Gravitational waves from Cosmic strings

Sachiko Kuroyanagi (Tokyo U. of Science)

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Based on
S. Kuroyanagi, K. Miyamoto, T. Sekiguchi, K. Takahashi, J. Silk,
PRD 86, 023503 (2012) and PRD 87, 023522 (2013)
Cosmic string?

One dimensional topological defects generated in the early universe

- have infinite length (= no end point)
- can be a form of loop
- affect matters only through gravitational force
- emits gravitational waves

\[ \mu : \text{tension (line density)} \]
\[ G \mu = \frac{\mu}{m_{\text{pl}}^2} \]
Generation mechanism 1: phase transition

The Universe has experienced symmetry breakings.

If you consider U(1) symmetry breaking...

High energy vacuum remains at the center

Tension $G \mu \sim$ the energy scale of the phase transition
Generation mechanism 2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation in superstring theory

**Difference from phase transition origin**

- reconnection probability: $p$
  - D-string: $p=0.1-1$
  - F-string: $p=10^{-3}-1$

- can have broad values of $G \mu$
- have Y junction
- mixed network of different $G \mu$

→ Cosmic strings provides insight into fundamental physics
Evolution of cosmic strings

energy density of cosmic strings
\[ \sim \frac{\text{line density} \times \text{length}}{\text{volume}} \]

constant \[ \propto a^{-2} \]

\[ \propto a^{-1} \quad \propto a^{-3} \]
radiation \[ \propto a^{-4} \]
matter \[ \propto a^{-3} \]

\[ L \propto a^1 \]
\[ V \propto a^3 \]

\[ \rightarrow \text{easily dominates the energy density of the Universe.} \]
\[ \text{not allowed by observation?} \]
Evolution of cosmic strings

**Scaling law**

$O(1)$ infinite strings in the Hubble horizon

Cosmic strings become loops via reconnection.
Loops lose energy by emitting gravitational waves.
Evolution of cosmic strings

**Scaling law**

O(1) infinite strings in the Hubble horizon

- Increase of infinite string length by the horizon growth
- Loss of infinite string length by generation of loops

- Higher reconnection rate
- More efficient generation of loops
- More energy release by the emission of GWs
Gravitational waves from cosmic strings

Strong GW emission from singular points called kinks and cusps

Rare Burst: GWs with large amplitude coming from close loops

Gravitational wave background (GWB): superposition of small GWs coming from the early epoch
Observations of GWs

GWs with large amplitude

GWs with small amplitude but numerous

Burst

GWB

Cross correlation analysis
Cross correlate the signals from two or more detector and extract stable GWs

→ provide different information on cosmic strings
Gravitational wave experiments

- Direct detection
  - Ground: Advanced-LIGO, KAGRA, Virgo, IndIGO
  - Space: eLISA/NGO, DECIGO

- Pulsar timing: SKA

- CMB B-mode polarization: Planck, CMBpol


PTA image (NRAO)

KAGRA image (http://gwcenter.icrr.u-tokyo.ac.jp/)

eLISA image (http://elisa-ngo.org/)

WMAP Three Year Polarized CMB Sky (http://wmap.gsfc.nasa.gov/)
Current constraints on cosmic string parameters

3 parameters to characterize cosmic string

- $G\mu (= \mu / M_{pl}^2)$: tension (line density)
- $\alpha$: initial loop size $L \sim \alpha H^{-1}$
- $p$: reconnection probability

- CMB temperature fluctuation: $G\mu < \sim 10^{-7}$
- Gravitational lensing: $G\mu < \sim 10^{-6}$ for infinite strings

- **Gravitational waves**
  - Pulsar timing: $G\mu < \sim 10^{-9}$ for loops $\alpha = 0.1$, $p = 1$
  - Direct detection (LIGO GWB & burst): $G\mu < \sim 10^{-6}$

What about future constraints?
Estimation of the GW burst rate

Initial number density of loops

\[ \frac{\text{length of infinite string going to loops}}{\text{initial length of loops} = \alpha t_i} \]

Evolution of infinite strings
- velocity-dependent one-scale model

\[ 2 \frac{dL}{dt} = 2HL(1 + v^2) + cv \]

energy conservation

energy goes to loops

\[ \frac{dv}{dt} = (1 - v^2) \left( \frac{k}{R} - 2Hv \right) \]

acceleration due to the curvature of the strings

Evolution of infinite strings
- random walk of straight strings

length L, velocity v

\[ \frac{d\rho_{\text{inf}}}{dt} \bigg|_{\text{loop}} = cv\frac{\rho_{\text{inf}}}{L} \]

for small p: \( c \to cp \)

damping due to the expansion

momentum parameter:

\[ k = \frac{2\sqrt{2}}{\pi} \left( \frac{1 - 8v^6}{1 + 8v^6} \right) \]
Estimation of the GW burst rate

Initial number density of loops (naive estimate)

\[
\text{Number of loops} = \frac{(\text{length to lose})}{(\text{initial length of loops})} = \frac{p^{-1} t}{\alpha t} = \frac{1}{p \alpha}
\]

Scaling law

O(1) infinite strings in the Hubble horizon

To satisfy the scaling law...

infinite strings
should lose O(1) Hubble length per 1 Hubble time \(\rightarrow\) more loops for small \(\alpha\)
= should reconnect O(1) times per Hubble time
\(\rightarrow\) for small \(p\), string density should increase to reconnect O(1) times
Estimation of the GW burst rate

Loop evolution depends on $G\mu$ and $\alpha$

**Initial loop length**

$$l_{i} = \alpha t_{i}$$

$t_{i}$: time when the loop formed

**GW power**

$$P = \Gamma G\mu^{2}$$

$\Gamma$: numerical constant $\sim 50-100$

From the energy conservation law
(energy of loop at time $t = \mu l$)

$$l(t, t_{i}) = \alpha t_{i} - \Gamma G\mu(t - t_{i})$$

**Loop length at time $t$**

**Lifetime of the loop**

$$= \frac{(\text{initial loop energy})}{(\text{energy release rate per time})}$$

$$= \frac{\mu \alpha t_{i}}{\Gamma G\mu^{2}} = \frac{\alpha t_{i}}{\Gamma G\mu}$$
Estimation of the GW burst rate

GW burst rate emitted at $t\sim t+dt$ from loops formed at $t_i\sim t_i+dt_i$

$$\frac{dR}{dtdt_i} = \frac{1}{4}\theta_m(f, z, l)^2 \frac{2c}{(1+z)l(t, t_i)} \frac{dn}{dt_i}(t, t_i) \frac{dV}{dt} dtdt_i \times \Theta(1 - \theta_m(f, z, l))$$

- **Beaming**
- **Time interval of GW emission**
- **Loop number**

$\theta_m \propto (\text{loop length at } t)^{1/3}$

$$(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$$

GW amplitude from loop of length $l$

$$h(f, z, l) \simeq 2.68 \frac{G \mu l}{((1+z)fl)^{1/3} r(z) f}$$
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G \mu = 10^{-7}, \alpha = 10^{-16}, p = 1 \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G \mu = 10^{-7}, \ \alpha = 10^{-16}, \ p = 1 \]

LIGO \sim 220Hz

220 oscillations per second

\[ = 7 \times 10^9 \text{ oscillations per year} \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G\mu = 10^{-7}, \alpha = 10^{-16}, p=1 \]

LIGO~220Hz

220 oscillations per second = \(7 \times 10^9\) oscillations per year

Small amplitude but numerous
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ \mu = 10^{-7}, \alpha = 10^{-16}, p = 1 \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

$G \mu = 10^{-7}, \alpha = 10^{-16}, p = 1$

$LIGO$
$h \sim 10^{-25} @ f \sim 220Hz$

rate (per year) $\downarrow$
$dR/d\log h$

GWB

rare burst
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[
G \mu = 10^{-7}, \quad \alpha = 10^{-16}, \quad p = 1
\]

LIGO
\[h \sim 10^{-25}@ f \sim 220\text{Hz}\]
The parameter dependences of the large burst (rare burst) and small burst (GWB) are different because they are looking at different epoch of the Universe → give different information on cosmic string parameters
dependence on $G\mu$

$\Omega_{GW}$

$G\mu = 10^{-8}, 10^{-10}, 10^{-12}, 10^{-14}, 10^{-16}$

$\alpha = 10^{-1}, p = 1$

GW power from cusps

$h^2 \propto (G\mu)^2$

life time of loops

$\propto (G\mu)^{-1}$
dependence on $\alpha$

loop size directly corresponds to the frequency of the GW

$\Omega_{GW}$

Spectrum of the GWB

$\mu = 10^{-8}$, $p=1$

$\alpha = 10^{-1}, 10^{-5}, 10^{-9}, 10^{-13}, 10^{-17}$

frequency [Hz]
small $p$ increases the number density of loops

dependence on $p$

$\Omega_{GW}$

$G\mu = 10^{-12}$, $\alpha = 10^{-1}$

$p = 1, 10^{-1}, 10^{-2}, 10^{-3}$

Spectrum of the GWB
Accessible parameter region (for p=1)
What if both bursts and GWB are detected by Advanced-LIGO?

\[ G_\mu = 10^{-7}, \, \alpha = 10^{-16}, \, p = 1 \]
**Constraint on parameters**

**Fisher information matrix**

log(Likelihood)

\[
\mathcal{F}_{ij} = -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle
\]

If the likelihood shape is sensitive to the parameter = easy to estimate the parameter

**Burst**

Observable : amplitude vs number

N is predictable by the rate \( \frac{dR}{dh} \)

\[
\mathcal{F}_{ij} \propto \frac{\partial (dR/dh)}{\partial p_i} \frac{\partial (dR/dh)}{\partial p_j}
\]
Constraint on parameters

Fisher information matrix

\[ F_{ij} = -\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \rangle \]

\[ F_{ij} \propto \frac{\partial \Omega_{GW}}{\partial p_i} \frac{\partial \Omega_{GW}}{\partial p_j} \]

If the likelihood shape is sensitive to the parameter = easy to estimate the parameter

GWB Observable: \( \Omega_{GW} \)
Predicted constraint on parameters

Different parameter dependence = different constraints on parameters

\[ G \mu = 10^{-7}, \alpha = 10^{-16}, p=1 \]

LIGO 3year Predicted constraint on parameters
Kuroyanagi et. al. PRD 86, 023503 (2012)

Black: Burst only
Red: Burst + GWB
Predicted constraint on parameters

Before marginalized over

Strong degeneracy seen in constraint from GWB since the observable is only $\Omega_{GW}$

$LIGO$ 3year

$G \mu = 10^{-7}, \alpha = 10^{-16}, p = 1$

Kuroyanagi et. al. PRD 86, 023503 (2012)

black : Burst only

dotted: GWB only

red : Burst + GWB
Constraints from other experiments?

- CMB
- pulsar timing
- direct detection
- KAGRA
- LIGO
- SKA
- eLISA
- Planck
- CMBpol
- DECIGO

amplitude of GWB $\Omega_{GW}$

- $10^{-5}$
- $10^{-10}$
- $10^{-15}$
- $10^{-20}$

observing GWs from different epochs

old

new

frequency [Hz]
CMB signals

Temperature

string motion + lensing

B-mode

Note: lensing > GW
If we combine CMB constraints...  

\[ G \mu = 10^{-7}, \ \alpha = 10^{-16}, \ p = 1 \]

LIGO 3 year + CMB B-mode

Kuroyanagi et. al. PRD 87, 023522 (2013)

black: LIGO Burst only
red: LIGO Burst + GWB
blue: LIGO + Planck
green: LIGO + CMBpol
orange: CMB pol only
Pulsar timing (SKA) + Advanced-LIGO burst search

$G \mu = 10^{-9}, \alpha = 10^{-9}, p=1$

dotted: Burst
solid: GWB

$G_\mu$ vs $\alpha$ graph with various lines and markers indicating different experimental sensitivities and theoretical predictions.
Direct detection + Pulsar timing

\[ G \mu = 10^{-9}, \alpha = 10^{-9}, p=1 \]

LIGO 3year (burst only) + SKA 10year

Kuroyanagi et. al. PRD 87, 023522 (2013)
Parameter constraint by eLISA

$G \mu = 10^{-9}$, $\alpha = 10^{-9}$, $p=1$

eLISA 3 year (burst only)

Kuroyanagi et al. PRD 87, 023522 (2013)
Summary

- Future GW experiments can be a powerful tool to probe cosmic strings.

- It could provide strong constraints on cosmic string parameters. If it is detected, it would determine cosmic string parameters, which can provide us with hints of fundamental physics such as particle physics or superstring theory.

- Two different kinds of GW observation (rare burst and GWB) provide different constraints on cosmic string parameters and lead to better accuracy in determining parameters.

- Combination with CMB or Pulser timing also helps to get stronger constraints, depending on the value of the parameters.

- Space GW missions are more powerful to probe cosmic strings.