

Tests of bi-gravity model with graviton oscillations using gravitational wave observations

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自己紹介

- 成川達也
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- 新学術領域「重力波天体の多様な観測による宇宙物理学の新展開」A04班 「多様な観測に連携する重力波探索データ解析の研究」
- この研究: A05班 理論班との共同研究
- 共同研究者: 上野昂、田越秀行、田中貴浩、神田展行、中村卓史

Outline of the talk

1. Vainshtein mechanism
2. Gravitational waves in Bi-gravity: Graviton oscillations
3. Constraints on Bi-gravity with a KAGRA type detector

1. Vainshtein mechanism

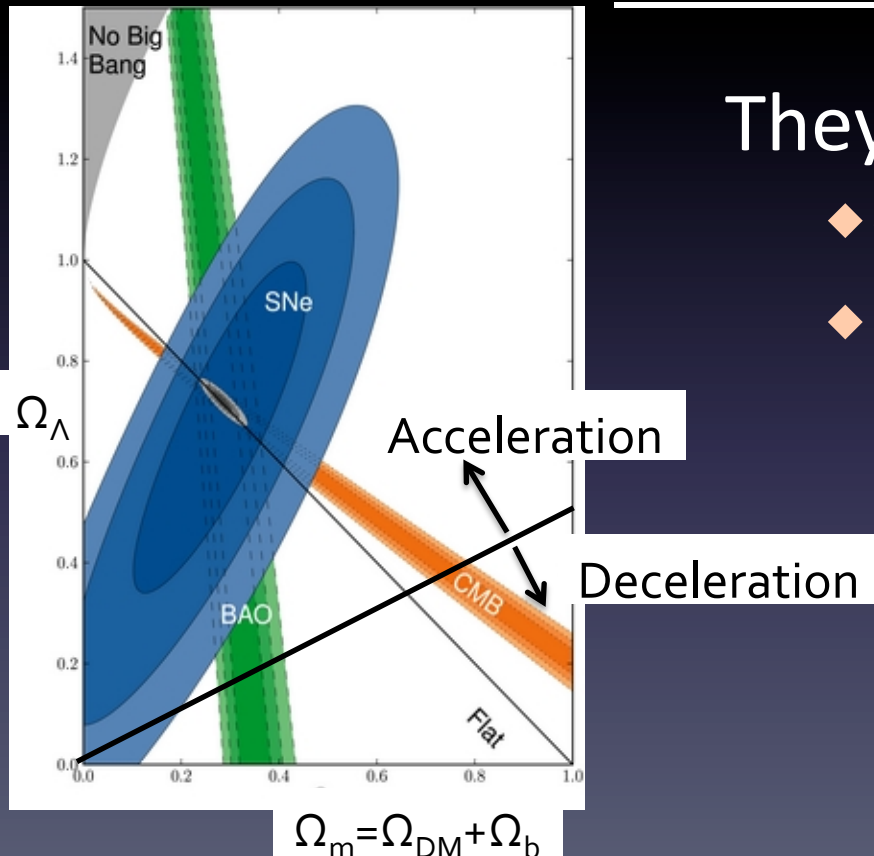
Why modify GR ?

Late time accelerated expansion of the Universe

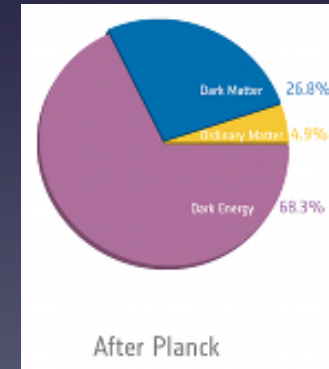
Combination of SNe, CMB & BAO

They are converging

- ◆ Cosmic acceleration
- ◆ Existence of Dark Energy



$$H_0 \sim 10^{-33} \text{eV}$$



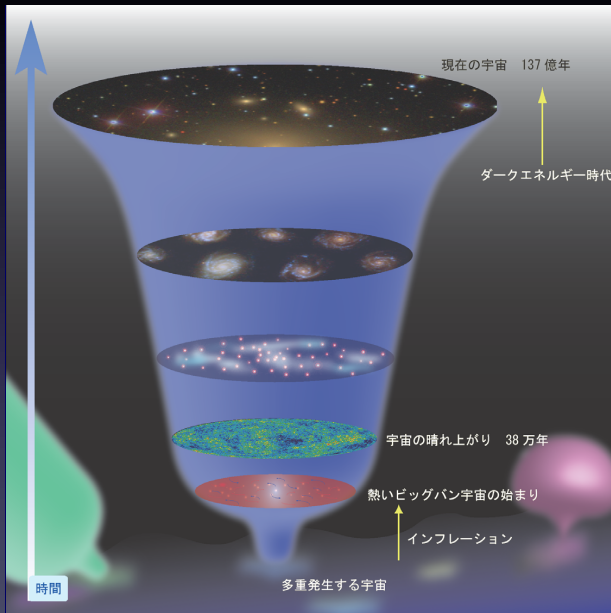
[Suzuki *et al.*, 1105.3470]

Why modify GR ?

Mystery of dark energy



Signs of the breakdown of GR on cosmological scales ?



[M. Nakashima (RESCEU)]

Modified gravity as an alternative to Dark energy

New d.o.f is added to cosmic accelerate

The effects of the additional d.o.f is hidden by the screening mechanism in the vicinity of a matter source.

→recover GR and pass solar-system tests

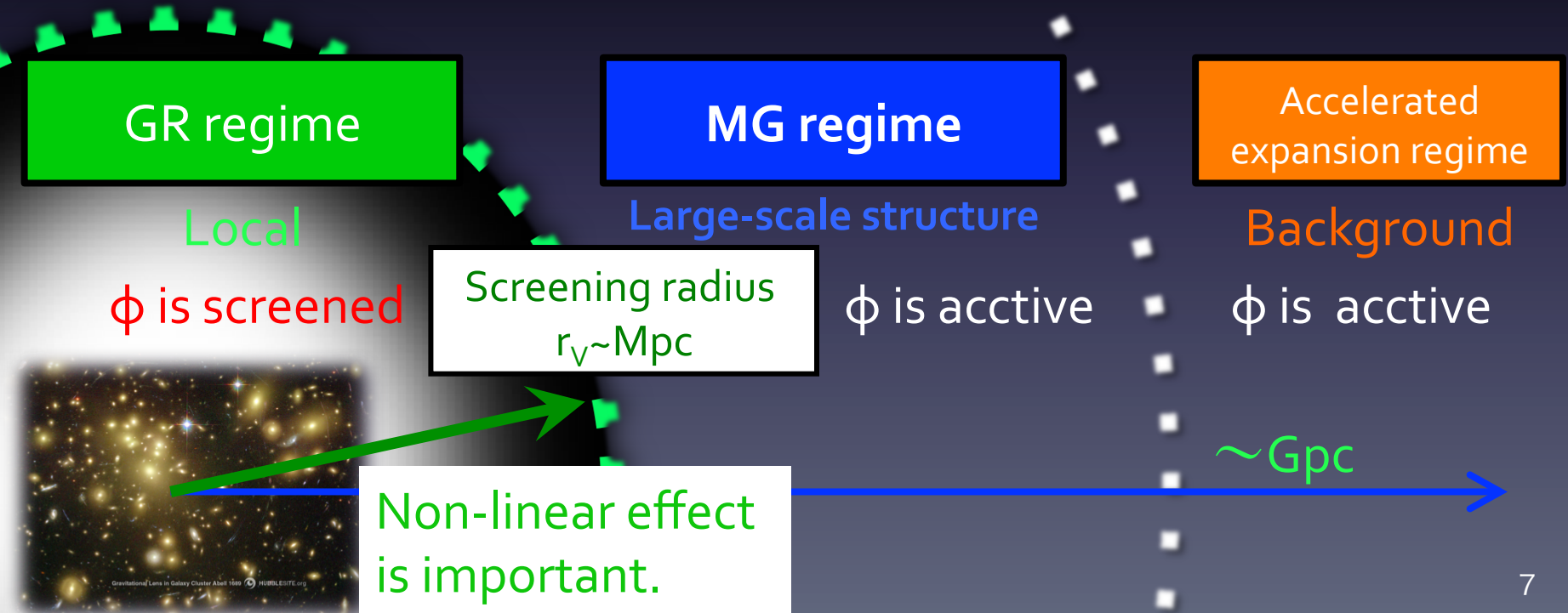
General description of Modified Gravity

Screening Mechanisms:

- **Kinetic type: Vainshtein mechanism** [Vainshtein, 1972]

Additional d.o.f is effectively weakly coupled to matter

- Cf. Potential type: Chameleon mechanism [Khoury & Weltman, 0309411]



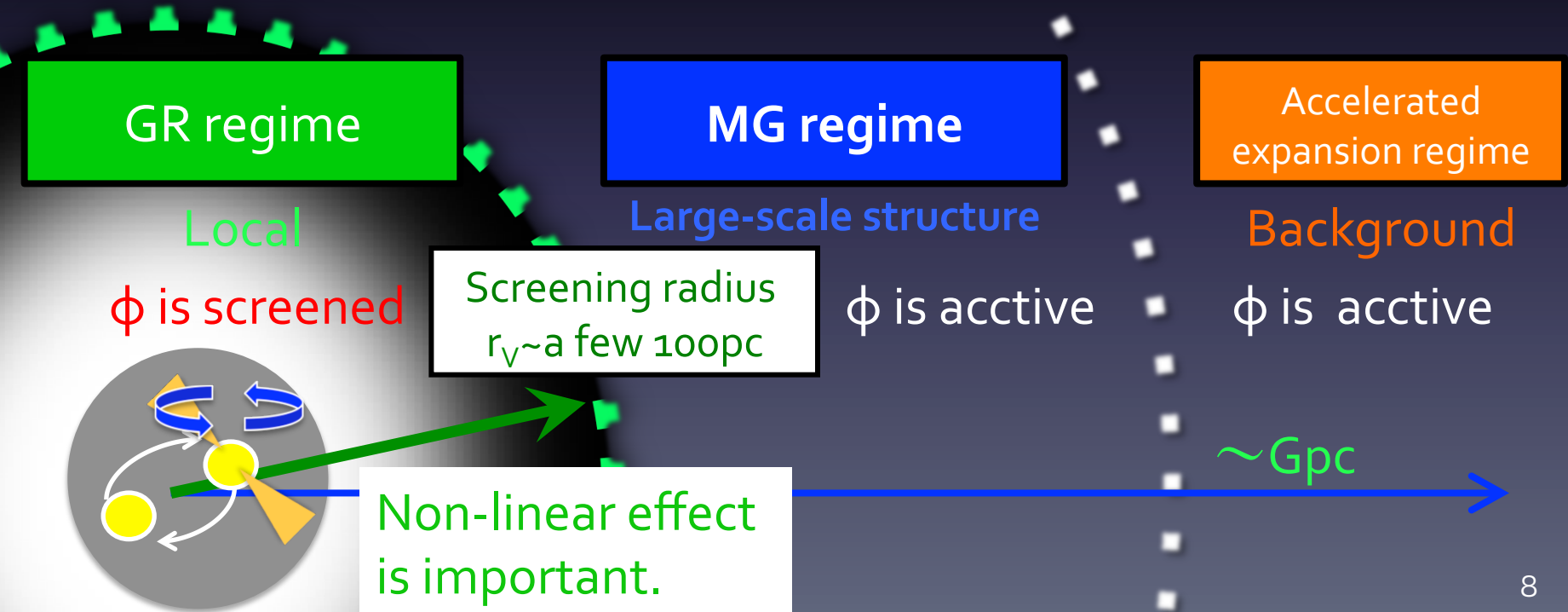
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Vainshtein mechanism in general scalar-tensor theory and massive gravity

[TN, Kobayashi, Yamauchi, & Saito, 1302.2311]

- Work in **General framework** (Horndeski's theory)
- Derive a screening condition to study **static, spherically symmetric configuration**
- Demonstrate how an effect of ϕ appears on **lensing signal $\Delta\Phi_+$** in the case that the Vainshtein screening works in modified gravity models
- Testing modified gravity models by comparing some model predictions **with cluster lensing data**

Start with Horndeski's general scalar-tensor gravity

$$\begin{aligned} \mathcal{L} = & K(\phi, X) - G_3(\phi, X)\square\phi \\ & + G_4(\phi, X)R + G_{4X} \times (\text{field derivatives}) \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & - \frac{1}{6}G_{5X} \times (\text{field derivatives}) \end{aligned}$$

[Horndeski (1974)] is equivalent to the Generalized Galileon [Deffayet+ (2011); Kobayashi, Yamaguchi, Yokoyama(2011)]

Static, spherically symmetric perturbations produced by a non-relativistic matter

$$ds^2 = -[1+2\Phi(r)]dt^2 + [1-2\Psi(r)]\delta_{ij}dx^i dx^j \quad \phi = \phi_0 + \varphi(r)$$

Combining metric EOM and ϕ EOM, we arrive at

Quintic Scalar-Field Equation

$$\begin{aligned} P(x, A) := & \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right)x + [\mu + 6\alpha\xi - 3\beta A(r)]x^2 \\ & + (\nu + 2\alpha^2 + 4\beta\xi)x^3 - 3\beta^2 x^5 = 0 \end{aligned}$$

where we define $x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}}\Lambda^3} \frac{M(r)}{8\pi r^3}$

Outer Solution where $A(r) \ll 1$: Asymptotically flat

$$x \approx x_f := -\frac{2\xi A(r)}{\eta + 6\xi^2}$$

Decaying solution in $1/r$

Inner Solution where $A(r) \gg 1$: Vainshtein screening

$$x \approx x_- := -\sqrt{\frac{\xi}{3\beta}} = \text{const.}$$

We have the Newtonian behavior:

$$\Psi'/r \simeq \Phi'/r \propto A$$

As a relevant example, decoupling limit of massive gravity
(Proxy theory of massive gravity [de Rham & Heisenberg 2011])

$$\eta = \mu = \nu = 0, \quad \xi = 1, \quad \alpha \neq 0, \quad \beta \neq 0$$

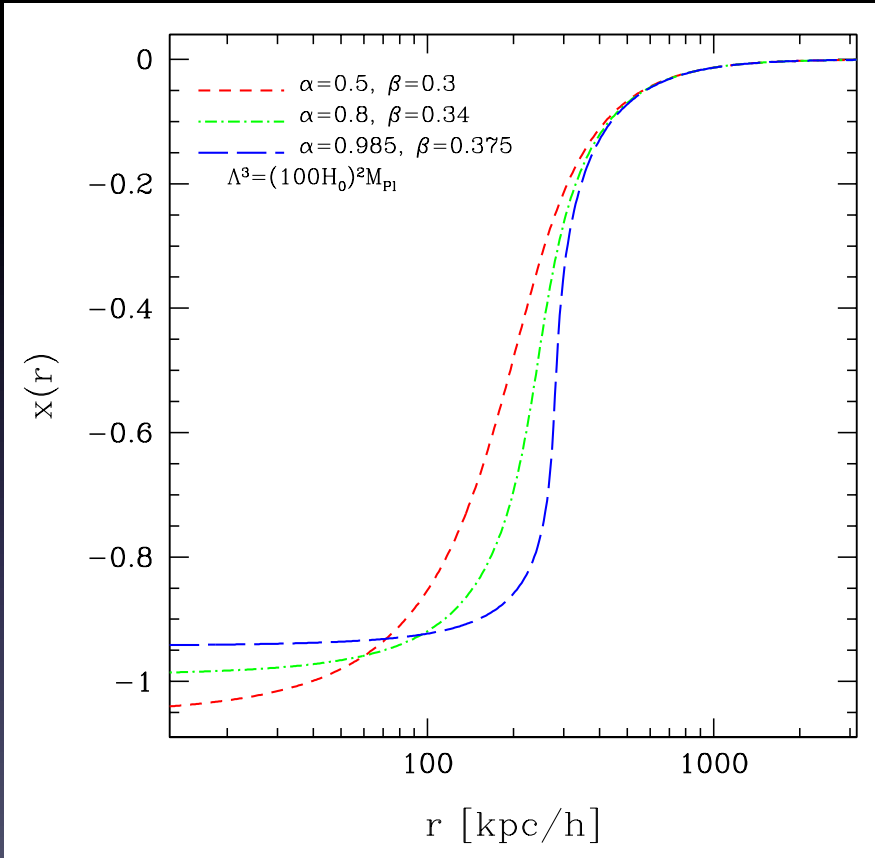
The condition of smooth matching of the two solutions:

$$\alpha < 0 \text{ or } \frac{\sqrt{\beta}}{\alpha} \geq \sqrt{\frac{5 + \sqrt{13}}{24}} \sim 0.6$$

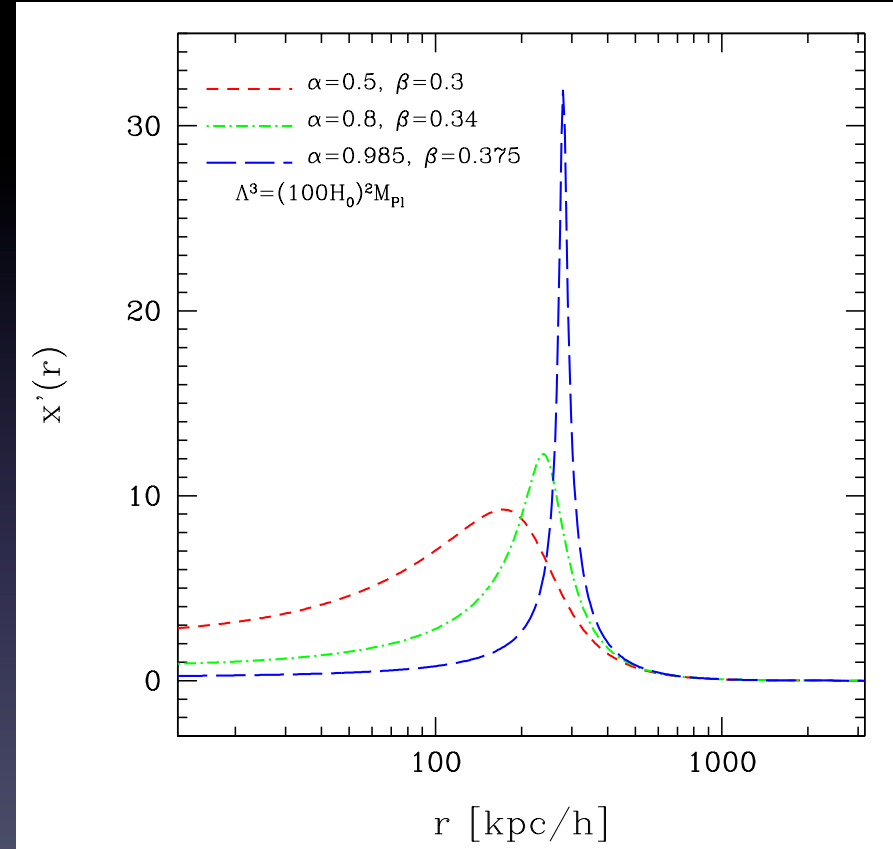
$x'(r)$ can be large at transition from screened to unscreened regions

$$x = \frac{1}{\Lambda^3} \frac{\phi'}{r}$$

$$\Delta\Phi_+ \propto r\phi'\phi''$$

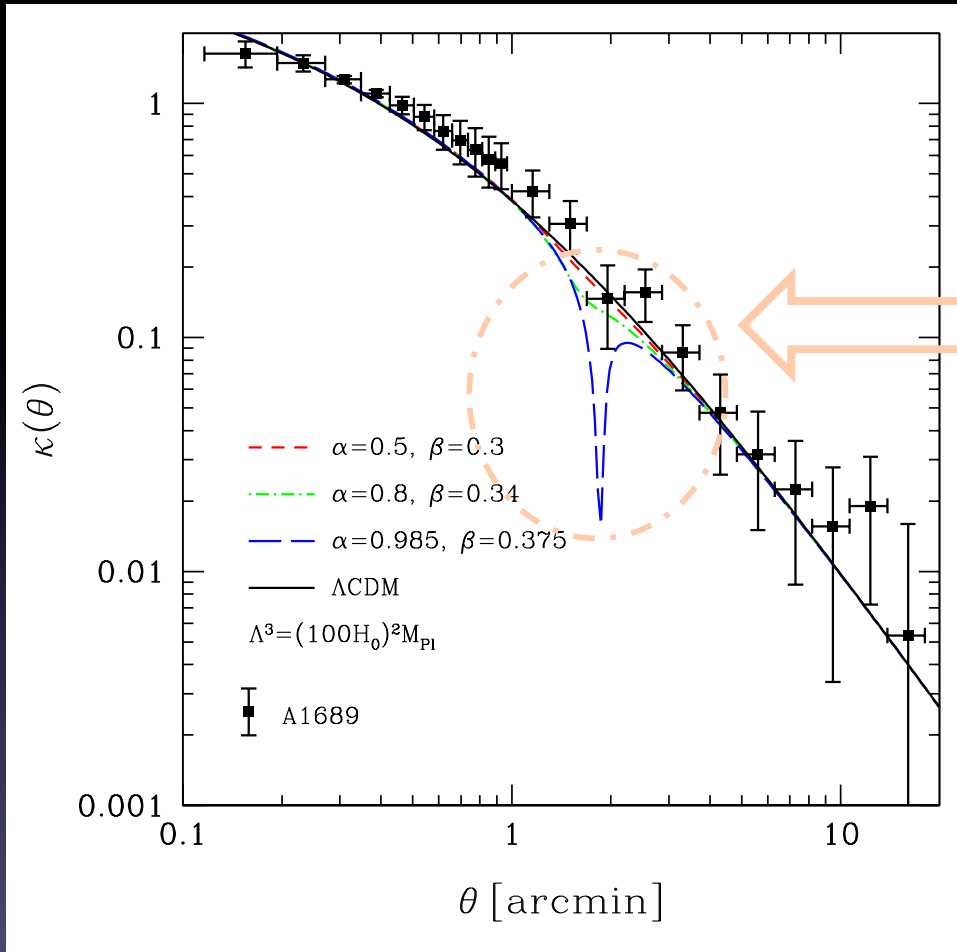


Smoothly matching

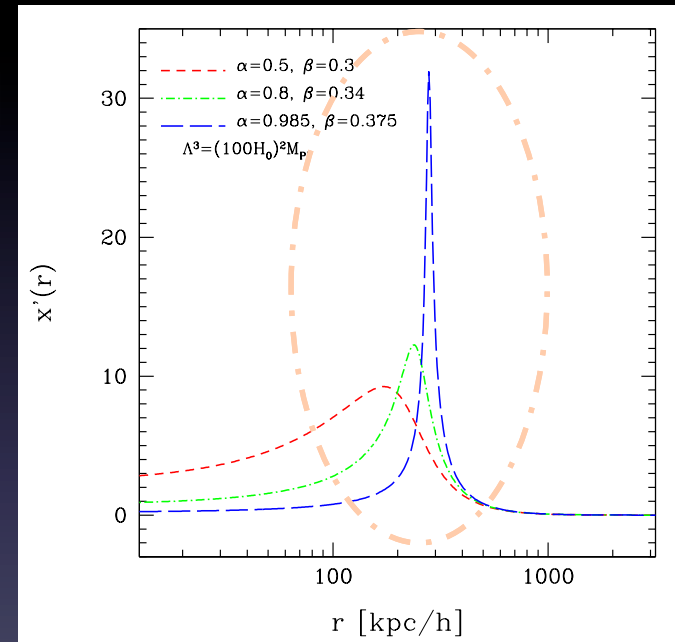


A dip appears.

Surface Mass Density in Modified Gravity



Parameters: $\{\alpha, \beta, \Lambda; \rho_s, r_s\}$
 $(\Lambda^3 = M_{Pl} / (0.01/H_0)^2, \rho_s, r_s \text{ fixed})$



An origin of the dip: $x'(\leftrightarrow \phi'')$

A dip appears at $r \sim r_V := (r_s M_{Pl} / \Lambda^3)^{1/3}$ in a typical case.

→ This allows us to put constraint.

Tests of gravity: Solar-system bounds on Parameterized Post-Newtonian parameters

γ : g_{ij} component

$$g_{ij} = (1 + 2\gamma U)\delta_{ij}$$

β : g_{00} component

$$g_{00} = -1 + 2U - 2\beta U^2$$

Weak field:

$$\epsilon := \frac{2GM}{Rc^2} \ll 1$$

[Will,gr-qc/0510072]

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed

Local gravity constraints are strong.

Einstein's GR is valid when gravity is weak.

ic

pulsar acceleration

4×10^{-20}

pulsar \dot{P} statistics

Vainshtein mechanism in Cubic Galileon

$$\mathcal{L} = \mathcal{L}_{\text{GR}} - X - \frac{1}{3\mu^2} X \square \phi + \mathcal{L}_m \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$

EOM for the modification to Φ :

$$\Delta\delta\Phi + \mu^{-2}(\partial\partial\delta\Phi)^2 = G_N\rho$$

In the vicinity of a matter source,
non-linearity becomes important

$$\rightarrow \delta\Phi/\Phi \sim \mu\sqrt{r^3/r_g}$$

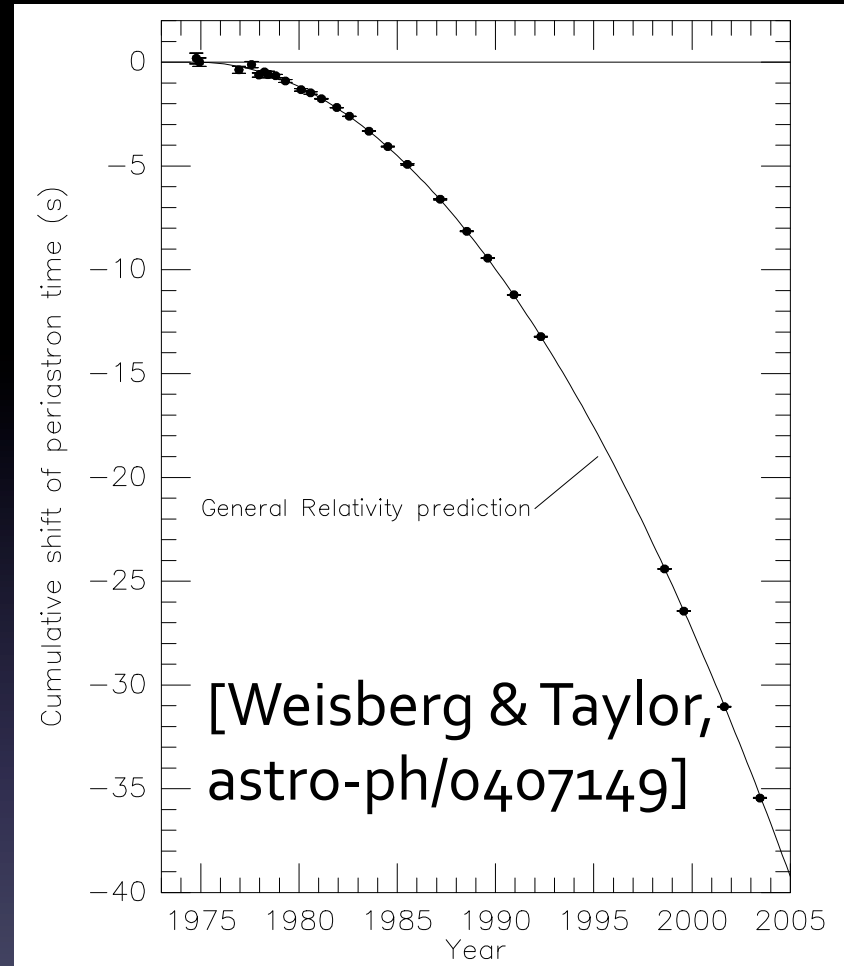
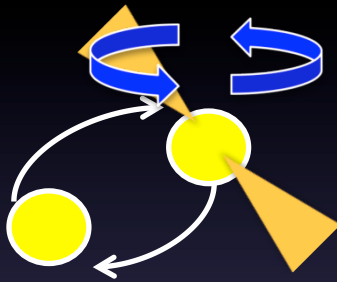
$$\rightarrow \mu^{-1} \geq 300\text{Mpc}$$

2. Gravitational waves in Bi-gravity: Graviton oscillations

Indirect detection of the GWs: Binary Pulsars

PSR B1913+16

Hulse-Taylor binary pulsar

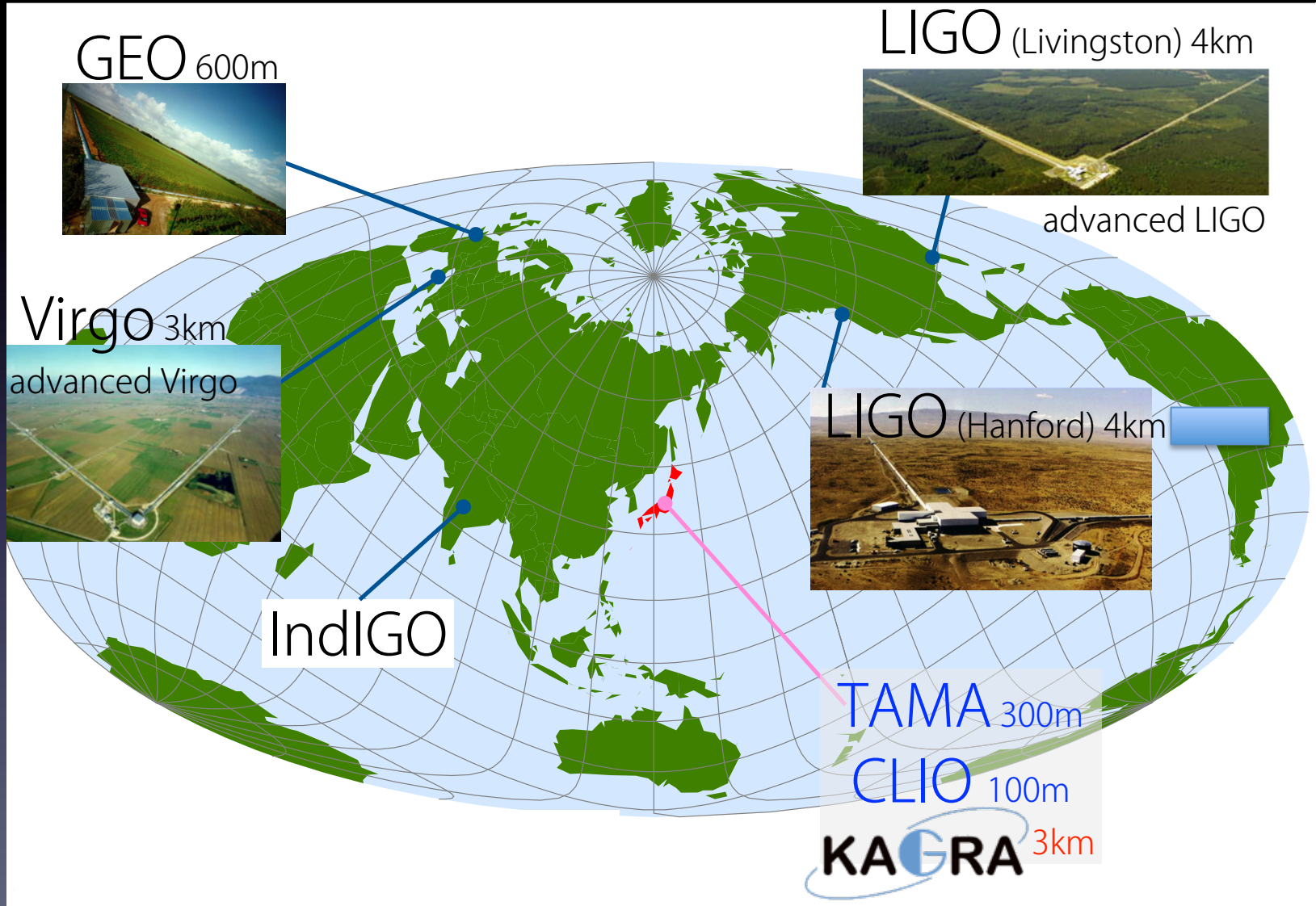


GR prediction curve agrees with the data points very well.

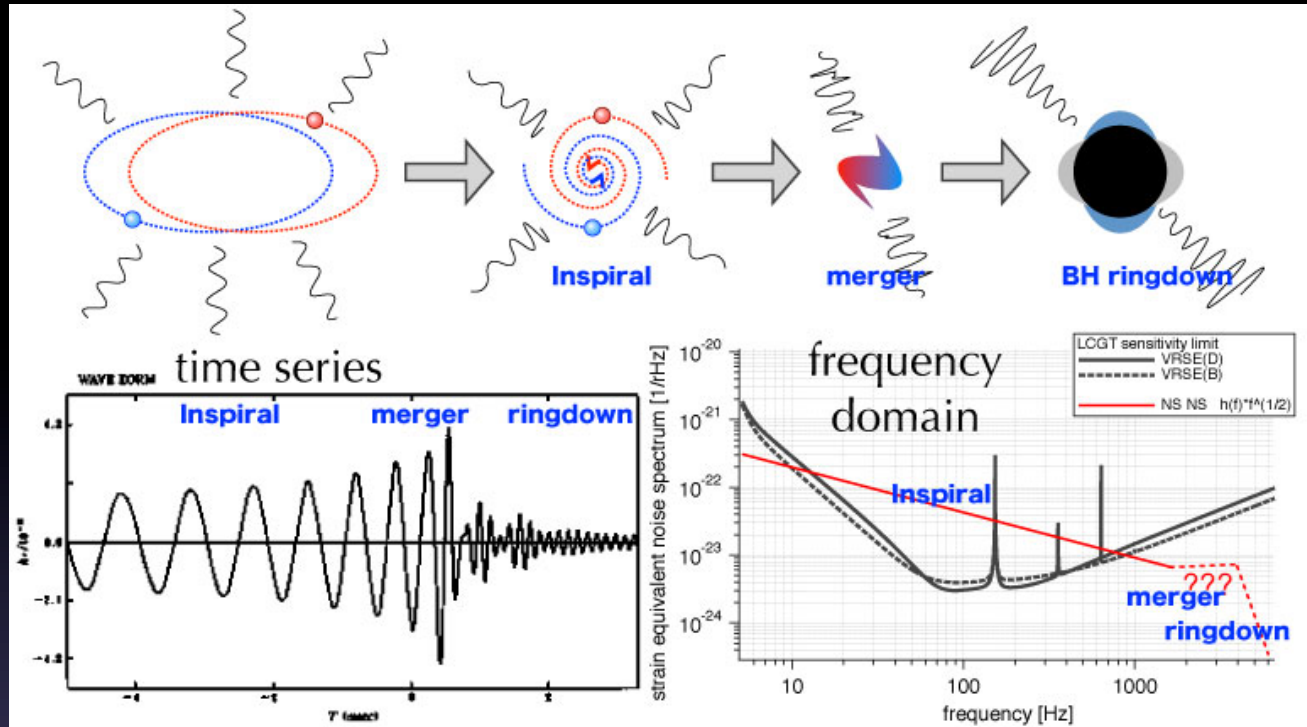
GWs are emitted from binaries.

GR is valid close to a matter source.

Progress of gravitational observations: laser interferometers



Compact Binary Coalescence (CBC)



Credit: Ao4

- ▷ Inspiral signal ← most promising source for KAGRA type detectors
 - Point particles → Clean system
 - Well-known waveform (the small number of parameters) are available.
 - Extracting Binary parameters: $\{M_c, \eta, D, t_c, \phi_c\}$
 - Testing GR and extended models of gravity

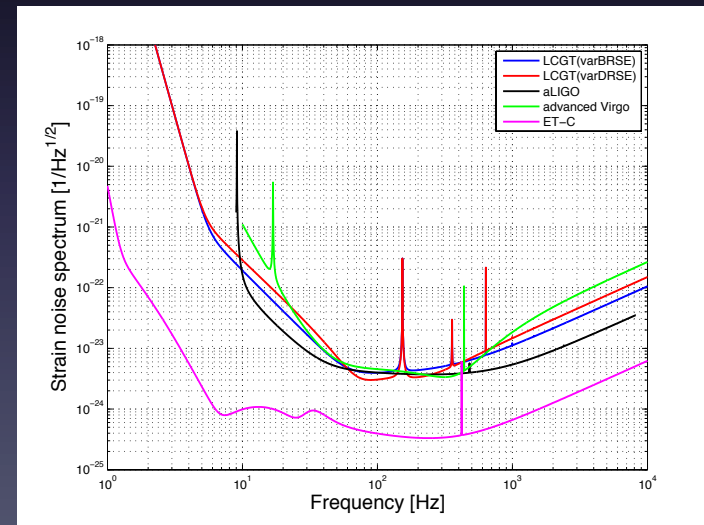
NS-NS merger rate

[Kim 2008, 2010; Lorimer 2008; Abadie et al. CQG 27, 173001 (2010); Kanda-san's lecture note (2013)]

Galactic merger rate: 3-190 events/Myr/galaxy

Number density of galaxies: 0.01-0.015 /Mpc³

→ ~10 events/yr for KAGRA/aLIGO



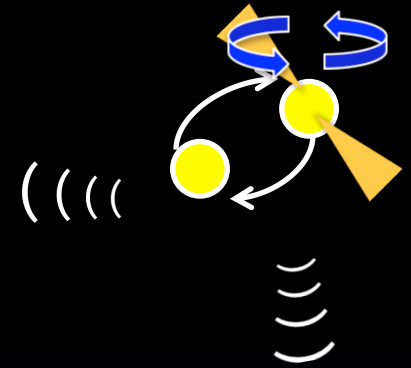
(0.4-400 events/yr for aLIGO [Abadie et al. (2010)])

Testing Gravitational theory with GWs from CBC

If $\frac{v}{c} \sim 1$ and $\epsilon \equiv \frac{v^2}{c^2} = \frac{2GM}{Rc^2} \sim 1$

High speed

Strong gravitational field



strong GWs are produced.

GWs from CBC give us a new probe to extended models of gravity on strong gravitational fields. Constraints by gravitational wave might be more stringent than current bound

[Will, gr-qc/9709011; Berti, Buonanno, & Will, gr-qc/0411129; Stavridis & Will, 0906.3602; Yagi & Tanaka, 0906.4269; Gair, et al. 1212.5575; Yunes & Siemens, 1304.3473].

Explore the possibility of testing cosmological viable extended models of gravity with GWs.

Test of GW propagation

So far, modified GW waveforms: Scalar-tensor gravity, Simple addition of mass to graviton, Chern-Simon, PPE test, etc. leave out related to cosmological viability.

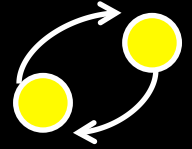
Actually, GW propagation in cosmological viable model of bi-gravity is interesting: graviton oscillations.

Two gravitons will oscillate like neutrino oscillations during propagation of GWs from binary inspirals.

[De Felice, Nakamura, & Tanaka, 1304.3920]

GW waveform for binary inspirals

[Blanchet, 1310.1528]



Post Newtonian approximation:
(v/c) expansion

GW waveform in Fourier space

$$h(f) \approx A f^{-7/6} e^{i\Phi(f)}$$

$$A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}}{D_L} \quad \mathcal{M} = \mu^{3/5} M^{2/5} \quad \eta = \frac{\mu}{M}$$

$$\Phi(f) = 2\pi f t_c - \phi_c + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \left[1 + \frac{20}{9} \left(\frac{743}{331} + \frac{11}{4} \eta \right) y^{2/3} - (16\pi - \beta)y + \dots \right]$$

$$y \equiv \pi \mathcal{M} f$$

1PN

1.5PN

Massive gravity

Motivation

- Can graviton have mass?
- Alternative to dark energy?

Consistent theory found in 2010.

Nonlinear massive bi-gravity

Based on dRGT massive gravity.

[Hassan & Rosen, 1109.3515]

\tilde{g} is promoted to a dynamical field.

$$\mathcal{L} = \underbrace{\frac{M_G^2}{2} \sqrt{-g} R}_{\text{massless}} + \underbrace{\frac{\kappa M_G^2}{2} \sqrt{-\tilde{g}} \tilde{R}}_{\text{massive}} + \underbrace{\frac{M_G^2}{2} \sqrt{-g} m^2 \sum_{n=0}^4 c_n V_n(g, \tilde{g})}_{\text{Mass term}} + \sqrt{-g} \mathcal{L}_m$$

Both massless and massive gravitons exist.

Mass term

Both g and \tilde{g} are dynamical metrics.

$$V_0=1, V_2=[Y], V_2=[Y]^2-[Y^2], \dots, [Y^n]:=Tr(Y^n),$$

$$Y_{\nu}^{\mu} = \sqrt{g^{\mu\alpha} \tilde{g}_{\alpha\nu}}$$

No helicity-0, Boulware-Deser ghost

[Hassan & Rosen, 2012]

FLRW background: cosmic acceleration

[Comelli, Crisostomi, Nesti & Pilo, 1111.1983]

Physical metric

$$ds^2 = a^2(t)(-dt^2 + dx^2)$$

Hidden metric

$$d\tilde{s}^2 = \tilde{a}^2(t)(-\tilde{c}^2 dt^2 + dx^2)$$

Friedmann equation for g:

$$H^2 = \frac{\rho_m + \rho_V}{3M_G^2} \quad \xi \equiv \tilde{a}/a$$

Mass term \rightarrow effective cosmological constant

$$\rho_V(\xi) \equiv M_G^2 m^2 (c_0 + 3\xi c_1 + 4c_2 \xi^2 + 6\xi^3 c_3)$$

$$\Gamma(\xi) \equiv c_1 \xi + 4c_2 \xi^2 + 6c_3 \xi^3$$

$$\nabla^\mu T_{\mu\nu}^{(\text{mass})} = 0 \rightarrow \underbrace{3\Gamma(\xi)}_{\text{Branch 1}} \underbrace{[\tilde{c}aH - (\dot{\tilde{a}}/\tilde{a})]}_{\text{Branch 2}} = 0$$

Branch 1

Branch 2

Branch 1: Pathological: unstable for the homogeneous anisotropic mode.

Branch 2: Healthy: All perturbation modes are equipped.

Branch 2 background

[Comelli, Crisostomi, Nesti & Pilo, 1111.1983]

An algebraic equation for ξ :

$$\frac{\rho_m}{M_G^2 m^2} = \left[\frac{c_1}{\kappa \xi} + \left(\frac{6c_2}{\kappa} - c_0 \right) + \left(\frac{18c_3}{\kappa} - 3c_1 \right) \xi + \left(\frac{24c_4}{\kappa} - 6c_2 \right) \xi^2 - 6c_3 \xi^3 \right].$$

$\xi \rightarrow \xi_c$, $\Gamma_c \rightarrow \Gamma(\xi_c)$ for $\rho_m \rightarrow 0$, i.e. $m^2 \gg \rho_m / M_G^2$

$$H^2 \approx \frac{\rho_m}{3(1 + \kappa \xi_c^2) M_G^2}$$

We arrive at

$$\tilde{c} \approx 1 + \frac{\kappa \xi_c^2 (\rho_m + P_m)}{\Gamma_c m^2 (1 + \kappa \xi_c^2) M_G^2}$$

$$\tilde{c} = 1 \text{ for } \rho_m \rightarrow 0.$$

Screening of modification works in local region

[De Felice, Nakamura & Tanaka, 1304.3920]

Static, spherically symmetric configuration:

$$\begin{aligned} ds^2 &= -e^{u-v} dt^2 + e^{u+v} (dr^2 + r^2 d\Omega^2), \\ d\tilde{s}^2 &= -\xi_c^2 e^{\tilde{u}-\tilde{v}} dt^2 + \xi_c^2 e^{\tilde{u}+\tilde{v}} (d\tilde{r}^2 + \tilde{r}^2 d\Omega^2) \end{aligned}$$

Erasing \tilde{u}, \tilde{v} , \rightarrow

EOM for massive scalar mode ϕ

$$(\Delta - \mu^2)u - \frac{3\bar{C}}{8\mu^2} [(\Delta u)^2 - (\partial_i \partial_j u)^2] = \frac{\kappa \xi_c^2}{3M_G^2} \rho_m$$

In local region ($r \ll r_V$), **non-linear term** becomes large,
Newtonian gravity is recovered. $r_V \approx (Cr_g \mu^{-2})^{1/3}$

$$C \equiv \frac{d(\log \Gamma)}{d(\log \xi)} \Big|_{\xi=\xi_c}$$

Constraints from solar-system: $\sqrt{\bar{C}} \mu^{-1} \geq 300 \text{Mpc}$

Propagation of the gravitational waves in Bi-gravity

[Comelli, Crisostomi, Nesti & Pilo, 1202.1986]

[De Felice, Nakamura, & Tanaka, 1304.3920]

The propagation equations

$$\ddot{h} - \Delta h + m^2 \Gamma_c (h - \tilde{h}) = 0$$

$$\ddot{\tilde{h}} - \tilde{c}^2 \Delta \tilde{h} + \frac{m^2 \Gamma_c}{\kappa \xi_c^2} (\tilde{h} - h) = 0$$

Propagation of the gravitational waves in Bi-gravity

[De Felice, Nakamura, & Tanaka, 1304.3920]

$$\begin{pmatrix} -\omega^2 + k^2 + m_g^2 & -m_g^2 \\ -\frac{1}{\kappa\xi^2} m_g^2 & -\omega^2 + c^2 k^2 + \frac{1}{\kappa\xi^2} m_g^2 \end{pmatrix} \begin{pmatrix} h \\ \tilde{h} \end{pmatrix} = 0$$

Short wavelength approximation: $k \gg m_g \gg H$

$$\longrightarrow k_{1,2}^2 = (2\pi f)^2 - \frac{\mu^2}{2} \left(1 + x \mp \sqrt{1 + 2x \frac{1 - \kappa\xi_c^2}{1 + \kappa\xi_c^2} + x^2} \right) \quad \text{: eigen wavenumbers}$$

The corresponding eigen function:

$$\begin{aligned} h_1 &= \cos \theta_g h + \sin \theta_g \sqrt{\kappa\xi_c} \tilde{h}, \\ h_2 &= -\sin \theta_g h + \cos \theta_g \sqrt{\kappa\xi_c} \tilde{h}. \end{aligned}$$

where

$$\mu^2 \equiv \lambda_\mu^{-2} = \frac{(1 + \kappa\xi_c^2) \Gamma_c m^2}{\kappa\xi_c^2}$$

$$x \equiv \frac{2(2\pi f)^2 (\tilde{c} - 1)}{\mu^2}$$

mixing angle

$$\theta_g = \frac{1}{2} \cot^{-1} \left(\frac{1 + \kappa\xi_c^2}{2\sqrt{\kappa\xi_c}} x + \frac{1 - \kappa\xi_c^2}{2\sqrt{\kappa\xi_c}} \right)$$

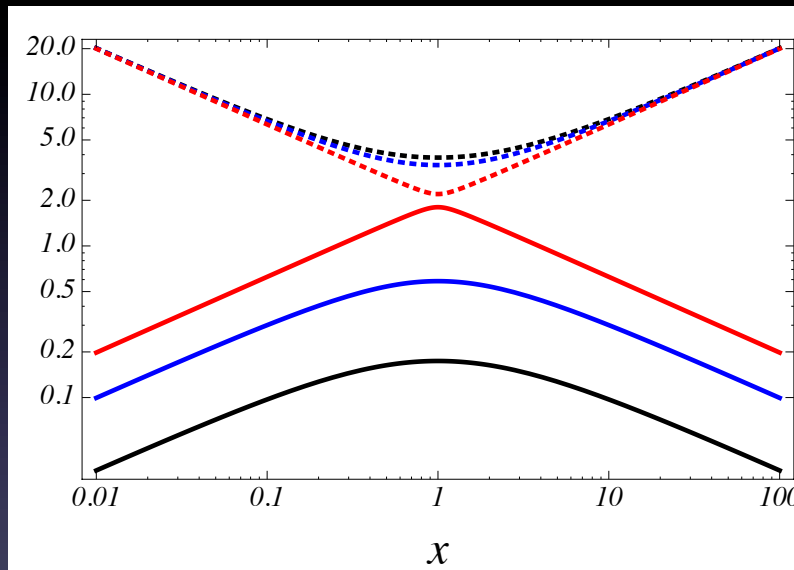
Phase shift

[De Felice, Nakamura, & Tanaka, 1304.3920]

$$\delta\Phi_{1,2} = -\frac{\mu D \sqrt{\tilde{c}-1}}{2\sqrt{2x}} \left(1 + x \mp \sqrt{1 + x^2 + 2x \frac{1 - \kappa\xi_c^2}{1 + \kappa\xi_c^2}} \right)$$

$$x \equiv \frac{2(2\pi f)^2(\tilde{c}-1)}{\mu^2}$$

depend on distance D and density Ω_0 .



Dashed: $\delta\Phi_2$

$\kappa\xi_c^2 = 100, 1, 0.2$

Solid: $\delta\Phi_1$

$x \ll 1$ and $x \gg 1$: the 1st mode becomes massless. The 2nd mode can be significantly large.

The average density of the Universe is much lower than one in galaxies, where binaries are embedded. Therefore, GW experience much lower value of x during the propagation.

Gravitational waveform from binary inspiral

[De Felice, Nakamura, & Tanaka, 1304.3920]

At the time of generation of GWs from coalescing binaries, both h and \tilde{h} are equally excited.

When we detect GWs, we sense h only.

The observed inspiral waveform in Fourier space

$$h(f) = A(f)e^{i\Phi(f)} \left[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)} \right]$$

GR's restricted waveform truncated at 3.5PN order.

$$\delta\Phi_{1,2} = -\frac{\mu D \sqrt{\tilde{c} - 1}}{2\sqrt{2x}} \left(1 + x \mp \sqrt{1 + x^2 + 2x \frac{1 - \kappa\xi^2}{1 + \kappa\xi^2}} \right) \quad \text{where} \quad x \equiv \frac{2(2\pi f)^2 (\tilde{c} - 1)}{\mu^2}$$

Phase shift depend on distance D .

B_1 and B_2 : degrees of mix ($B_1 + B_2 = 1$)

GW oscillations

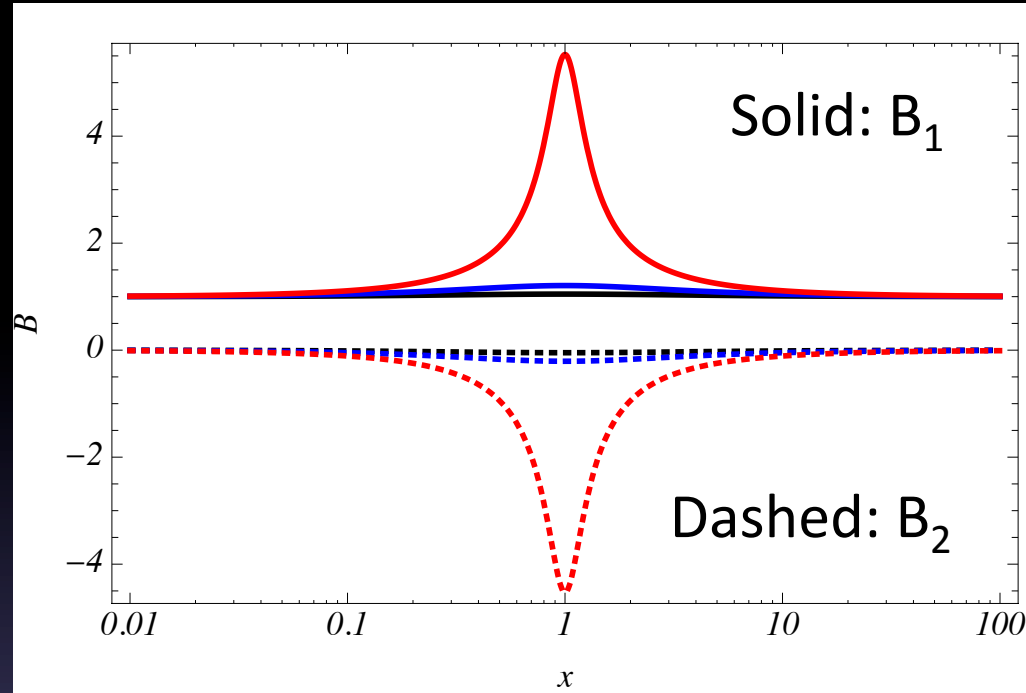
[De Felice, Nakamura, & Tanaka, 1304.3920]

$$h(f) = A(f)e^{i\Phi(f)} \left[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)} \right]$$

B_1 and B_2 : degrees of mix
($B_1 + B_2 = 1$)

$$x \equiv \frac{2(2\pi f)^2(\tilde{c} - 1)}{\mu^2}$$

$$\kappa\xi_c^2 = 100, 1, 0.2$$



$x \ll 1$: only the 1st mode is excited.

$x \sim 1$: Both modes are excited.

→ Both modes can be observed.

$x \gg 1$: only the 1st mode is excited.

Graviton oscillations occurs near $x \sim 1$.

Three Bi-gravity parameters

1. μ : ``graviton mass'' of 2nd mode in Minkowski limit ($x \rightarrow 1$).
2. $\tilde{c} - 1$: difference between propagation speed of hidden graviton, and light speed of physical sector
3. $\kappa \bar{\xi}_c^2$: difference between hidden sector and physical sector for gravitational constant*(scale factor)²

Modification region: $x \sim 1 \Leftrightarrow \tilde{c} - 1 \propto \mu^2$
 $\kappa \bar{\xi}_c^2$ determines amplitude of excitation for 1st mode.

Target values of Bi-gravity parameters

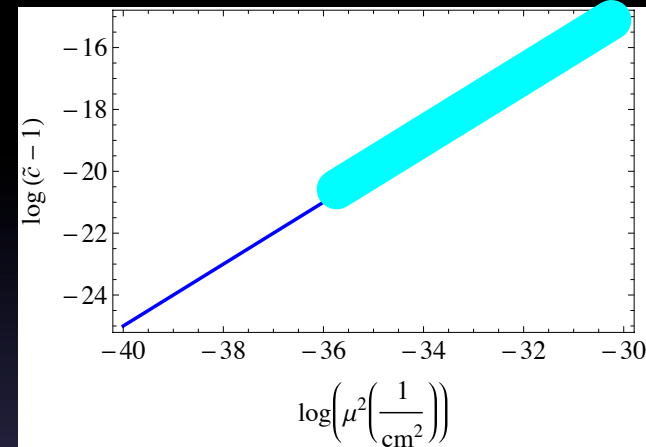
Modification region: $x \sim 1$ at $f \sim 100$ Hz

Deviation from GR appears for parameters near $x \sim 1$.

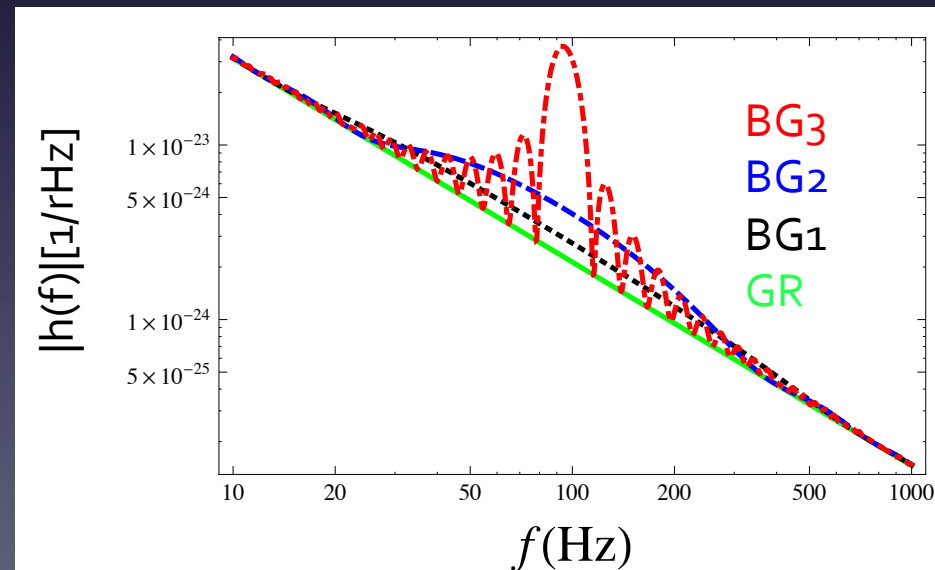
$$\frac{\mu^2}{\tilde{c} - 1} \simeq 10^{-15} \text{cm}^{-2}$$

GR limits of waveform:

1. $\kappa \rightarrow 0$: degrees of mix
2. $\mu \rightarrow 0$: phase shift



	$\kappa^{1/2} \xi_c$	$\mu[\text{cm}^{-1}]$	$\tilde{c}-1$
BG1	50	$1.0 * 10^{-17}$	$1.3 * 10^{-19}$
BG2	100	$1.4 * 10^{-17}$	$2.6 * 10^{-19}$
BG3	1000	$4.5 * 10^{-17}$	$2.6 * 10^{-18}$



3. Constraints on Bi-gravity with a KAGRA type detector

Data analysis method

Fisher matrix analysis

inner product

$$(h_A|h_B) \equiv 2 \int_0^\infty \frac{h_A^* h_B + h_B^* h_A}{S_n(f)} df$$

where $S_n(f)$ is the noise power spectrum for KAGRA type.

SNR

$$\rho[h] \equiv S/N[h] = (h|h)^{1/2}.$$

Fisher information matrix

$$\Gamma_{ab} \equiv \left(\frac{\partial h}{\partial \theta^a} \middle| \frac{\partial h}{\partial \theta^b} \right)$$

The measurement error of each parameter

$$\Delta \theta^a \equiv \sqrt{\langle (\theta^a - \langle \theta^a \rangle)^2 \rangle} = \sqrt{\Sigma^{aa}}$$

where Σ is the corresponding component of the inverse of the covariance matrix.

A unified model-comparison performance analysis

is valid for sufficiently-loud signals

yields the detection SNR required for a statistically detection of GR violations as a function of the fitting factor

The FF measures the extent to which one can reabsorb modified-GR effects by varying standard-GR parameters from their true values.

$$\text{FF}(\theta_{\text{AG}}) = \max_{\theta_{\text{GR}}} \frac{(h_{\text{GR}}(\theta_{\text{GR}}), h_{\text{AG}}(\theta_{\text{AG}}))}{|h_{\text{GR}}(\theta_{\text{GR}})| |h_{\text{AG}}(\theta_{\text{AG}})|}$$

Testing GR with GWs via Fitting Factor

[Vallisneri, 1207.4759]

Fitting Factor characterizes the deviation between GR and AG waveforms.

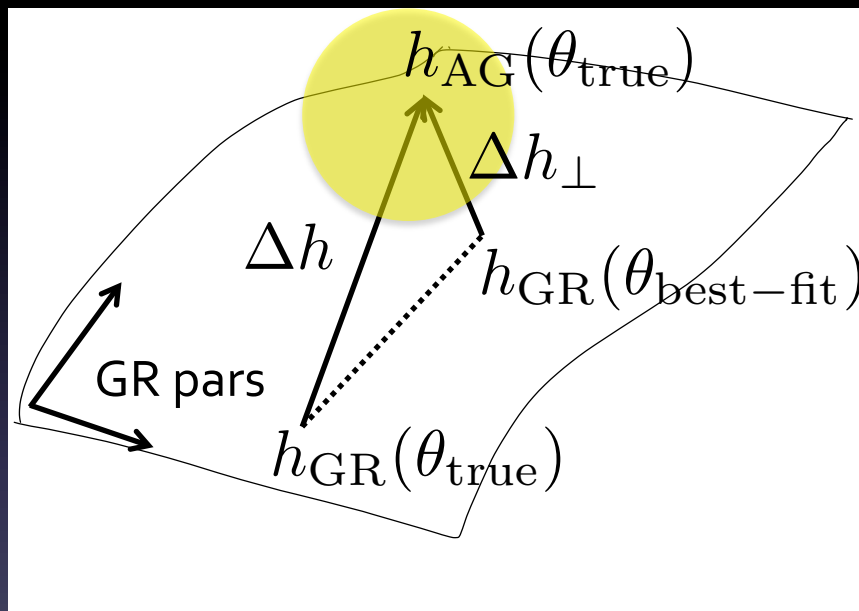
Distance to closest GR waveform:

$$|\Delta h_{\perp}|^2 = 2|h_0|^2(1 - FF)$$

Fitting Factor:

$$FF = \frac{(h_{AG}(\theta_{\text{true}}), h_{GR}(\theta_{\text{best-fit}}))}{|h_{AG}(\theta_{\text{true}})| \cdot |h_{GR}(\theta_{\text{best-fit}})|}$$

maximized by $\theta_{\text{b-f}}$.



Manifold of GR waveforms

We can distinguish Alternative-Gravity corrections when the modified waveform is sufficiently distant from manifold of GR waveforms. If $FF < 0.9$, 1yr-KAGRA can distinguish AG.

Decision scheme (AG or GR?)

[Vallisneri, 1207.4759]

with the Bayesian odds ratio \mathcal{O} , is designed as the detection statistic.

When $\mathcal{O}_{AG} > \mathcal{O}_{thr}$, we claim detection.

Odds ratio

prior

$$\mathcal{O} = \frac{P(\text{AG}|s)}{P(\text{GR}|s)} = \frac{P(\text{AG}) \int p(s|\theta^{i,a})p(\theta^{i,a})d\theta^{i,a}}{P(\text{GR}) \int p(s|\theta^i)p(\theta^i)d\theta^i}$$

$$SNR_{AG} \equiv SNR\sqrt{1 - FF}$$

Evidence for AG and GR

False-alarm probability: $F = P(\mathcal{O}_{GR} > \mathcal{O}_{thr})$

Efficiency: $E = P(\mathcal{O}_{AG} > \mathcal{O}_{thr})$

$$E = 1 - \text{erf}(-SNR_{AG} + \text{erfc}^{-1}(F)) - \text{erf}(-SNR_{AG} - \text{erfc}^{-1}(F))/2$$

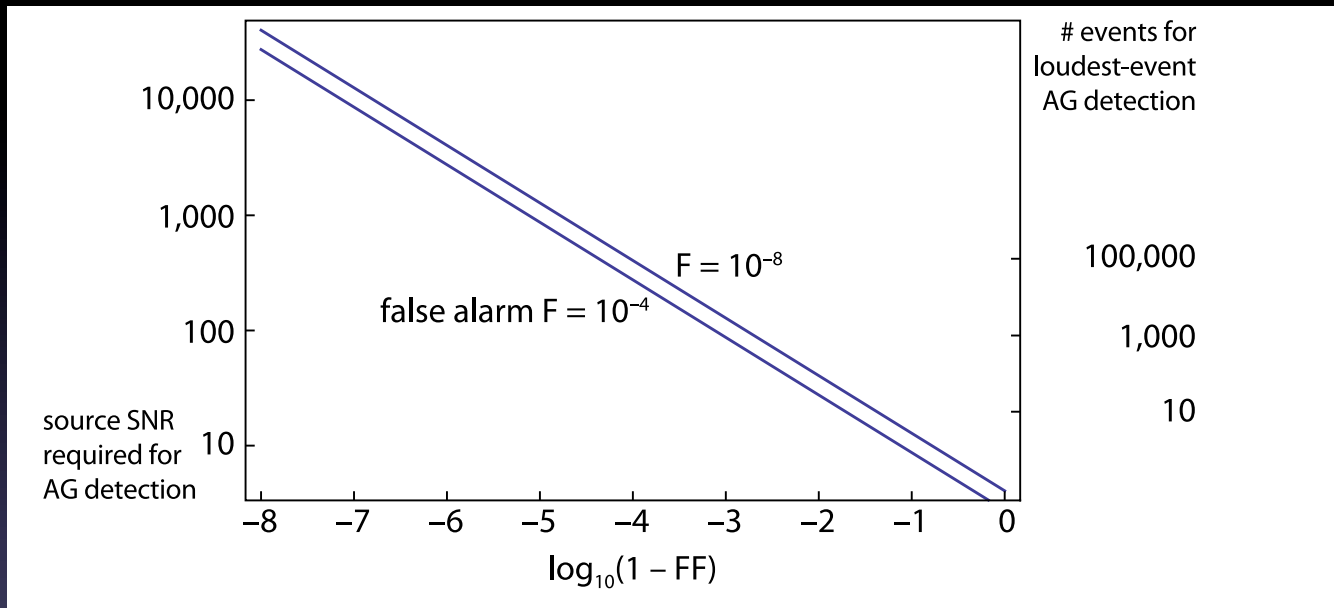
Solving $E=1/2$ yields the SNR_{AG} required for confident AG detection as a function of F .

Decision scheme (AG or GR?)

[Vallisneri, 1207.4759]

$$SNR \propto (1 - FF)^{-1/2}$$

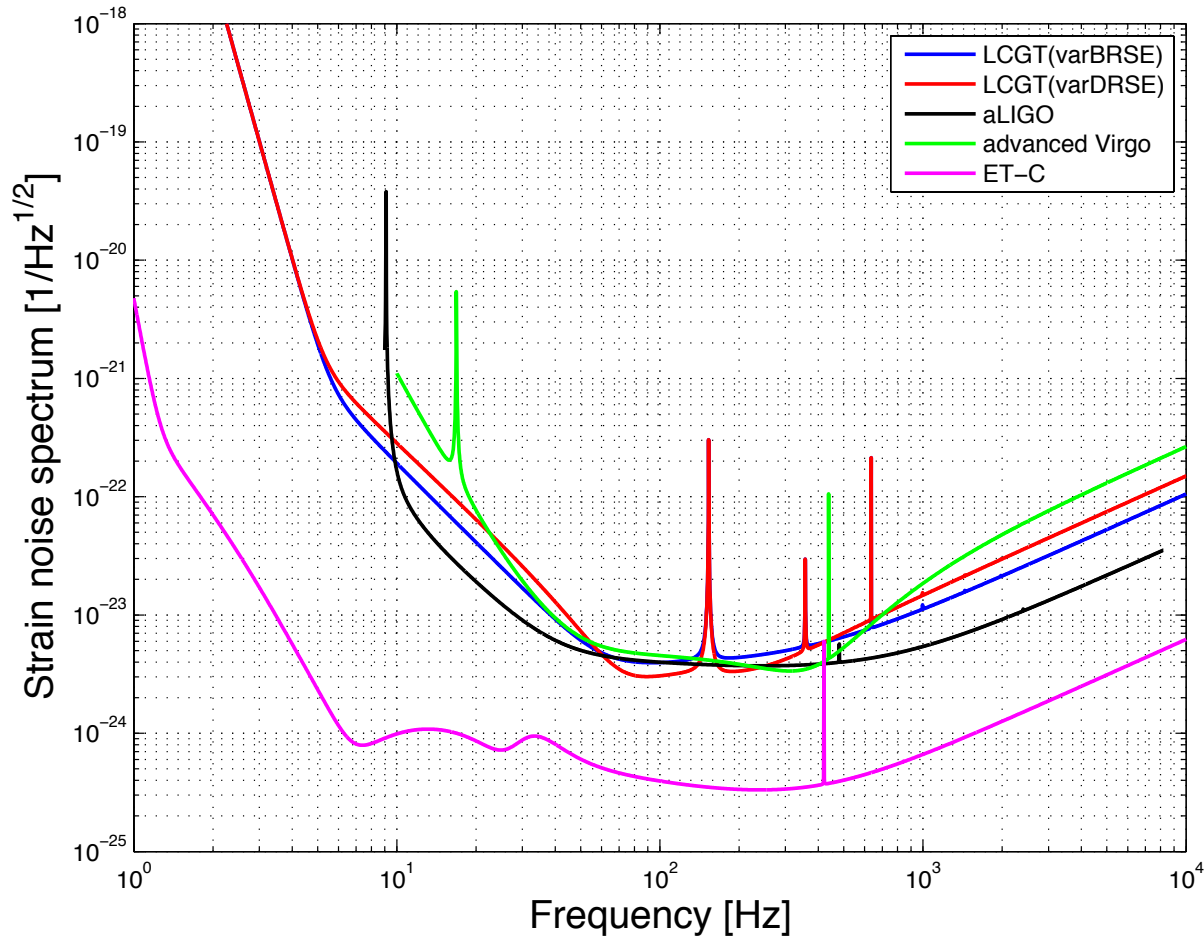
$$\# \propto (1 - FF)^{-3/2}$$



SNR required for AG detection with $E=1/2$ as a function of FF .

events for loudest-event AG detection

Strain noise spectrum



($f_{\min}=15\text{Hz}$, $f_{\max}=f_{\text{ISCO}}=1570\text{Hz}$ for (1.4, 1.4))

Results: Possibility of Constraints on Bi-gravity

Bi-gravity parameters $\{\mu, \tilde{c}^{-1}, \text{ \& } \kappa \xi_c^2\}$

GR parameters $\{M_c, \eta, t_c, \phi_c\}$

chirp mass: $M_c = (m_1 m_2)^{5/3} / (m_1 + m_2)^{1/5}$

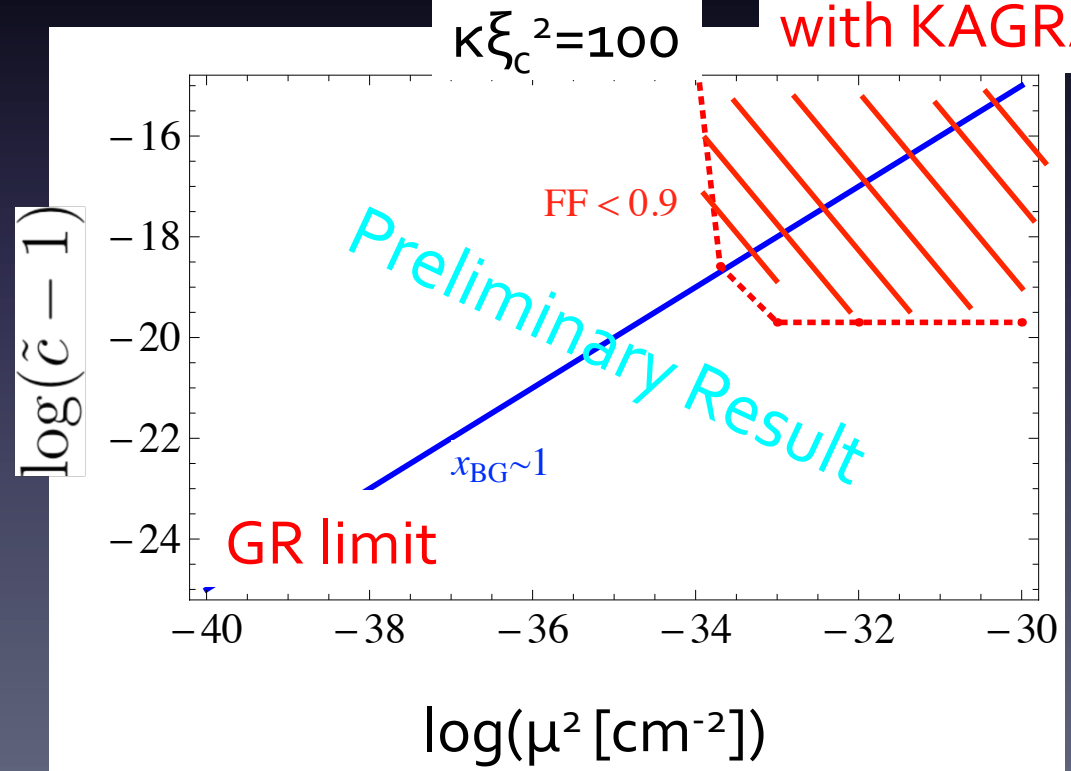
reduced mass ratio: $\eta = m_1 m_2 / (m_1 + m_2)^2$

Coalescence time: t_c

Distance: $r = 100$ Mpc

KAGRA would detect Bi-gravity corrections to GR waveforms for red shaded region.

$$FF(\theta_{AG}) = \max_{\theta_{GR}} \frac{(h_{GR}(\theta_{GR}), h_{AG}(\theta_{AG}))}{|h_{GR}(\theta_{GR})| |h_{AG}(\theta_{AG})|}$$



Summary

- ★ Cosmological viable modified gravity: ex) Bi-gravity
 - ★ Cosmic acceleration
 - ★ Screening Mechanisms to recover GR
 - ★ Modified GW propagation in Bi-gravity
- ★ Strong field tests of bi-gravity with gravitational waves from binary inspirals
- ★ Constraints on Bi-gravity with KAGRA via Fitting Factor
 - ★ KAGRA would detect Bi-gravity corrections to GR waveforms for a parameter region.