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Gauss-Bonnet braneworld redux: A novel scenario for the bouncing universe

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based on

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Contents

Introduction

Relativistic cosmology

- Purpose: Understand the history & fate of our universe
- Zeroth-order approximation of our universe
  - Friedmann-Robertson-Walker cosmological spacetime

$$-d\tau^2 + a(\tau)^2 \left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$f_0(\chi) = \chi,\ f_1(\chi) = \sin\chi,\ \text{and}\ f_{-1}(\chi) = \sinh\chi$$

flat, closed, open

  - Homogeneous & isotropic space, characterized by its curvature k
  - Only 1 dynamical degree of freedom: scale factor a(t)

Dynamics of the FRW universe

- Einstein equation with a perfect fluid:

Energy density

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 T_{\mu\nu}$$ where $$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho, p, p, p)$$

Cosmological constant

pressure

- Equation of state: $p = (\gamma - 1)\rho$
- Energy-momentum conservation gives $\rho = \frac{\rho_0}{a^{3\gamma}},$
  - Weak energy condition (WEC): Energy density observed is non-negative
    - g=[0,2] with non-negative ρ_0
  - Strong energy condition (SEC): Gravity is attractive
    - WEC+SEC: g =[2/3,2] with non-negative ρ_0
- Einstein equation gives the master equation for a(t)

$$H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$$ where $$H := \dot{a}/a$$

Friedmann equation

1-dim Potential problem

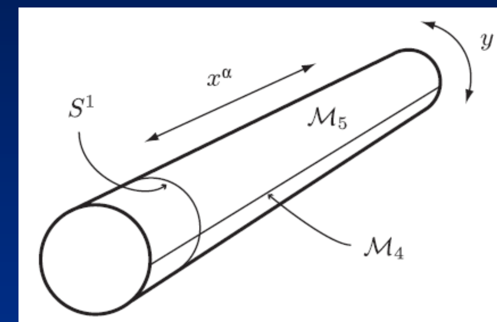
- Friedmann equation: $H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$ =:V(a)
  - One-dimensional potential problem
  - Allowed domain of a(t) is given by V(a)>0
- a=0 corresponds to the Big Bang (Big Crunch) singularity
  - Zero spatial volume
  - GR breaks down and Quantum effect of gravity dominates
- Qualitative analysis of the evolution is possible
  - Big Bounce (if a=0 not allowed): from contraction to expansion
    - g =[0,2/3) with k=1 required
  - Bouncing point is given by V(a)=0
  - FACT: No bounce for ordinary matter (WEC + SEC)

Motivation: Initial singularity problem

- Singularities are generic in GR (Hawking-Penrose, Geroch, 70’s)
- Should be cured, but no full quantum gravity available
- (Super)string/M-theory is a strong candidate toward QG
- String-inspired Randall-Sundrum braneworld (`99) is an interesting toy model of the higher-dim universe with a large extra dimension
  - The universe as a domain wall embedded in the 5-dim bulk spacetime
  - Original motivation is for the hierarchy problem (why gravity is so weak)
  - Dynamics of the early universe is modified
- Q. Does the braneworld solve the initial singularity problem?

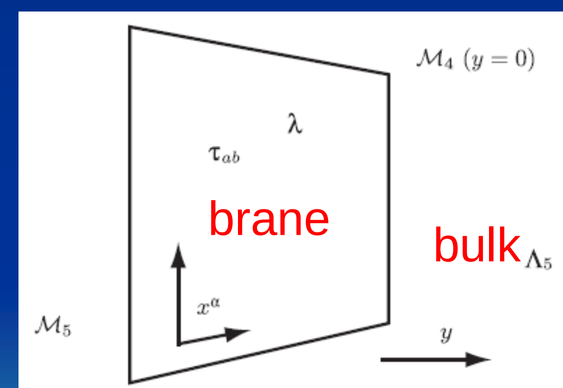
Randall-Sundrum braneworld

- Kaluza-Klein compactification
  - Small & compact extra dimension



Kaluza-Klein

- Randall-Sundrum brane-world (‘99)
  - A (3+1)-dim. timelike hypersurface (brane) in the (4+1)-dim. (asymptotically) AdS bulk spacetime
  - AdS => Newton’s law on the brane
    - Volcanic potential
  - RS2 model: single-brane model
    - Newtonian gravity on the brane is recovered even with infinitely large extra dimension



Braneworld

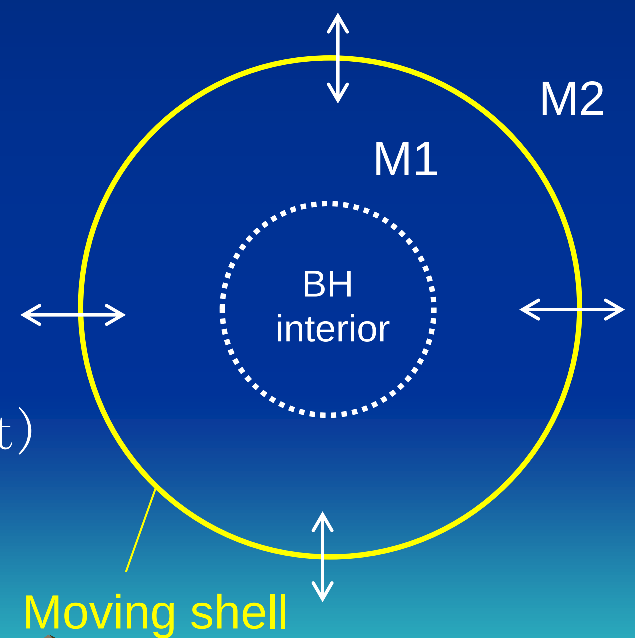
Shell dynamics in GR for warming up

- Consider a spherically symmetric thin-shell in the Schwarzschild vacuum spacetime: $$ds^2 = -\left(1 - \frac{M}{r}\right)dt^2 + \left(1 - \frac{M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$
  - M=M1 for inside the shell and M=M2(>M1) for outside
- Israel junction condition (=Einstein equation) gives the dynamical equation for the shell

$$[K^a{}_b]_\pm - \delta^a{}_b[K]_\pm = -\kappa_4^2 \tau^a{}_b \qquad [X]_\pm := X^+ - X^-,$$

↑ Extrinsic curvature of the shell

↑ Energy-momentum tensor on the shell



- The orbit of the shell is given as r=a(τ), t=T(τ)
  - A timelike hypersurface in the spacetime
  - τ is a proper time of the shell

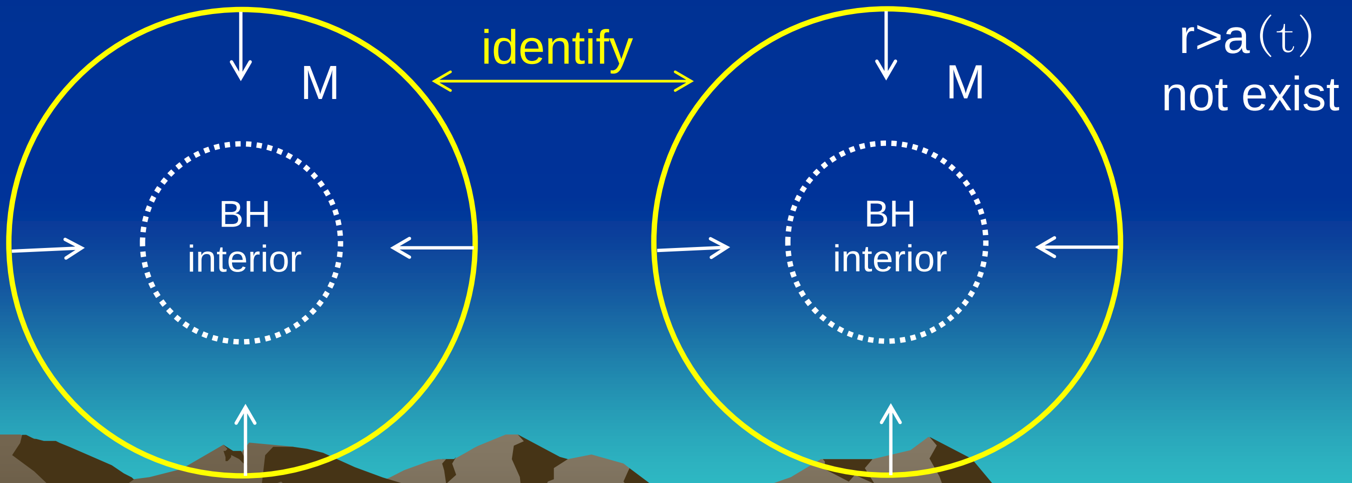
Simplest brane-universe in GR

- Consider a thin-shell (=brane) in the 5-dim Schwarzschild-Tangherlini-AdS vacuum spacetime (Binetruy-Deffayet-Langlois `00 , Kraus `99, Binetruy-Deffayet-Ellwanger-Langlois `00, Ida `00)

$$-h(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right].$$

$$h(r) = k - \frac{\mu}{8r^2} - \frac{1}{6}\Lambda r^2.$$

  - Position of the brane: r=a（t）, t=T（t）
  - Take r<a（t） and paste with the same copy



Friedmann equation on the brane

- Induced metric on the brane => FRW universe

$$-d\tau^2 + a(\tau)^2 \left[d\chi^2 + f_k(\chi)^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$f_0(\chi) = \chi,\ f_1(\chi) = \sin\chi,$ and $f_{-1}(\chi) = \sinh\chi$

- Israel junction condition gives the dynamical equation for a(t)
  - Consistent with a perfect fluid and the EOS p=(g-1)r
  - Energy-momentum conservation is the same: $\rho = \frac{\rho_0}{a^{3\gamma}},$
- Modified Friedmann equation:

$$H^2 = \frac{\kappa_5^4}{36}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2 - \frac{k}{a^2} + \frac{\mu}{8a^4} + \frac{1}{6}\Lambda =: V_{\rm GR}(a).$$

Brane tension

Modified Friedmann equation

- Modified Friedmann equation

$$H^2 = \frac{\kappa_5^4}{36}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2 - \frac{k}{a^2} + \frac{\mu}{8a^4} + \frac{1}{6}\Lambda =: V_{\rm GR}(a).$$

quadratic

Dark radiation

  - Standard 4-dim GR case: $H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$
  - Existence of the ``dark radiation'' (from the mass term in the bulk)
  - Early-time evolution modified
- Q. Bouncing is possible with perfect fluid satisfying WEC＋SEC?
- A. Still not

Bouncing brane universe?

- Yes, with matter in the bulk
  - U(1) gauge field (Barcelo-Visser `00): but inner horizon in the bulk is unstable (Hovdebo-Myers `03)
  - SU(2) Yang-Mills field (Okuyama-Maeda `04): non-singular bouncing universe is possible with a solitonic bulk spacetime
- In this talk, I consider VACUUM bulk but in modified gravity
  - Einstein-Gauss-Bonnet gravity

$$I = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left(R - 2\Lambda + \alpha L_{GB} \right) + I_{\partial M},$$

where

$$L_{GB} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

  - Still 2^{nd}-order theory
  - Low-energy limit of heterotic string theory is 10-dim E-GB with a dilaton
  - We assume a>0, L<0, and $1 + 4\alpha\Lambda/3 \geq 0$

Preliminaries

Bulk solution

- Field equation: $G^{\mu}{}_{\nu} + \alpha H^{\mu}{}_{\nu} + \Lambda \delta^{\mu}{}_{\nu} = 0,$

$$H_{\mu\nu} := 2\left(RR_{\mu\nu} - 2R_{\mu\alpha}R^{\alpha}{}_{\nu} - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\right) - \frac{1}{2}g_{\mu\nu}L_{GB}.$$

- Vacuum solution (Boulware-Deser `85, Wheeler `86, Cai `02):

$$-h(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$h(r) = k + \frac{r^2}{4\alpha}\left(1 \mp \sqrt{1 + \frac{\alpha\mu}{r^4} + \frac{4}{3}\alpha\Lambda}\right),$$

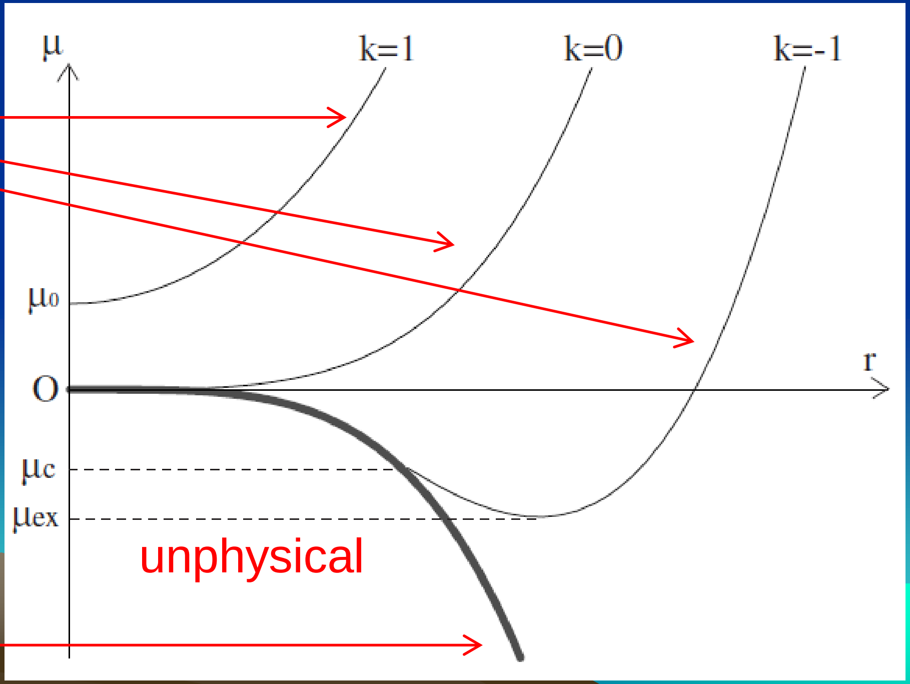
- Two branches of solutions: GR (-) and non-GR (+) branches
  - Only GR branch has the GR limit
  - non-GR branch is dynamically unstable
  - We consider only GR-branch

Horizons and singularities in the bulk

- m>0: Central singularity (at r=0)
- **m<0: Non-central ``branch'' singularity (at finite r)**
  - Metric is finite
  - Metric becomes complex and unphysical for r<r_b
- Number of horizons depends on k and m

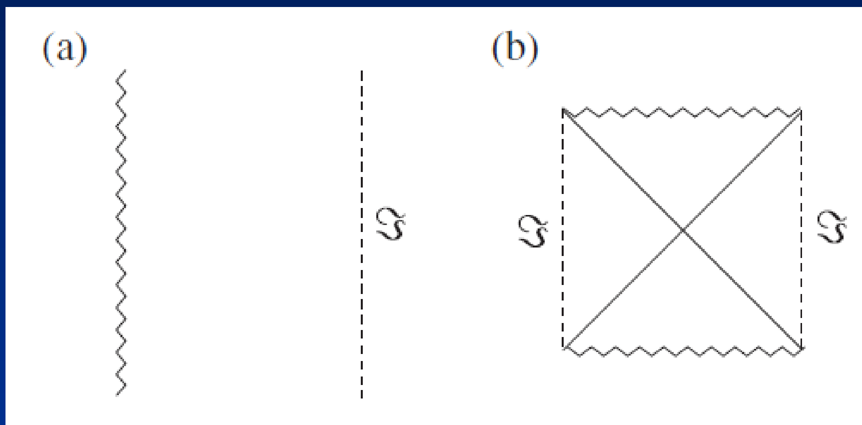
m-r diagram

Horizon curves

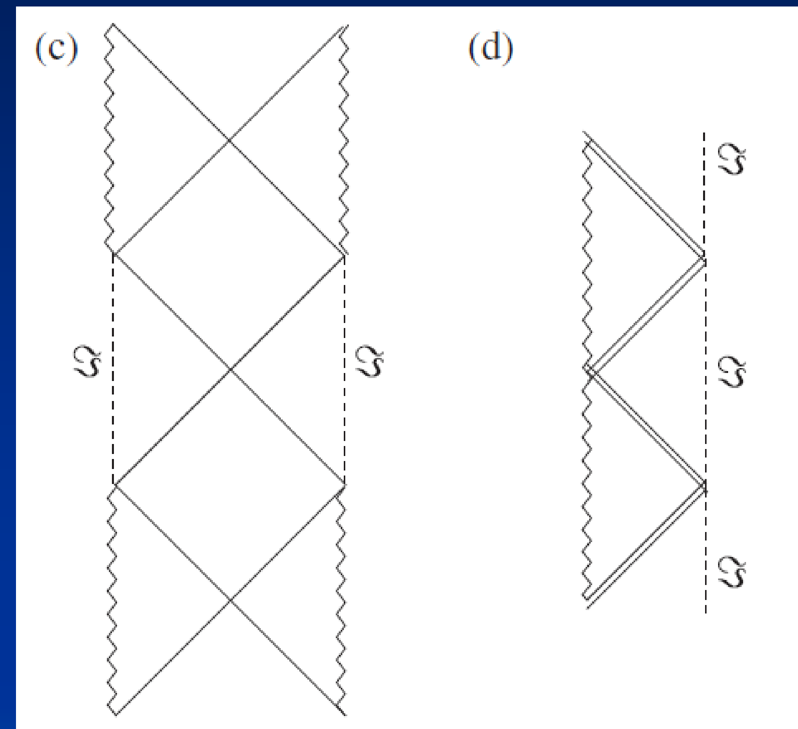


Branch singularity curve

Global structure of the bulk for m<0



k=1,0



k=-1 depending on m (<0)

Friedmann equation in the GB braneworld

- Junction condition in EGB (Gravanis-Willison `03, Davis `03)
  - e=1 (timlike brane), -1(spacelike brane)

$$[K^a{}_b]_\pm - \delta^a{}_b[K]_\pm + 2\alpha\left(3\varepsilon[J^a{}_b]_\pm - \varepsilon\delta^a{}_b[J]_\pm - 2P^a{}_{dbf}[K^{df}]_\pm\right) = -\varepsilon\kappa_5^2\tau^a{}_b, \qquad [X]_\pm := X^+ - X^-,$$

$$\begin{aligned} J_{ab} :=& \frac{1}{3}\left(2KK_{ad}K^d{}_b + K_{df}K^{df}K_{ab} - 2K_{ad}K^{df}K_{fb} - K^2K_{ab}\right) \\ P_{adbf} :=& R_{adbf} + 2h_{a[f}R_{b]d} + 2h_{d[b}R_{f]a} + Rh_{a[b}h_{f]d}. \end{aligned}$$

- Modified Friedmann equation (cubic for H^2) (Charmousis-Dufaux `02)

$$\frac{\kappa_5^4}{36}(\rho+\sigma)^2 = \left(\frac{h(a)}{a^2} + H^2\right)\left[1 + \frac{4\alpha}{3}\left(\frac{3k - h(a)}{a^2} + 2H^2\right)\right]^2$$

  - We assume a perfect fluid on the brane with linear EOS

$$\tau^a{}_b = \mathrm{diag}(-\rho, p, p, p) + \mathrm{diag}(-\sigma, -\sigma, -\sigma, -\sigma,),$$

Modified Friedmann equation in the GB braneworld

$$H^2 = V_{\mathrm{GB}(+)}(a),$$

$$V_{\mathrm{GB}(+)}(a) := \frac{1}{8\alpha}\left[-\frac{8k\alpha}{a^2} - 2 + \left\{ A^{3/2} + 256\alpha^3 P^2 + 16\sqrt{2\alpha^3 P^2 \left(128\alpha^3 P^2 + A^{3/2} \right)} \right\}^{1/3} \right.$$

$$\left. + A\left\{ A^{3/2} + 256\alpha^3 P^2 + 16\sqrt{2\alpha^3 P^2 \left(128\alpha^3 P^2 + A^{3/2} \right)} \right\}^{-1/3} \right],$$

$$A := 1 + \frac{\alpha\mu}{a^4} + \frac{4}{3}\alpha\Lambda,$$

$$P^2 := \frac{\kappa_5^4}{256\alpha^2}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma \right)^2.$$

Still one-dimensional potential problem

Neo bouncing brane universe

New scenario for the Big Bounce

- Dynamics is complicated but still 1-dim potential problem
  - For $m>0$, $a=0$ corresponds to the Big-Bang singularity on the brane
  - For $m<0$, $a=0$ is NOT allowed since branch singularity is at $a=a_b$ (>0)
- Q. What happens if the brane hits the branch singularity?
  - A singularity on the brane? **NO**.
  - My claim: It is a totally new type of the Big Bounce
- This claim is supported by
  - Fact 1: Collision of the brane with the branch singularity is generic
  - Fact 2: All the curvature invariants on the brane do NOT blow up
  - Fact 3: This bulk singularity is weak

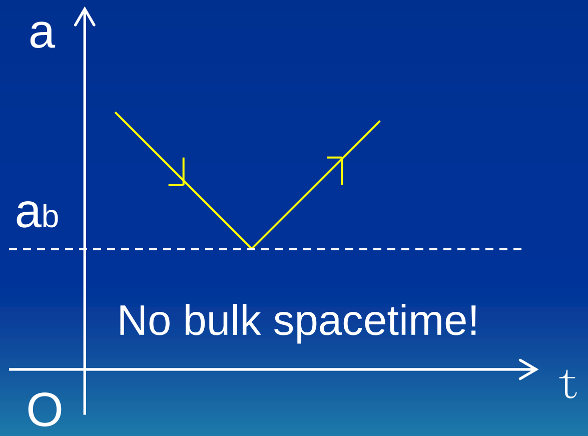
Fact 2: Regularity on the brane

- Asymptotic behavior near $a=a_b$:
  - Curvatures remain finite near $a=a_b$

$$
\begin{aligned}
a &\simeq a_{\rm b} + a_1(\tau - \tau_{\rm b}) + O((\tau - \tau_{\rm b})^2), \\
a_1^2 &:= V_{\rm GB}(a_{\rm b}),
\end{aligned}
$$

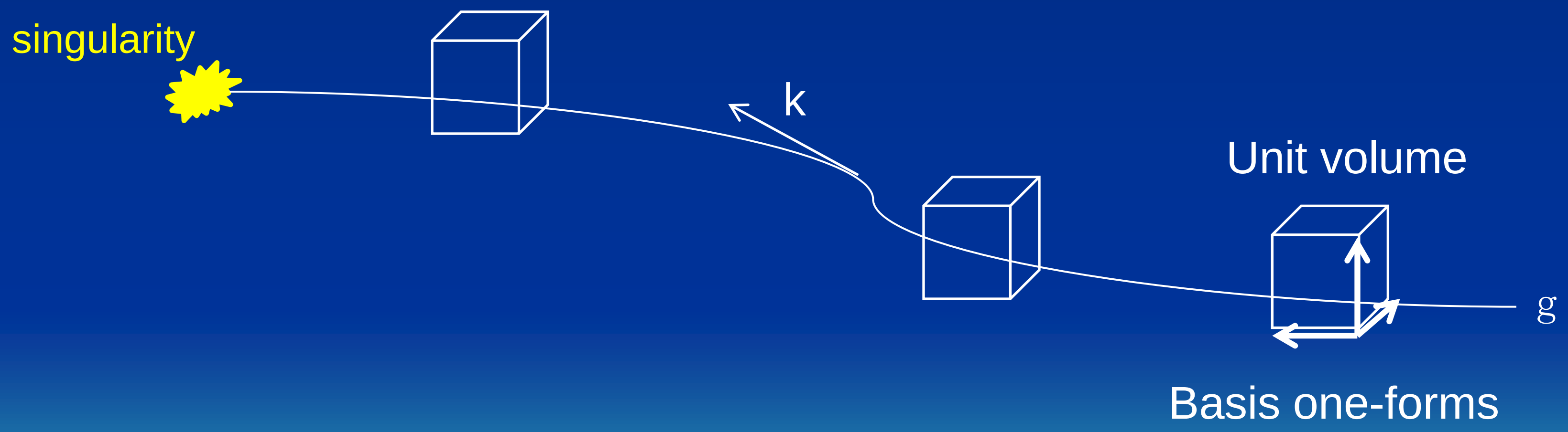
$$
V_{\rm GB}(a_{\rm b}) = \frac{1}{8\alpha a_{\rm b}^2}\left[a_{\rm b}^2\left\{2\kappa_5^4\alpha\left(\frac{\rho_0}{a_{\rm b}^{3\gamma}}+\sigma\right)^2\right\}^{1/3} - 2(a_{\rm b}^2 + 4k\alpha)\right]
$$

- Derivative of $a(t)$ is discontinuous at $a=a_b$:
  - Sudden transition from the collapsing phase to the expanding phase
  - On the brane, there appears an instantaneous matter field
  - With a fine-tuning $a_1=0$, smooth bounce is realized but it is non-generic



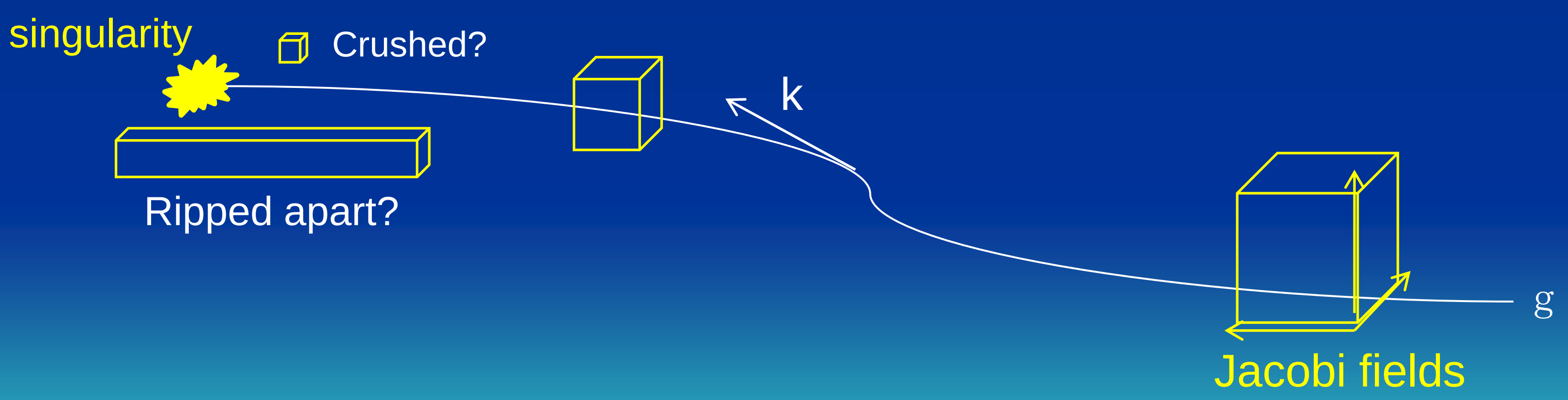
For the Fact 3: Strength of the singularity

- Let g a causal geodesic to or from the singularity and let k its tangent vector
- Consider a parallel propagated unit orthonormal frame along g
  - Unit orthonormal basis 1-forms orthogonal to k define a unit volume



For the Fact 3:
Strength of the singularity

- Consider a set of Jacobi fields, proportional to the basis 1-forms
  - Jacobi fields are governed by the Jacobi equation
- Definition (Tipler '77)
  A singularity is Tipler weak (strong) if the volume made of the Jacobi fields is finite (zero or infinite) at the singularity
- Tipler weak singularity => Harmless for a finite body

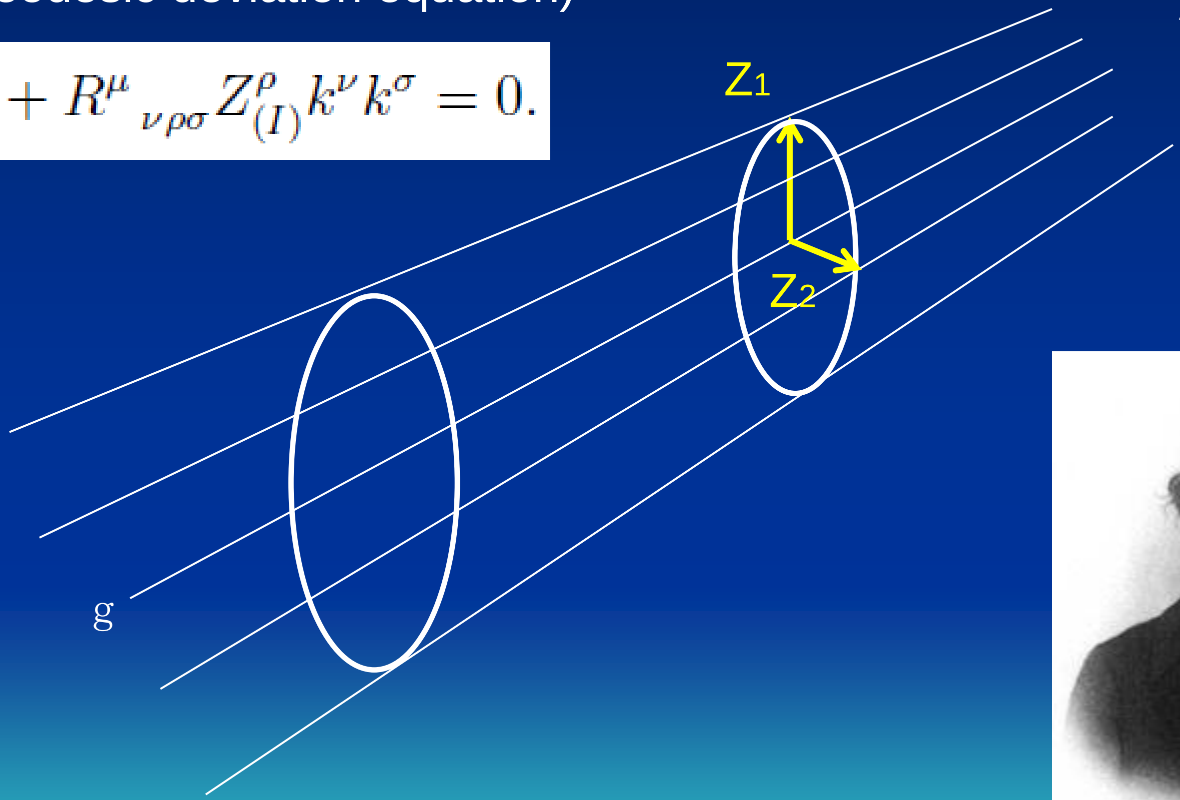


Jacobi field

Jacobi equation
(or geodesic deviation equation)

$$\ddot{Z}^{\mu}_{(I)} + R^{\mu}{}_{\nu\rho\sigma} Z^{\rho}_{(I)} k^{\nu} k^{\sigma} = 0.$$

A bundle of geodesics

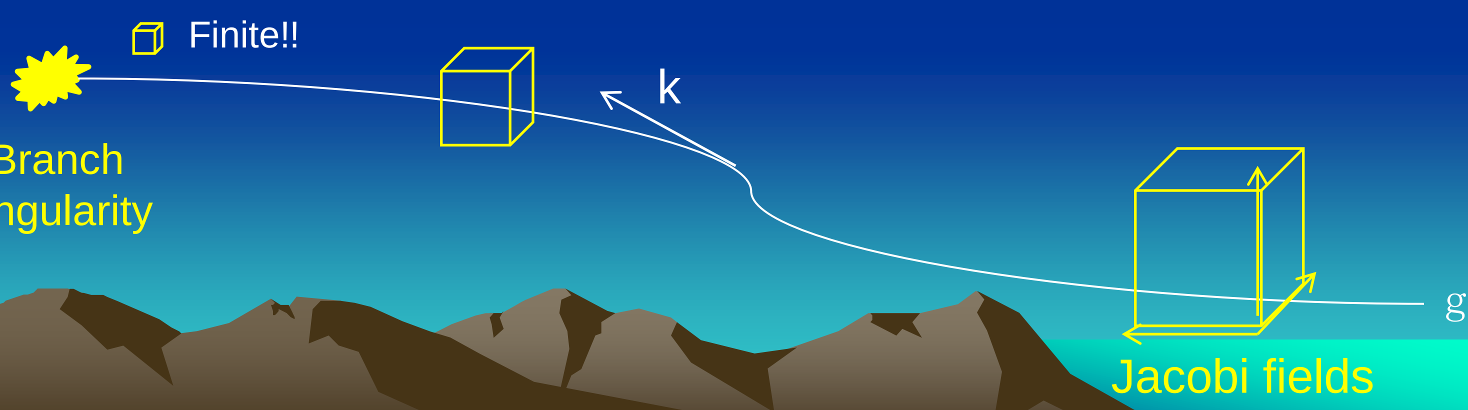


Fact 3: Branch singularity is weak

- Jacobi fields: $Z^{\mu}_{(I)}\frac{\partial}{\partial x^{\mu}} := l_{(I)}(\lambda)\underbrace{\eta^{\mu}_{(I)}\frac{\partial}{\partial x^{\mu}}}_{\text{Basis vector in the orthonormal frame}} \quad (I = 1, 2, 3, 4),$

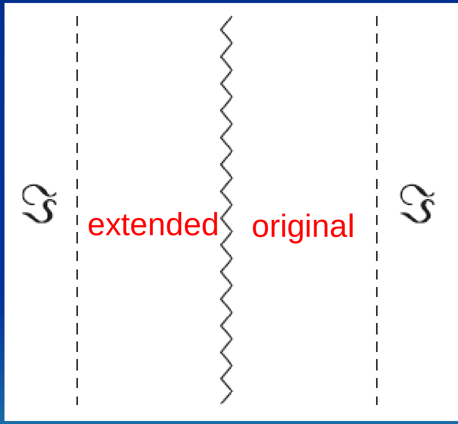
  Basis vector in the orthonormal frame

- Volume made of the Jacobi dual 1-forms: $V := |l_{(1)}||l_{(2)}||l_{(3)}|l_{(4)}|.$
  - Tipler weak: V is finite
  - Deformationally weak (Ori `00): all l's are finite
- Fact 3: the branch singularity is deformationally weak along radial causal geodesics



C^0 extension beyond the branch singularity

- Branch singularity is not the end of the world in the brane universe
- Q. What is the extended region beyond the branch singularity?
  - C^0 extension (because metric is finite)
- A. The same Boulware-Deser-Wheeler spacetime
  - Because of (Birkhoff's theorem) + (dynamical stability)



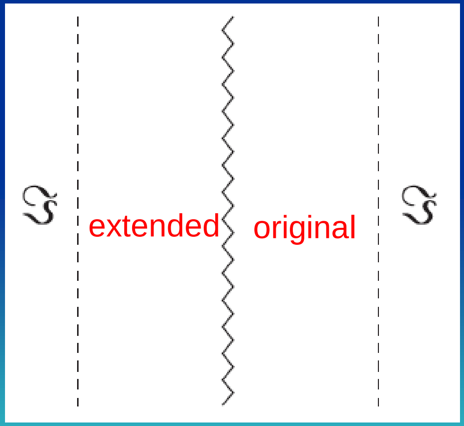
An example of the C^0 extended spacetime

Branch singularity as a massive thin-shell

- Derivative of the metric diverges there, but…
- EGB junction condition shows the finite energy-momentum tensor on the branch singularity $\tau^a{}_b = \mathrm{diag}(-\rho_\mathrm{b}, p_\mathrm{b}, p_\mathrm{b}, p_\mathrm{b}),$

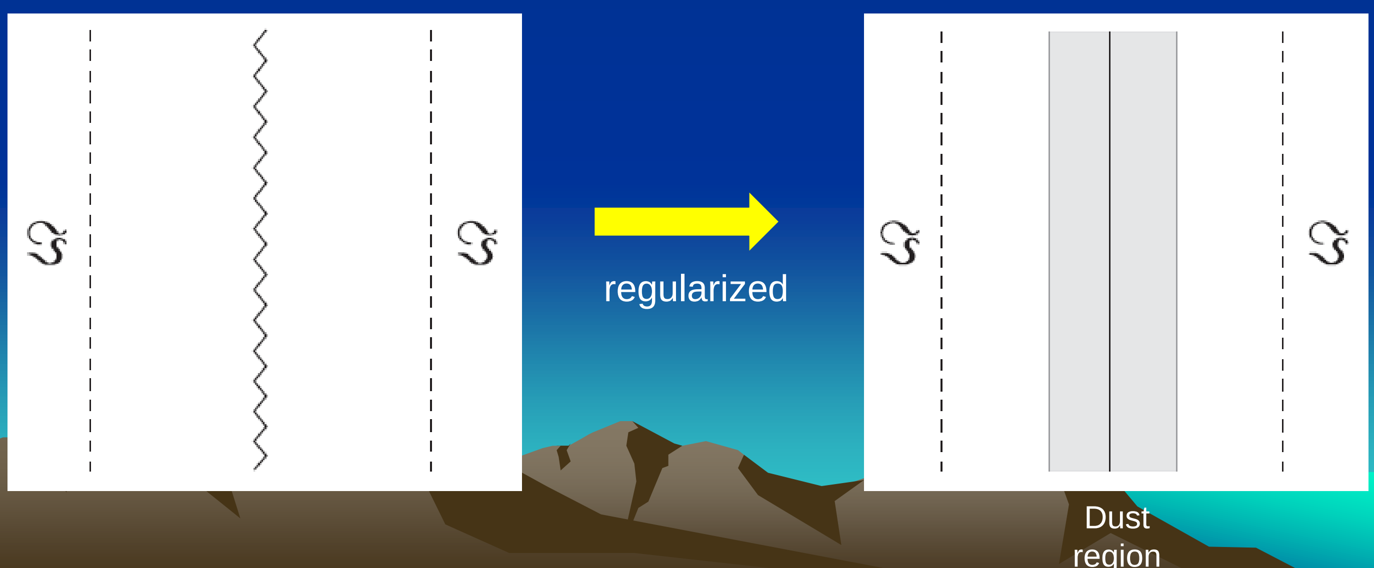
  where $$\rho_\mathrm{b} = -16\alpha \frac{h(r_\mathrm{b})^{3/2}}{r_\mathrm{b}^3}, \quad p_\mathrm{b} = \frac{8h(r_\mathrm{b})r_\mathrm{b}^2 - \mu}{2\sqrt{h(r_\mathrm{b})}r_\mathrm{b}^3},$$

  - WEC is violated
- So, the branch singularity may be considered as a massive thin-shell (=another brane)
- 3-brane arriving branch singularity = collision of two branes

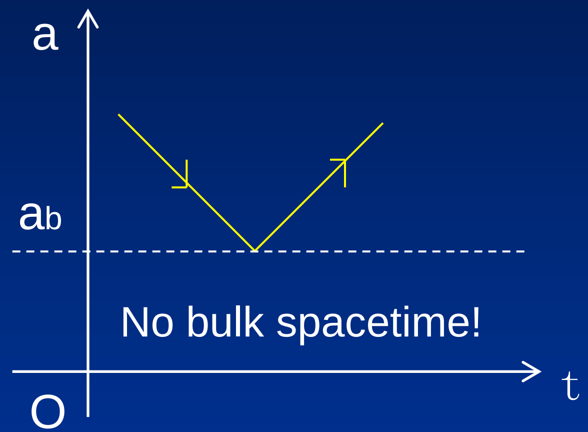


A regularized model

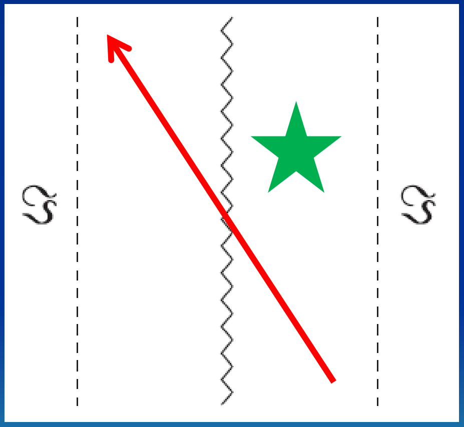
- Branch singularity is (naked) singular anyway
  - It causes a problem when we consider the perturbations in the bulk
  - No criterion for the boundary condition there
- But it is believed that the singularity is cured by the quantum-gravity effect
- So, in reality, the bulk should be given by a regularized solution where effective matter (by the QG effect) fills around the branch singularity
- An exact model is available for k=-1 with dust with negative energy density
  - Effective energy-momentum tensor from the QG effect could violate the energy conditions



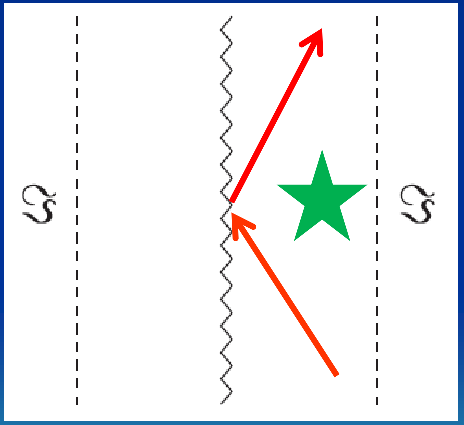
Evolution after the collision



(a)



(b)

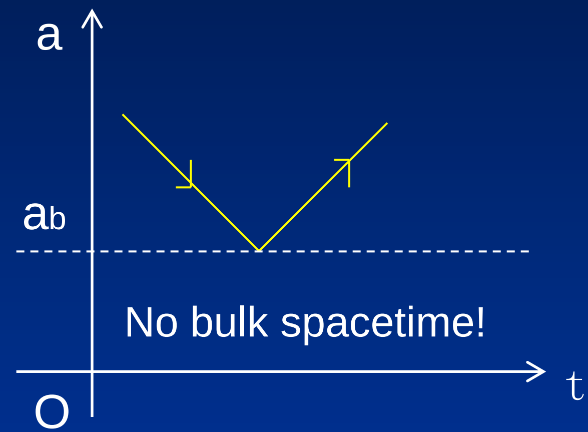


Red line:
Orbit of the brane

Right-hand side of the orbit (region with a star) is removed by the Z_2-symmetry

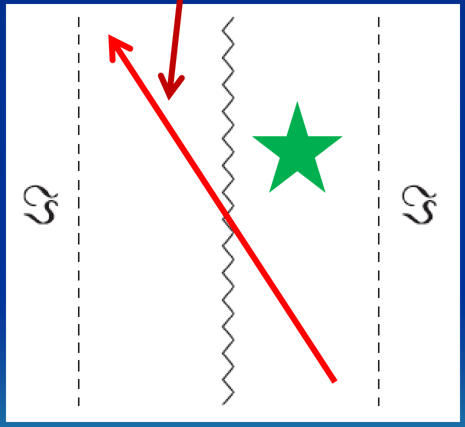
Which is the natural evolution after the collision?

The answer is (b)

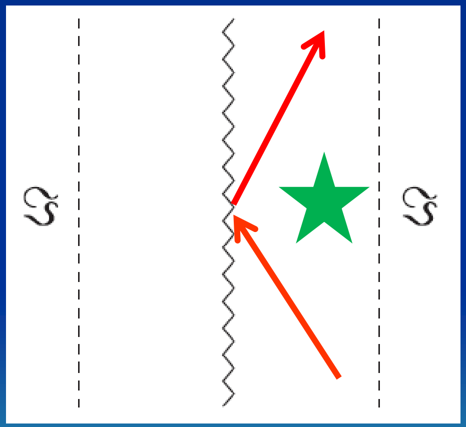


Negative energy density here

(a)



(b)



However, back reaction to the branch singularity should be considered

Summary

Summary

- In the braneworld, the bulk branch singularity is NOT the end of the world
- It is another massive thin-shell
  - Note: For k=-1 (open FRW universe), the branch singularity can be spacelike (S(pacelike)-brane)
- 3-brane reaches there = collision of two branes
- New scenario for the Big-Bounce

Branch singularity in higher-curvature gravity

- The branch singularity maybe generic in any higher-curvature gravity
  - In EGB with a U(1) gauge field, there is no central singularity and the branch singularity is generic

- **Q. More consequence/applications in cosmology or BH physics?**

FIN