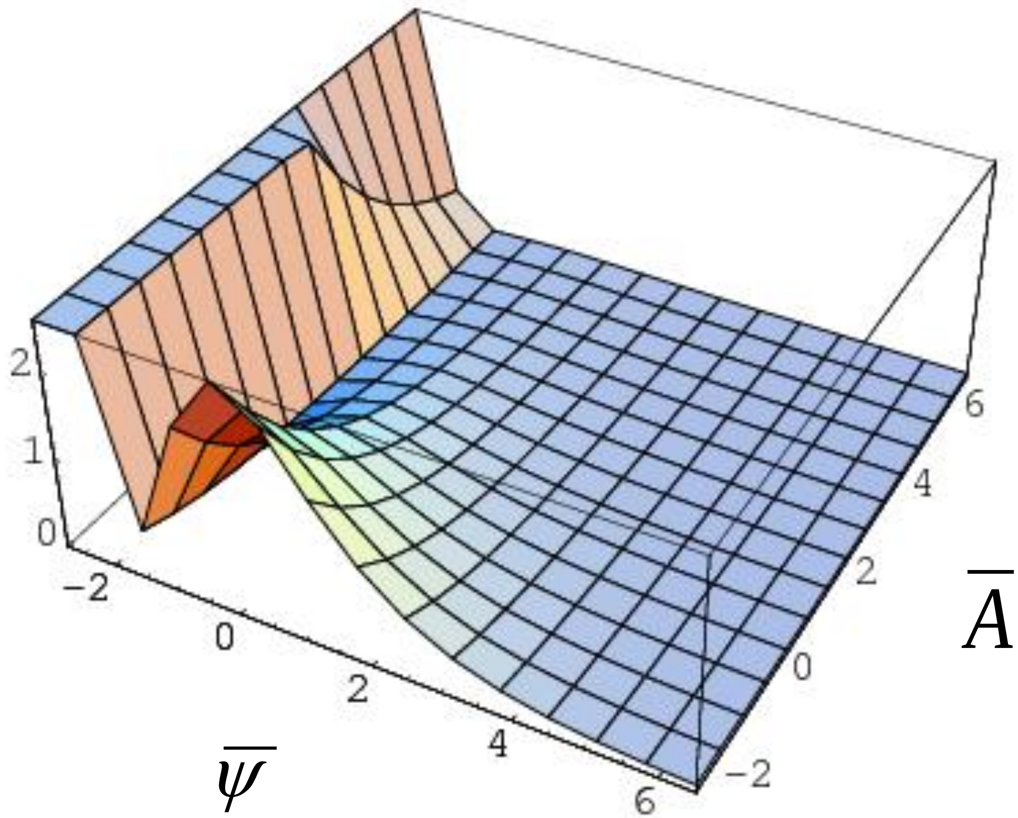
$$n = 4 \quad p = 5 \quad \bar{A} = 1 \quad f = .17 \quad \alpha = 0.5 \quad \Lambda = 1$$



$\bar{A}$ cannot be fixed by the background

$\bar{\psi}$ can be fixed

$$n = 4 \quad p = 5 \quad \bar{A} = 1 \quad f = .17 \quad \alpha = 0.5 \quad \Lambda = 1$$


2
1
0
−2
0
2
4
6
$\overline{\psi}$
6
4
2
0
−2
$\overline{A}$

Stability

$$ds_D^2 = e^{2A(y)}\Big(\big(1+2\phi_1\big)q_{\mu\nu}dx^\mu dx^\nu + (1+2\phi_2)dy^2 + (1+2\phi_3)\omega_{ab}dz^a dz^b\Big)$$

$$\delta G_{AB} = \kappa^2 \delta T_{AB}$$

- no 4D anisotropic stress

  keeping 4D symmetry

- no excitation along the compact internal space $\partial_a = 0$

mode decomposition $\phi_2 - \phi_3 = \int dm g_m(y)\psi_m(x)$

$$\left[\Delta_4 + 6H^2 - m^2\right]\psi_m(y) = 0$$

The lowest mode $m = 0$ leads to a tachyonic effective mass

$$M^2 = -6H^2$$

The result is irrespective of the existence and kinds of the bulk matter

Summary

What have been investigated

1) Dynamical branes

0-brane : Milne universe

1-brane : no regular collisions in D>4
regular collision in D=3

cosmological solutions in 6D Nishino-Salam-Sezgin model

expansion but too slow

intersecting branes

giving too slow expansion of the 3D space

2) Warped de Sitter compactifications

Brane world models

no excitations of light modes on the brane

moduli instability

Thank you.

Case I Maeda, Ohta and Uzawa (2009)

Ansatz
$$e^{\phi} = h_r^{\varepsilon_r c_r/2} h_s^{\varepsilon_s c_s/2}$$
$$F_{p_r+2} = d\left[h_r^{-1}(x,z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$
$$F_{p_s+2} = d\left[h_s^{-1}(x,z)\right] \wedge \Omega(X) \wedge \Omega(Y_1)$$

- Only one of two functions can depend on both x and z.
- For Euclidean brane/ transverse space geometries

$$h_r(x,z) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left|\vec{z} - \vec{z}_\ell\right|^{D+p-p_r-p_s-3}}$$
$$h_s(z) = C + \sum_\ell \frac{M_c}{\left|\vec{z} - \vec{z}_c\right|^{D+p-p_r-p_s-3}}$$

Intersecting rules in ten dimensions

$$p = \frac{1}{2}\left(p_r + p_s - 4\right)$$

Case II

Ansatz $$e^{\phi} = h_r^{\varepsilon_r c_r/2} h_s^{\varepsilon_s c_s/2}$$

$$F_{p_r+2} = d\left[h_r^{-1}(x,y)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

$$F_{p_s+2} = d\left[h_s^{-1}(x,z)\right] \wedge \Omega(X) \wedge \Omega(Y_1)$$

- Only one of two functions can depend on both x and z.
- For Euclidean brane/ transverse space geometries

$$h_r(x,y) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left|\vec{y} - \vec{y}_\ell\right|^{p_s - p - 2}}$$

$$h_s(z) = C + \sum_\ell \frac{M_c}{\left|\vec{z} - \vec{z}_c\right|^{D + p - p_r - p_s - 3}}$$

Intersecting rule in ten dimensions

$$p = \frac{1}{2}(p_r + p_s - 4)$$

Case III

Ansatz $$e^{\phi} = h_r^{\varepsilon_r c_r/2} h_s^{\varepsilon_s c_s/2}$$

$$F_{p_r+2} = h_s d\left[h_r^{-1}(x,y)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$
$$F_{p_s+2} = h_r d\left[h_s^{-1}(x,v)\right] \wedge \Omega(X) \wedge \Omega(Y_1)$$

•Only one of two functions can depend on both x and z.

• For Euclidean brane/ transverse space geometries

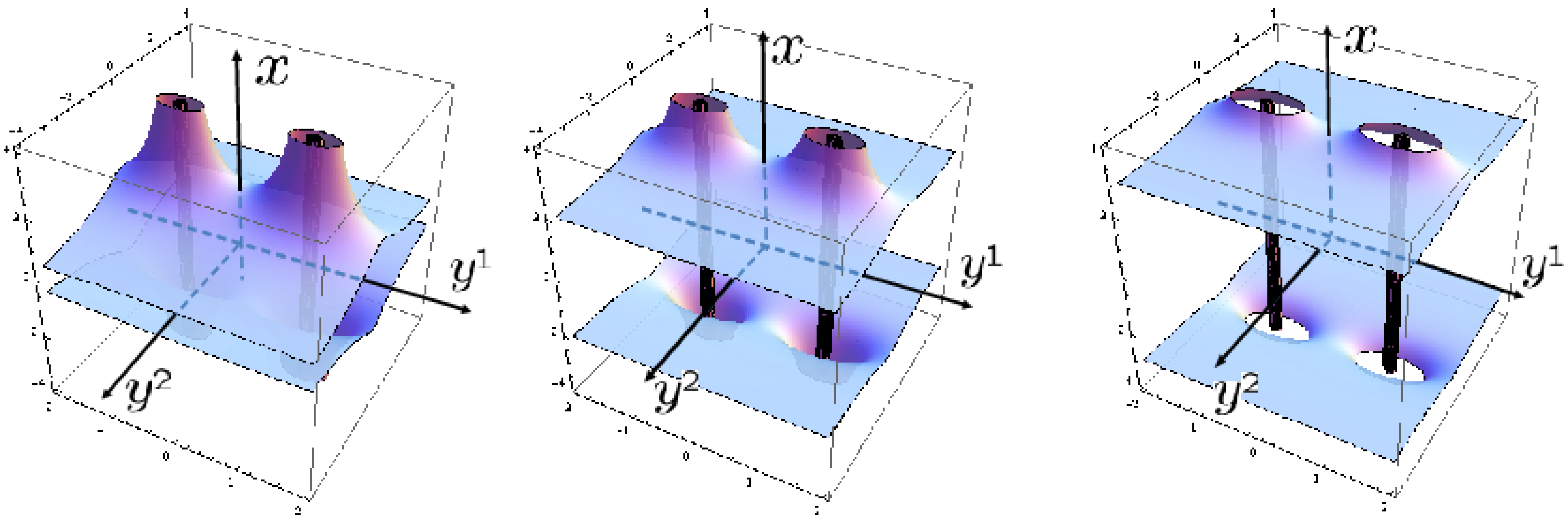
$$h_r(x,y) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left|\vec{y} - \vec{y}_\ell\right|^{p_s - p - 2}}$$
$$h_s(v) = C + \sum_\ell \frac{M_c}{\left|\vec{v} - \vec{v}_c\right|^{p_r - p - 2}}$$

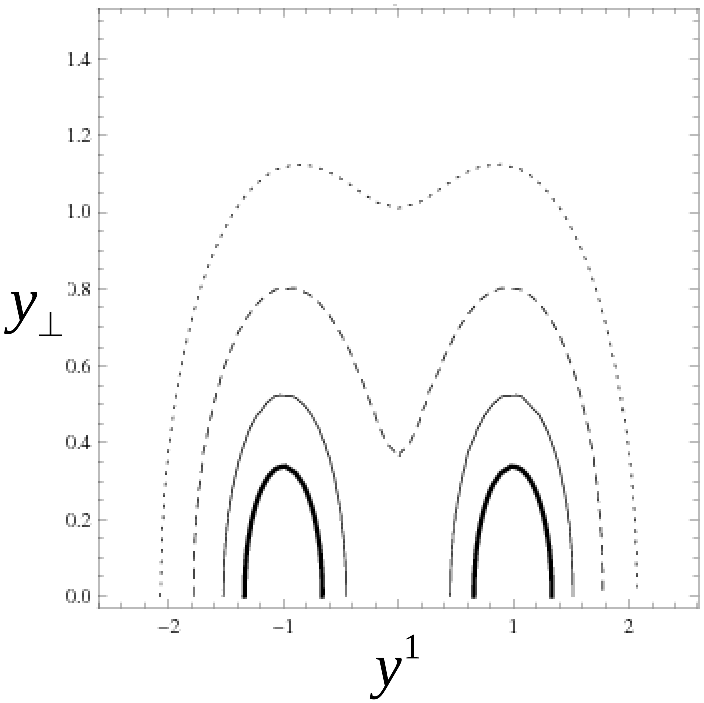
Intersecting rule in ten dimensions

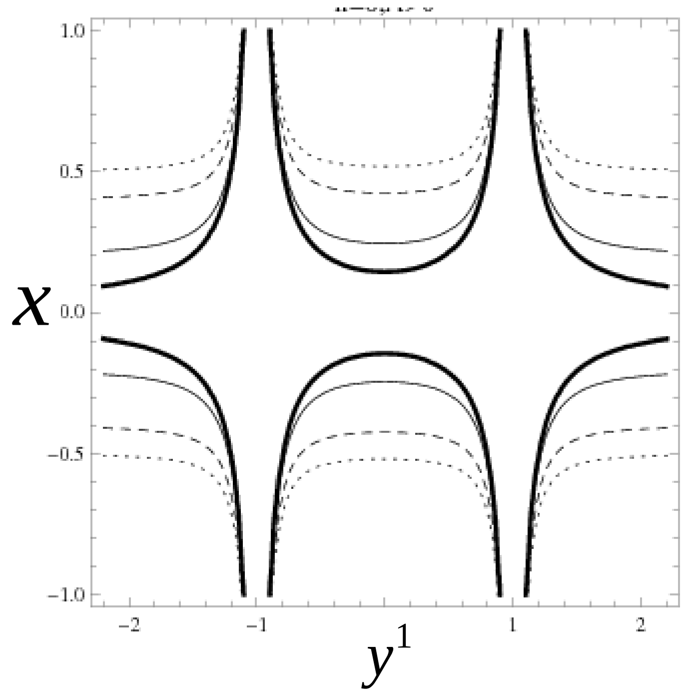
$$p = \frac{1}{2}\left(p_r + p_s - 8\right)$$

- Brane dynamics $D > 4$ $\Lambda > 0$

regular region $h(t, x, \vec{y}) = h_1(\vec{y}) + \frac{1}{2}\Lambda(t^2 - x^2) > 0$

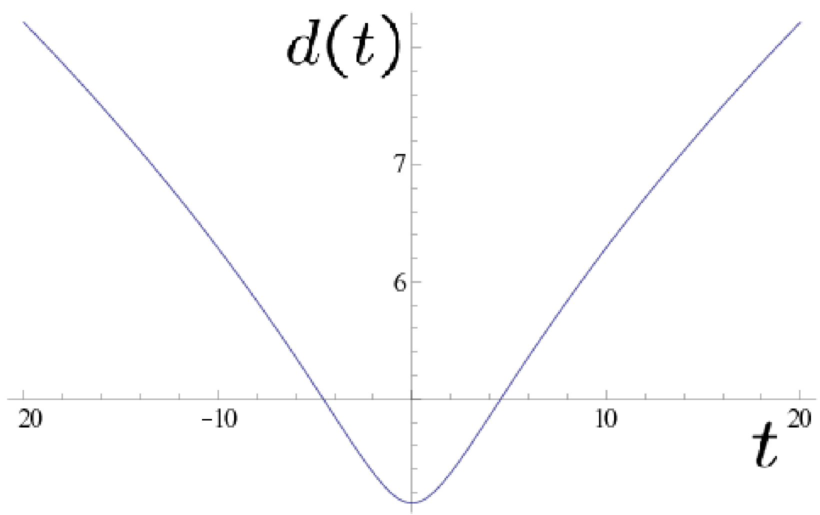


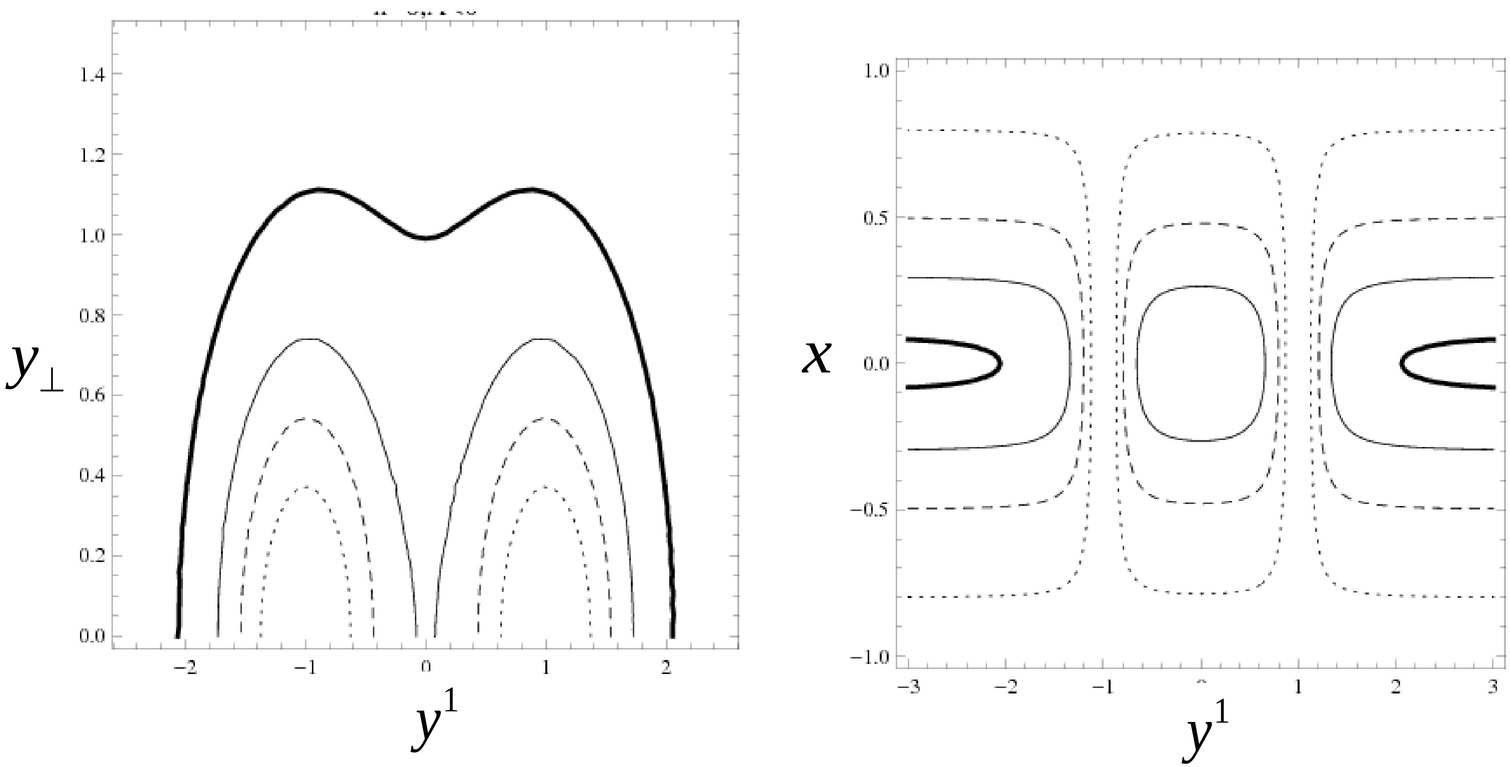




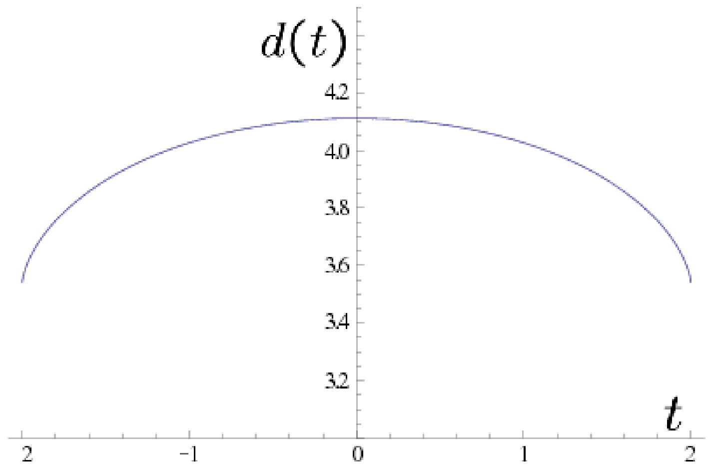
Proper distance

$$d(t) = \int_{-L}^{L} dy^1 \left[\frac{\Lambda}{2}t^2 + \frac{M}{|y^1+L|^{D-4}} + \frac{M}{|y^1-L|^{D-4}}\right]^{\frac{1}{D-2}}$$




$y_\perp$
y^1
x
y^1

Proper distance



A singularity is formed before two branes meet.

- Asymptotically Milne solution $c \neq 0$

Scalar field is dynamical

Solution exists only for N=2 and p=0

Solution exists only for p=0

$$h(t,z) = h_0(t) + h_1(z)$$

$$h_0(t) = \pm\sqrt{2\Lambda}t + B \qquad \Delta_Z h_1 = 0 \qquad R_{ab}(Z) = 0$$

no solution for $\Lambda < 0$

For $u_{ab} = \delta_{ab}$

$$ds^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-(D-2)} h_1(z)\right]^{-\left(\frac{2(D-3)}{D-2}\right)} \left[-d\tau^2 + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-(D-2)} h_1(z)\right\}^2 \left(\frac{\tau}{\tau_0}\right)^2 \delta_{ab} dz^a dz^b\right]$$

- approaching Milne universe in the far brane limit

Maki and Shiraishi (93), case (II) $\alpha = -c = \sqrt{\frac{2}{D-2}}$

- NSS model with vanishing 3-form field strength for $D = 6 \quad \Lambda = 0$

- Two kinds of field strengths

$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2(p_r+2)!}e^{\varepsilon_r c_r\phi}F_{p_r+2}^2-\frac{1}{2(p_s+2)!}e^{\varepsilon_s c_s\phi}F_{p_s+2}^2\right]$$

$$c_I^2=4-\frac{2(p_I+1)(D-p_I-3)}{D-2}$$

$$\varepsilon_I=\begin{cases}+ & \text{electric brane}\\ - & \text{magnetic brane}\end{cases}$$

supergravities in 10 or 11-dim

The case of $D=5$

$$ds^2 = A(y)^2\left(-dt^2 + c_0 e^{2Ht}\left(\delta_{ij} + h_{ij}\right)dx^i dx^j + dy^2\right)$$

transverse-traceless (gravitational wave perturbations) $h^{ij}_{,j} = h^i_i = 0$

$$h_{ij} = \int dm d^3k \ A^{-3/2} X_m(y)\varphi_{m,k}(t)e^{ik_j x^j}\hat{e}_{ij}$$

eigen-equation $$\left[-\frac{d^2}{dy^2} + V(y)\right]X_m(y) = m^2 X_m(y)$$

$$V(y) = \frac{9}{4}H^2 + \frac{360|\Lambda|H^4}{(24H^2 e^{H|y|} - |\Lambda|e^{-H|y|})^2} - 3H\sqrt{1 + \frac{|\Lambda|}{6H^2}}\delta(y - y_0)$$

$m = 0$ zero (localized) mode

no excitations of light modes $0 < m < \dfrac{3}{2}H$

Intersecting rules for case I) and II)

- intersecting rules in 11D

$$p = \frac{1}{9}(p_r + 1)(p_s + 1) - 1$$

$$M2 \cap M2 = 0 \quad M5 \cap M5 = 3 \quad M2 \cap M5 = 1$$

- intersecting rules in 10D

$$p = \frac{1}{2}(p_r + p_s - 4)$$

$$\mathrm{D}p \cap \mathrm{D}p = p - 2 \quad \mathrm{D}(p-2) \cap \mathrm{D}p = p - 3 \quad \mathrm{D}(p-4) \cap \mathrm{D}p = p - 4$$

$$\mathrm{F1} \cap \mathrm{NS5} = 1 \quad \mathrm{NS5} \cap \mathrm{NS5} = 3$$

$$\mathrm{F1} \cap \mathrm{D}p = 0 \quad \mathrm{D}p \cap \mathrm{NS5} = p - 1 \ (p \leq 6)$$

Intersecting rules for case III)

- intersecting rules in 11D

$$p = \frac{1}{9}(p_r + 1)(p_s + 1) - 3$$

$$\mathrm{M2} \cap \mathrm{M2} = 0 \quad \mathrm{M5} \cap \mathrm{M5} = 1 \quad \mathrm{M2} \cap \mathrm{M5} = 0$$

- intersecting rules in 10D

$$p = \frac{1}{2}(p_r + p_s - 8)$$

$$\mathrm{Dp} \cap \mathrm{Dp} = \mathrm{p} - 4 \quad \mathrm{D}(\mathrm{p} - 2) \cap \mathrm{Dp} = \mathrm{p} - 5 \quad \mathrm{D}(\mathrm{p} - 4) \cap \mathrm{Dp} = \mathrm{p} - 6$$

$$\mathrm{D}(\mathrm{D} - 6) \cap \mathrm{Dp} = \mathrm{p} - 7 \quad \mathrm{D}(\mathrm{p} - 8) \cap \mathrm{Dp} = \mathrm{p} - 8 \quad \mathrm{NS5} \cap \mathrm{NS5} = 1$$

$$\mathrm{Dp} \cap \mathrm{NS5} = \mathrm{p} - 3\,(\mathrm{p} \le 8)$$

- Ansatz

$$ds^2 = h^a(x,z)q_{\mu\nu}(X)dx^\mu dx^\nu + h^b(x,z)u_{ab}(Z)dz^a dz^b$$

$$a = -\frac{4(D-p-3)}{N(D-2)} \qquad b = \frac{4(p+1)}{N(D-2)}$$

$$e^\phi = h^{2\varepsilon c/N} \qquad F_{p+2} = \frac{2}{\sqrt{N}}\sqrt{-q}\,d\left(h^{-1}\right)\wedge\left(dx^0\wedge\cdots\wedge dx^p\right)$$

Cosmological evolutions 2

- D-dimensional solution

$$ds^2 = -hdt^2 + ds^2\left(\tilde{X}\right) + ds^2\left(Y_1\right) + ds^2\left(Y_2\right) + ds^2\left(Z\right)$$

$$ds^2\left(\tilde{X}\right) = h\delta_{PQ}d\theta^P d\theta^Q \qquad ds^2\left(Y_1\right) = h_r^{\alpha}h_s^{\beta-1}\gamma_{ij}\left(Y_1\right)dy^i dy^j$$

$$ds^2\left(Y_2\right) = h_r^{\alpha-1}h_s^{\beta}w_{mn}\left(Y_1\right)dv^m dv^n \quad ds^2\left(Z\right) = h_r^{\alpha}h_s^{\beta}u_{ab}\left(Z\right)dz^a dz^b$$

$$h = h_r^{\alpha-1}h_s^{\beta-1} \qquad \left\{ \begin{array}{l} h_r = At + h_1 \\ \partial_\mu h_r = 0 \end{array} \right.$$

- Compactifying $d = d_1 + d_2 + d_3 + d_4$ dimensions

$$ds^2 = \underbrace{ds^2\left(M\right)}_{(D-d)-\mathrm{dim}} + \underbrace{ds^2\left(N\right)}_{d-\mathrm{dim}}$$

Cosmological evolutions 3

- conformal transformation to (D-d)-dim Einstein frame

$$ds^2(M) = h_r^A h_s^B ds^2(\overline{M}) \qquad B = \frac{-\alpha d + d_1 + d_3}{D-d-2} \qquad C = \frac{-\beta d + d_2 + d_4}{D-d-2}$$

$$\begin{aligned} ds^2(\bar{\mathrm{M}}) \;=\; & h_s^{C'} \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right]^{B'} \left[-d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{2B'/(B'+2)} \delta_{P'Q'}(\tilde{\mathrm{X}}')d\theta^{P'}d\theta^{Q'} \right. \\ & + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right\} \left(\frac{\tau}{\tau_0}\right)^{2(B'+1)/(B'+2)} \gamma_{k'l'}(\mathrm{Y_1}')dy^{k'}dy^{l'} \\ & + h_s \left(\frac{\tau}{\tau_0}\right)^{2B'/(B'+2)} w_{m'n'}(\mathrm{Y_2}')dv^{m'}dv^{n'} \\ & \left. + h_s \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)} h_1\right\} \left(\frac{\tau}{\tau_0}\right)^{2(B'+1)/(B'+2)} u_{a'b'}(\mathrm{Z}')dz^{a'}dz^{b'} \right], \end{aligned}$$

$$B' = -B + \alpha - 1 \qquad C' = -C + \beta - 1$$

- reading the expansion rate after compactification / Brane world

No-Go theorem for de Sitter compactifications

no solution realizing 4D de Sitter spacetime after compactifications under the following assumptions. Maldacena and Nunez (02)

(1) no higher–derivative / higher curvature corrections

(2) scalar potential takes a nonpositive value

(3) fields with positive kinetic terms

(4) 4-dimensional effective gravitational constant is finite.
The internal space is compact.

$\longrightarrow$ We relax the condition (4) and assume the existence of a noncompact direction.

※ The original proof assumes time-independent metric.

time-dependent, hyperbolic internal space $\longrightarrow$ acceleration.

e.g., Townsend and Wohlfarth (03), Ohta (03), Garriga and Emparan (03)

No-Go theorem

Maldacena and Nunez (02)

D-dimensional metric

$$ds^2 = A(y)^2 \left[q_{\mu\nu}(X) dx^\mu dx^\nu + u_{ij}(Y) dy^i dy^j \right]$$

$$n-\mathrm{dim} \qquad\qquad (D-n)-\mathrm{dim}$$

(1) no higher–derivative / higher curvature corrections

(2) scalar potential takes a nonpositive value

(3) fields with positive kinetic terms

(4) 4-dimensional effective gravitational constant is finite.
The internal space is compact.

Einstein equations

$$\frac{n}{(D-2)A^{D-2}}\Delta_Y A^{D-2} = R(X) + A^2\bar{T}$$

$$\bar{T} = -T_\mu^\mu + \frac{d}{D-2}T$$

Multiplying A^{D-2}
and integrating over the *compact* internal space

$$0 = \frac{n}{(D-2)}\int_Y d^{D-n}y\sqrt{u}\Delta_Y A^{D-2} = \int_Y d^{D-n}y\sqrt{u}A^{D-2}\left[R(X) + A^2\bar{T}\right]$$

If $\bar{T} > 0$ $\longrightarrow$ $R(X) < 0$

no de Sitter solution

(1) nonpositive potential

$$T_{MN} = -Vg_{MN}$$

$$\longrightarrow \quad \bar{T} = -\frac{2n}{D-2}V \geq 0$$

$R(X) < 0$ no de Sitter solution

(2) p-form field strength

$$\bar{T} = \frac{1}{2(p-1)!}\left(-F_{\mu P_1 \cdots P_{p-1}} F^{\mu P_1 \cdots P_{p-1}} + \frac{n(p-1)}{(D-2)p} F^2\right)$$

- If p-form field has no component in de Sitter space $\bar{T} > 0$
- If p-form field has component on de Sitter space

$$\bar{T} = -\frac{n(D-p-1)}{2(D-2)p!} F^2 \geq 0$$

$R(X) < 0$ no de Sitter solution

de Sitter braneworld

$$ds^2 = e^{2A(y)}\left(-dt^2 + e^{2Ht}\delta_{pq}dx^p dx^q + dy^2\right)$$

$$e^{-A(y)} = e^{H|y|} - \frac{|\Lambda| e^{-H|y|}}{24H^2}$$

Putting brane at $y = y_0$

$$e^{Hy_0} - \frac{|\Lambda| e^{-Hy_0}}{24H^2} = 1$$

$$ds^2_{ind} = -dt^2 + c_0 e^{2Ht}\delta_{pq}dx^p dx^q$$

Brane tension $\kappa^2\sigma = \sqrt{6(6H^2 + |\Lambda|)}$

Stability

$$ds_D^2 = e^{2A(y)}\left((1+2\phi_1)q_{\mu\nu}dx^\mu dx^\nu + (1+2\phi_2)dy^2 + (1+2\phi_3)\omega_{ab}dz^a dz^b\right)$$

$$\delta G_{AB} = \kappa^2 \delta T_{AB}$$

: no 4D anisotropic stress

: no excitation along the compact internal space $\partial_a = 0$

<u>(1) vacuum</u> $A(y) = e^{-H\sqrt{\frac{3}{D-2}}(y-y_0)}$

$$\left[\left(\Box_4 + 6H^2\right) + \partial_Y^2 - \sqrt{3(D-2)}H\partial_Y\right](\phi_2 - \phi_3) = 0$$

mode decomposition $\phi_2 - \phi_3 = \int dm g_m(y)\psi_m(x)$

$$\left[\Delta_4 + 6H^2 - m^2\right]\psi_m(y) = 0$$

$m = 0$ zero (localized) mode

$$\left[\Delta_4 + 6H^2\right]\psi_0(y) = 0$$ tachyonic

(2) with a scalar field

$$A(y) = e^{-H\sqrt{\frac{3\left(\frac{2}{D-2}+c\right)}{c(D-2)}}(y-y_0)} \qquad \phi(y) = 2H\sqrt{\frac{3}{c(D-2)}}(y-y_0)$$

$$\left[\left(\Box_4 + 6H^2\right) + \partial_y^2 - \sqrt{3(D-2)\left(1+\frac{2}{c(D-2)}\right)}H\partial_y\right](\phi_2 - \phi_3) = 0$$

mode decomposition $\phi_2 - \phi_3 = \int dm g_m(y)\psi_m(x)$

$$\left[\Delta_4 + 6H^2 - m^2\right]\psi_m(y) = 0$$

$m = 0$ zero (localized) mode

$$\left[\Delta_4 + 6H^2\right]\psi_0(y) = 0$$ tachyonic

(3) with scalar and p-form fields

$$\left[\left(\Box_4+6H^2\right)+\partial_y^2-\sqrt{3(D-2)\left(1+\frac{2(D-6)}{(D-2)\beta}\right)}H\partial_y\right](\phi_2-\phi_3)=\frac{f^2}{2}\left(2\phi_3+\alpha\delta\phi\right)$$

The field strength contribution induces a source term.

Taking the homogeneous part, the lowest mode becomes tachyonic with mass $-6H^2$

Universal tachyonic mass $-6H^2$

Cosmological evolutions 1

The time dependence in the metric comes from only one brane in the intersections. To find an expanding universe, we have two approaches.

(i) In the original higher-dimensional theory

(ii) To find solutions in the lower-dimensional effective theory after compactifying the bulk space as well as the brane world model, fixing our universe at a position in the transverse space.

τ the cosmic time.