

Gravitational waves from Cosmic strings

Sachiko Kuroyanagi (Tokyo U. of Science)

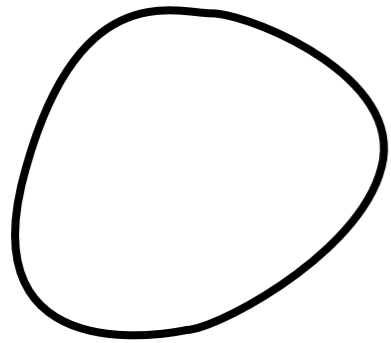
2013/12/3 関西合同セミナー

Based on

S. Kuroyanagi, K. Miyamoto, T. Sekiguchi, K. Takahashi, J. Silk,
PRD 86, 023503 (2012) and PRD 87, 023522 (2013)

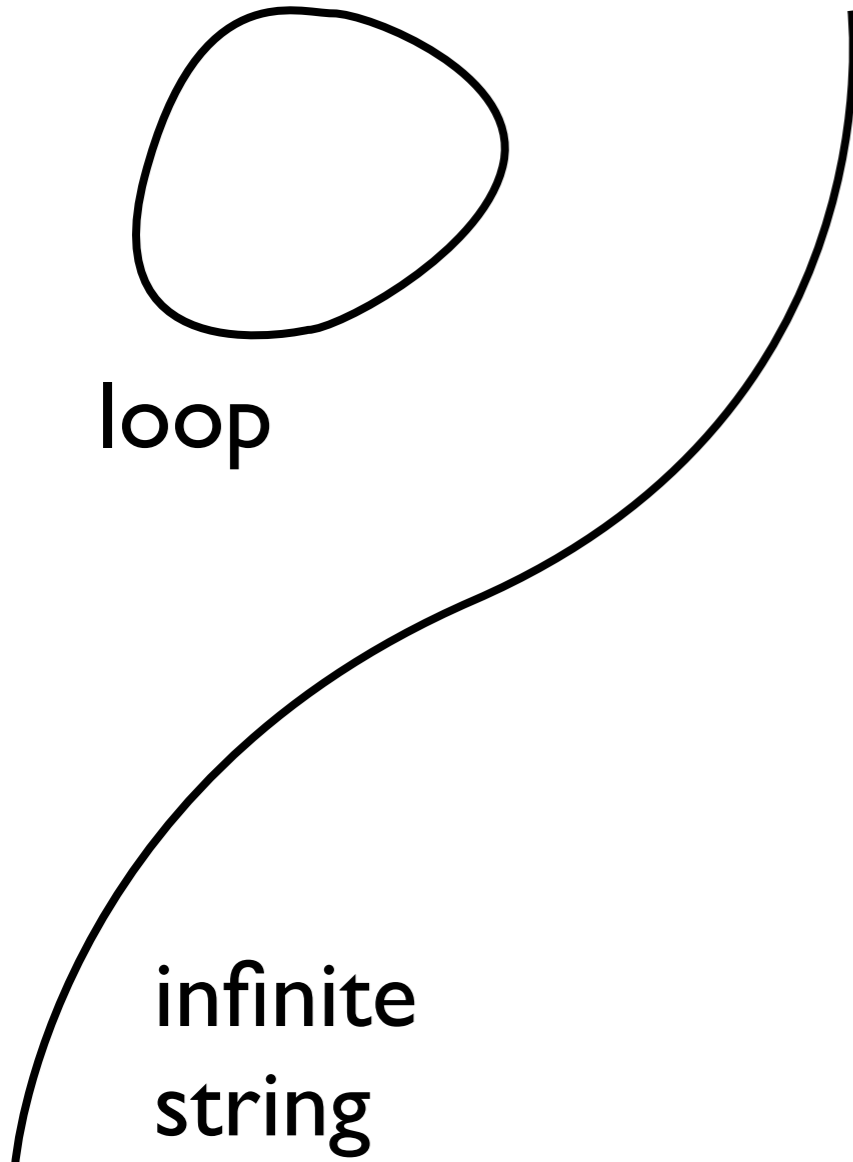
Cosmic string ?

One dimensional topological defects
generated in the early universe



loop

- have infinite length (= no end point)
- can be a form of loop
- affect matters only through gravitational force
- emits gravitational waves



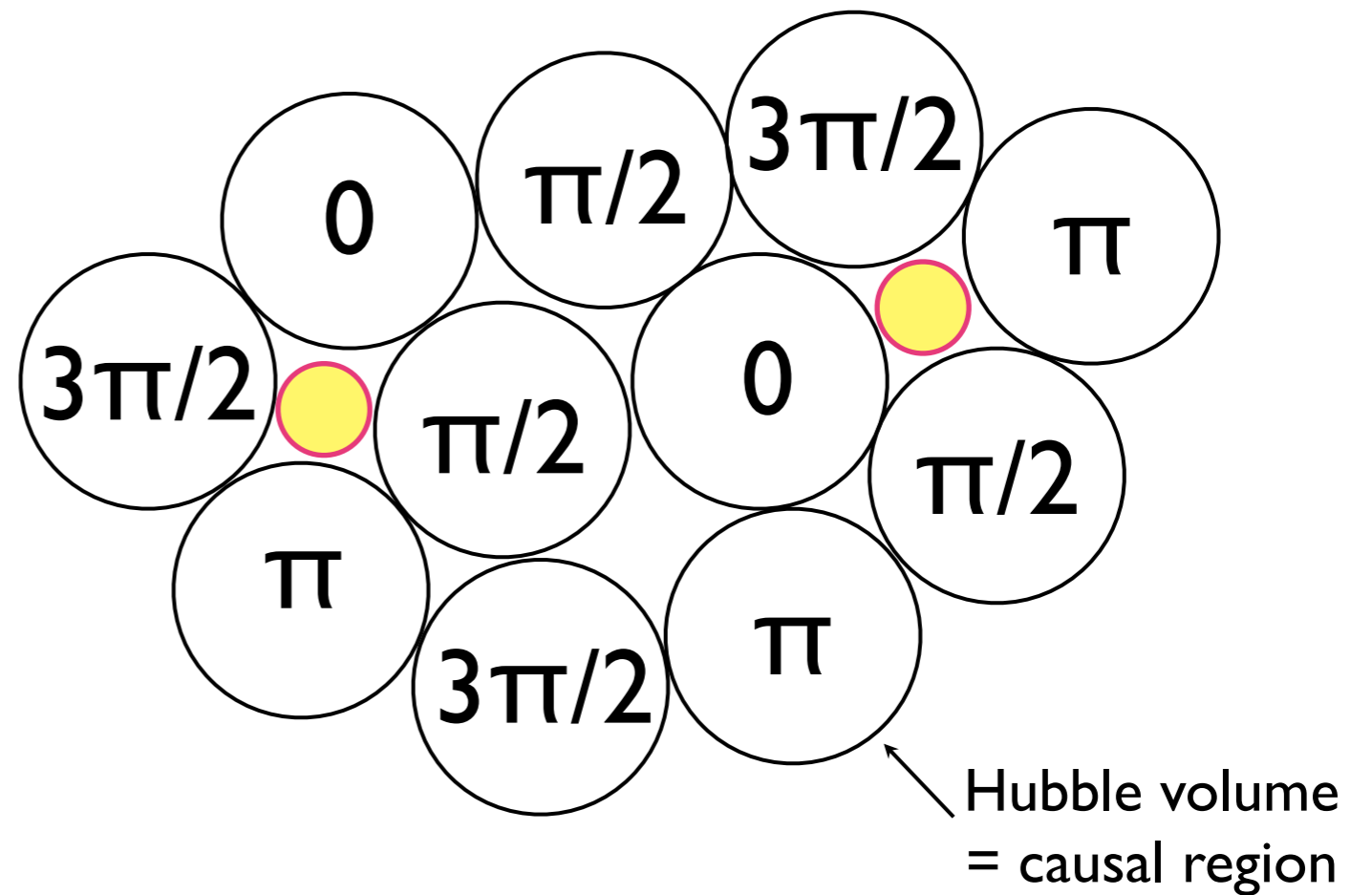
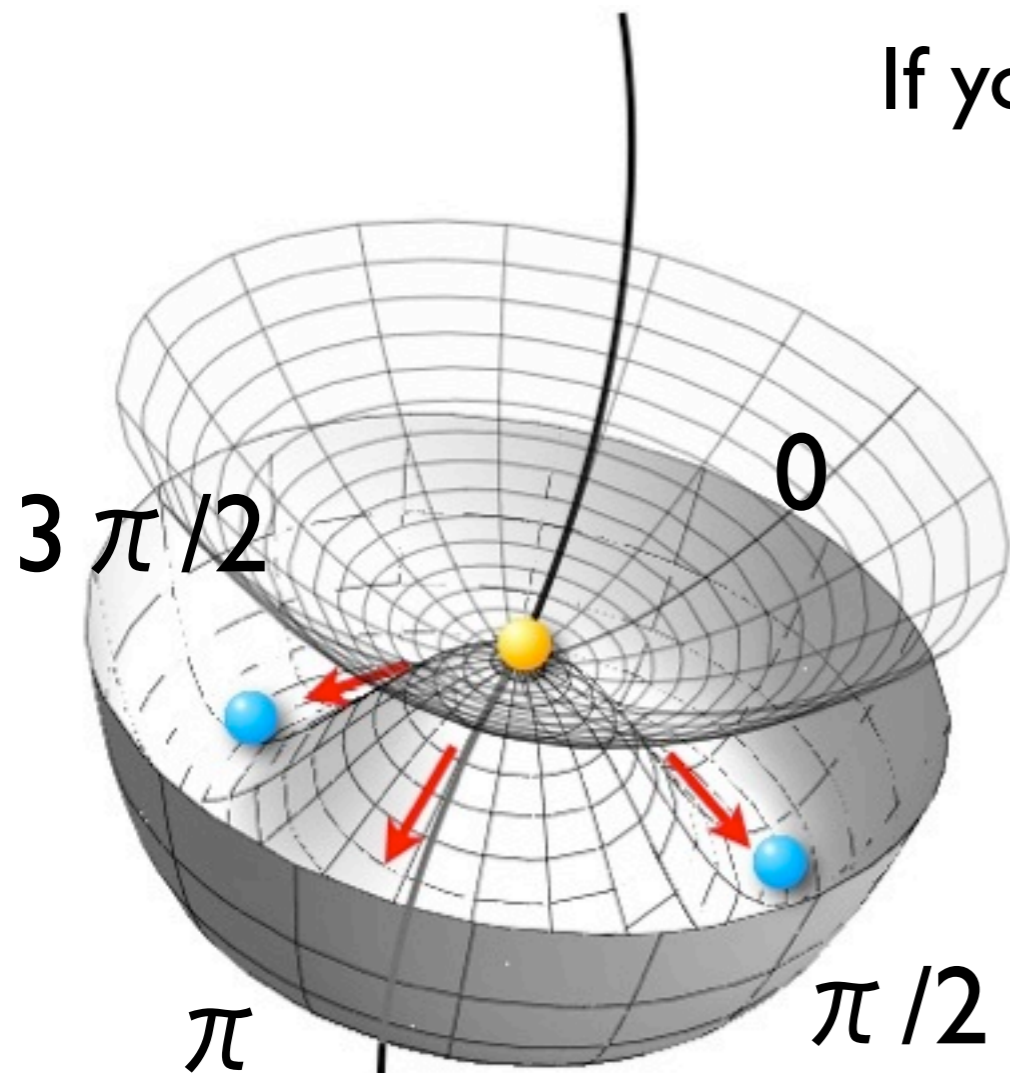
infinite
string

μ : tension (line density)
 $G\mu = \mu / m_{\text{pl}}^2$

Generation mechanism I: phase transition

The Universe has experienced symmetry breakings.

If you consider $U(1)$ symmetry breaking...



High energy vacuum remains at the center

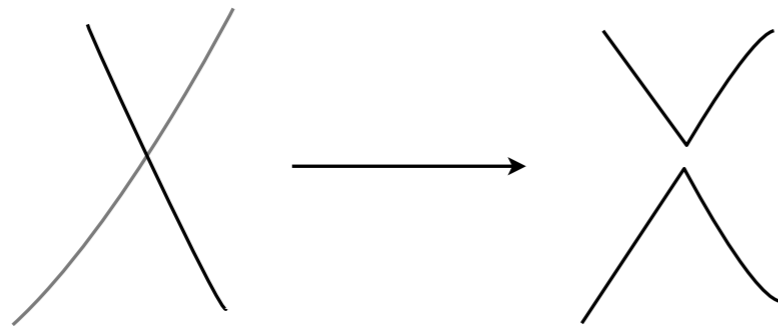
Tension $G\mu \sim$ the energy scale of the phase transition

Generation mechanism 2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation in superstring theory

Difference from phase transition origin

- reconnection probability: p



Phase transition origin: $p=1$

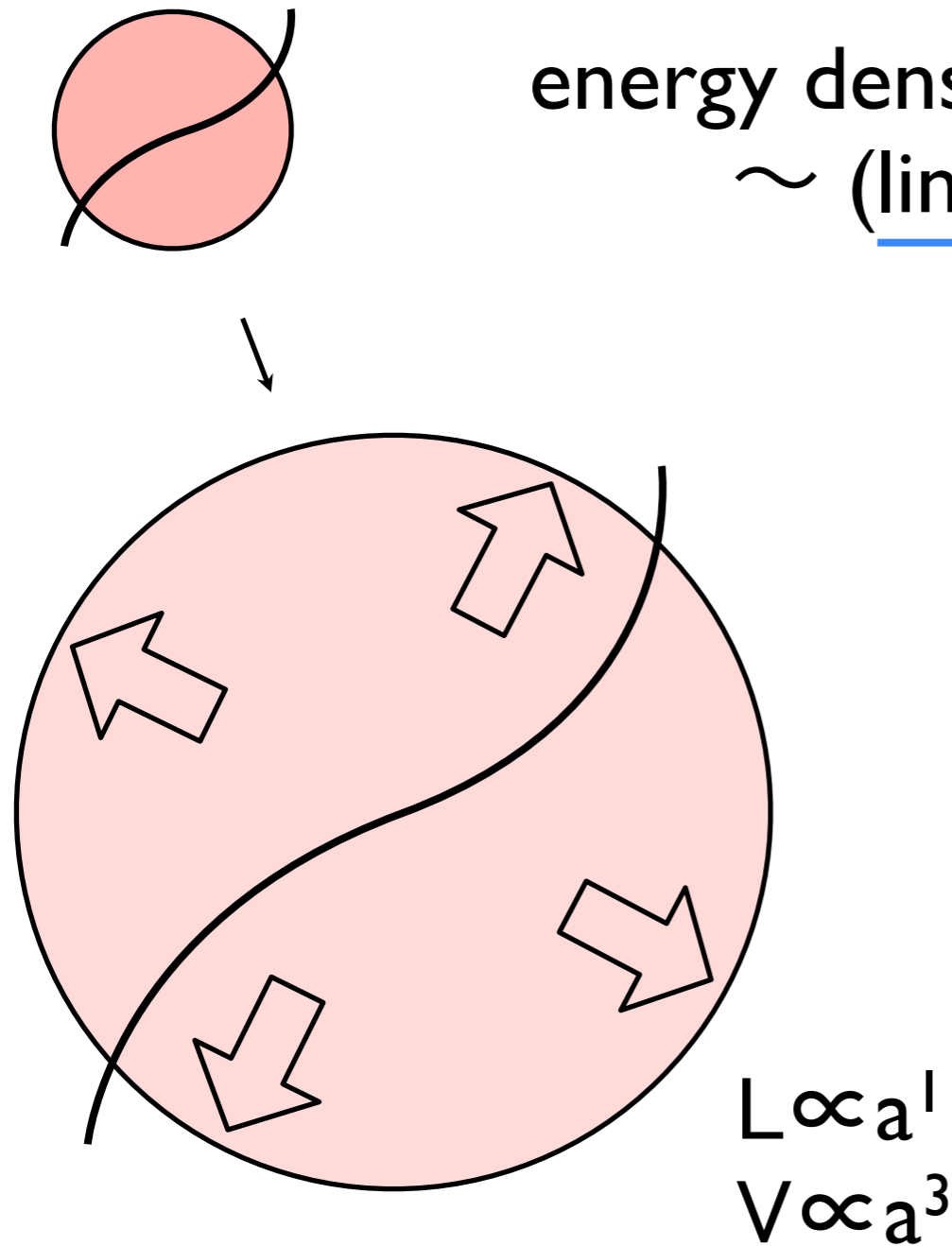
D-string: $p=0.1-1$

F-string: $p=10^{-3}-1$

- can have broad values of $G\mu$
- have Y junction
- mixed network of different $G\mu$

→ **Cosmic strings provides insight into fundamental physics**

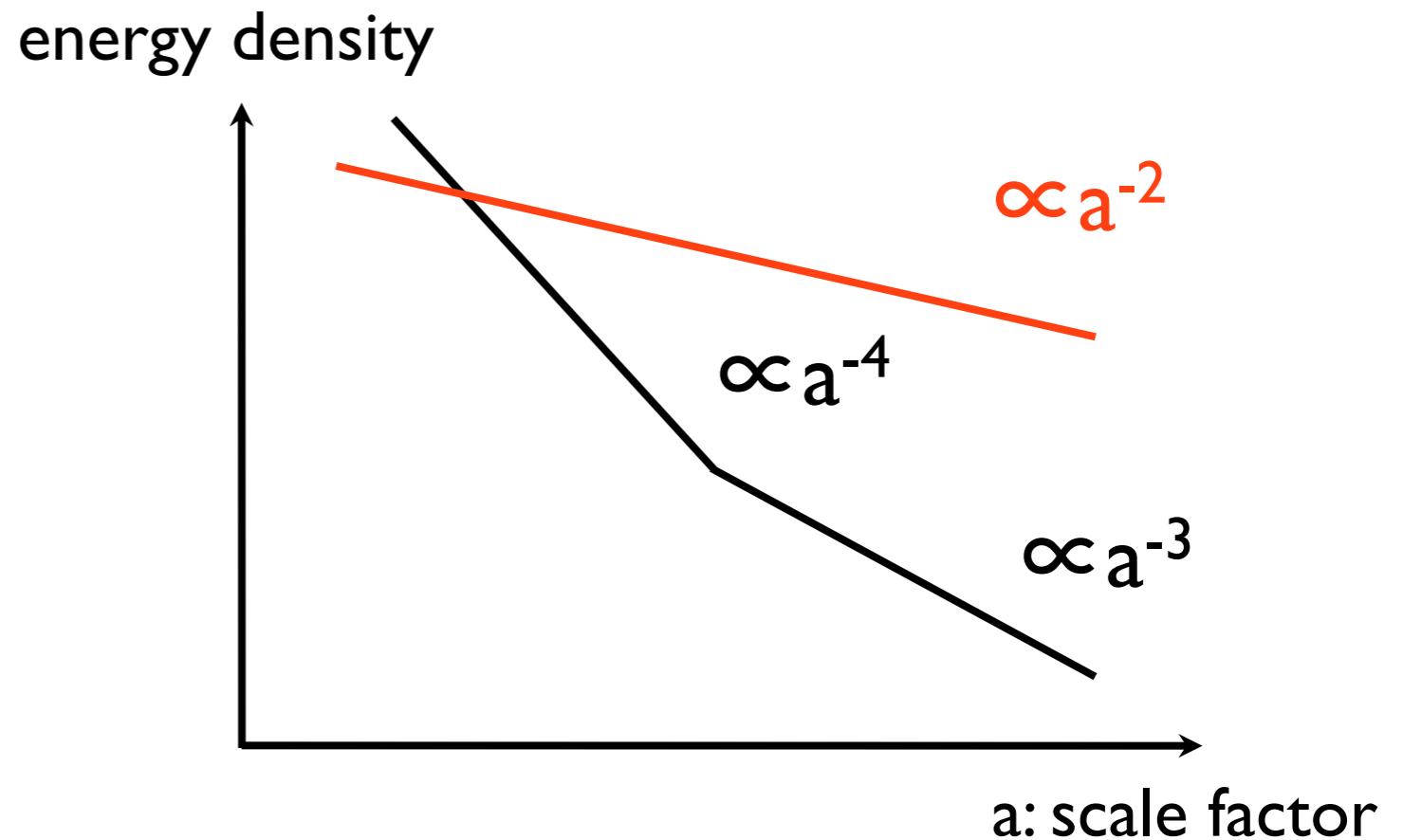
Evolution of cosmic strings



energy density of cosmic strings $\propto a^{-2}$
 $\sim \frac{\text{line density} \times \text{length}}{\text{volume}}$
 constant $\propto a^1$ $\propto a^{-3}$

radiation $\propto a^{-4}$

matter $\propto a^{-3}$

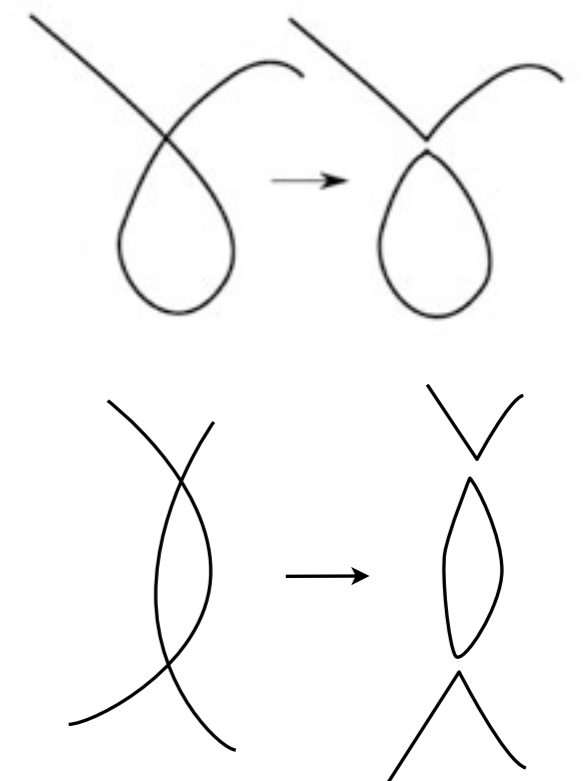
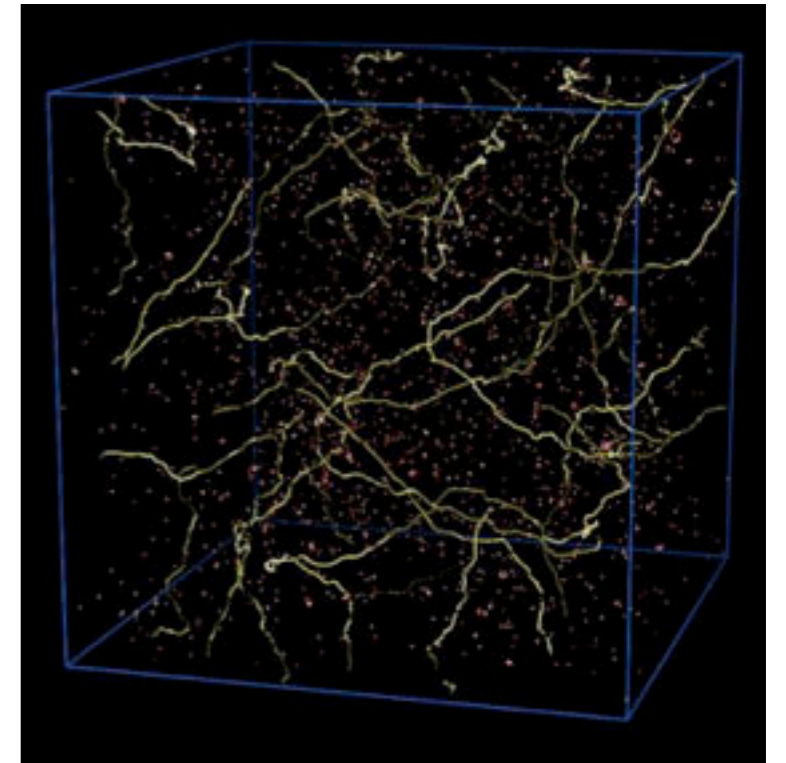
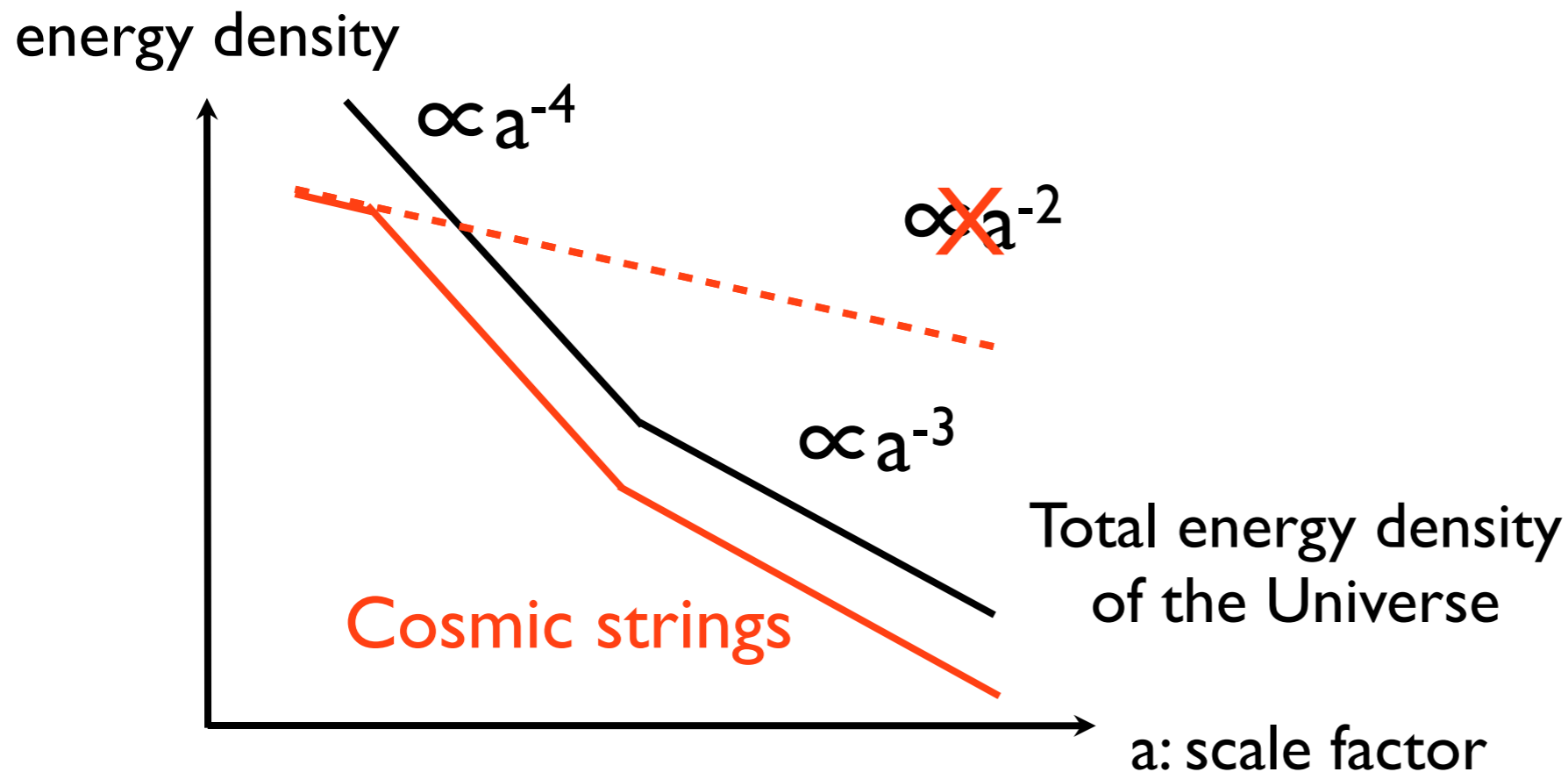


→ easily dominates the energy density of the Universe.
 not allowed by observation?

Evolution of cosmic strings

Scaling law

$O(1)$ infinite strings in the Hubble horizon



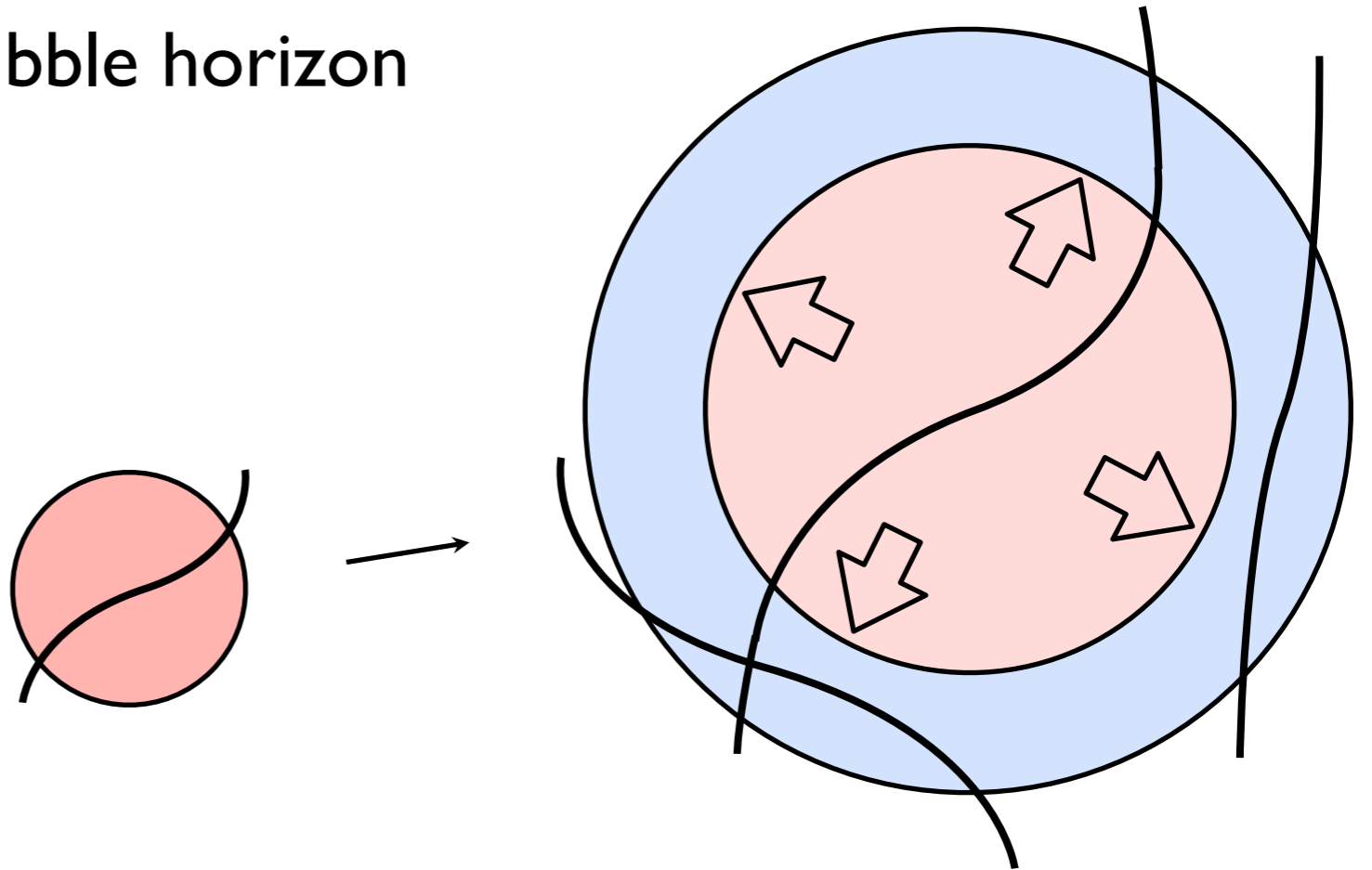
Cosmic strings become loops via reconnection.

Loops lose energy by emitting **gravitational waves**.

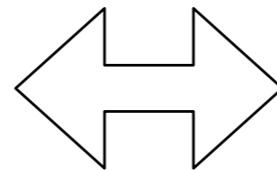
Evolution of cosmic strings

Scaling law

$O(1)$ infinite strings in the Hubble horizon



Increase of infinite string length
by the horizon growth

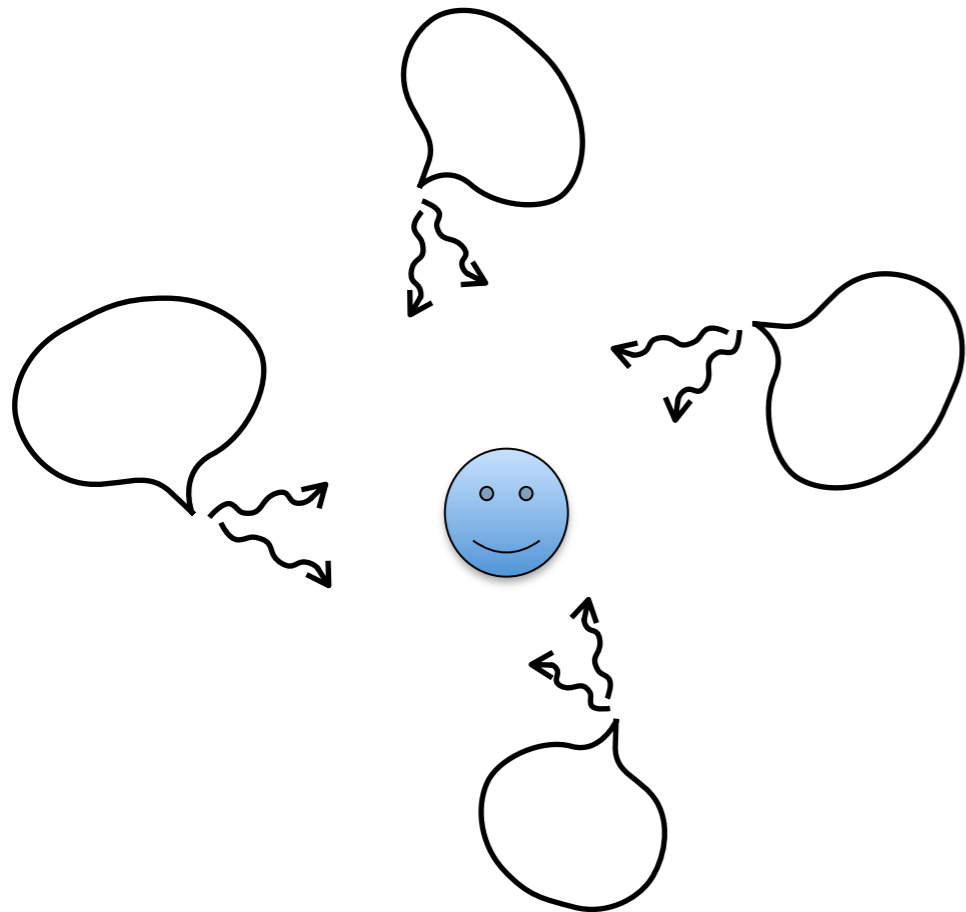
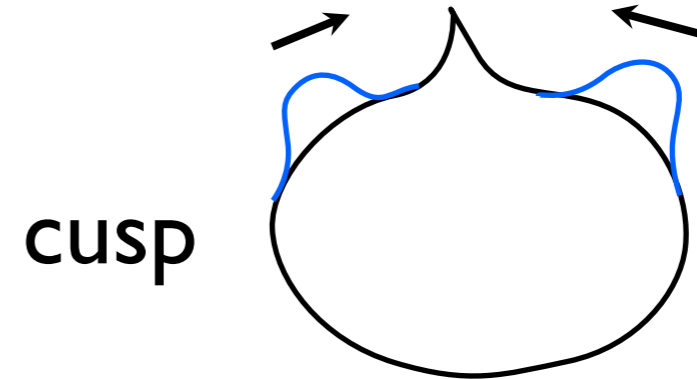
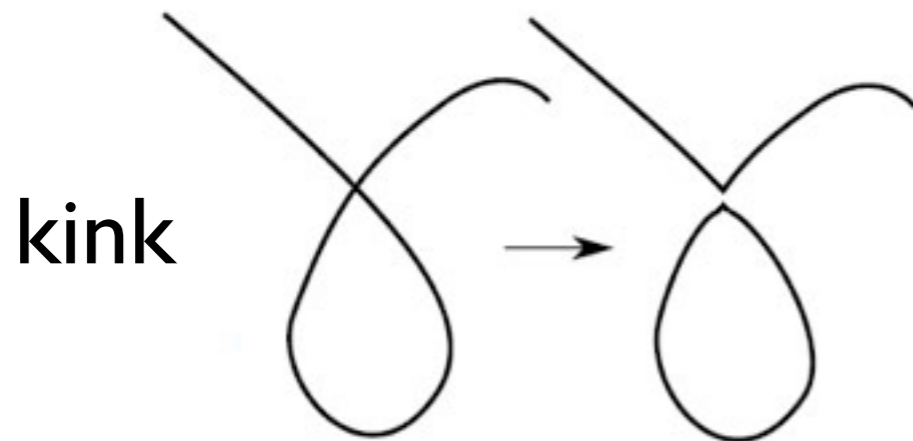


Loss of infinite string length
by generation of loops

Higher reconnection rate
more efficient generation of loops
more energy release by the emission of GWs

Gravitational waves from cosmic strings

Strong GW emission from singular points called **kinks** and **cusps**



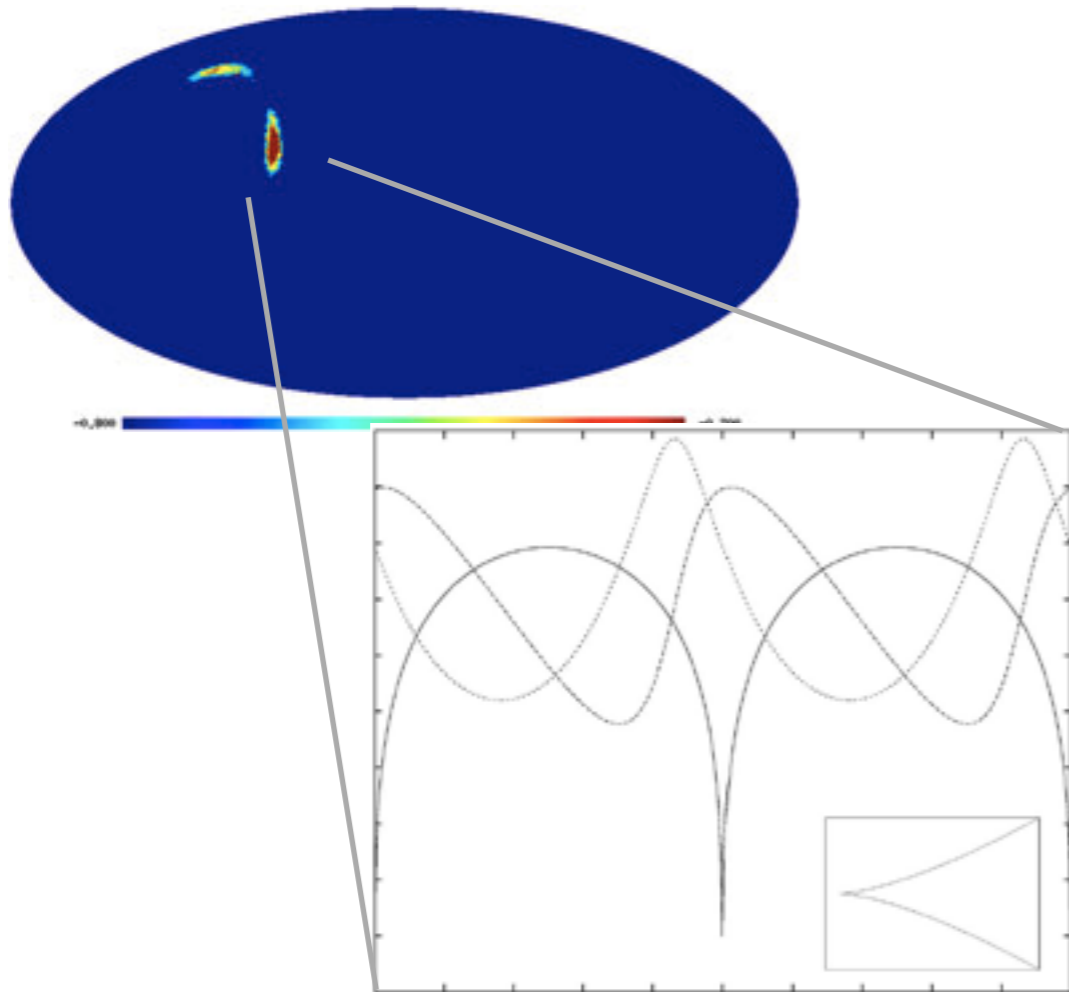
Rare Burst: GWs with large amplitude coming from close loops

Gravitational wave background (GWB): superposition of small GWs coming from the early epoch

Observations of GWs

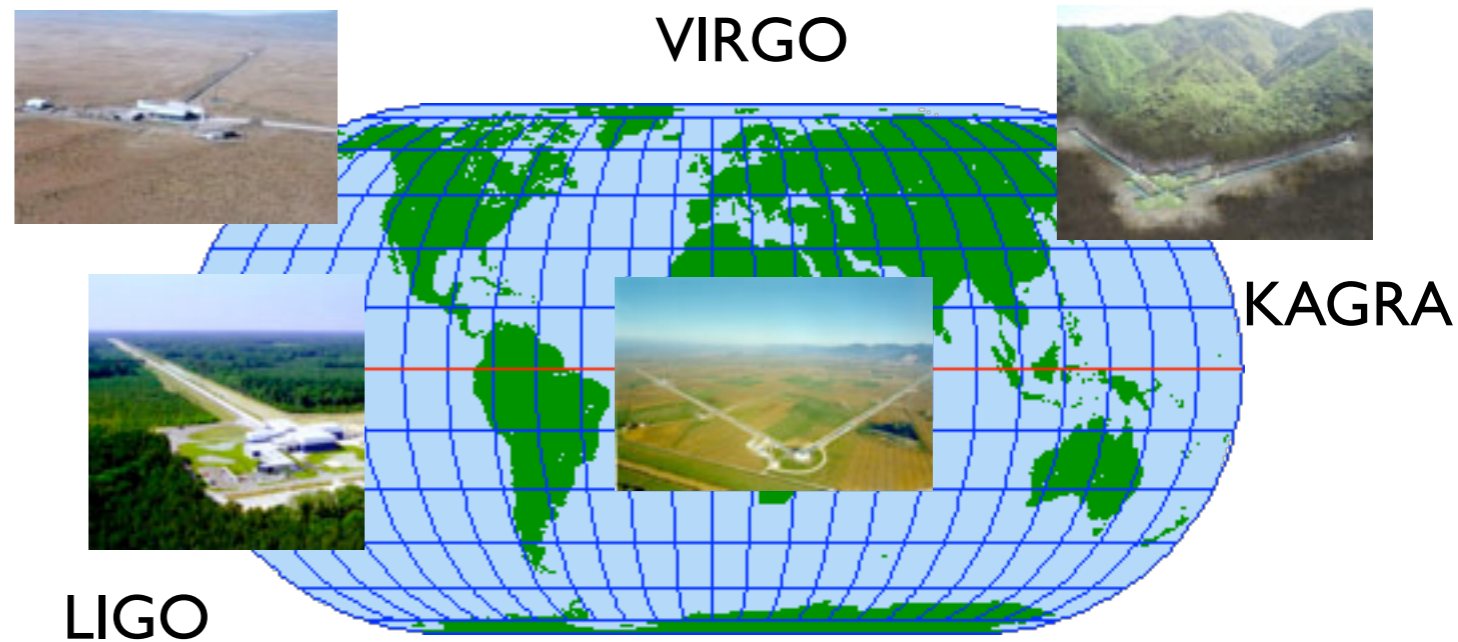
GWs with large amplitude

Burst



GWs with small amplitude
but numerous

GWB



Cross correlation analysis

Cross correlate the signals from two or more detector and extract stable GWs

→ provide different information on cosmic strings

Gravitational wave experiments

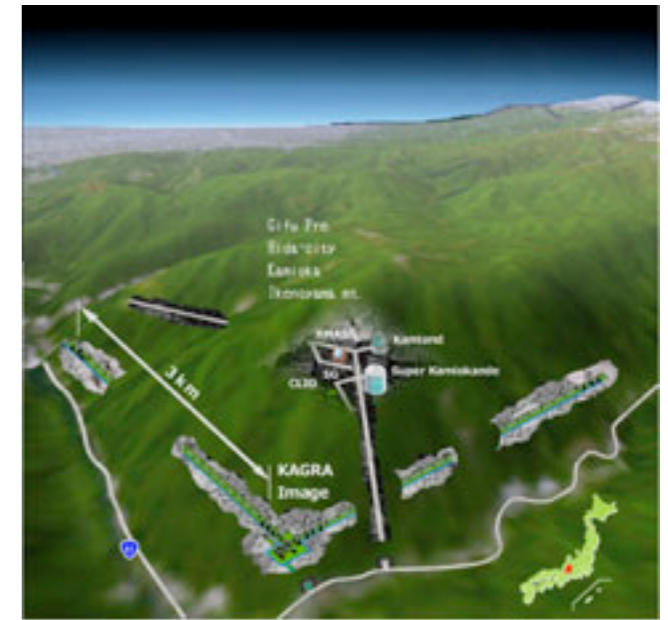
- Direct detection

Ground : **Advanced-LIGO, KAGRA, Virgo, IndIGO**

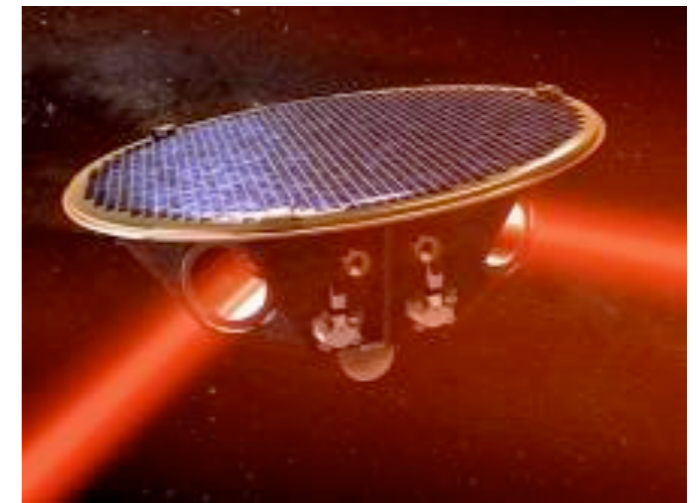
Space : **eLISA/NGO, DECIGO**

- Pulsar timing: **SKA**

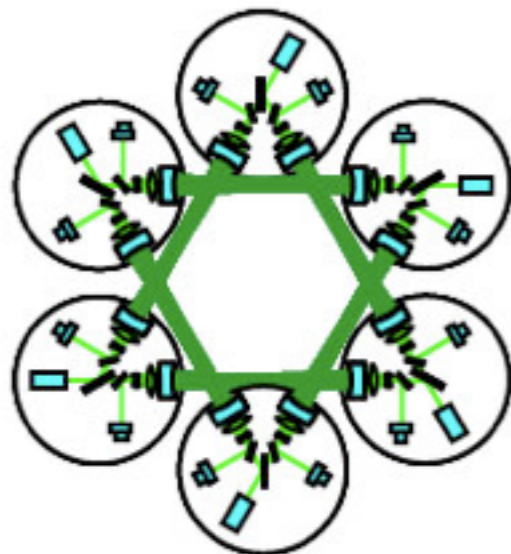
- CMB B-mode polarization: **Planck, CMBpol**



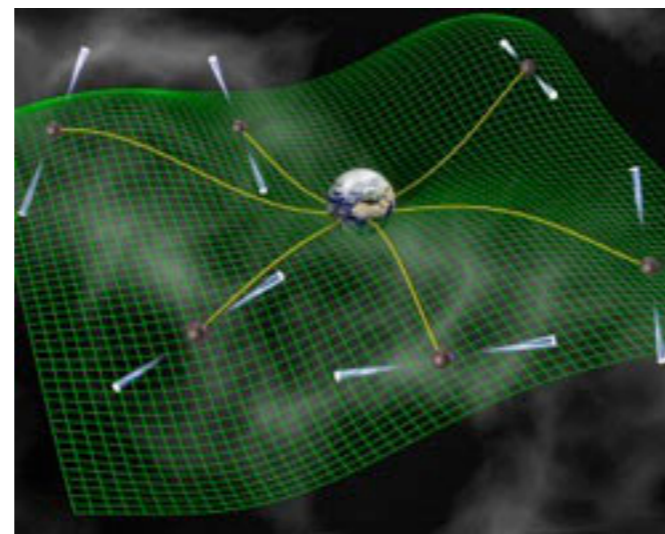
KAGRA image (<http://gwcenter.icrr.u-tokyo.ac.jp/>)



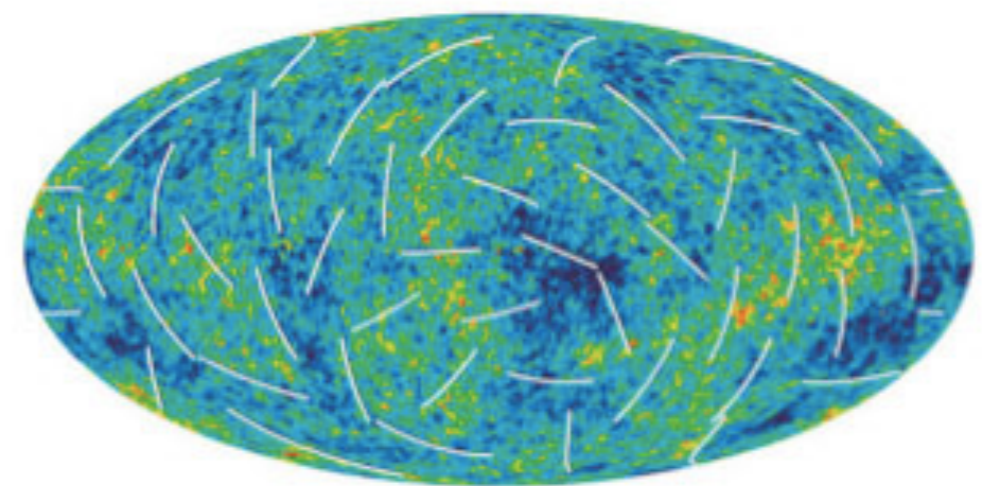
eLISA image (<http://elisa-ngo.org/>)



DECIGO image, S. Kawamura et al, J. Phys.: Conf. Ser. 122, 012006 (2006)



PTA image (NRAO)



WMAP Three Year Polarized CMB Sky (<http://wmap.gsfc.nasa.gov/>)

Current constraints on cosmic string parameters

3 parameters to characterize cosmic string

- $G\mu$ ($= \mu / M_{\text{pl}}^2$) : tension (line density)
- α : initial loop size $L \sim \alpha H^{-1}$
- p : reconnection probability

- CMB temperature fluctuation: $G\mu < \sim 10^{-7}$
- Gravitational lensing: $G\mu < \sim 10^{-6}$

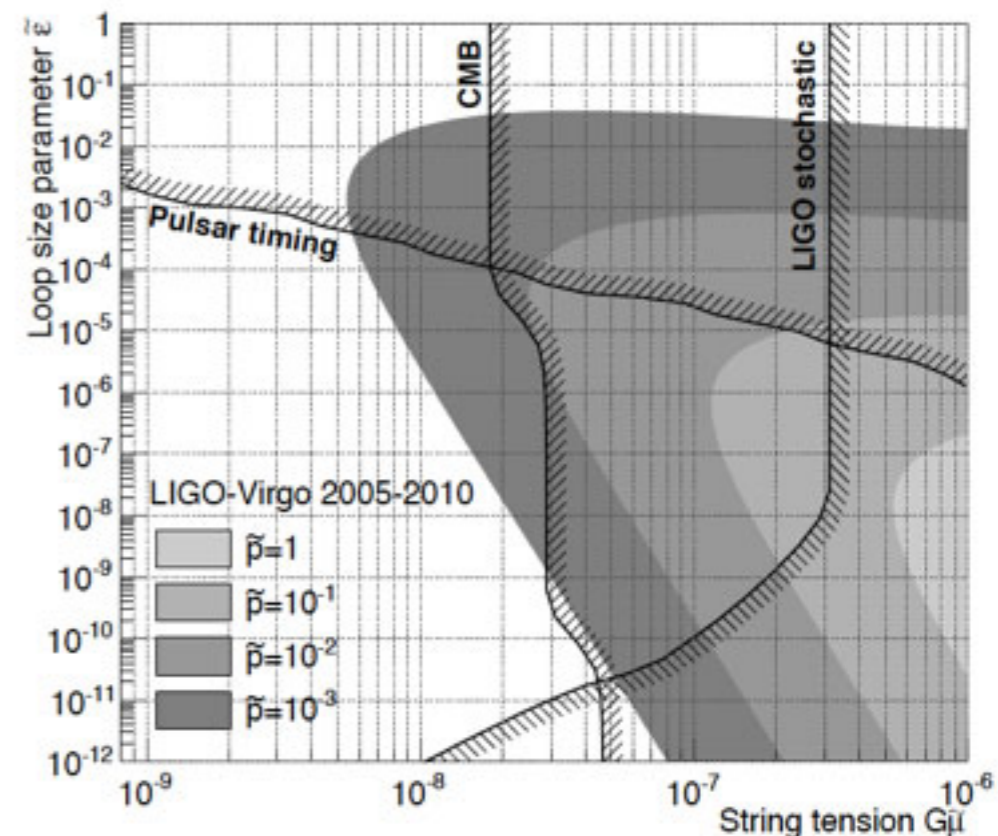
for infinite strings

• Gravitational waves

Pulsar timing: $G\mu < \sim 10^{-9}$
for loops $\alpha = 0.1$, $p = 1$

Direct detection
(LIGO GWB & burst): $G\mu < \sim 10^{-6}$

What about future constraints ?



Estimation of the GW burst rate

Initial number density of loops

depends on α and p

$$= \frac{\text{(length of infinite string going to loops)}}{\text{(initial length of loops = } \alpha t_i)}$$

Evolution of infinite strings

- velocity-dependent one-scale model

$$2 \frac{dL}{dt} = 2HL(1 + v^2) + \underline{cv}$$

energy conservation

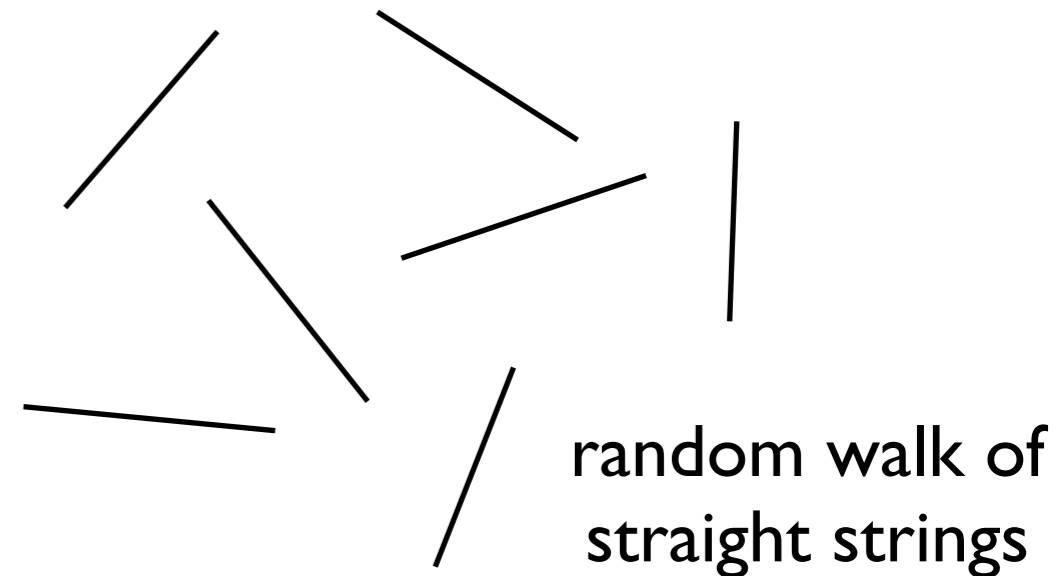
energy goes to loops

$$\frac{dv}{dt} = (1 - v^2) \left(\frac{k}{R} - 2Hv \right)$$

acceleration due to the curvature of the strings

damping due to the expansion

momentum parameter: $k = \frac{2\sqrt{2}}{\pi} \left(\frac{1 - 8v^6}{1 + 8v^6} \right)$



random walk of straight strings
length L , velocity v

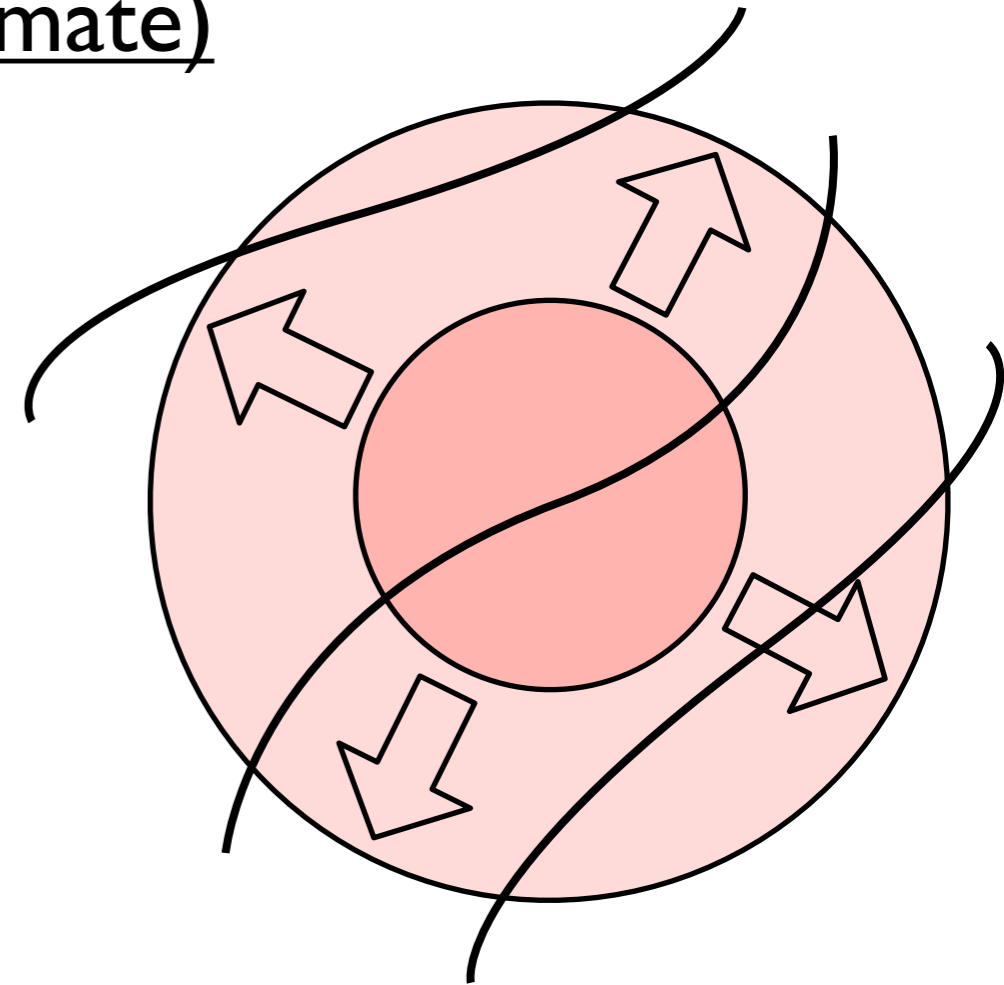
$$\left. \frac{d\rho_{inf}}{dt} \right|_{loop} = cv \frac{\rho_{inf}}{L}$$

for small p : $c \rightarrow cp$

Estimation of the GW burst rate

Initial number density of loops (naive estimate)

$$\begin{aligned}\text{Number of loops} &= \frac{\text{(length to lose)}}{\text{(initial length of loops)}} \\ &= \frac{p^{-1}t}{\alpha t} = \frac{1}{p\alpha}\end{aligned}$$



Scaling law

$O(1)$ infinite strings in the Hubble horizon

To satisfy the scaling law...

infinite strings

should lose $O(1)$ Hubble length per 1 Hubble time → more loops for small α

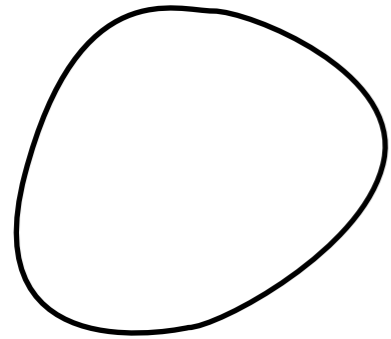
= should reconnect $O(1)$ times per Hubble time

→ for small p , string density should increase to reconnect $O(1)$ times

Estimation of the GW burst rate

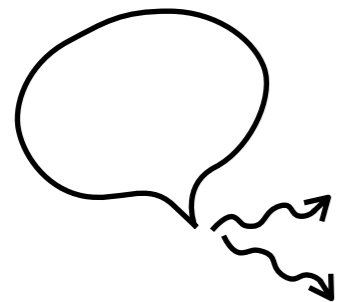
loop evolution

depends on $G\mu$ and α



Initial loop length = αt_i

t_i : time when the loop formed



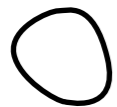
GW power $P = \Gamma G\mu^2$ Γ : numerical constant $\sim 50-100$

From the energy conservation law
(energy of loop at time $t = \mu l$)

$$= (\text{initial energy of the loop} = \mu\alpha t_i) - (\text{energy released to GWs} = P\Delta t)$$



Loop length at time t $l(t, t_i) = \alpha t_i - \Gamma G\mu(t - t_i)$



Lifetime of the loop = $\frac{(\text{initial loop energy})}{(\text{energy release rate per time})}$

$$= \frac{\mu\alpha t_i}{\Gamma G\mu^2} = \frac{\alpha t_i}{\Gamma G\mu}$$

0



Estimation of the GW burst rate

GW burst rate emitted at $t \sim t+dt$ from loops formed at $t_i \sim t_i+dt_i$

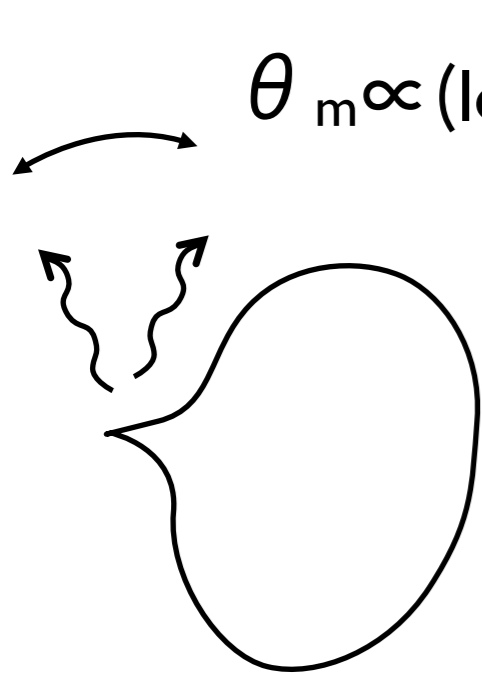
$$\frac{dR}{dt dt_i} = \underbrace{\frac{1}{4} \theta_m(f, z, l)^2}_{\text{Beaming}} \underbrace{\frac{2c}{(1+z)l(t, t_i)}}_{\text{Time interval of GW emission}} \underbrace{\frac{dn}{dt_i}(t, t_i) \frac{dV}{dt}}_{\text{Loop number}} \times \underbrace{\Theta(1 - \theta_m(f, z, l))}_{\text{Heaviside step function}}$$

Beaming

Time interval of GW emission

Loop number

$$\propto (\text{loop length at } t)^{-1}$$



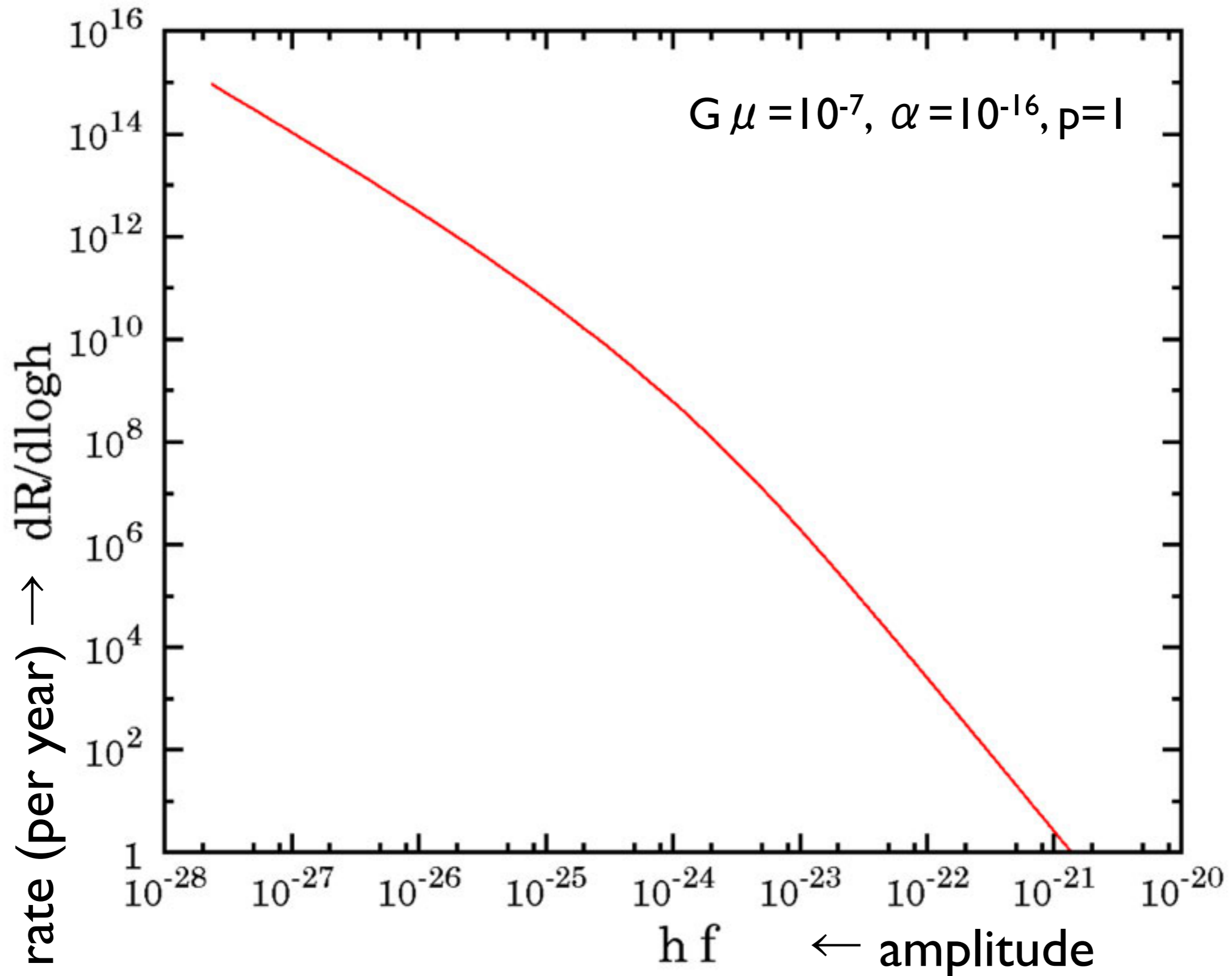
$$\theta_m \propto (\text{loop length at } t)^{1/3}$$

$$l(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$$

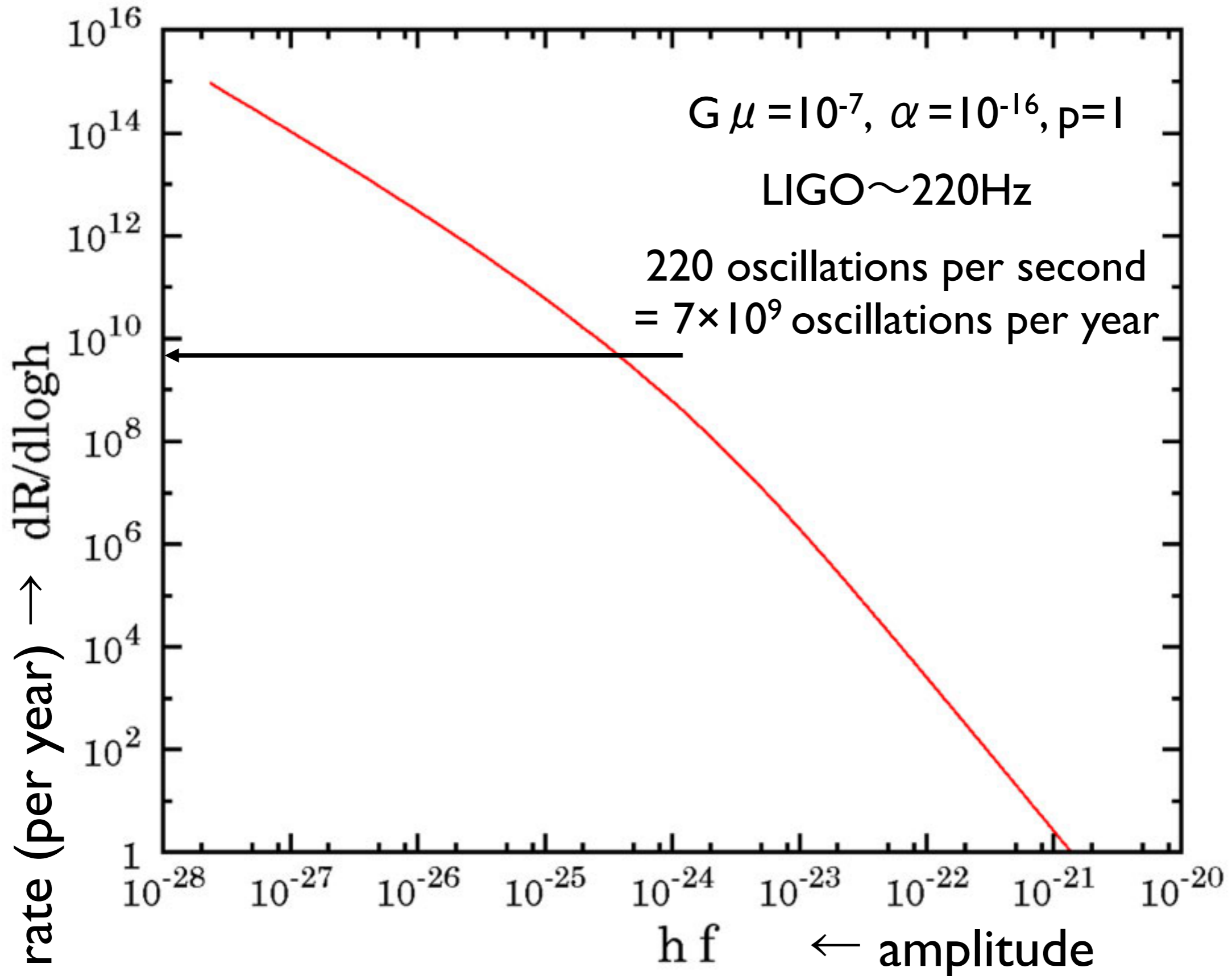
GW amplitude from loop of length l

$$h(f, z, l) \simeq 2.68 \frac{G \mu l}{((1+z)fl)^{1/3} r(z) f}$$

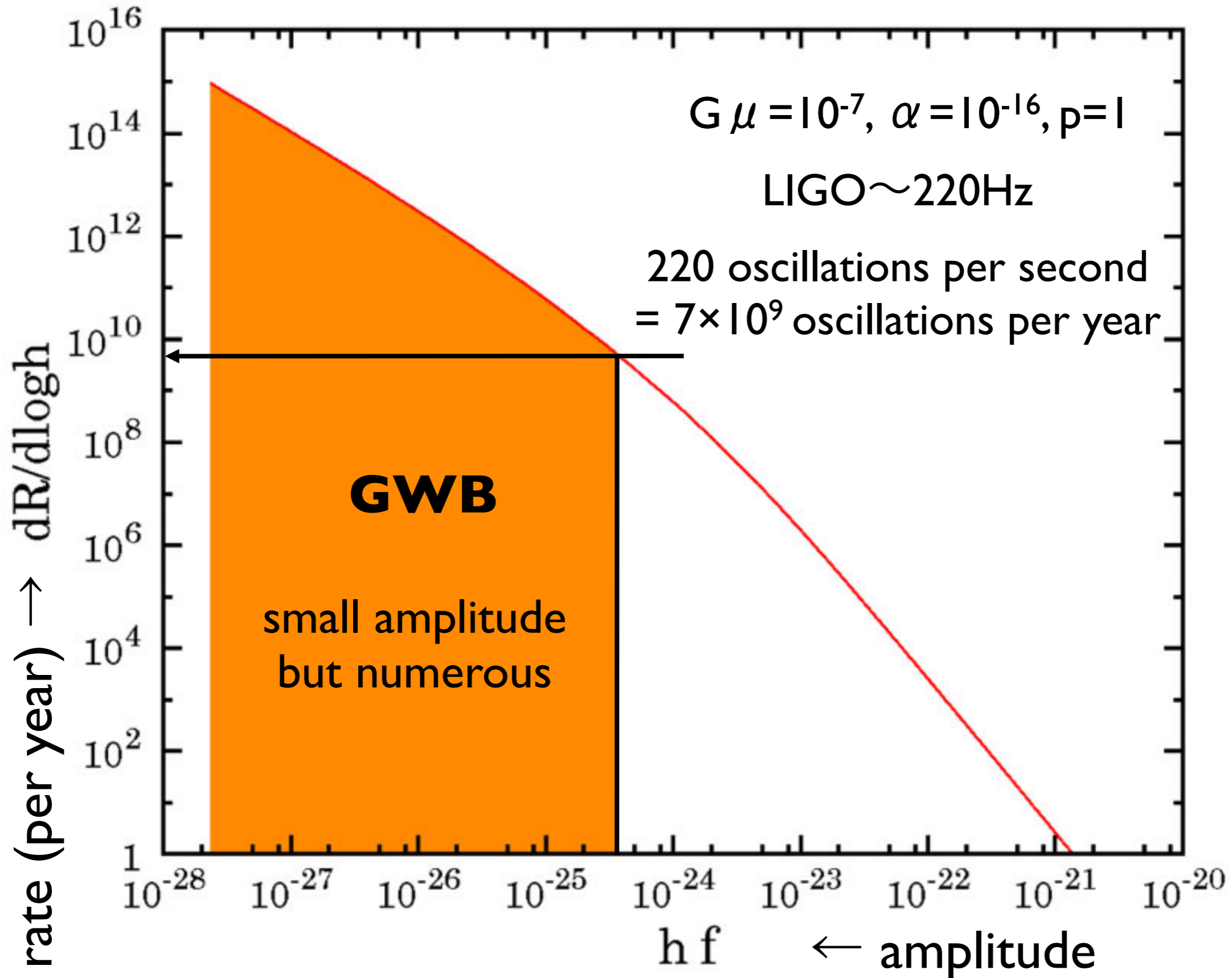
How many cosmic string bursts are coming to the earth per year?
(plotted as a function of the amplitude for the fixed frequency @220Hz)



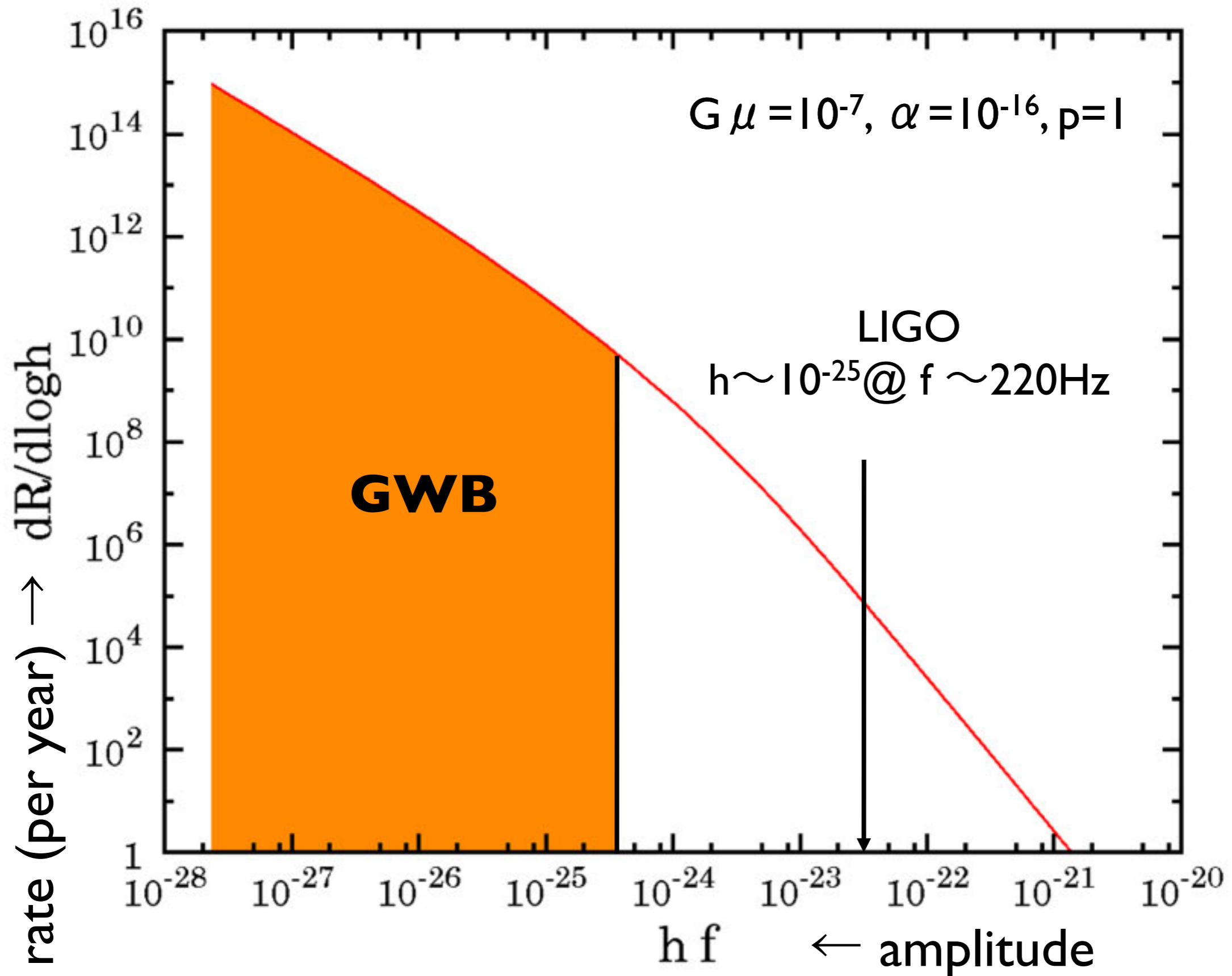
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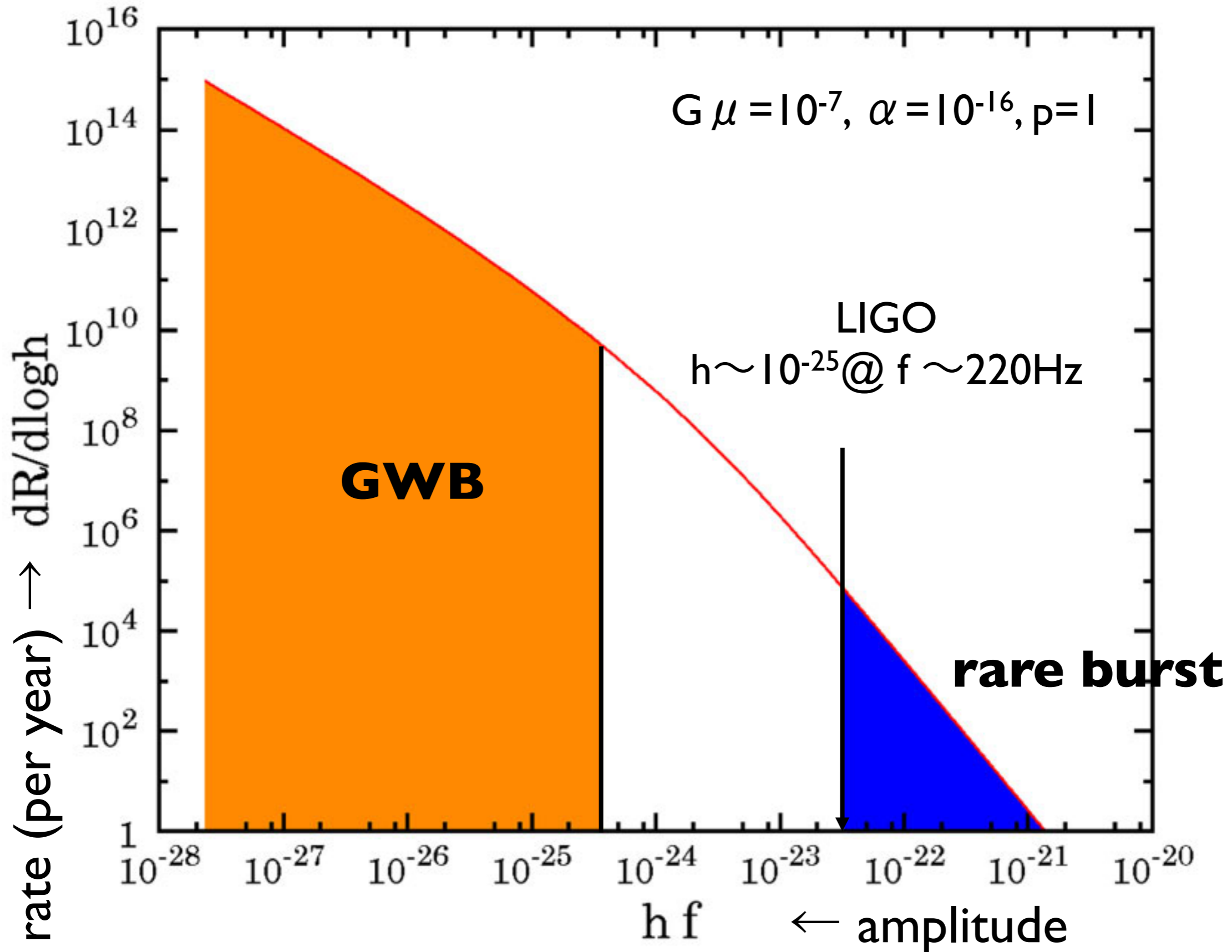
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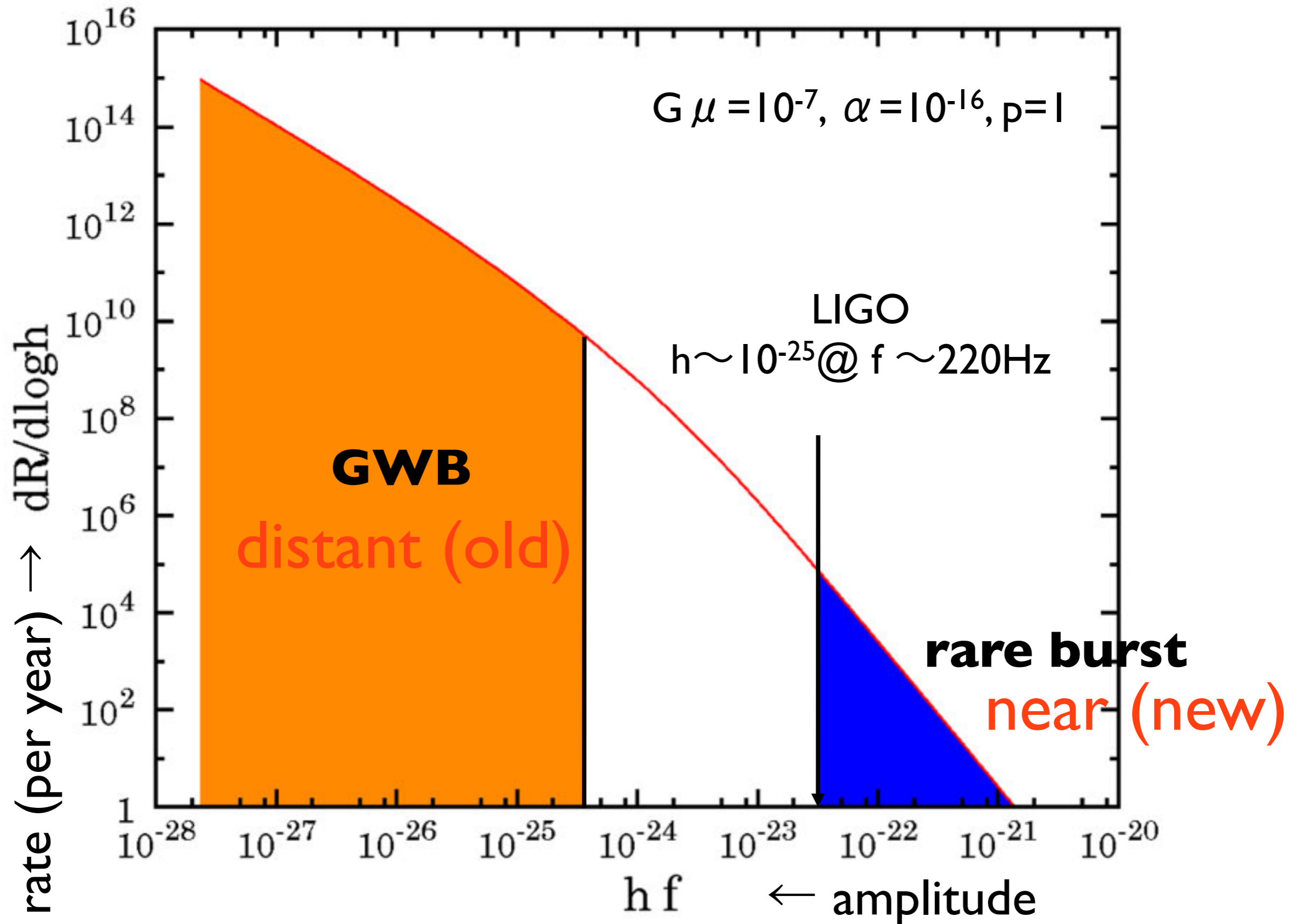
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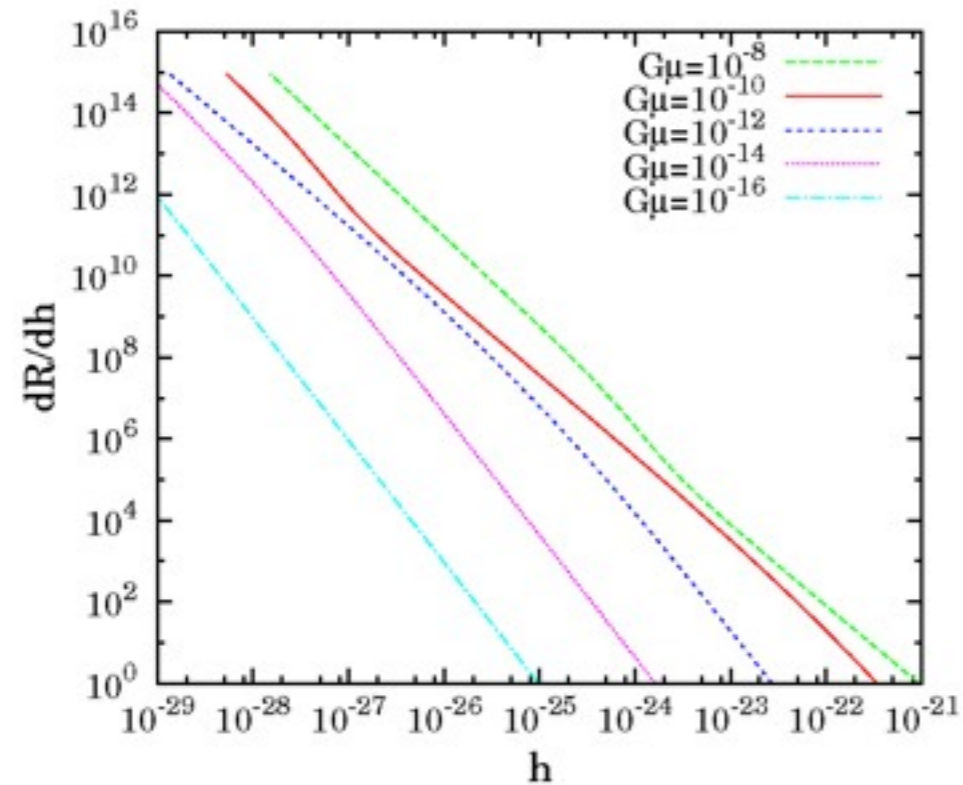


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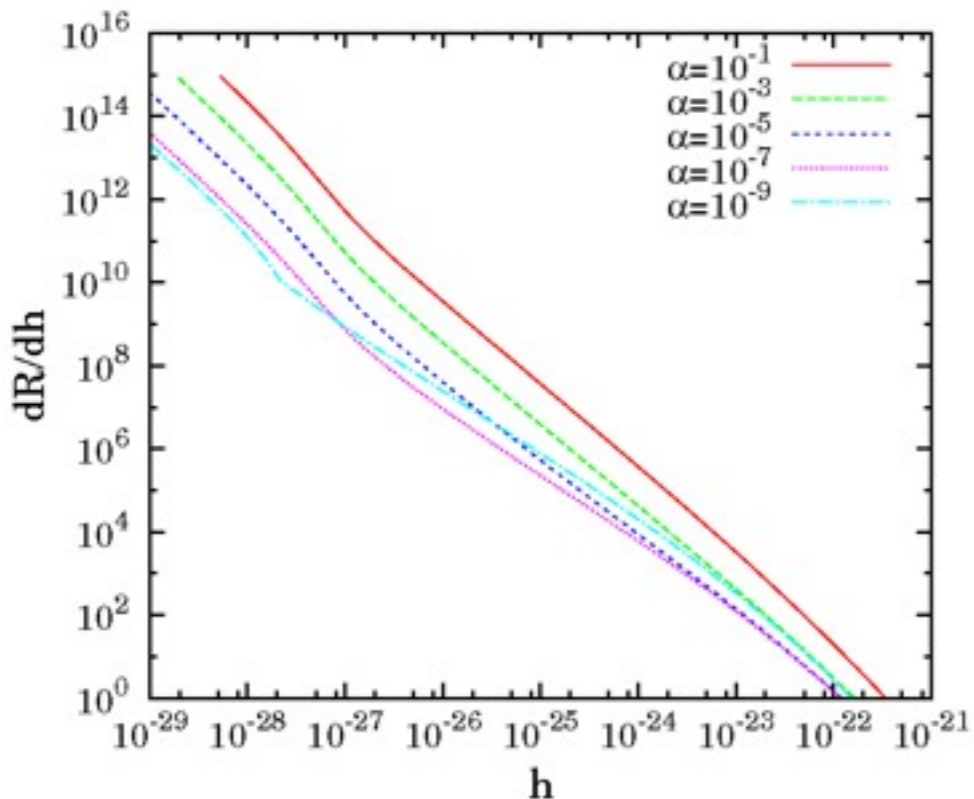
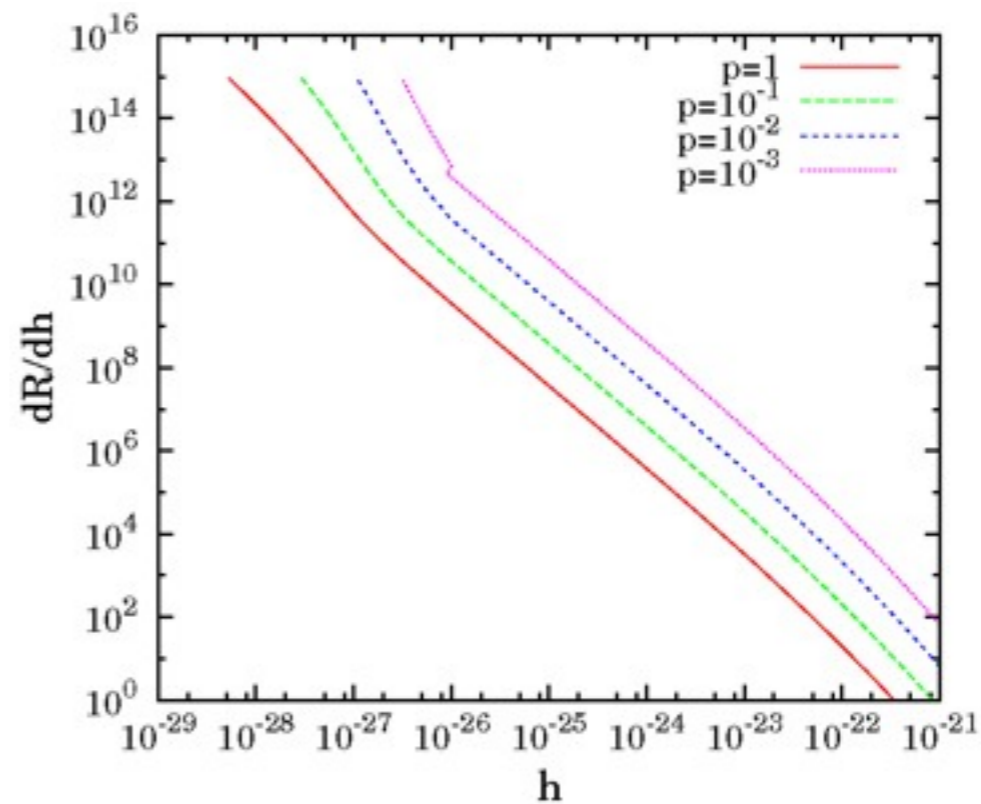


Parameter dependences of the rate

$G\mu$



p



α

The parameter dependences of the large burst (rare burst) and small burst (GWB) are different because they are looking at different epoch of the Universe

→ give different information on cosmic string parameters

Spectrum of the GWB

dependence
on $G\mu$

new



old

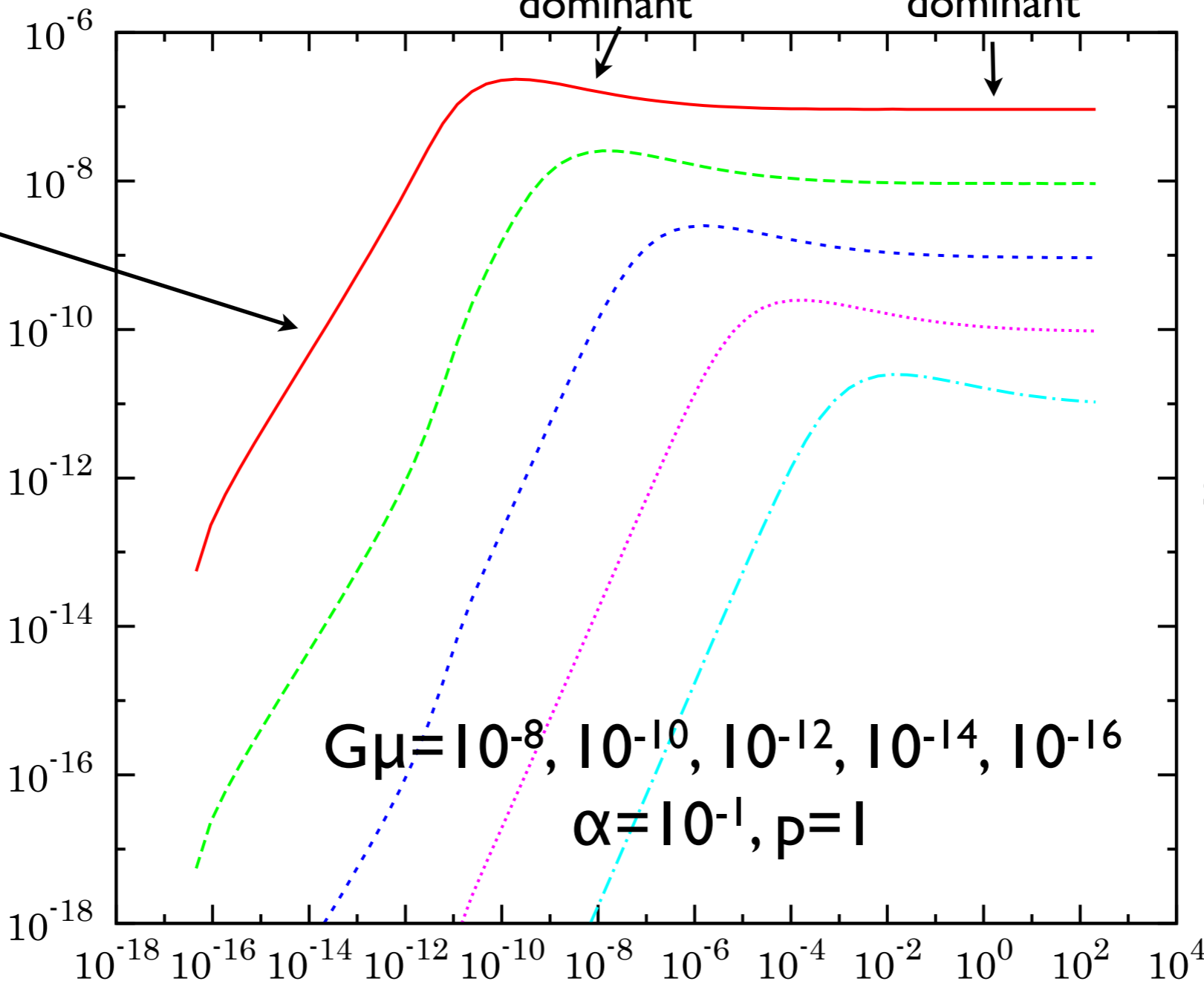
Matter
dominant

Radiation
dominant

not so many
large loops



Ω_{GW}



large $G\mu$



small $G\mu$

$G\mu = 10^{-8}, 10^{-10}, 10^{-12}, 10^{-14}, 10^{-16}$
 $\alpha = 10^{-1}, p = 1$

GW power
from cusps
 $h^2 \propto (G\mu)^2$

life time of loops
 $\propto (G\mu)^{-1}$

frequency [Hz]

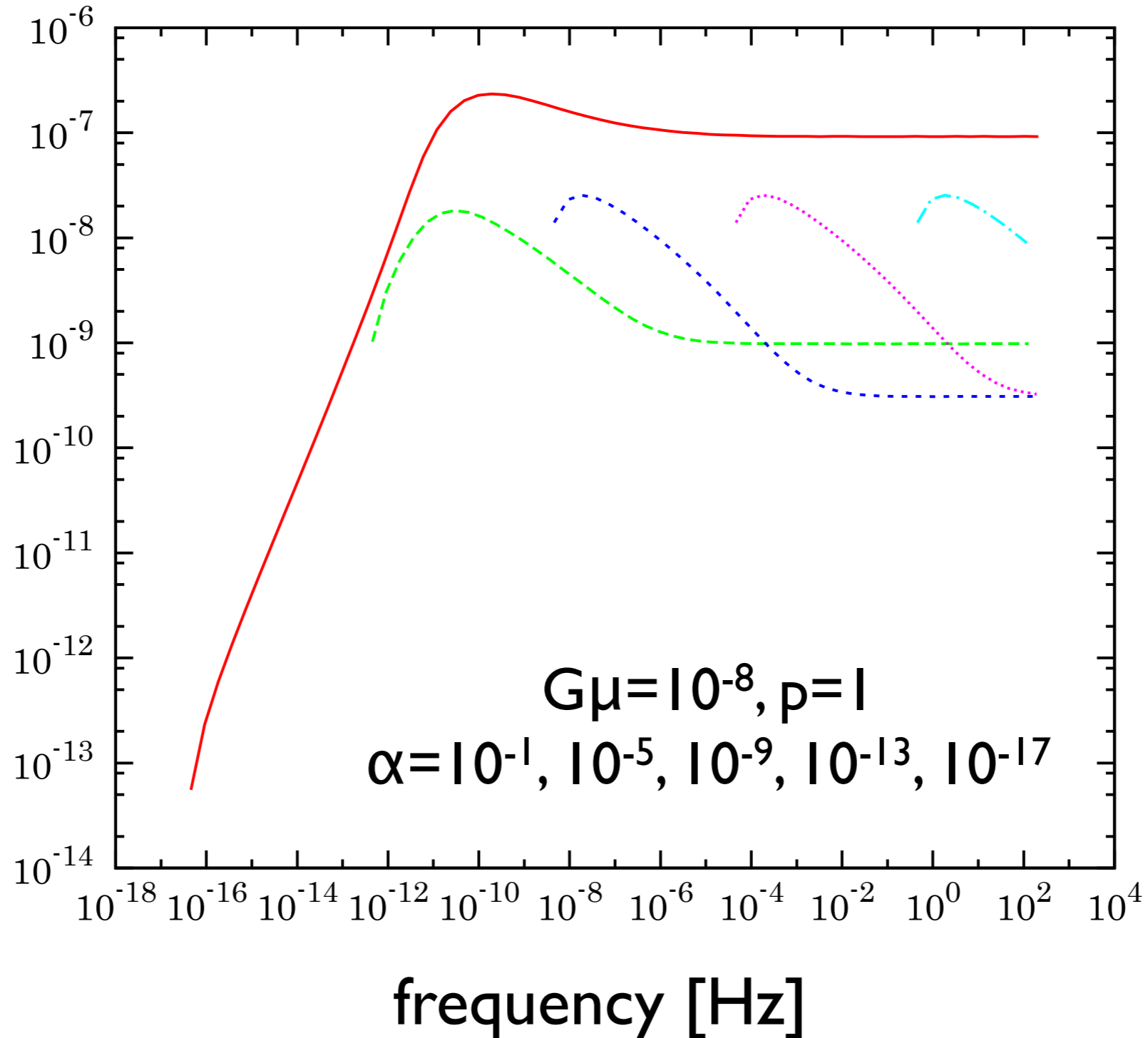
Spectrum of the GWB

dependence
on α

loop size directly corresponds to the frequency of the GW

→ small α

Ω_{GW}

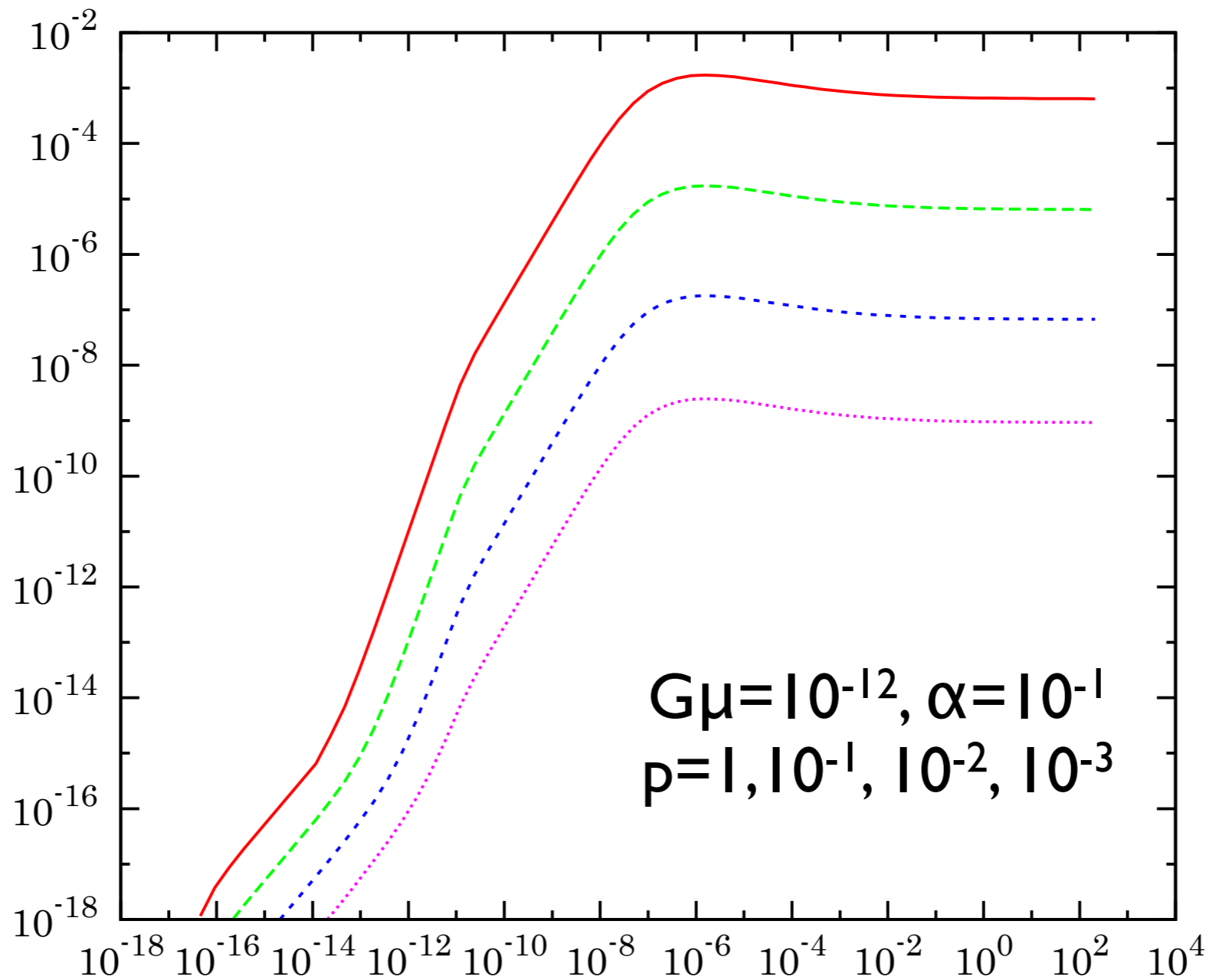


Spectrum of the GWB

dependence
on ρ

small ρ increases the number density of loops

Ω_{GW}

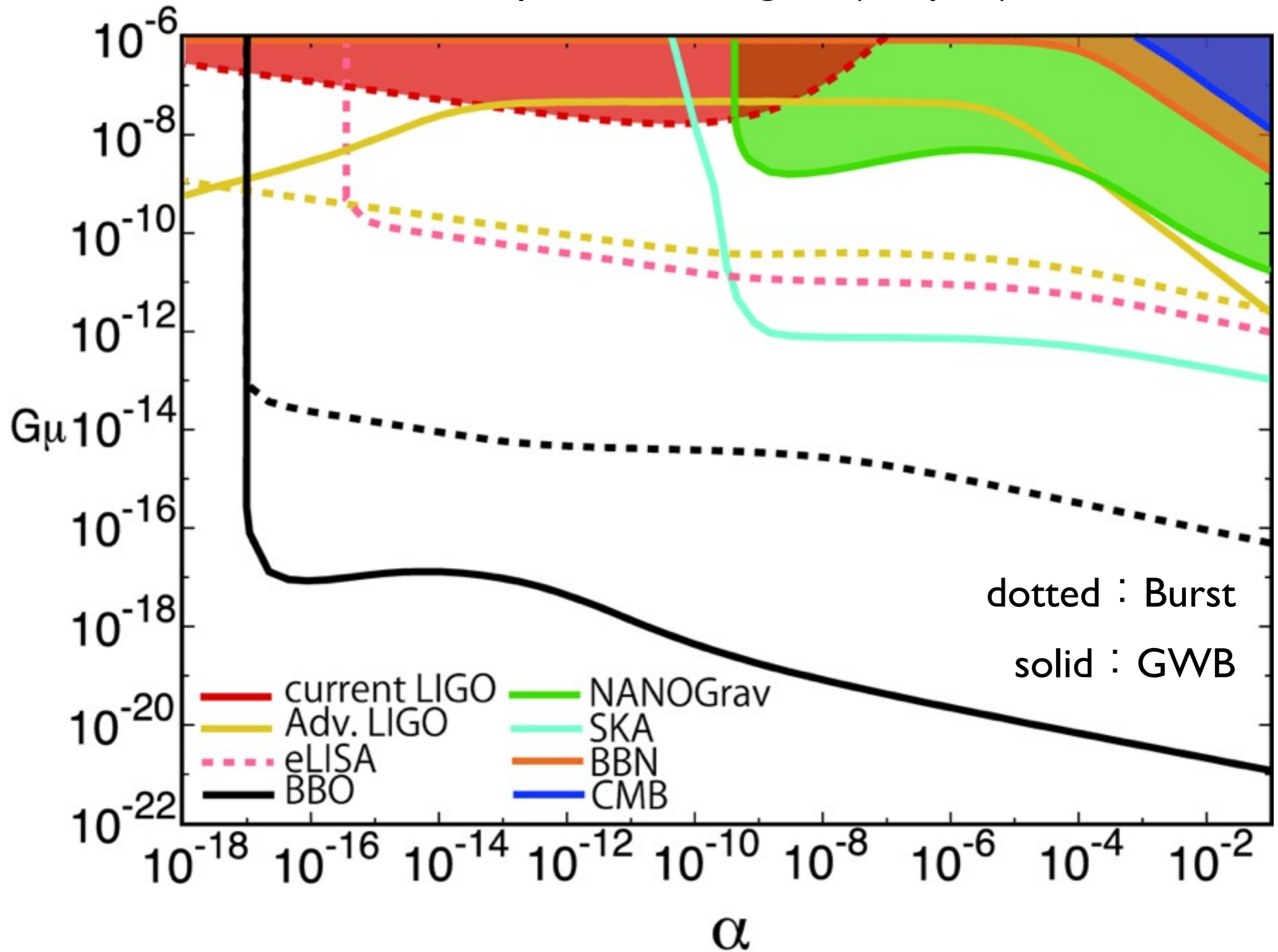


small ρ

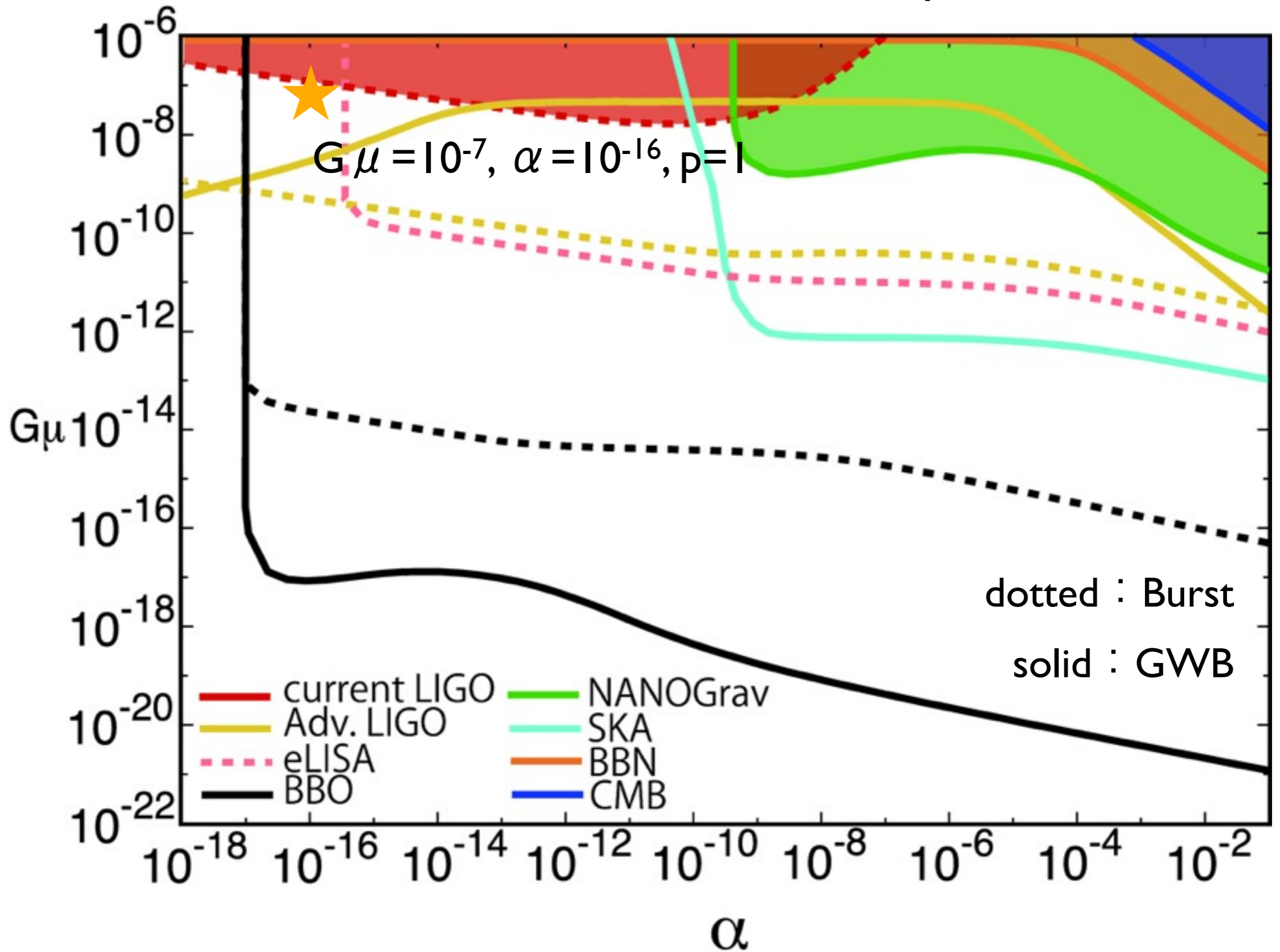
large ρ

frequency [Hz]

Accessible parameter region (for $p=1$)



What if both bursts and GWB are detected by Advanced-LIGO?

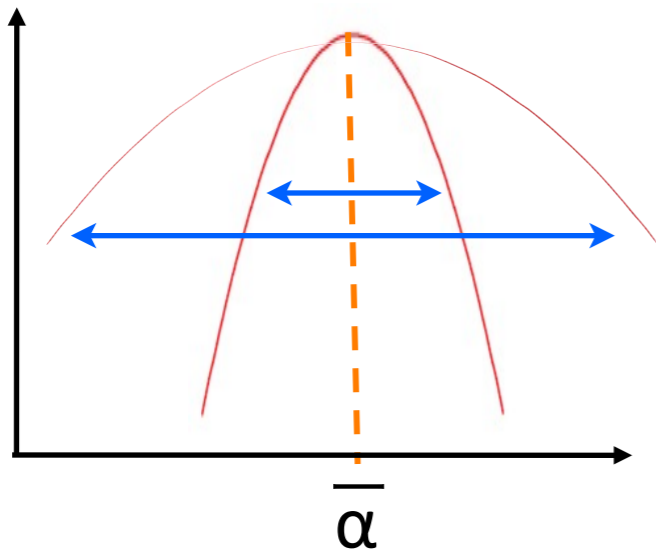


Constraint on parameters

Fisher information matrix

log(Likelihood)

$$\mathcal{F}_{ij} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle$$



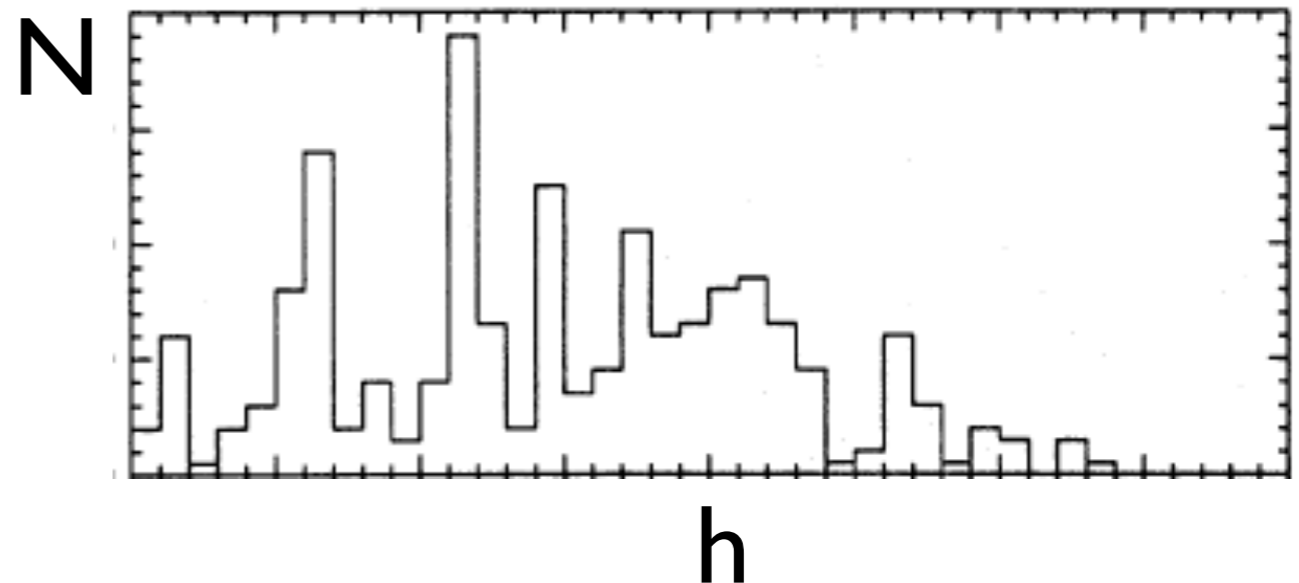
If the likelihood shape is sensitive to the parameter
= easy to estimate the parameter

Burst

Observable : amplitude vs number

N is predictable by the rate dR/dh

$$\mathcal{F}_{ij} \propto \frac{\partial(dR/dh)}{\partial p_i} \frac{\partial(dR/dh)}{\partial p_j}$$

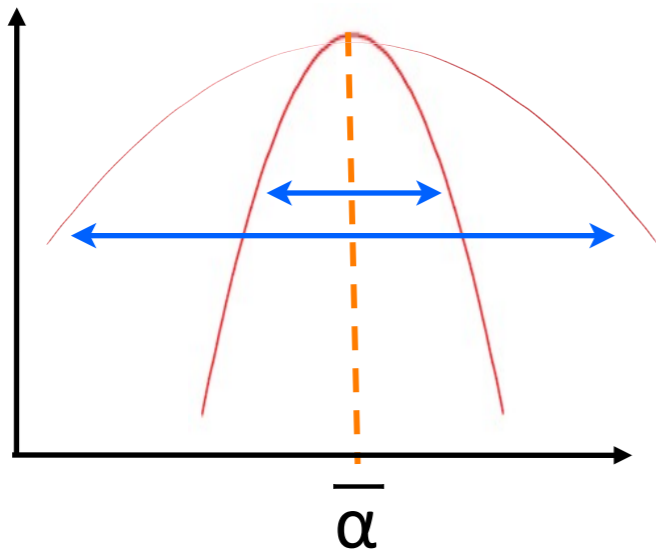


Constraint on parameters

Fisher information matrix

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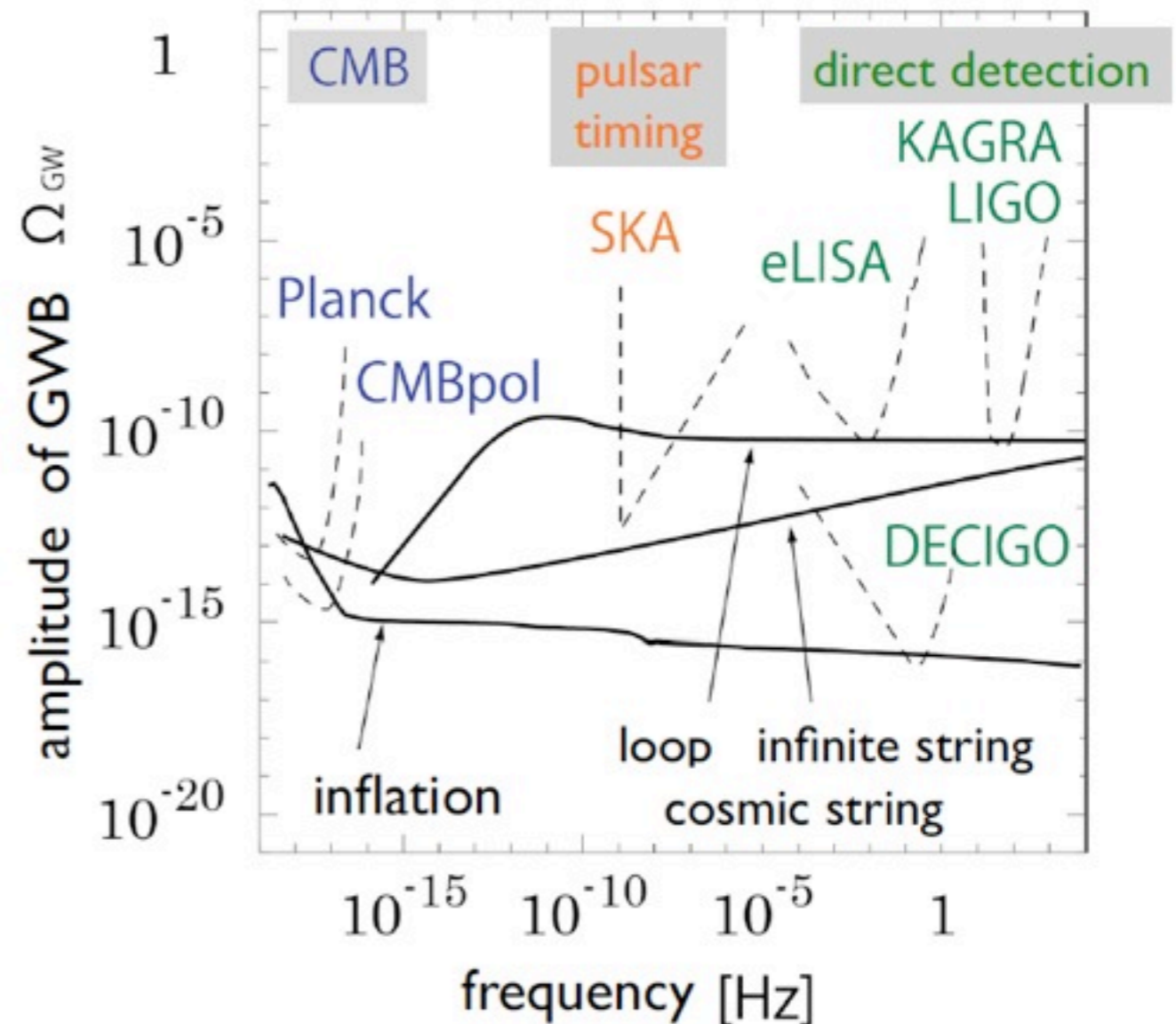
$$\mathcal{F}_{ij} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle$$



If the likelihood shape is sensitive to the parameter
= easy to estimate the parameter

GWB Observable : Ω_{GW}

$$\mathcal{F}_{ij} \propto \frac{\partial \Omega_{\text{GW}}}{\partial p_i} \frac{\partial \Omega_{\text{GW}}}{\partial p_j}$$

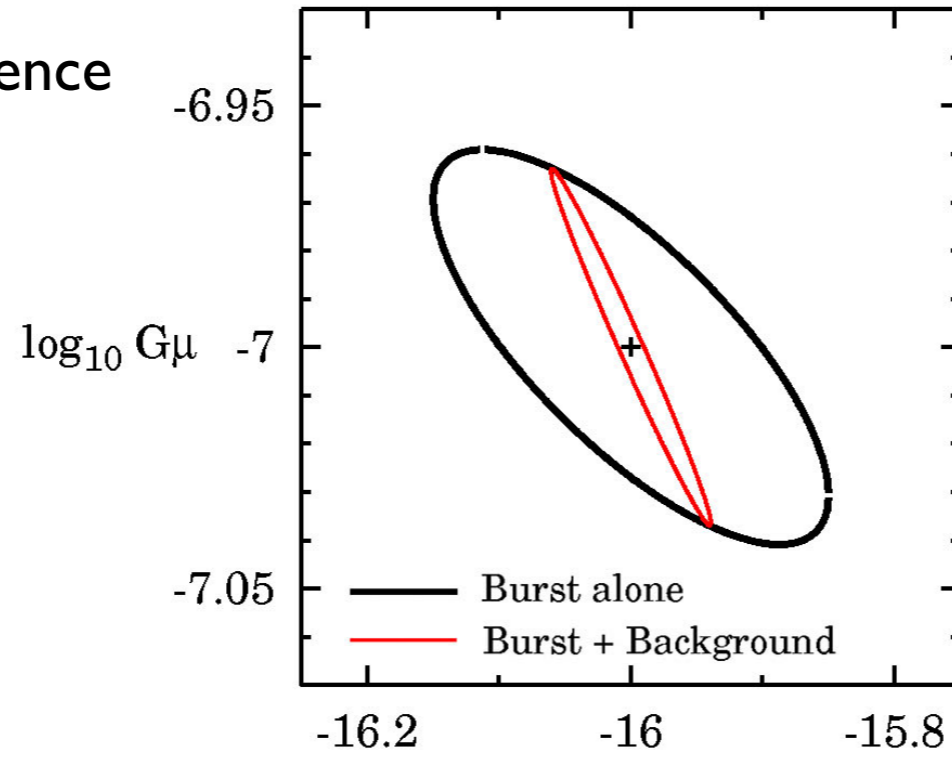


Predicted constraint on parameters

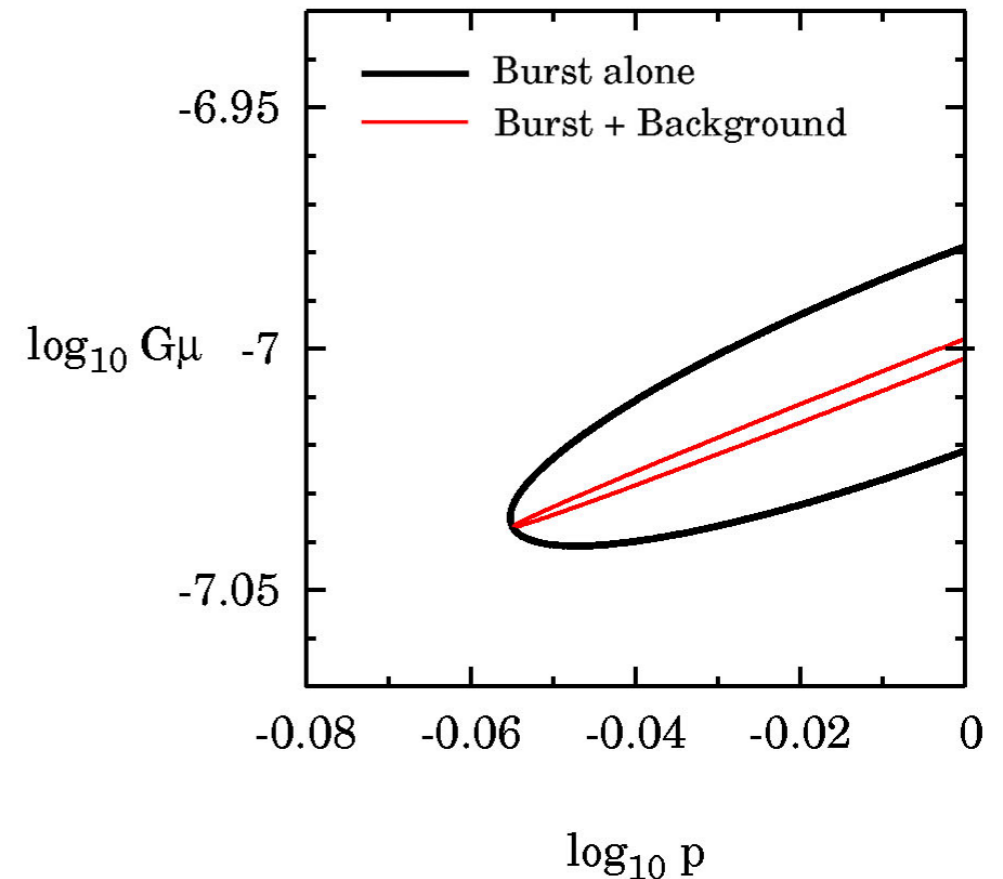
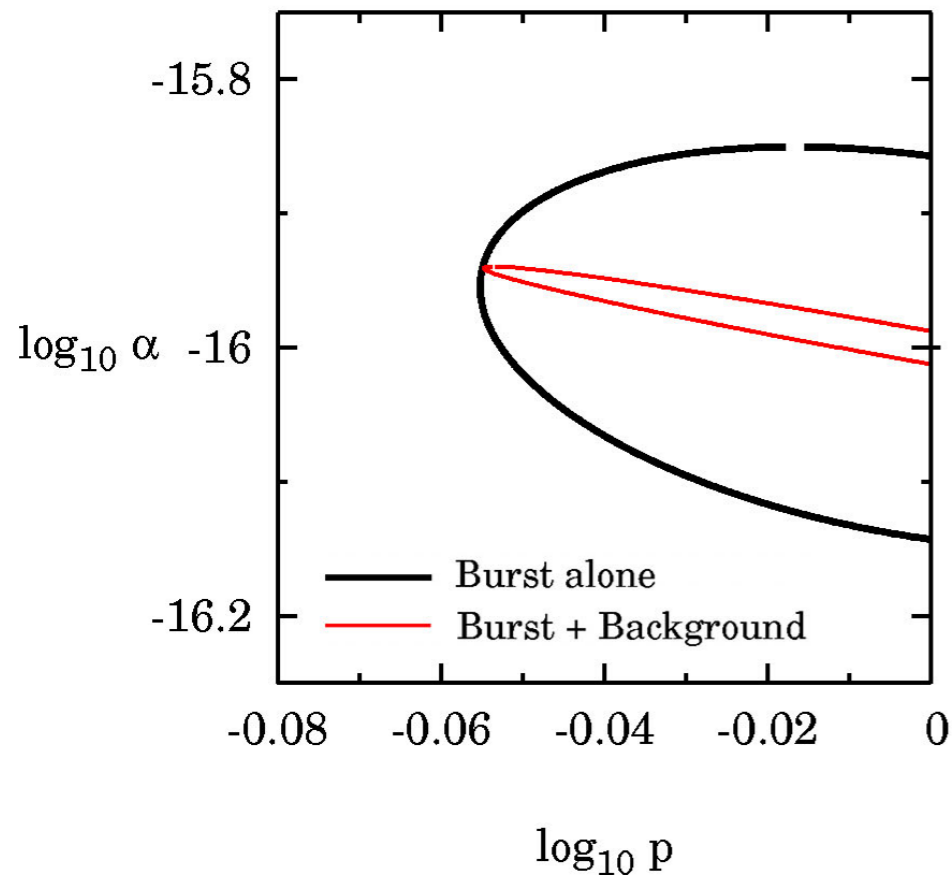
$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year

Kuroyanagi et. al. PRD 86, 023503 (2012)

different parameter dependence
= different constraints on
parameters



black : Burst only
red : Burst + GWB

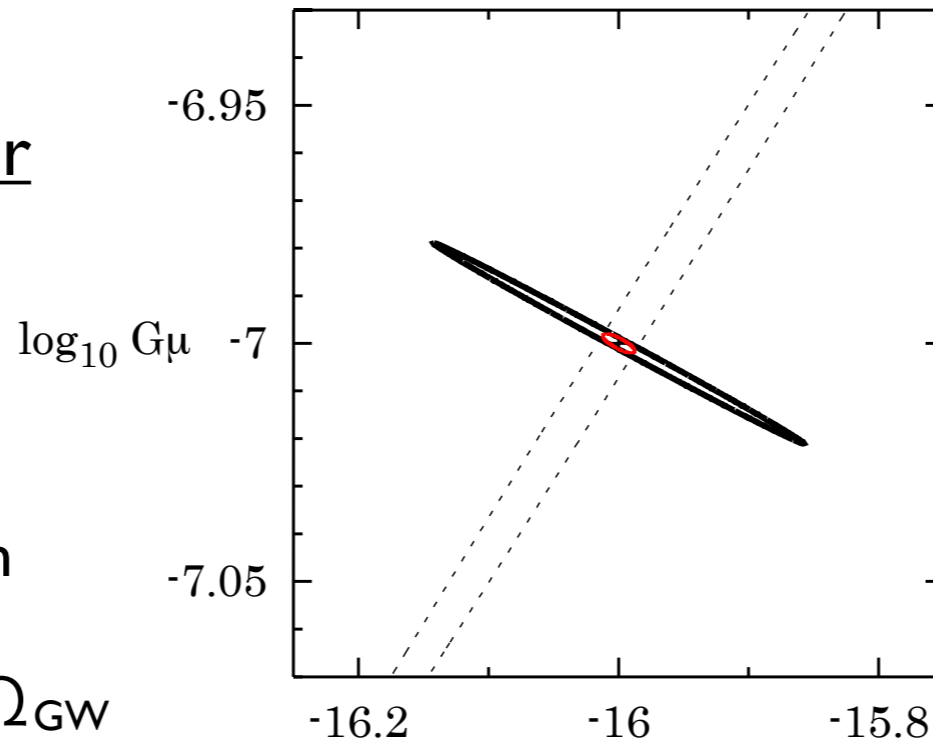


Predicted constraint on parameters

$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year

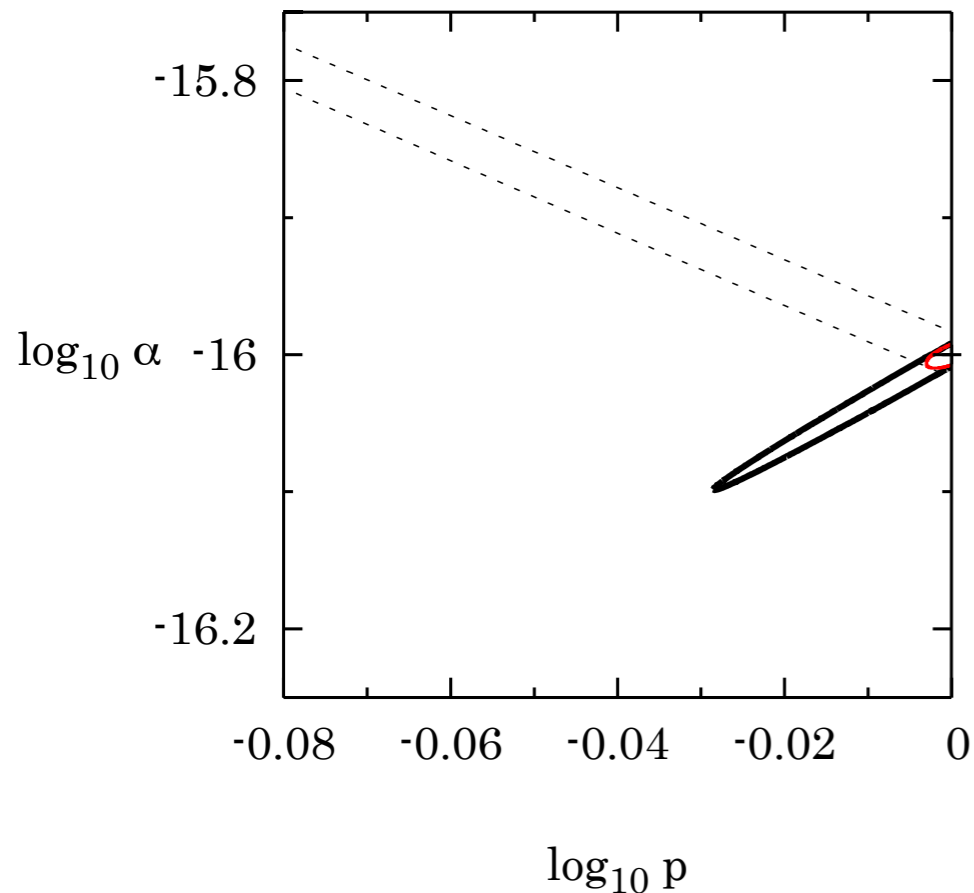
Kuroyanagi et. al. PRD 86, 023503 (2012)

Before marginalized over

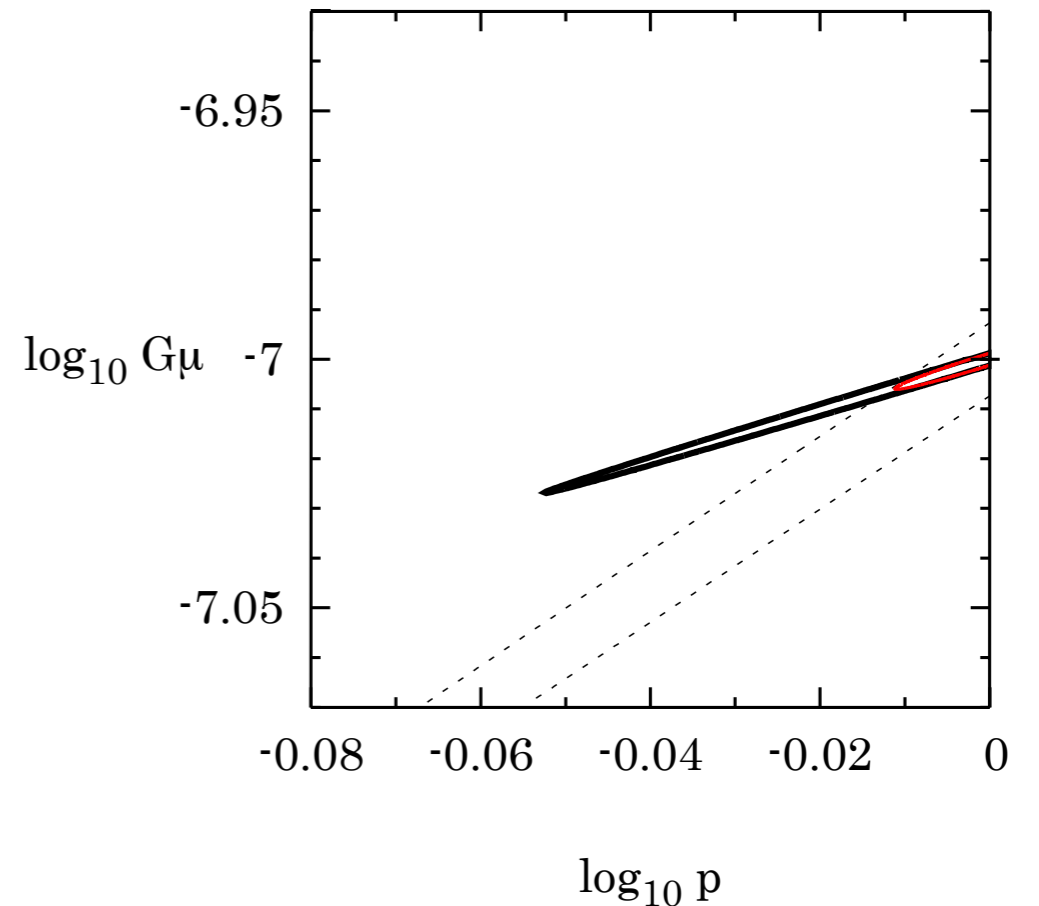


black : Burst only
dotted: GWB only
red : Burst + GWB

Strong degeneracy seen in
constraint from GWB
since the observable is only Ω_{GW}



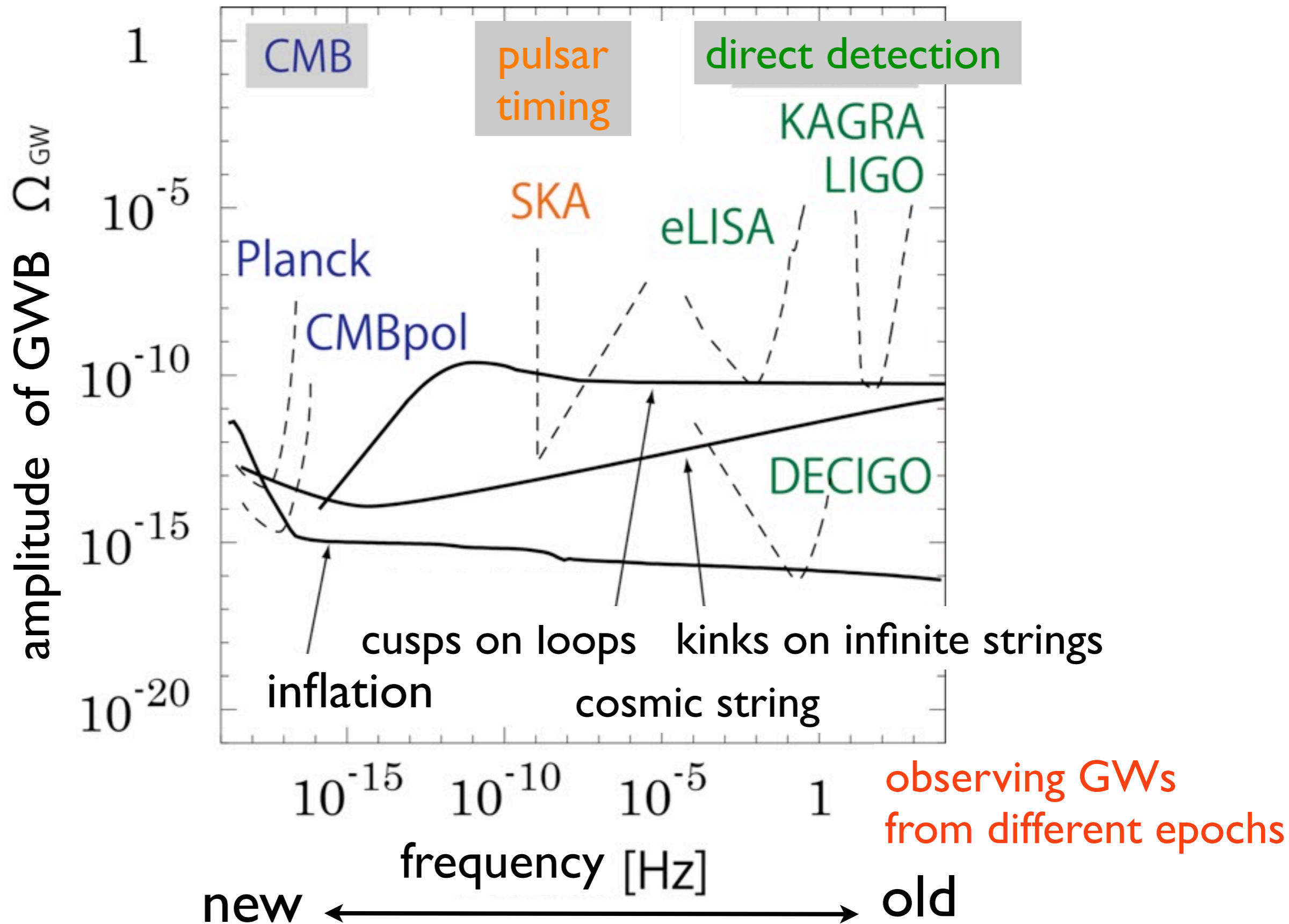
$\log_{10} \alpha$



$\log_{10} G\mu$

$\log_{10} p$

Constraints from other experiments?

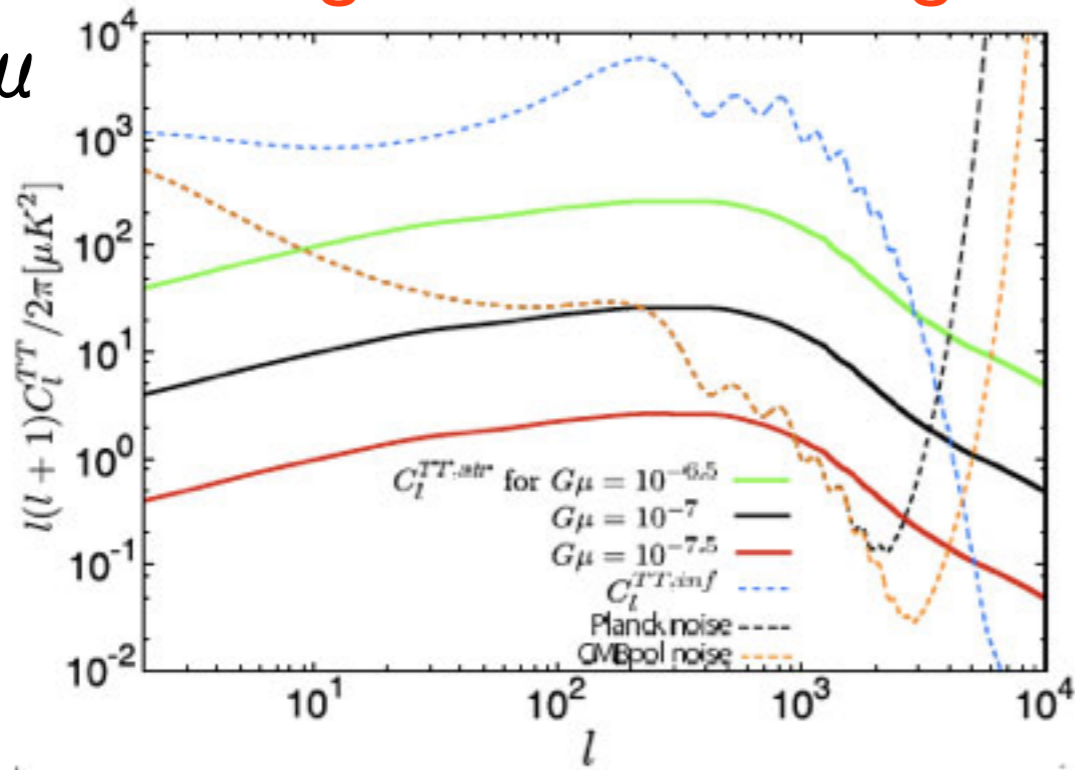


CMB signals

Temperature

string motion + lensing

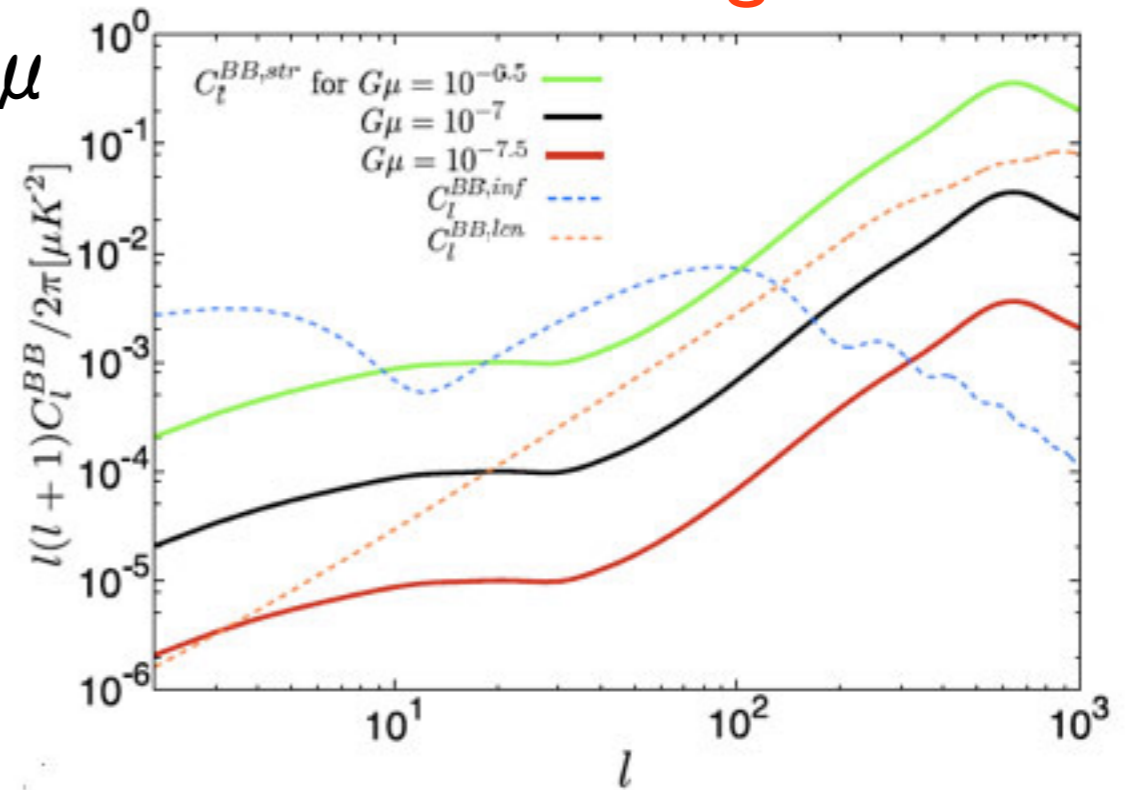
$G\mu$



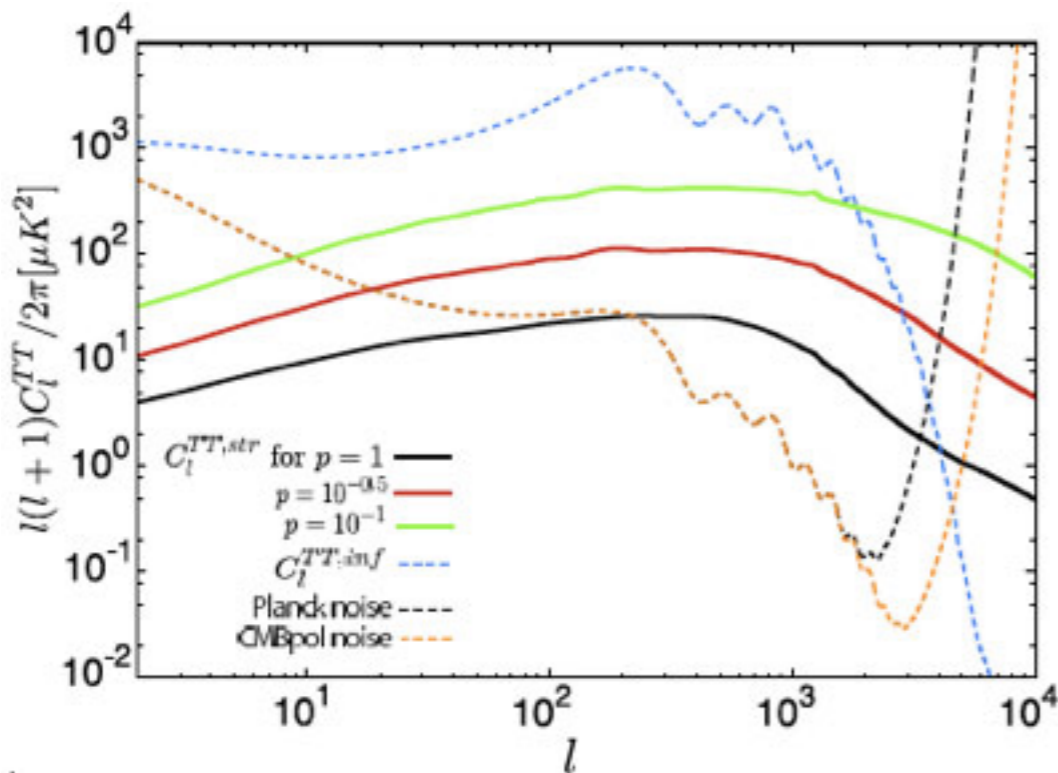
B-mode

Note: lensing > GW

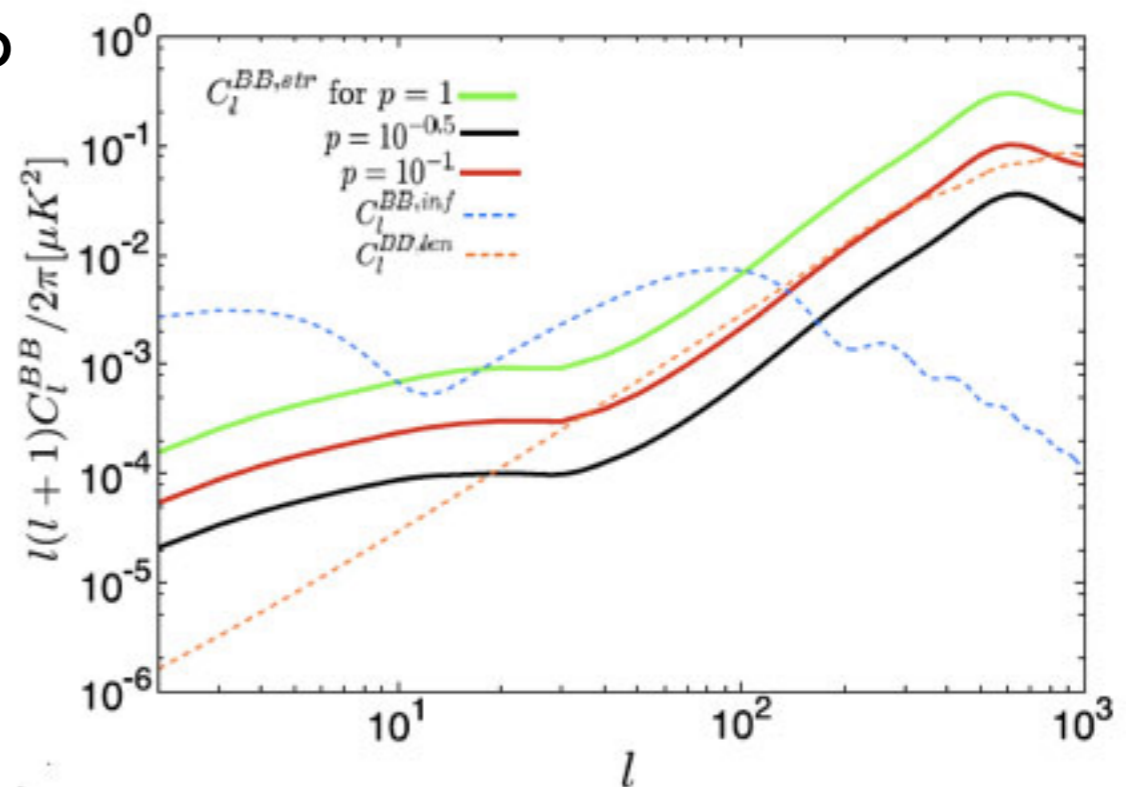
$G\mu$



p



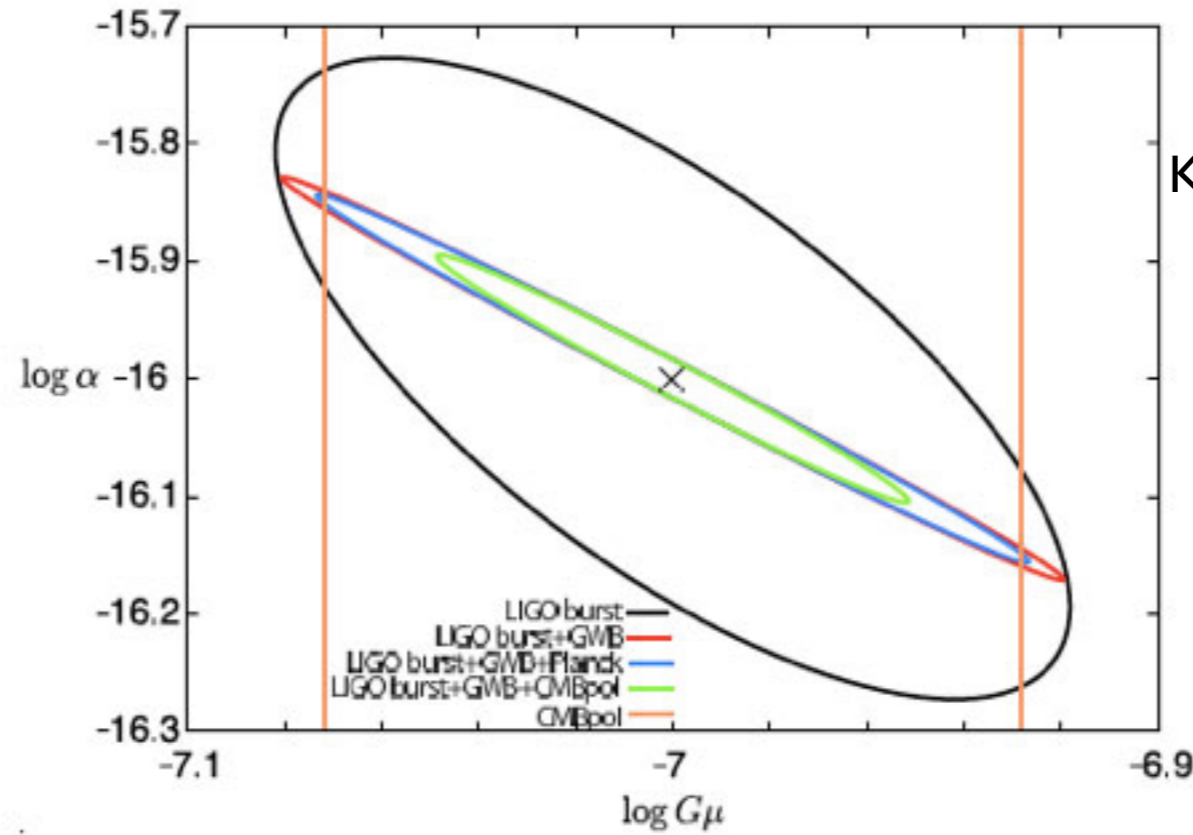
p



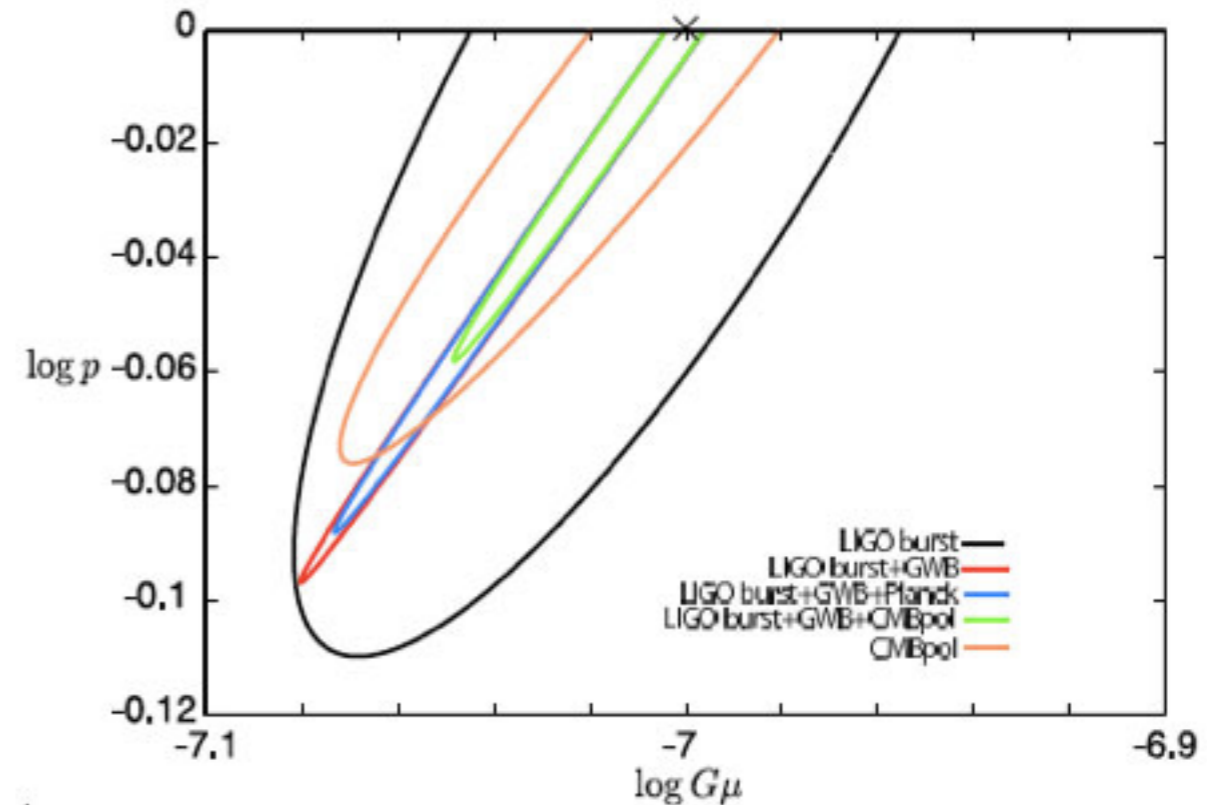
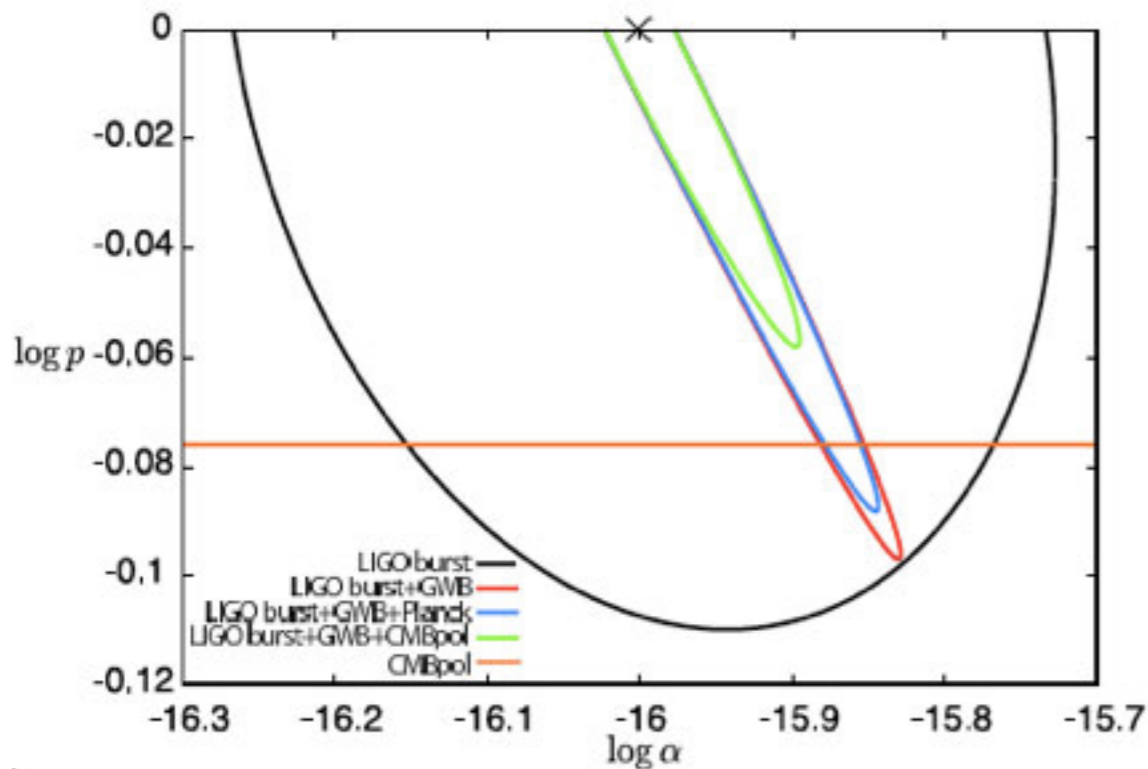
If we combine CMB constraints...

$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year
+ CMB B-mode

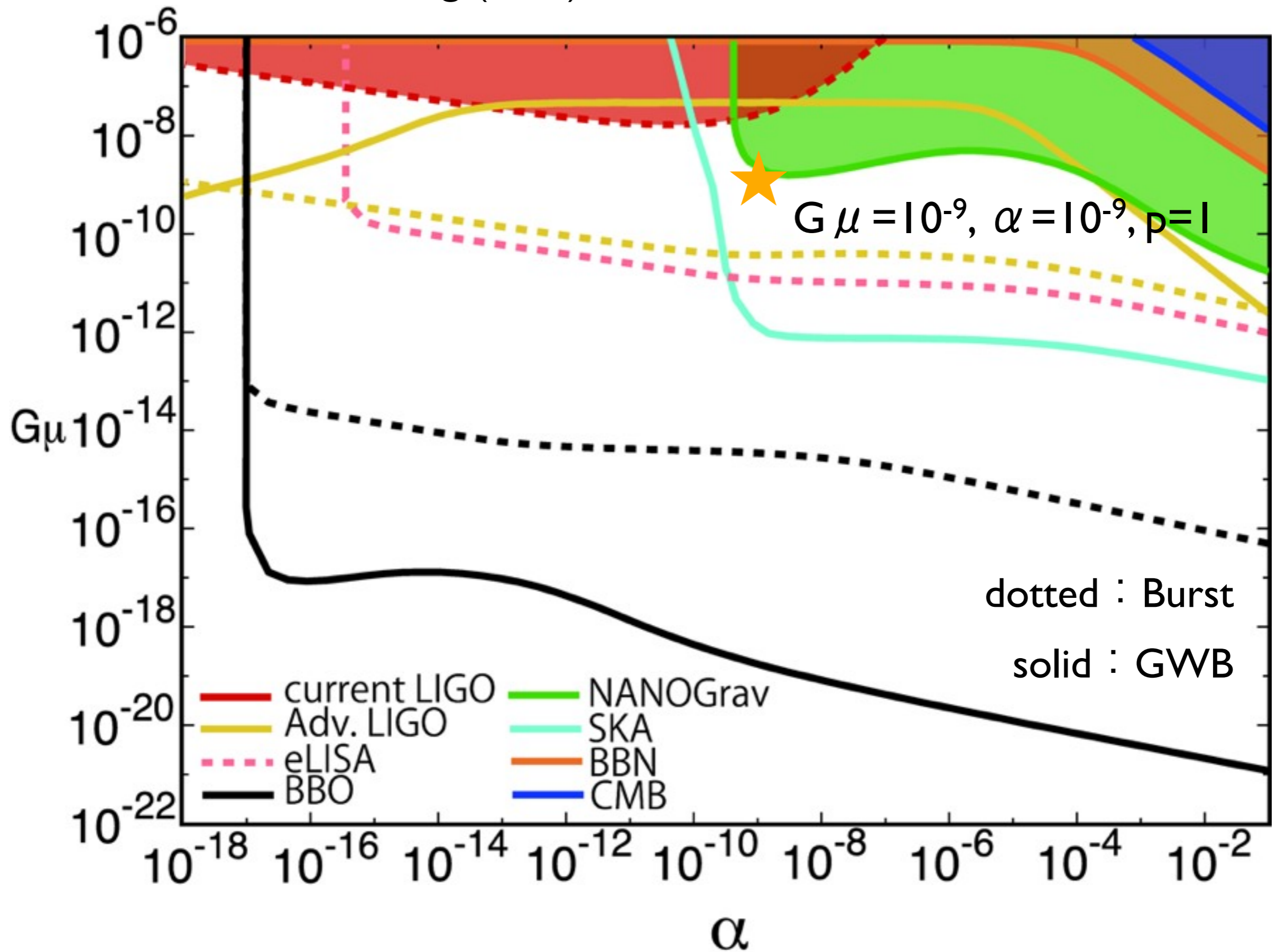
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black : LIGO Burst only
red : LIGO Burst + GWB
blue: LIGO + Planck
green: LIGO+CMBpol
orange: CMB pol only



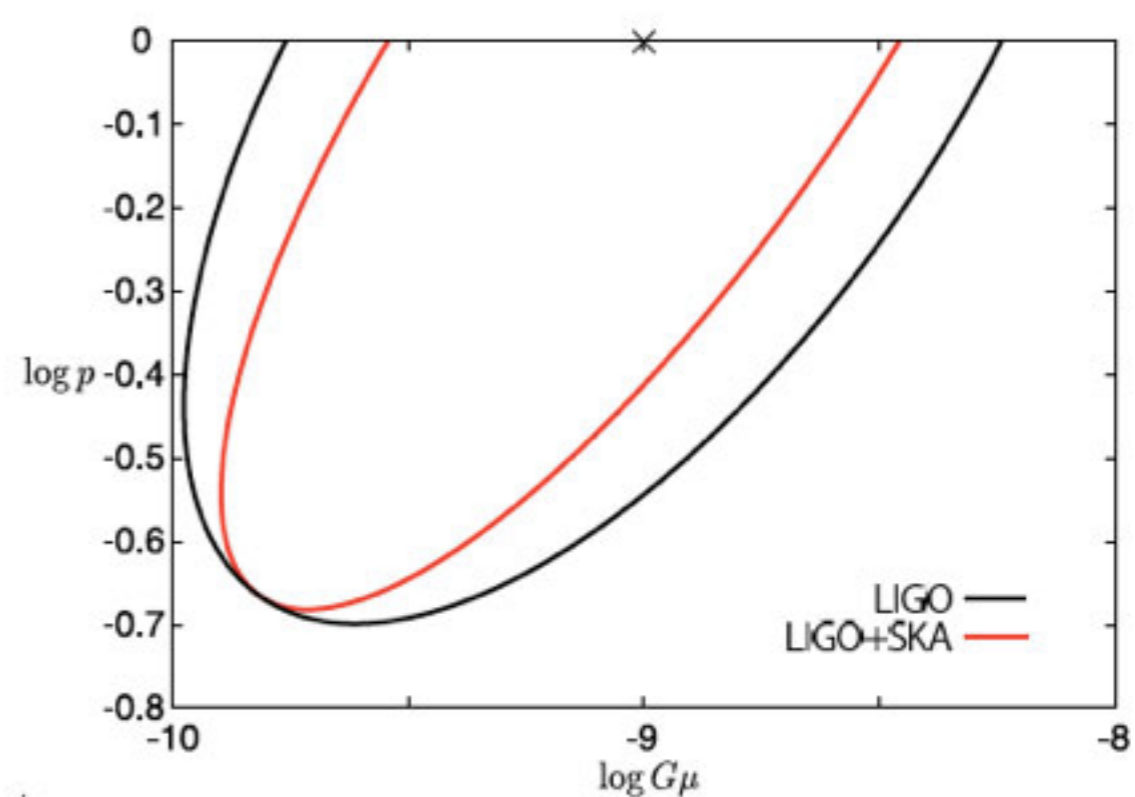
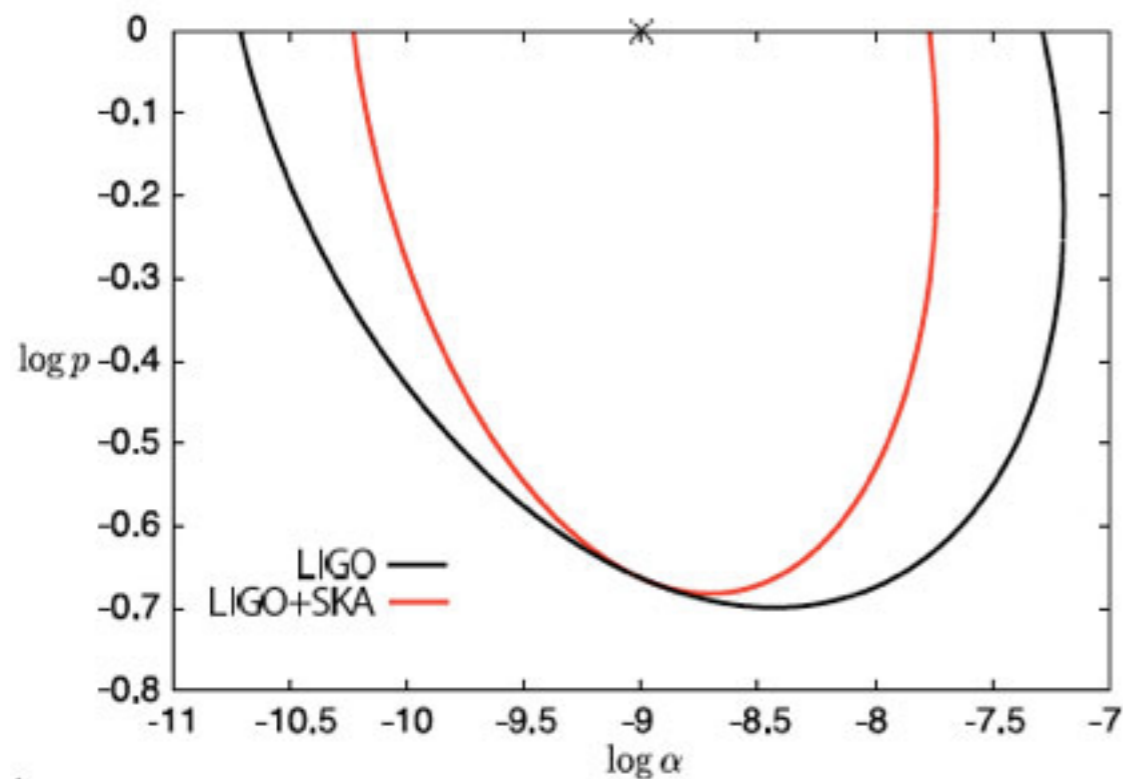
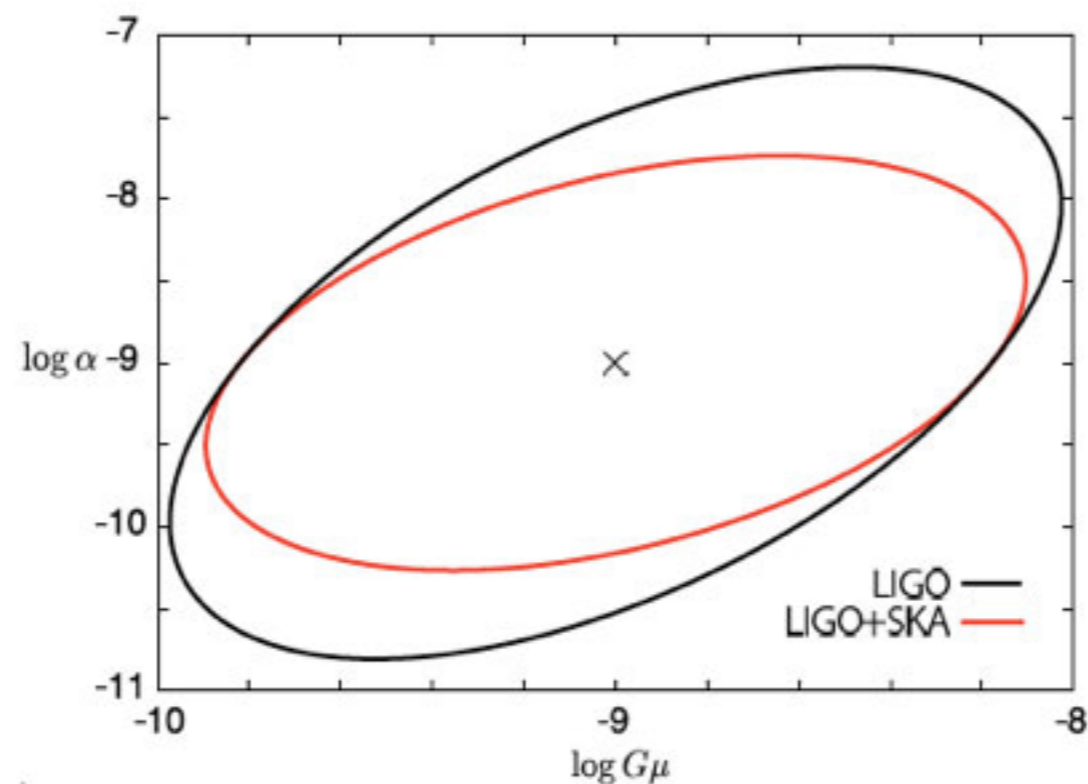
Pulsar timing (SKA) + Advanced-LIGO burst search



Direct detection + Pulsar timing

$G\mu = 10^{-9}$, $\alpha = 10^{-9}$, $p=1$
LIGO 3year (burst only)
+ SKA 10year

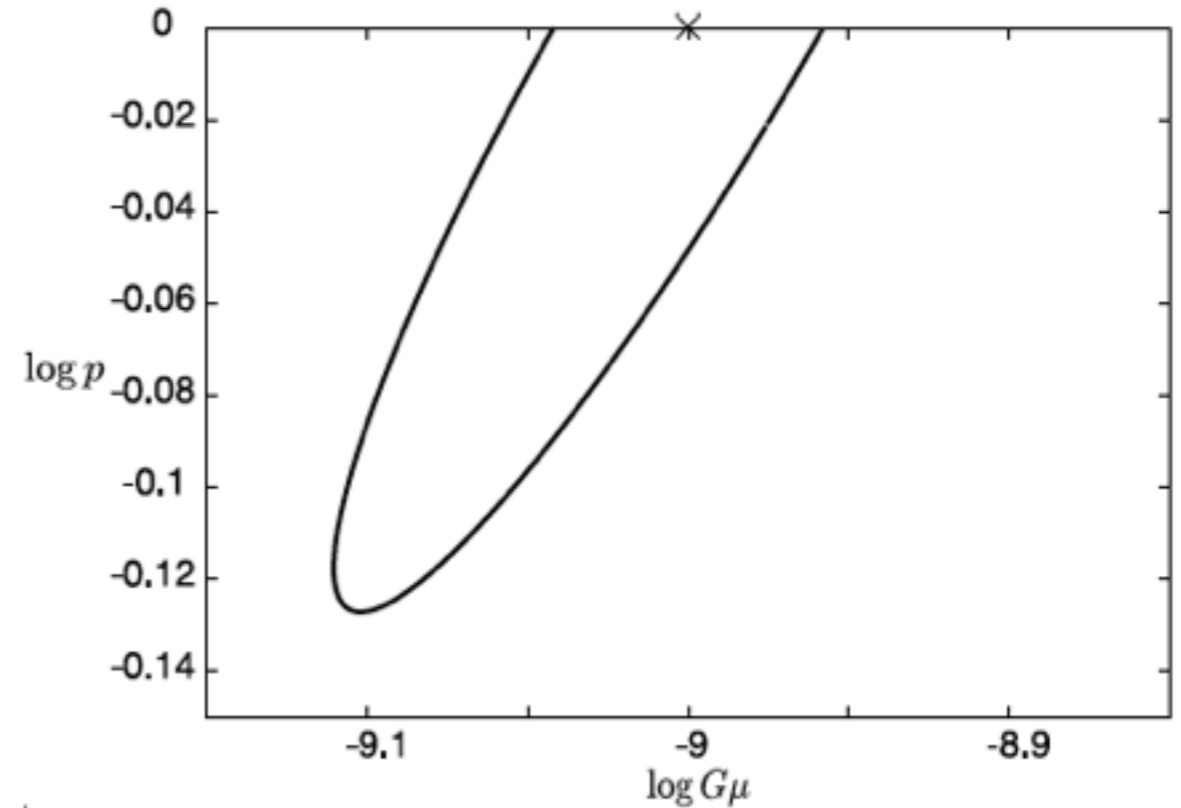
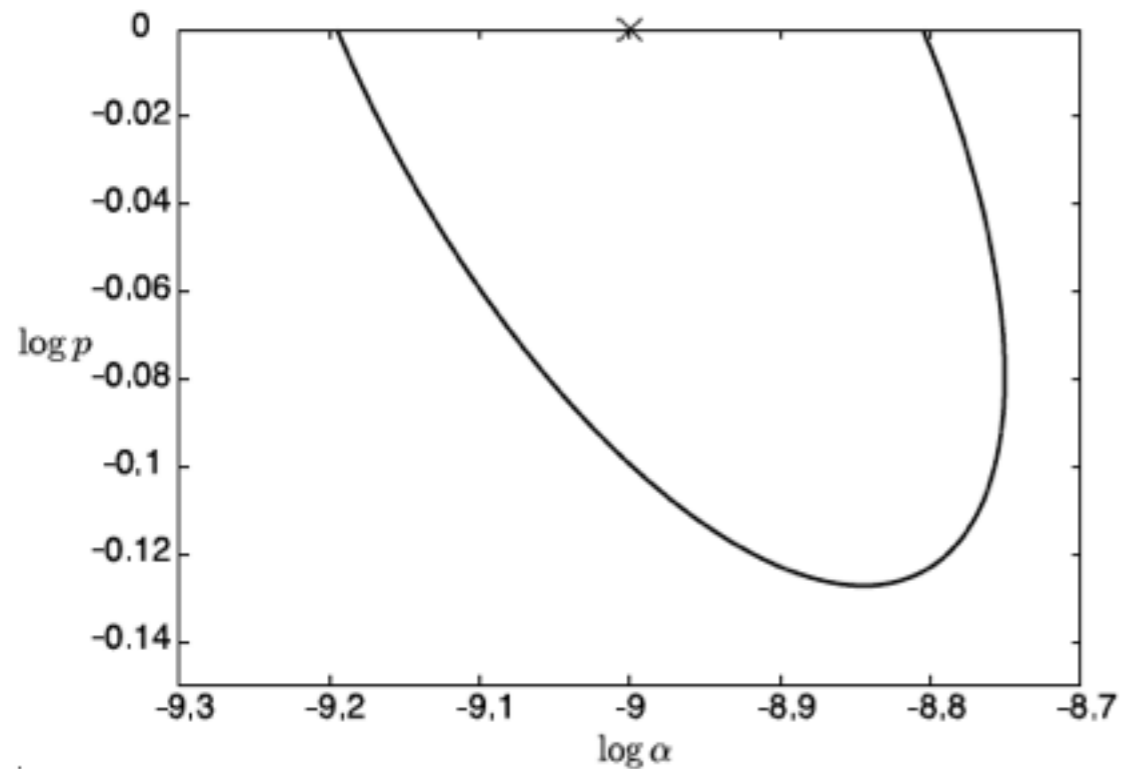
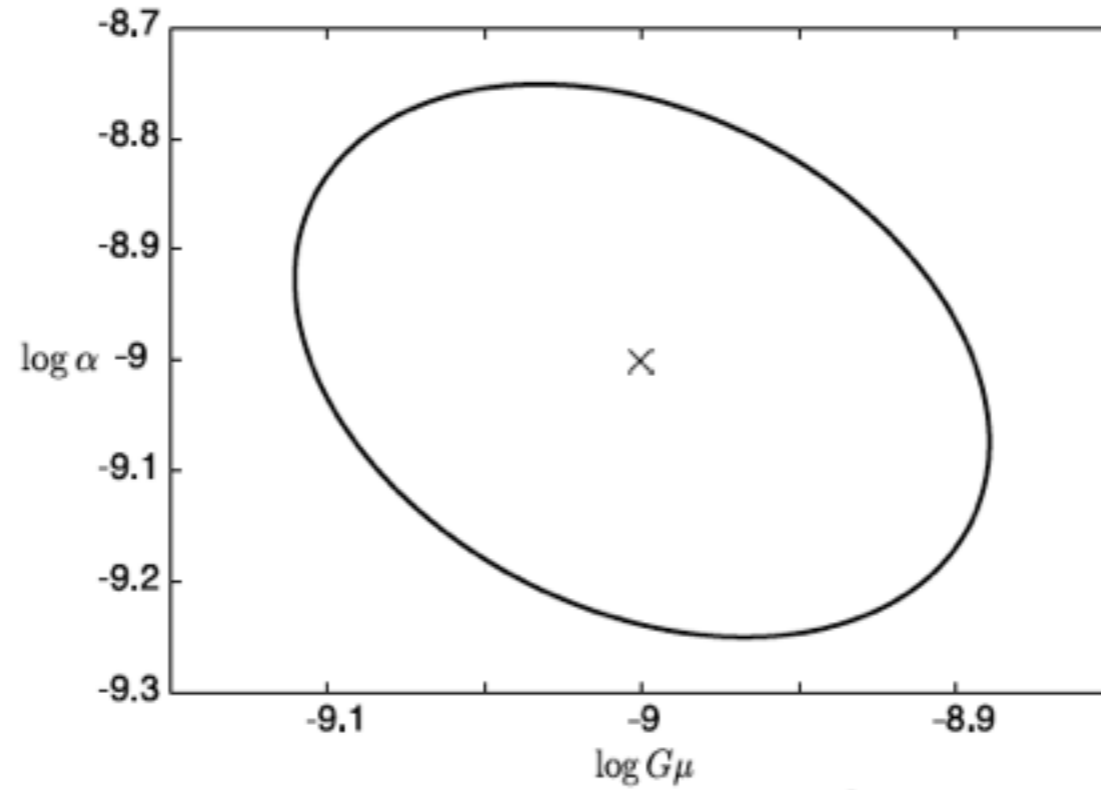
Kuroyanagi et. al. PRD 87, 023522 (2013)



Parameter constraint by eLISA

$G\mu = 10^{-9}$, $\alpha = 10^{-9}$, $p=1$
eLISA 3year
(burst only)

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Summary

- Future GW experiments can be a powerful tool to probe cosmic strings.
- It could provide strong constraints on cosmic string parameters. If it is detected, it would determine cosmic string parameters, which can provide us with hints of fundamental physics such as particle physics or superstring theory.
- Two different kinds of GW observation (rare burst and GWB) provide different constraints on cosmic string parameters and lead to better accuracy in determining parameters.
- Combination with CMB or Pulsar timing also helps to get stronger constraints, depending on the value of the parameters.
- Space GW missions are more powerful to probe cosmic strings.