Interactions between Inflaton and gauge fields

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Introduction

- Inflation as the standard paradigm of cosmology

$$\epsilon = -\frac{H'}{H^2} \sim \frac{1}{2}\left(\frac{V_\varphi}{V}\right)^2 \ll 1$$

- Interaction of the scalar field with the rest of the world is a key to uncovering its identity

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V + \mathcal{L}_{\rm i}[\varphi, \cdots]$$

Introduction

- Multiple scalar field models have been well-studied

$$\mathcal{L}_i = \mathcal{L}_{\mathrm{multi}}[\varphi, \psi_1, \psi_2, \cdots]$$

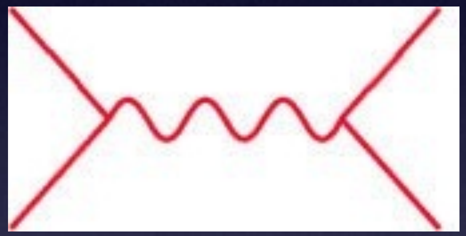
- Why not with gauge fields?

$$\mathcal{L}_i = \mathcal{L}_{\mathrm{gauge}}[\varphi, A_\mu]$$

Introduction

- One of the motivations to consider multiple scalar was interest in non-Gaussianity

$$\mathcal{L}_i \propto \varphi^2 \psi \quad \Rightarrow$$



- Potentially novel phenomenology with gauge fields; e.g. anisotropy, gravitational wave

$$\mathcal{L}_i \propto \varphi \partial A \partial A \quad \Rightarrow \quad ?$$

Introduction

- Earliest attempts to drive inflation by vector fields failed

$$\mathcal{L} = -\frac{1}{4}F^2 - V(A^2) \Rightarrow \text{Instabilities e.g. ghosts}$$

- Gauge symmetry Imperative!

Introduction

- In this talk, consider two types of action

$$\mathcal{L}_{\rm g-k} = -\frac{1}{4} f(\varphi)^2 F_{\mu\nu} F^{\mu\nu} \quad \text{or}$$

$$\mathcal{L}_{\rm C-S} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4}\lambda(\varphi) F_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}$$

- Look at particular scenarios with U(1) and SU(2)

Test field dynamics

- Examine behaviour of gauge fields in a single scalar background

$$\varphi'' + 3H\varphi' + m^2\varphi = 0$$

$$3M_{pl}^2 H^2 = \frac{1}{2}\varphi'^2 + \frac{1}{2}m^2\varphi^2 \sim \frac{1}{2}m^2\varphi^2 \ , \quad \frac{M_{pl}^2}{\varphi^2} \ll 1$$

- For the rest of talk, $M_{pl} = 1$

U(1) gauge kinetic

- General U(1) potential

$$A_\mu = (A_0, \mathbf{A})$$

- Equations of motion in Coulomb gauge

$$a^2 \left(\partial_t + H + \partial_t \ln f^2\right) \partial_t \mathbf{A} - \nabla^2 \mathbf{A} = 0$$

U(1) gauge kinetic

- Rewrite in terms of energy density

$$a\left(\partial_t + 4H + \partial_t \ln f^2\right)\rho_E + f^2 \mathbf{E}\cdot \mathrm{curl}\ \mathbf{B} = 0$$

$$a\left(\partial_t + 4H - \partial_t \ln f^2\right)\rho_B - f^2 \mathbf{B}\cdot \mathrm{curl}\ \mathbf{E} = 0$$

- Note

$$\rho_E = \frac{1}{2} f^2 E^2\ , \quad \rho_B = \frac{1}{2} f^2 B^2$$

- Gauge field can survive inflation if

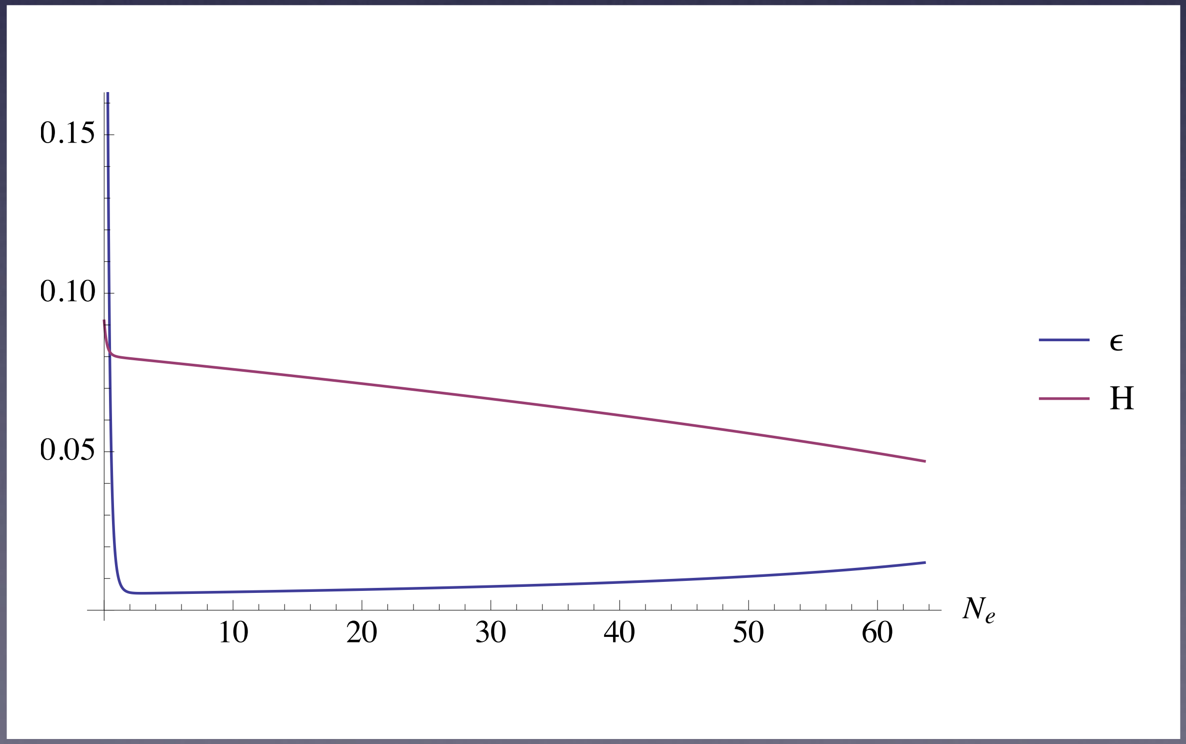
$$\left(\partial_t \ln f^2\right)^2 > 16H^2 \Leftrightarrow f^2 \propto a^{\mp(p+4)}\ , \quad p > 0$$

U(1) gauge kinetic

- Can be realised with exponential

$$f(\varphi)^2 = f_0^2 e^{\kappa\varphi} \ , \quad \kappa > 2\sqrt{\frac{2}{\epsilon}}$$

- Numerical results for $\kappa = \pm 50 \ , m = 10^{-2} \ , \epsilon \sim 10^{-2}$



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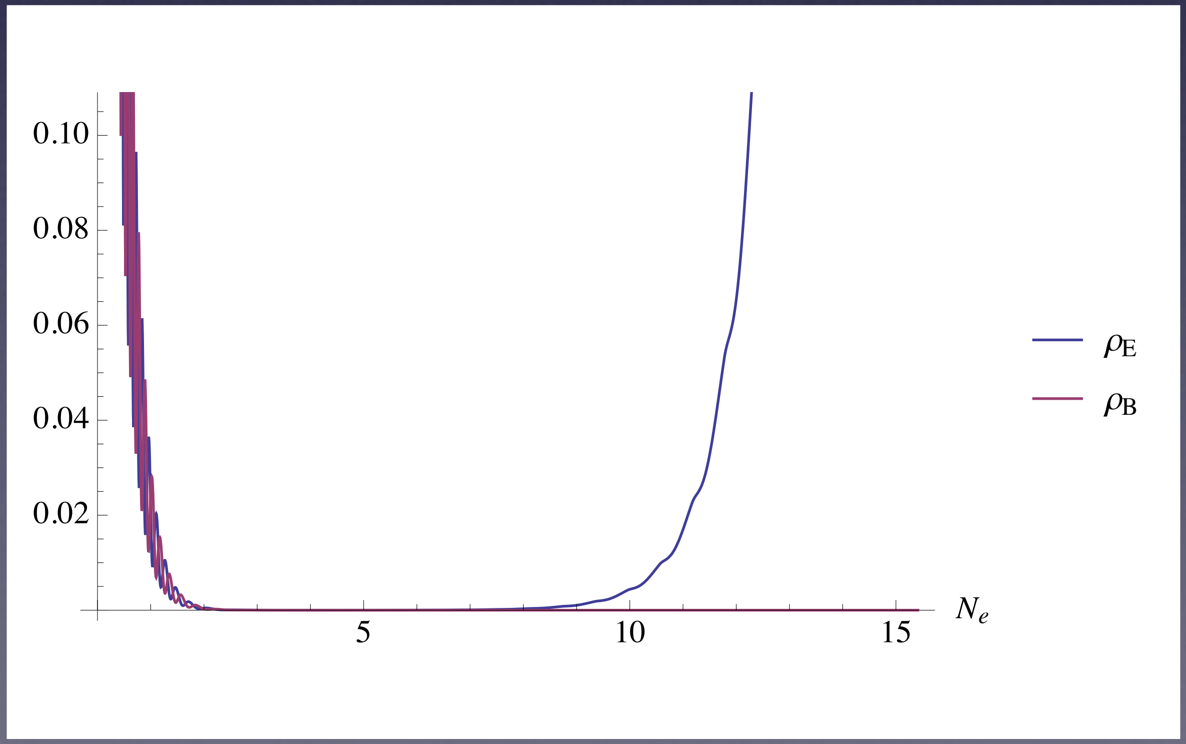

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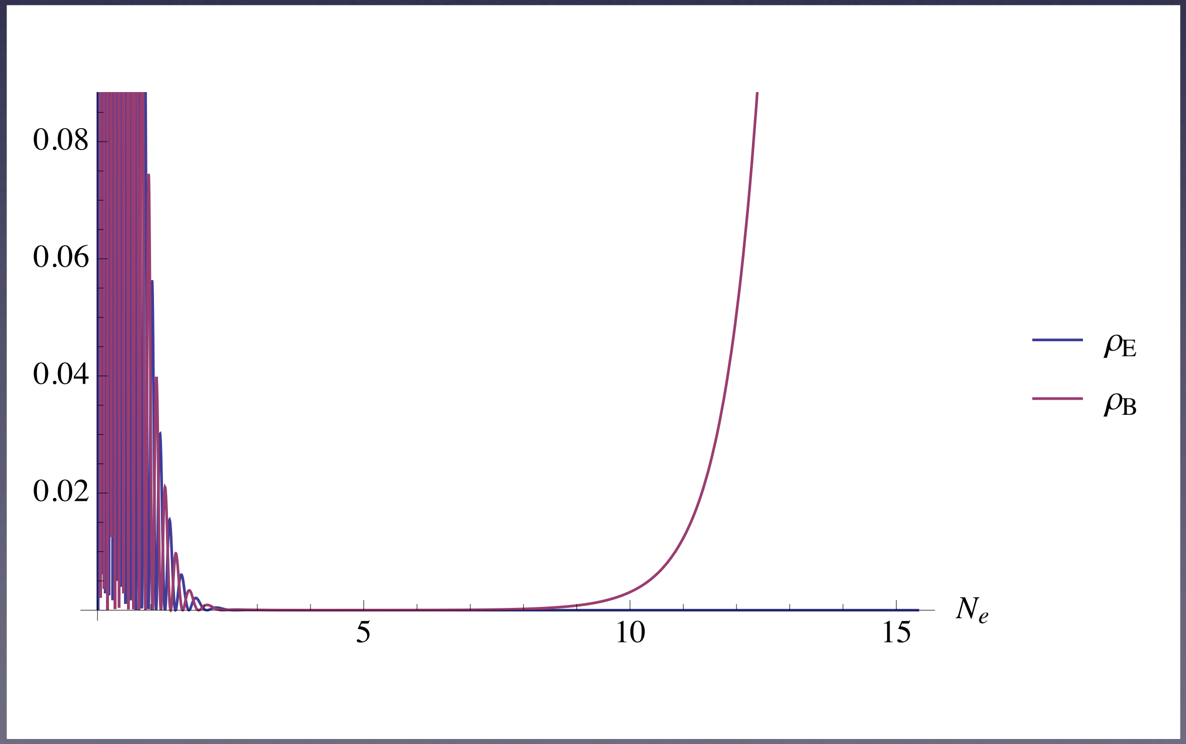


SU(2) gauge kinetic

- Three U(1) + nonlinear interactions $A_\mu = A_\mu^a \frac{\sigma^a}{2}$
- Diagonal setting

$$\mathbf{A}^1 = (A_1, 0, 0)\ , \quad \mathbf{A}^2 = (0, A_2, 0)\ , \quad \mathbf{A}^3 = (0, 0, A_3)$$

- Ignoring spatial dependence

$$a\left(\partial_t + 2H + (\ln f^2)'\right) E_1 + g\left(A_3 B_2 + A_2 B_3\right) = 0\ ,$$

$$a\left(\partial_t + 2H\right) B_1 - g\left(A_3 E_2 + A_2 E_3\right) = 0 \quad \text{etc.}$$

- Instability driven by U(1) contribution

SU(2) gauge kinetic

- When electric $f^2 \propto a^{-(4+p)}$

$$E \propto a^{2+p} \; , B \propto a^{-2} \; ,$$

$$A \sim H^{-1} aE \propto a^{3+p} \; , \quad a^{-1} AB \propto a^{1+p}$$

- Electric -> U(1) approximation breaks down

- When magnetic $f^2 \propto a^{4+p}$

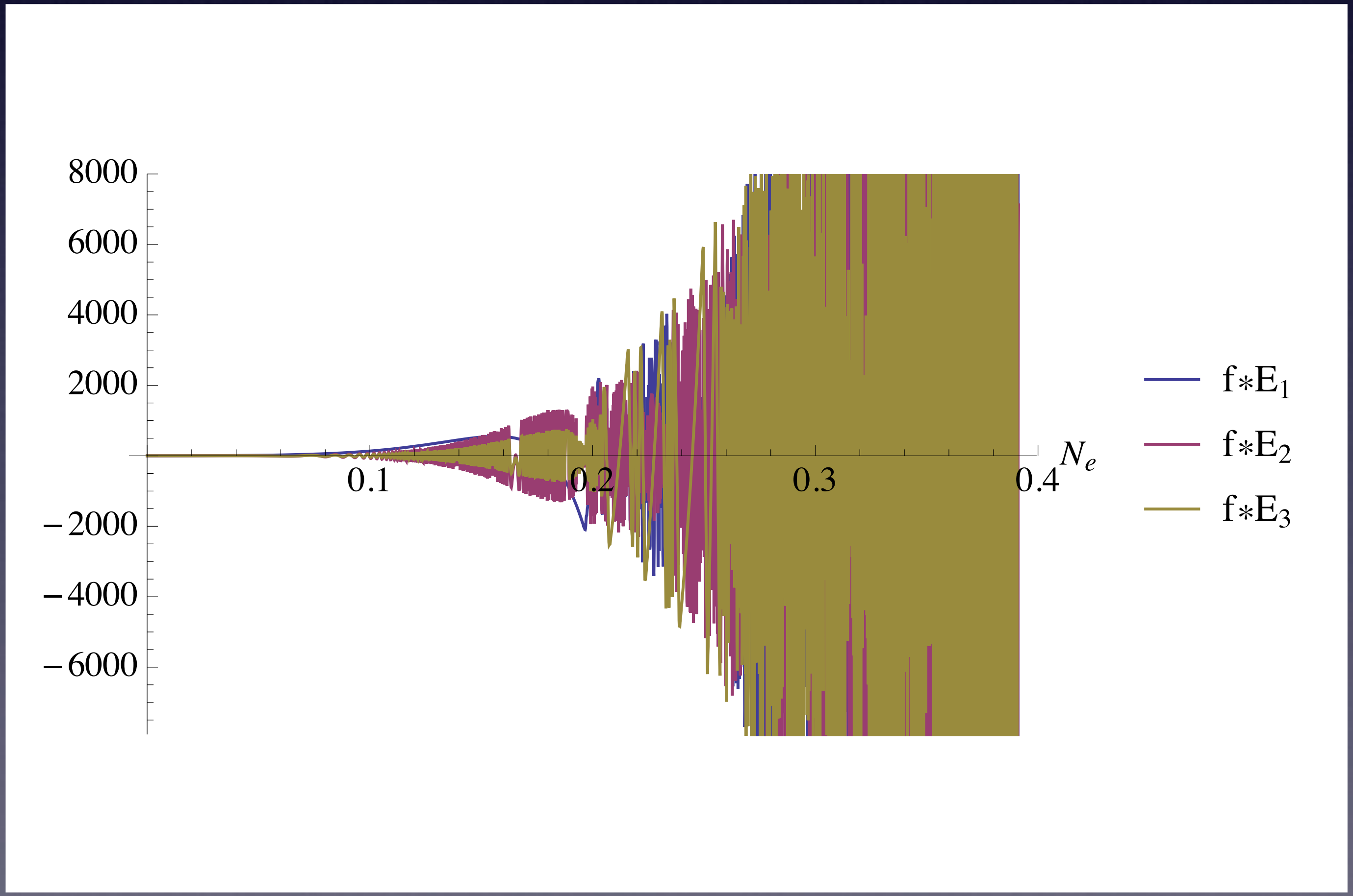
$$B \propto a^{-2} \; , E \propto a^{-(6+p)} \; ,$$

$$A \sim \sqrt{a^2 B} \rightarrow \text{const} \; , \quad a^{-1} AE \propto a^{-(7+p)}$$

- Magnetic -> can safely assume U(1) dynamics

SU(2) gauge kinetic

- SU(2) oscillation ($\kappa = 40$, $g = 1$, $\epsilon \sim 10^{-2}$)



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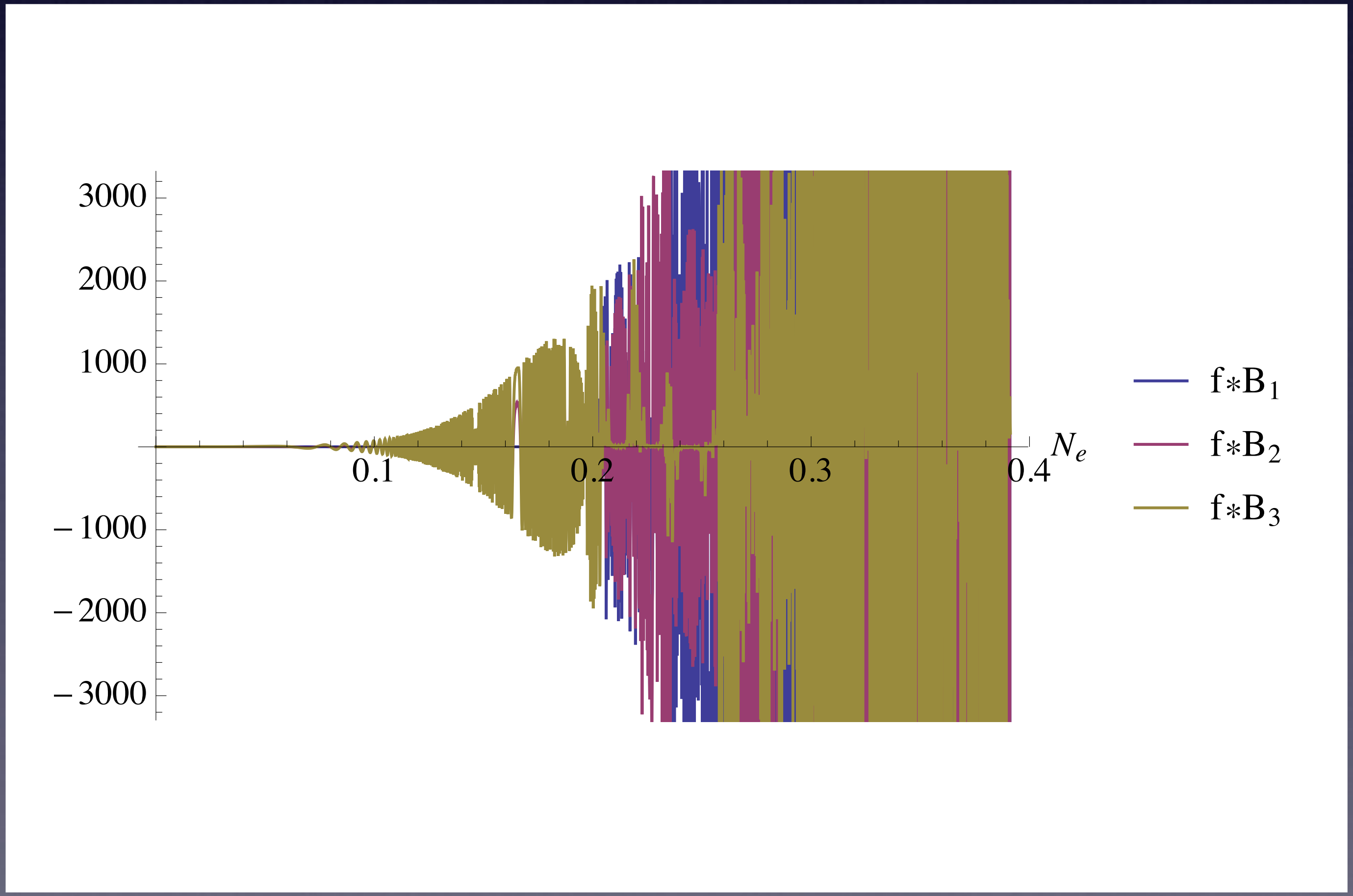
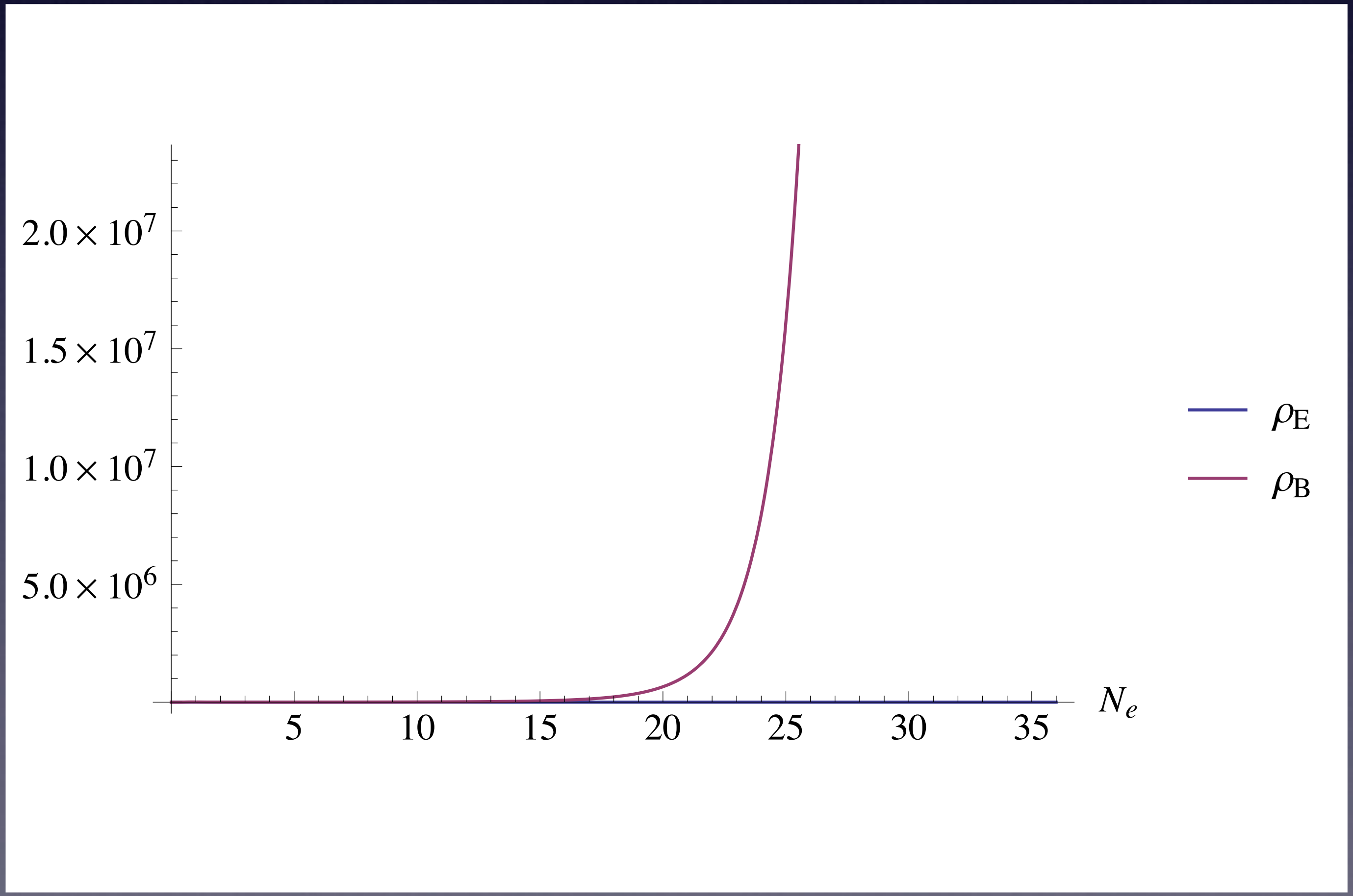


SU(2) gauge kinetic

- SU(2) magnetic field growth ($\kappa = -40\ , \epsilon \sim 10^{-2}$)



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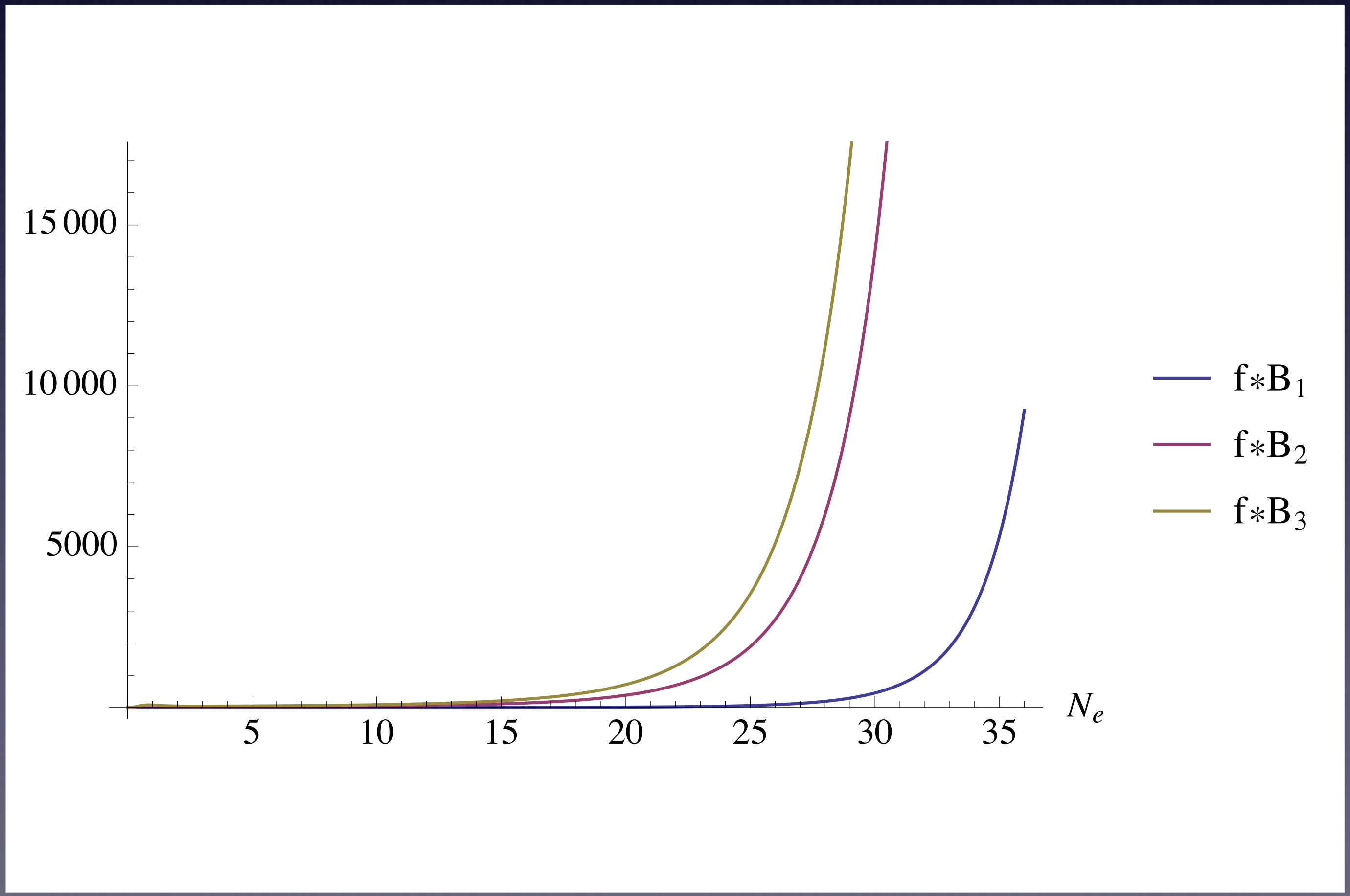

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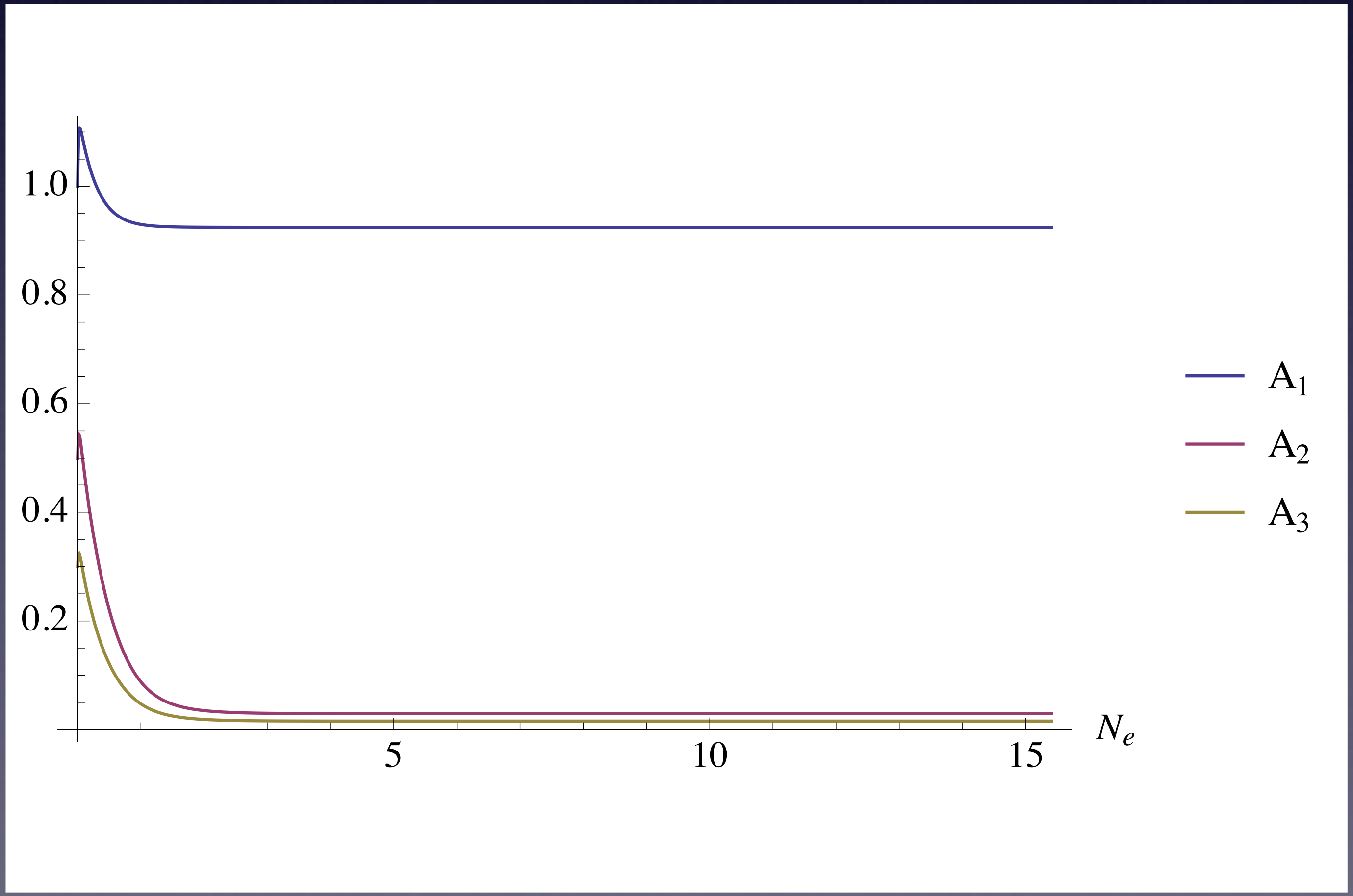


U(1) Chern-Simons

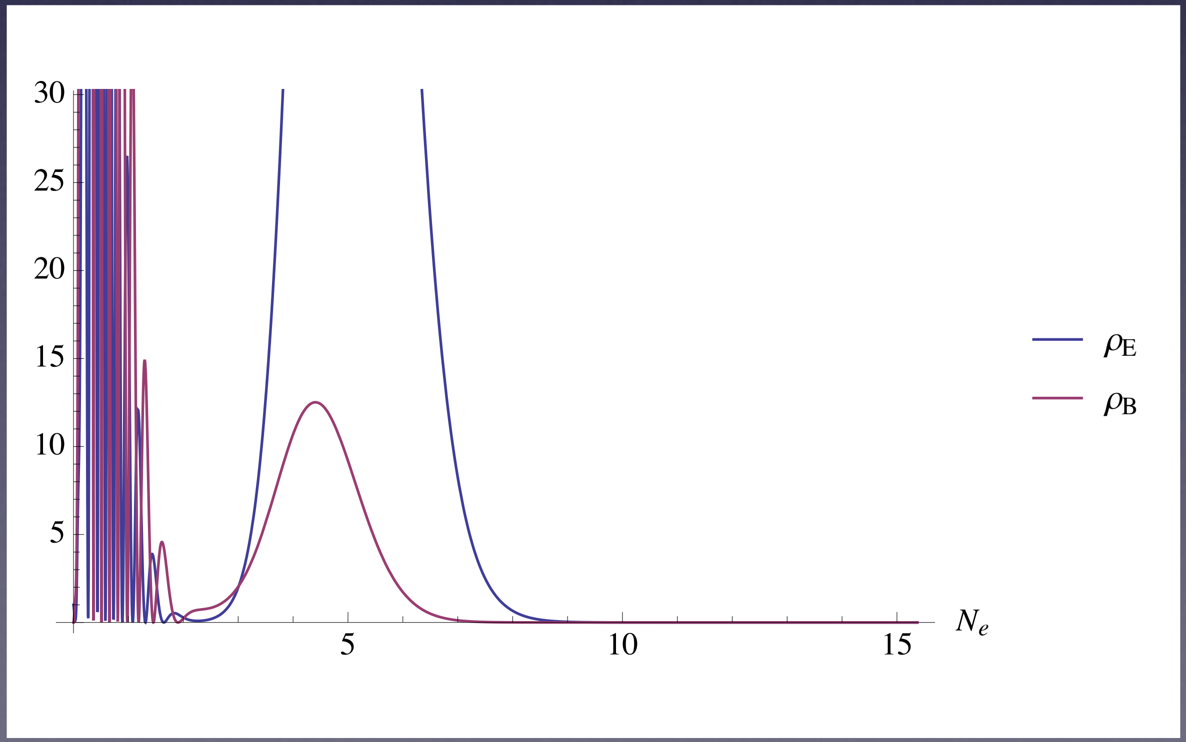
- Equation in Coulomb gauge

$$a^2\left(\partial_t + H\right)\partial_t \mathbf{A} - \nabla^2 \mathbf{A} - a\left(\partial_t \lambda\right) \mathrm{curl}\ \mathbf{A} = 0$$

- Essentially a skew mass-term
- Skew -> (almost) always a negative eigenvalue
- The effect goes away after $\dfrac{k}{a} \sim \dfrac{H^2}{\partial_t \lambda}$

U(1) Chern-Simons

- Chern-Simons term $\lambda(\varphi) = \lambda\varphi$
- For superhorizon effect, $\lambda > \sqrt{2\epsilon}^{-1}$
- Numerical



SU(2) Chern-Simons

- Same setting as gauge kinetic

$$a\,(\partial_t + 2H)\,E_1 + a\lambda' B_1 + g\,(A_3 B_2 + A_2 B_3) = 0\ ,$$
$$a\,(\partial_t + 2H)\,B_1 - g\,(A_3 E_2 + A_2 E_3) = 0$$

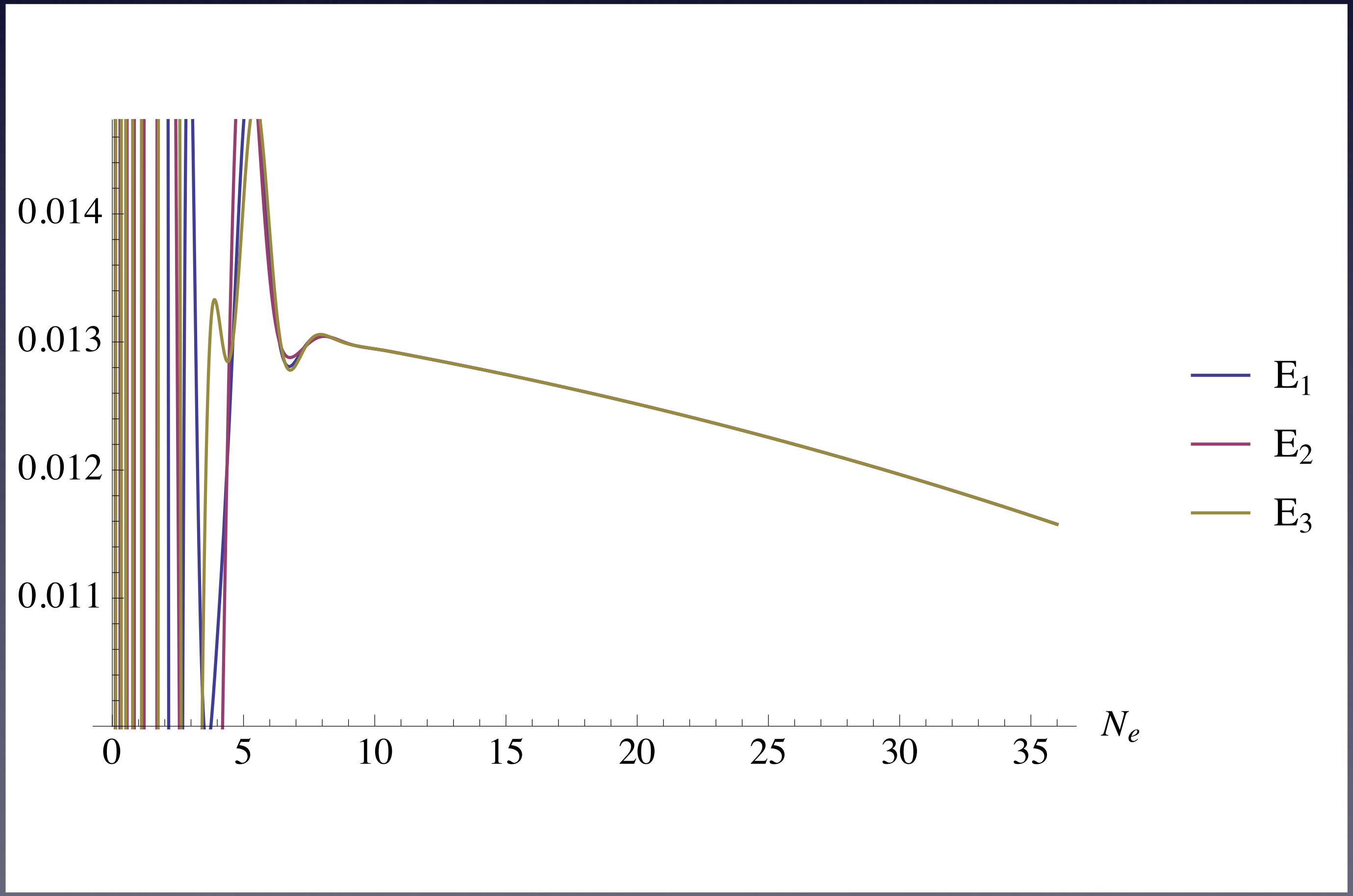
- Compare to U(1) equations

$$a\,(\partial_t + 2H)\,E_1 + a\lambda' B_1 + (\partial_y B_3 - \partial_z B_2) = 0\ ,$$
$$a\,(\partial_t + 2H)\,B_1 - (\partial_y E_3 - \partial_z E_2) = 0$$

- Similar effect expected

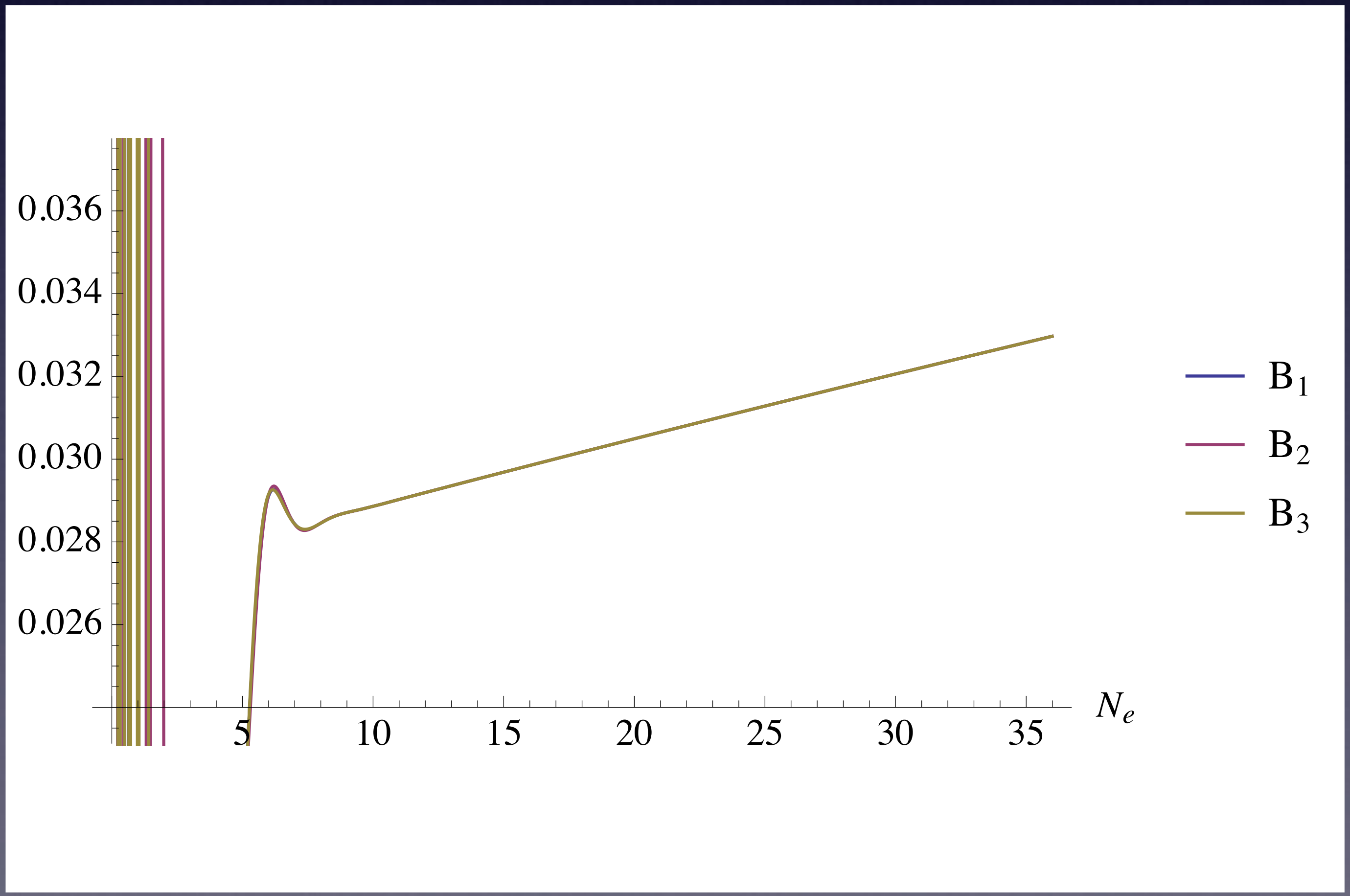
SU(2) Chern-Simons

- Self-induced isotropy ($\lambda = 50$, $g = 1$, $\epsilon \sim 10^{-2}$)



SU(2) Chern-Simons

- Self-induced isotropy ($\lambda = 50$, $g = 1$, $\epsilon \sim 10^{-2}$)



SU(2) Chern-Simons

- The CS interaction tends to parallelise E and B

$$\mathbf{E}' \sim \lambda\phi'\mathbf{B} \quad \longrightarrow \quad \mathbf{E} \sim r\mathbf{B}$$

- E and B are related through the potential A

$$E_1 = a^{-1}A_1' \sim a^{-1}HA_1 \ , \quad B_1 = ga^{-2}A_2A_3$$

- The solution is isotropic

$$A_1^2 \sim A_2^2 \sim A_3^2 \sim \left(\frac{aH}{gr}\right)^2$$

Test field summary

	Instability	Growth	Anisotropy
U(1) g-k	$\lvert\kappa\rvert \gtrsim \sqrt{\epsilon}^{-1}$	Exponential	Axisymmetric
SU(2) g-k	$\lvert\kappa\rvert \gtrsim \sqrt{\epsilon}^{-1}$	Exponential (B)/ Oscillatory (E)	General
U(1) C-S	$\lvert\lambda\rvert \gtrsim aH/(k\sqrt{\epsilon})$	Temporary	Axisymmetric
SU(2) C-S	$\lvert\lambda\rvert \gtrsim \sqrt{\epsilon}^{-1}$	Almost constant	Isotropic

Backreaction

- Instability -> Linear analysis breaks down
- Scalar dynamics is affected
- Gauge kinetic: $\varphi'' + 3H\varphi' + m^2\varphi - \frac{\kappa}{2}\left(\rho_E - \rho_B\right) = 0$
- Chern-Simons: $\varphi'' + 3H\varphi' + m^2\varphi - \lambda \mathbf{E}\cdot\mathbf{B} = 0$

Backreaction

- Always halt instability

  $\kappa\varphi' < 0$ -> E grows -> φ slows down

$$3H\varphi' + m^2\varphi - \frac{\kappa}{2}\rho_E \sim 0$$

  $\lambda\varphi' > 0$ -> E and B anti-parallel -> φ slows down

$$3H\varphi' + m^2\varphi - \lambda\mathbf{E}\cdot\mathbf{B} \sim 0$$

- New attractor solution expected

Anisotropic cosmology

- Assume simple anisotropic metric

$$ds^2 = -\,dt^2 + a^2\left(e^{4\beta_+}dx^2 + e^{-2(\beta_+ - \beta_-)}dy^2 + e^{-2(\beta_+ + \beta_-)}dz^2\right)$$

- Explicitly solve the Einstein-Maxwell or Einstein-Yang-Mills equations

Anisotropic cosmology

- Use the diagonal ansatz for SU(2) gauge field
- Einstein equations look the same for U(1) and SU(2)….

$$3H^2 = 3\beta_+^2 + \beta_-^2 + \frac{1}{2}\left(\varphi'^2 + m^2\varphi^2\right) + \frac{1}{2}\left(\rho_E + \rho_B\right) \ ,$$

$$\beta_+'' + 3H\beta_+' = \frac{f^2}{6}\left(E_2^2 + E_3^2 - 2E_1^2 + B_2^2 + B_3^2 - 2B_1^2\right) \ ,$$

$$\beta_-'' + 3H\beta_-' = \frac{f^2}{2}\left(E_3^2 - E_2^2 + B_3^2 - B_2^2\right)$$

Anisotropic cosmology

- Except the off-diagonals $G_{12} = 0 = T_{12}$

$$T_{12} = E_1 E_2 + B_1 B_2 \quad \text{for U(1)}$$

$$T_{12} = 0 \quad \text{for diagonal SU(2)}$$

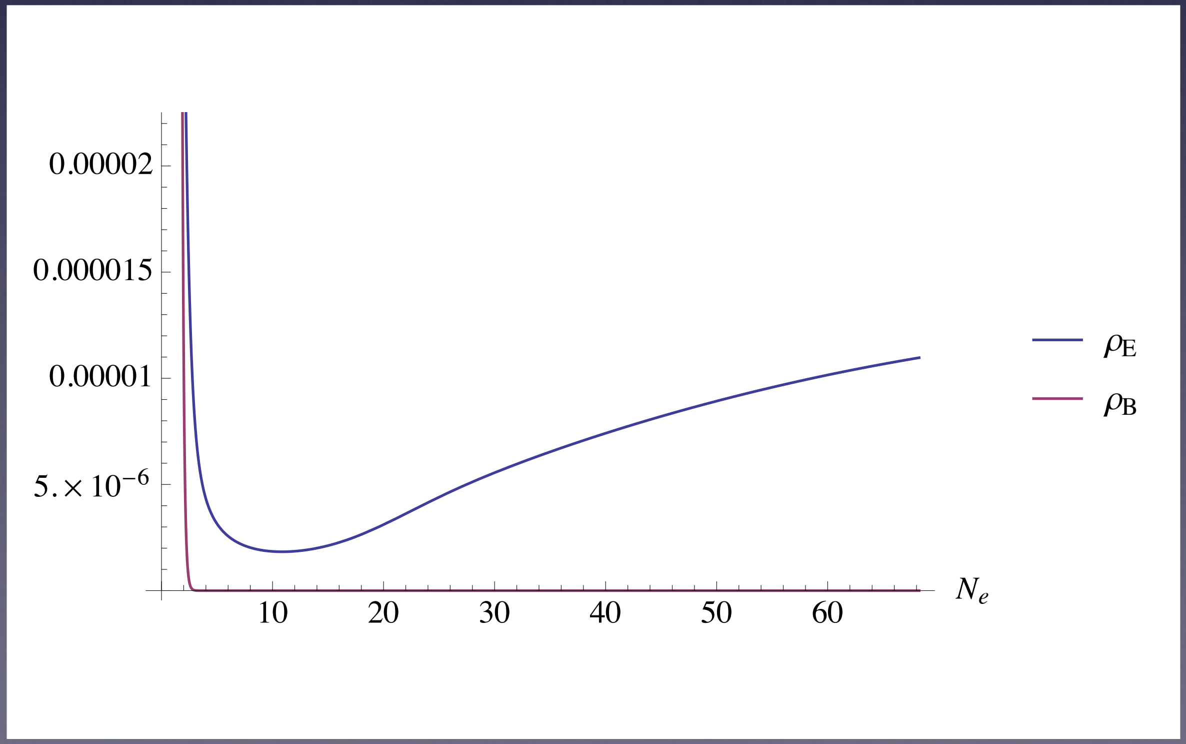
- U(1) has to be aligned to one of the coordinate axes

U(1) gauge kinetic

- Anisotropy only slightly modifies Maxwell eqs

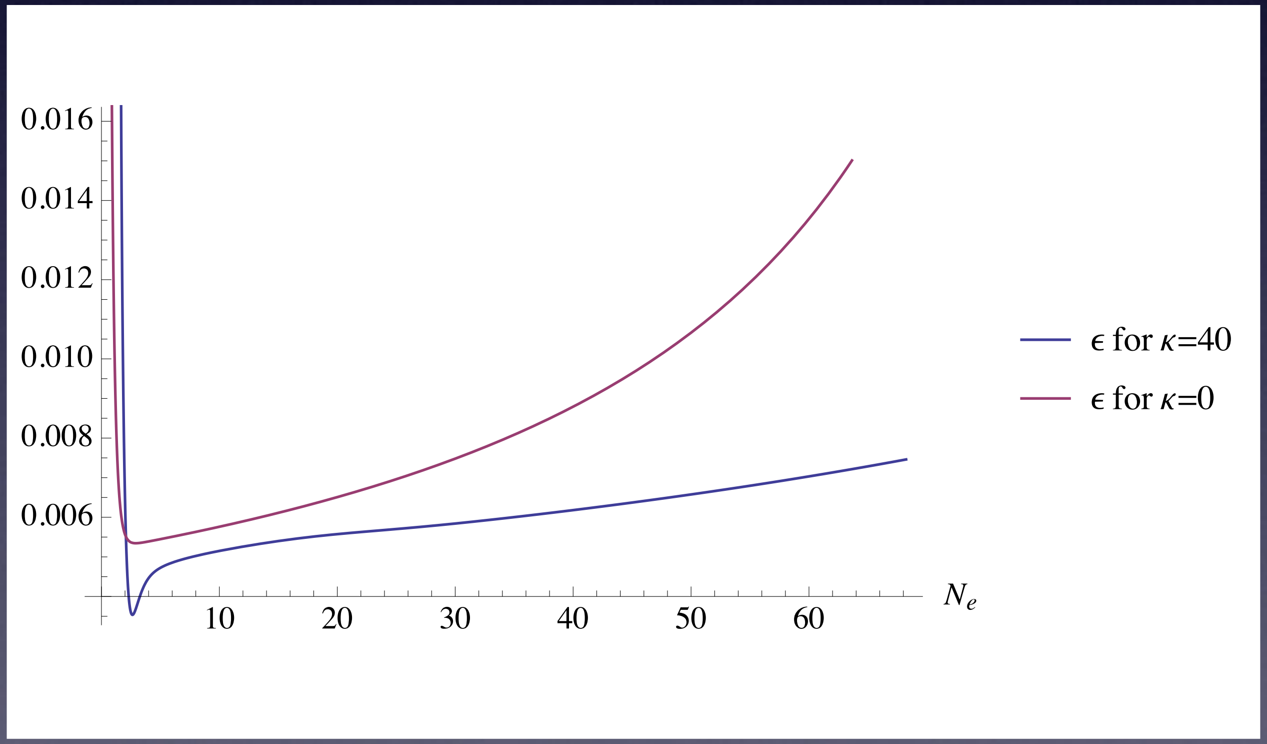
$$E_1' + \left(2H - 2\beta_+' + \kappa\varphi'\right) E_1 = 0 \ , \text{etc.}$$

- Obtain rather obvious anisotropic inflation



U(1) gauge kinetic

- Supportive role of gauge field in realising accelerated expansion

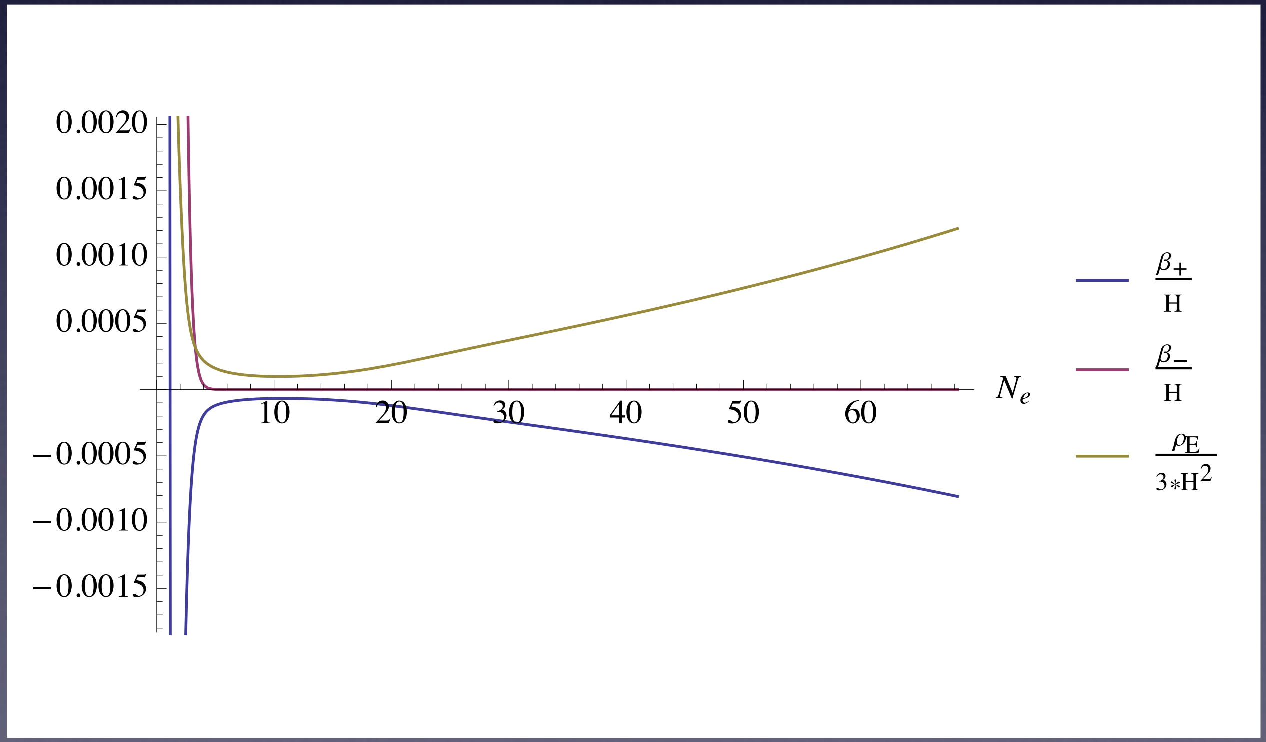


- Result of the back reaction

U(1) gauge kinetic

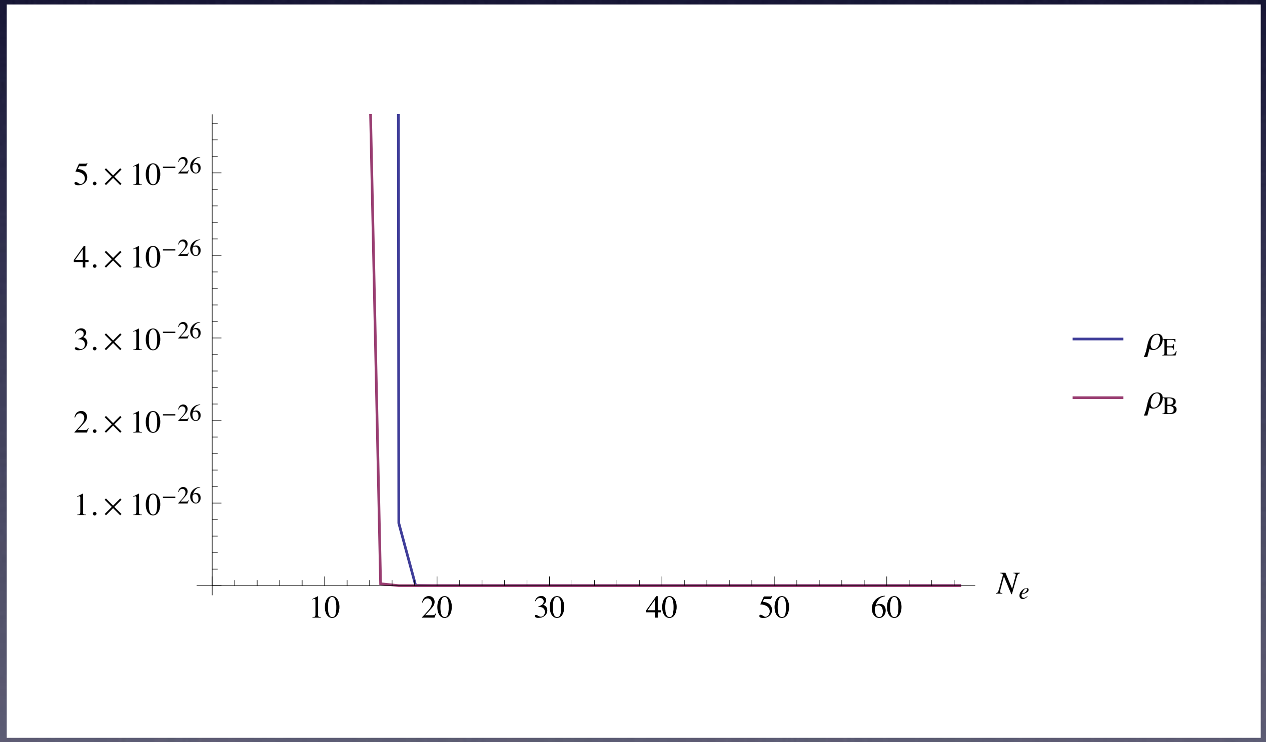
- Anisotropy bounded by slow-roll

$$3H\beta'_{\pm} \sim O(1)\rho_E \Rightarrow \frac{\beta_{\pm}}{H} \sim O(1)\frac{\rho_E}{3H^2} < \epsilon$$



U(1) Chern-Simons

- No appreciable effect (as expected)



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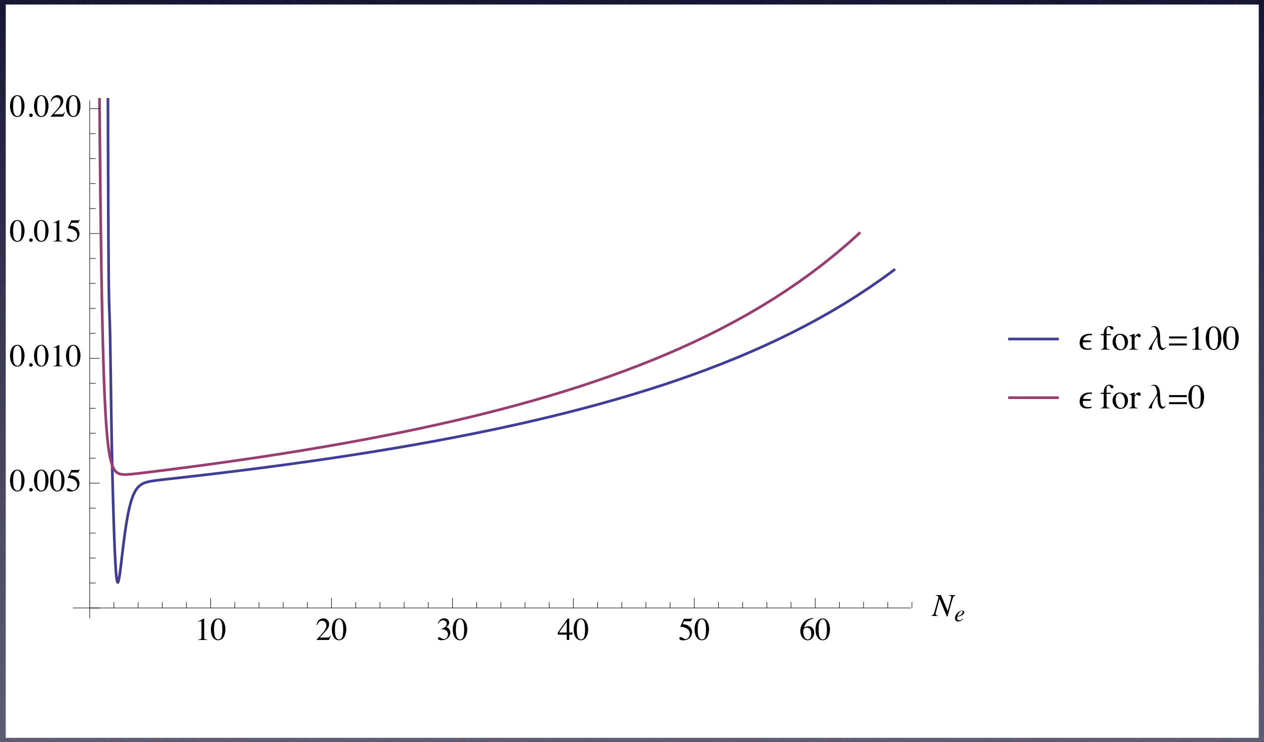


SU(2) gauge kinetic

- Abelian part expected to dominate for $\kappa\varphi' > 0$

$$\rho'_{B_1} + \left(4H - 4\beta'_+ - \kappa\varphi'\right)\rho_{B_1} = O(g)$$

$$\rho'_{B_2} + \left(4H + 2\beta'_+ - 2\beta'_- - \kappa\varphi'\right)\rho_{B_2} = O(g)$$

- If $|B_1| \gg |B_2|$

$$\beta'_+ \sim -\frac{2}{9}\frac{\rho_{B_1}}{H}\,, \quad 4H = 4\beta'_+ + \kappa\varphi'$$

- Instability of B_2

$$\rho'_{B_2} \sim -4\beta'_+\rho_{B_2}$$

SU(2) gauge kinetic

- Time-scale of instability $|\beta_+'^{-1}| \gtrsim O((\epsilon H)^{-1})$
- To see it before the end,

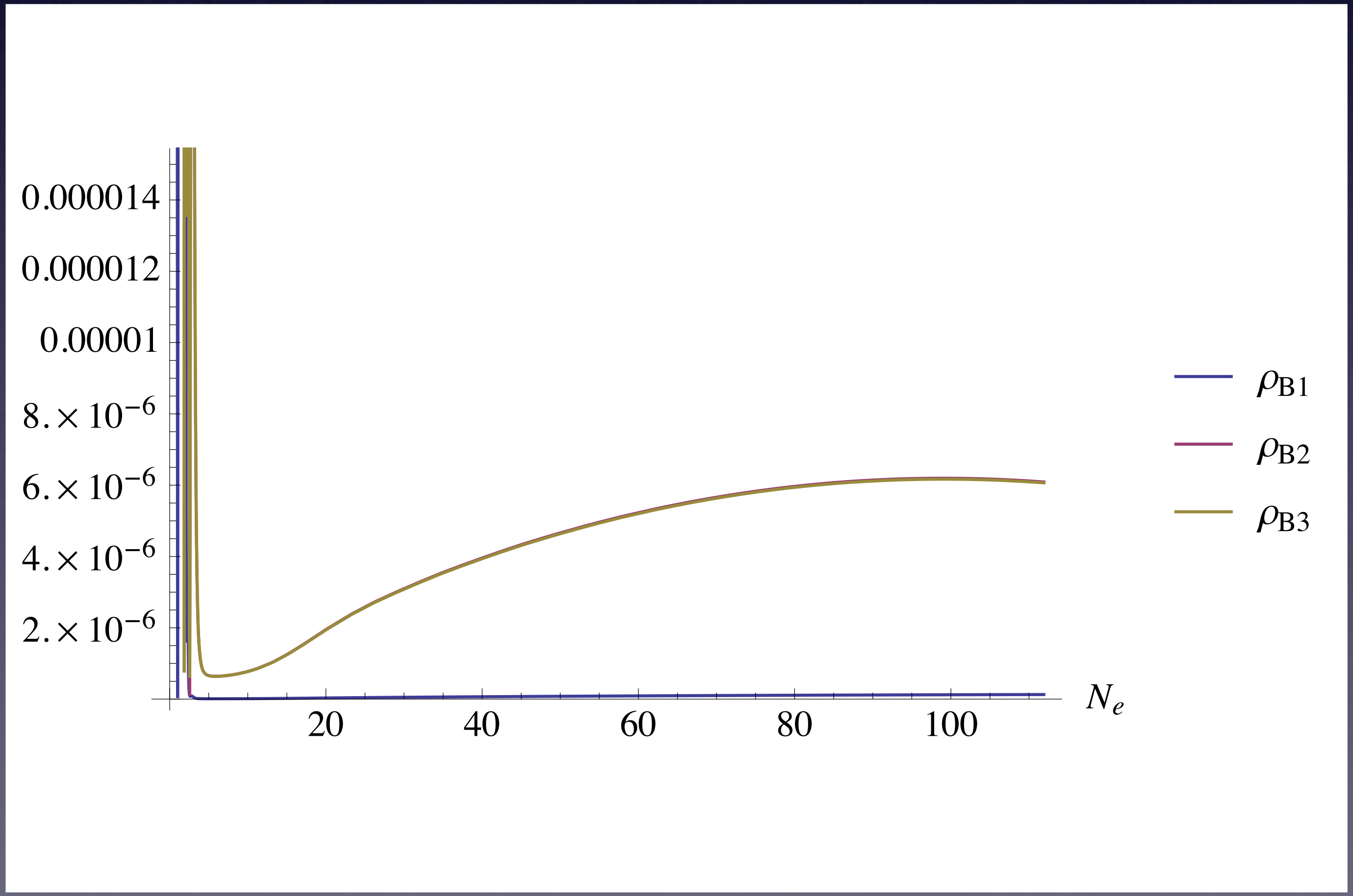
$$|\beta_+'| \gtrsim |\eta| H \ , \quad \eta = \frac{\epsilon'}{\epsilon H} \Rightarrow \epsilon \gtrsim \eta$$

- For quadratic potential,

$\epsilon \sim \eta \sim \varphi^{-2}$ → Will never see it happen

SU(2) gauge kinetic

- Numerical confirmation $\kappa = -40$



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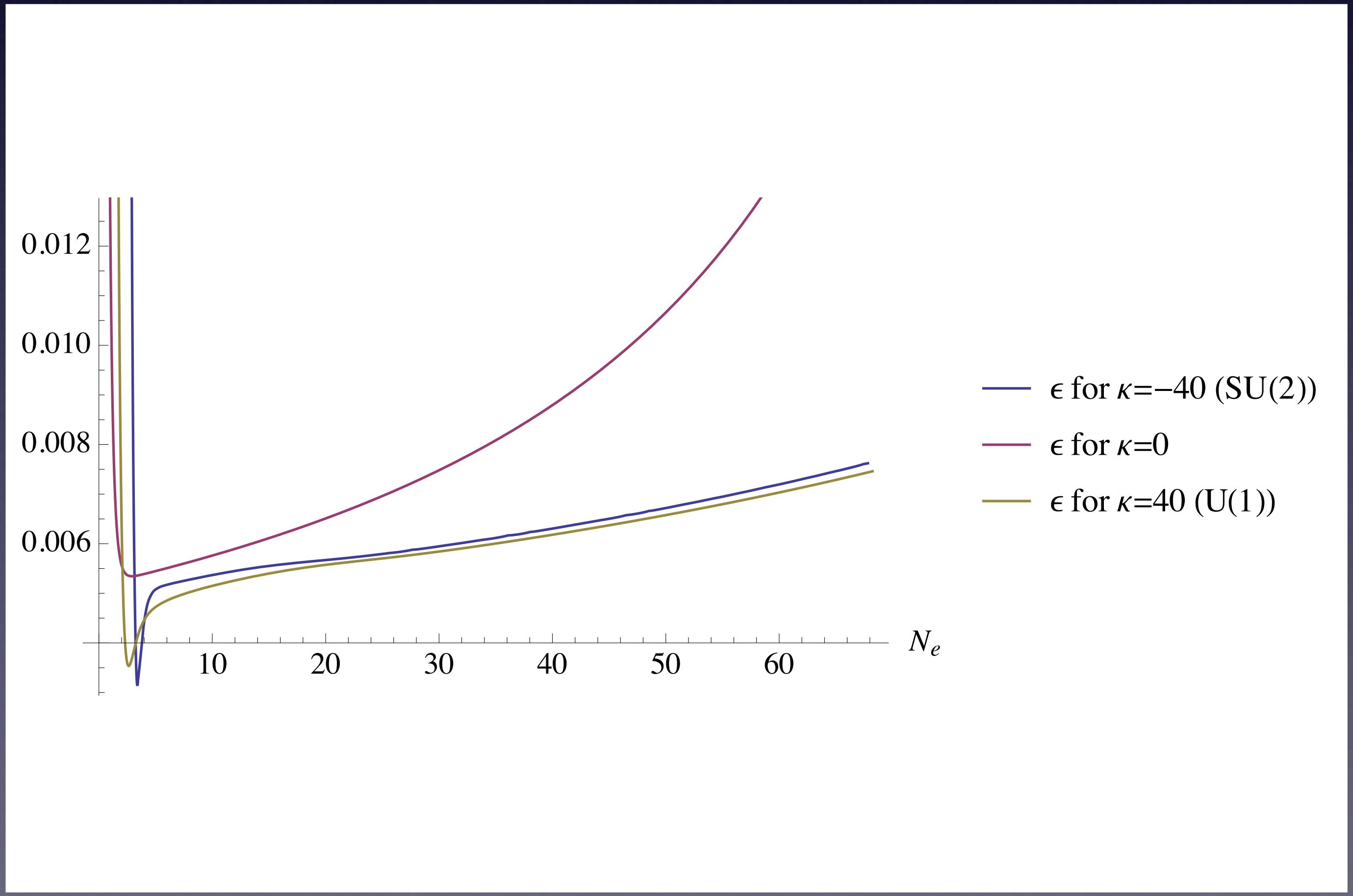
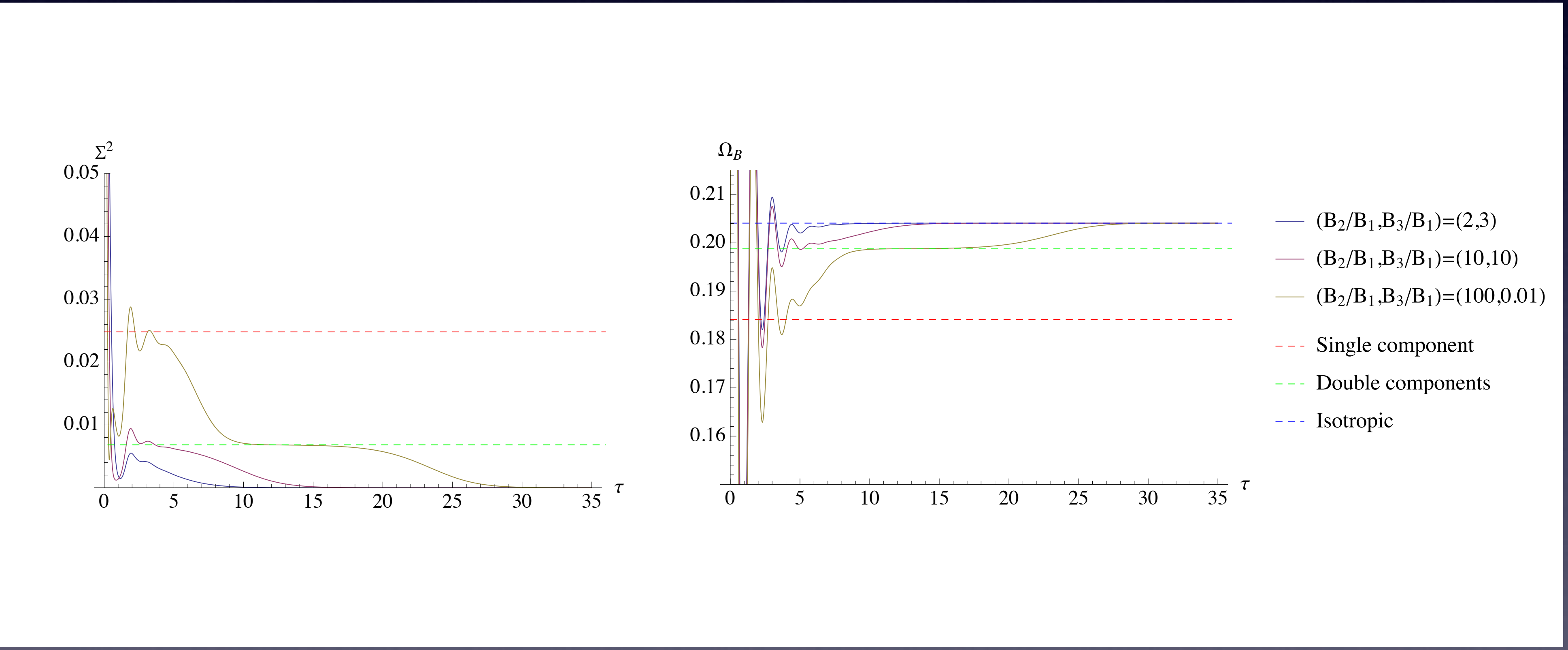


SU(2) gauge kinetic

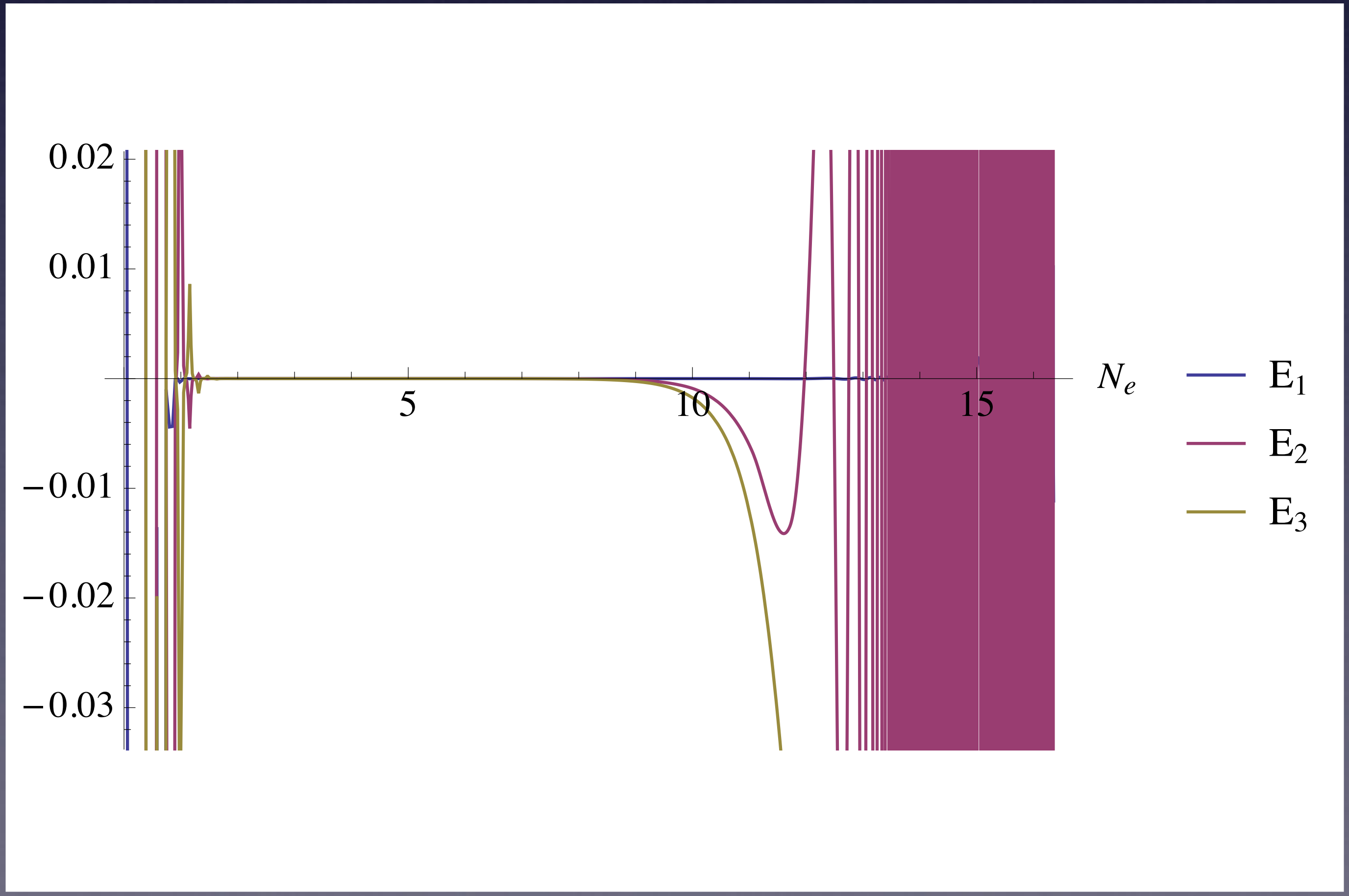
- Example of isotropy $V(\varphi) = V_0 e^{-\alpha\varphi} \Rightarrow \eta \sim \epsilon^2$



$\alpha = 2\ , \kappa = 5\ , \epsilon \sim 0.4$

SU(2) gauge kinetic

- Generically non-Abelian for $\kappa\varphi' < 0$
- Attractor appears to be oscillatory



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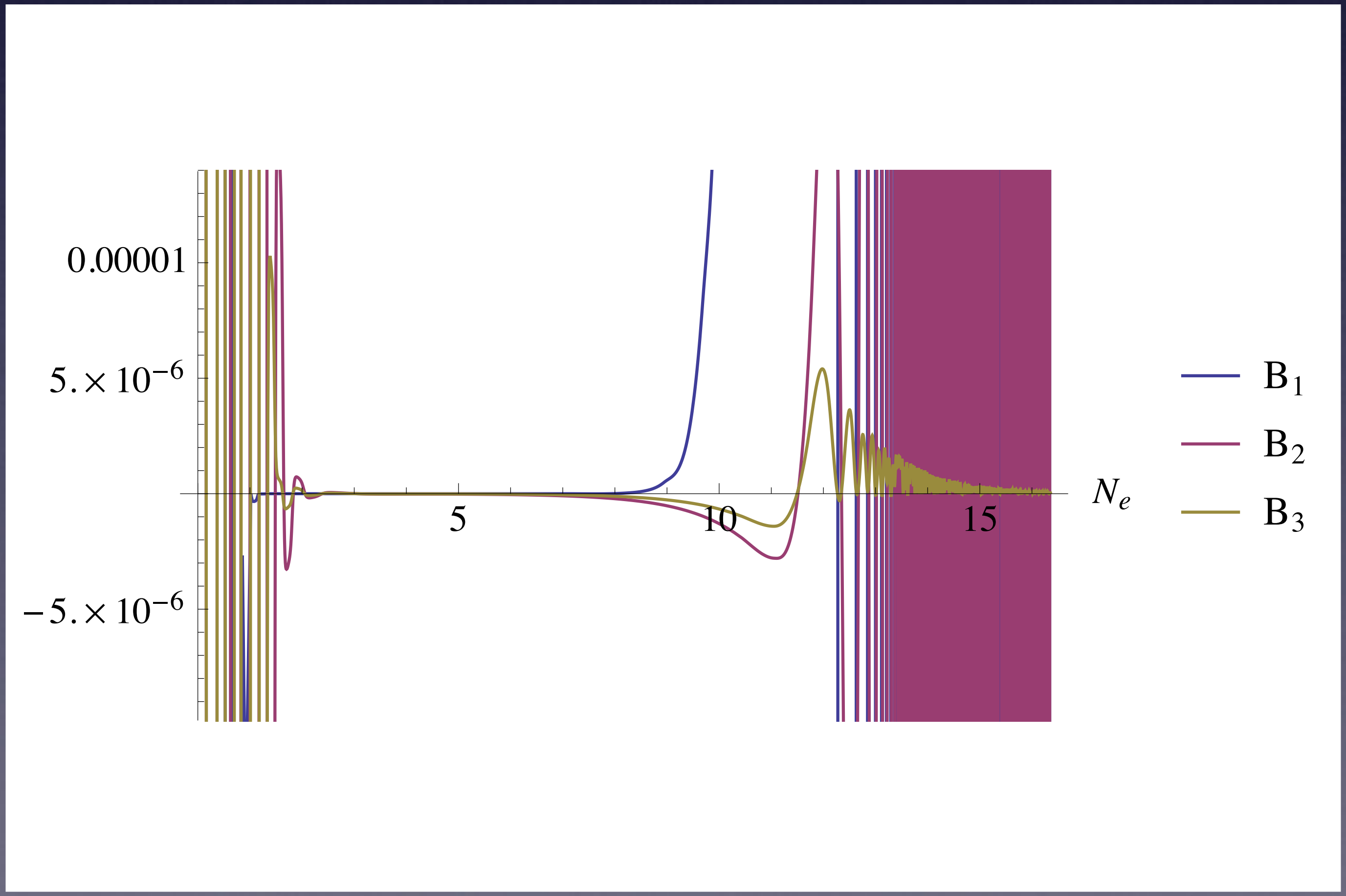

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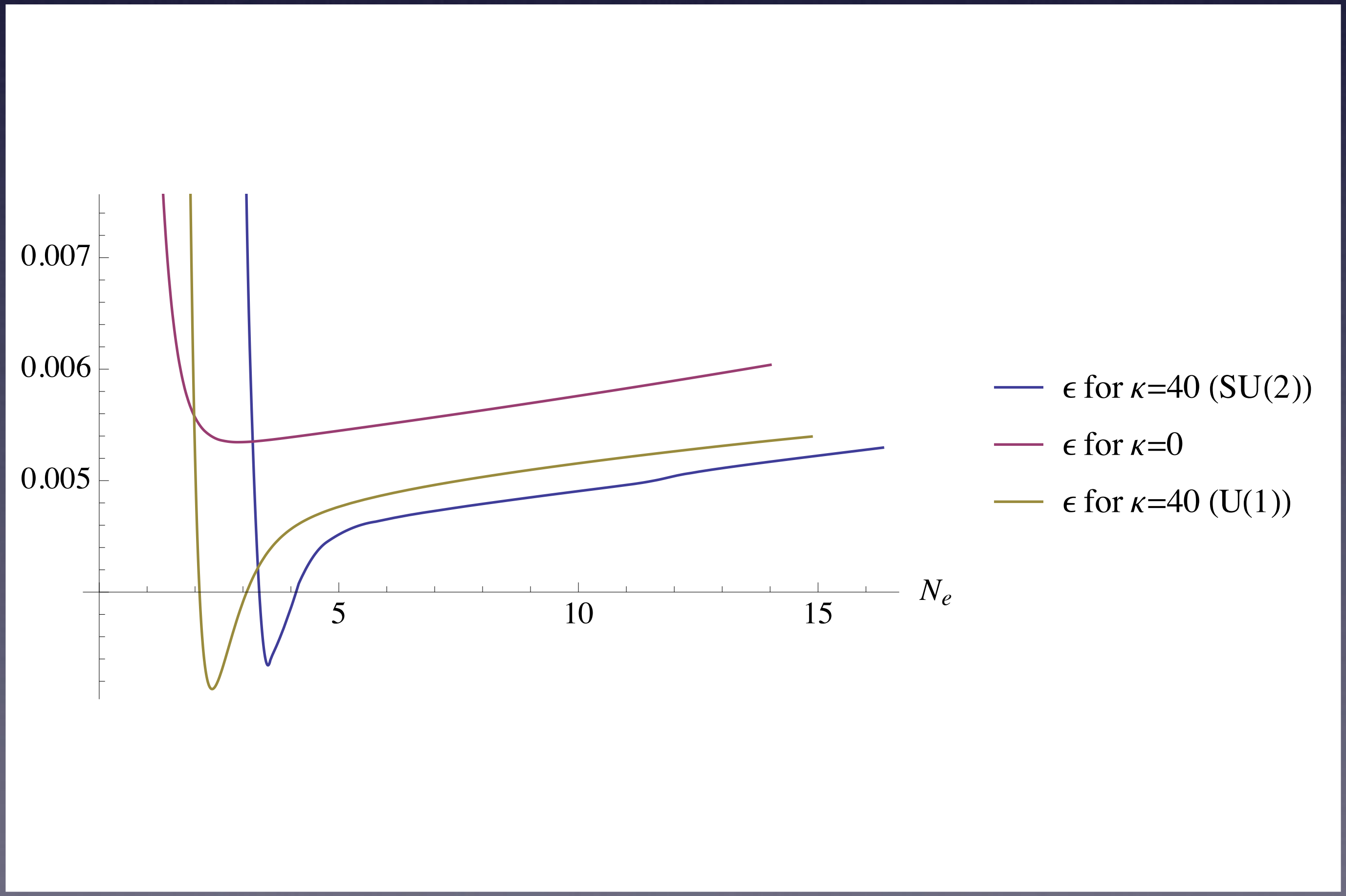


SU(2) Chern-Simons

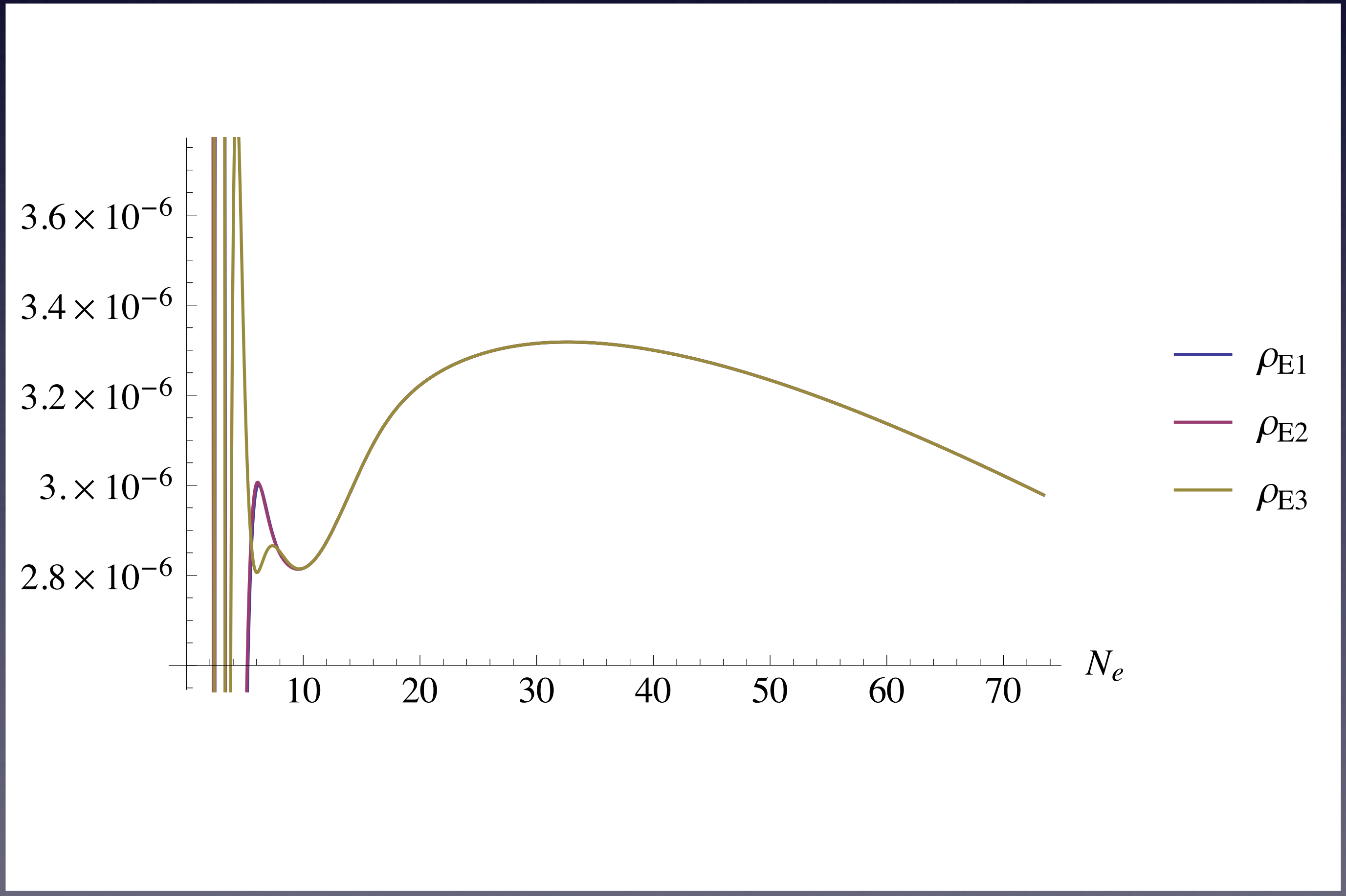
- The argument for isotropy hardly affected

$$a^{-1}e^{-2\beta_+}HA_1 \sim E_1 \sim rB_1 = rga^{-2}e^{2\beta_+}A_2A_3\ ,$$
$$a^{-1}e^{\beta_+-\beta_-}HA_2 \sim E_2 \sim rB_2 = rga^{-2}e^{-\beta_++\beta_-}A_1A_3$$
$$\Rightarrow e^{-4\beta_+}A_1^2 \sim e^{2(\beta_+-\beta_-)}A_2^2 \sim e^{-2(\beta_++\beta_-)}A_3^2$$

- Einstein equations not modified
- Gauge field in equilibrium by itself
- Backreaction slows inflaton

SU(2) Chern-Simons

- Isotropic inflation ($\lambda = 80$)



SU(2) Chern-Simons

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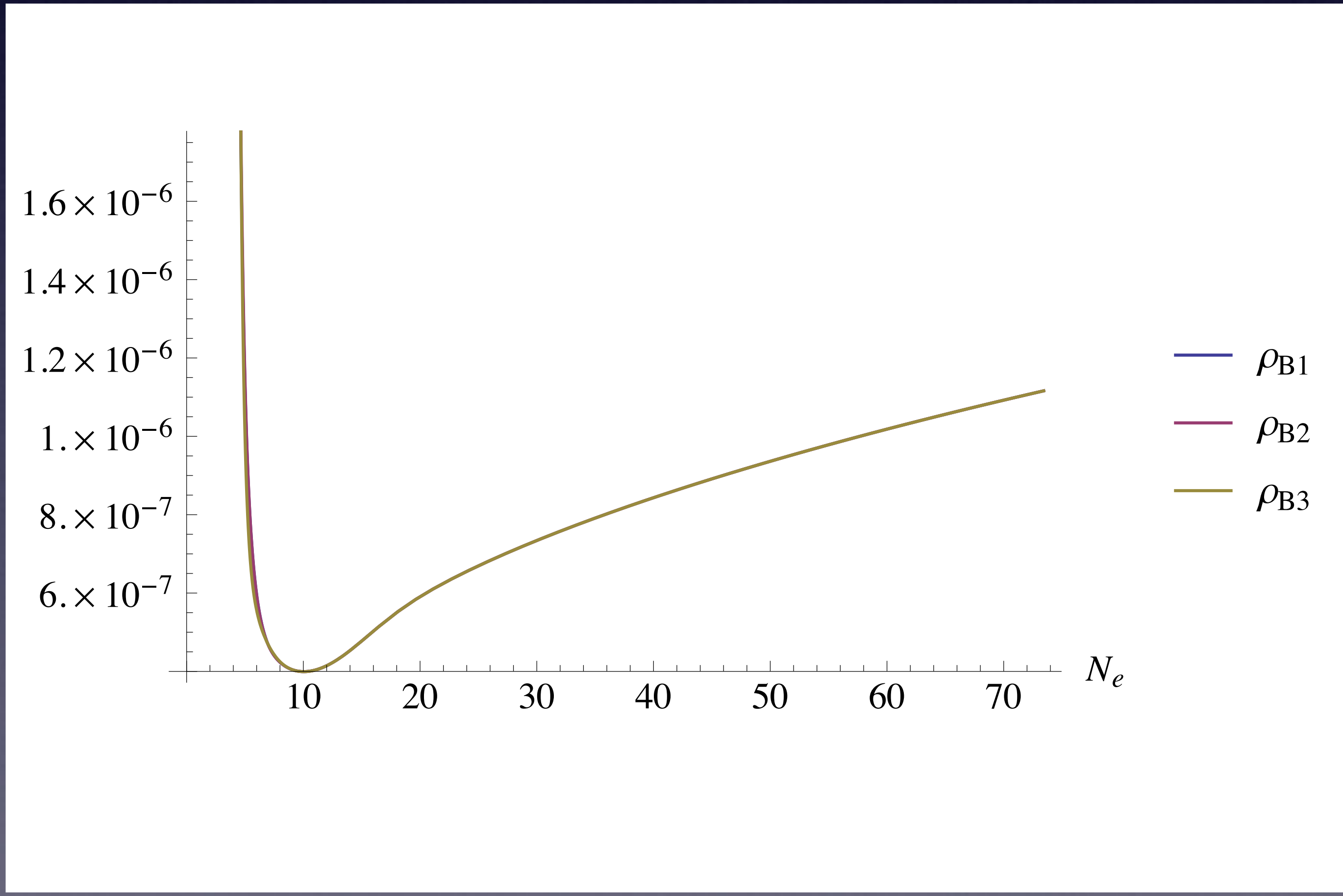

SU(2) Chern-Simons

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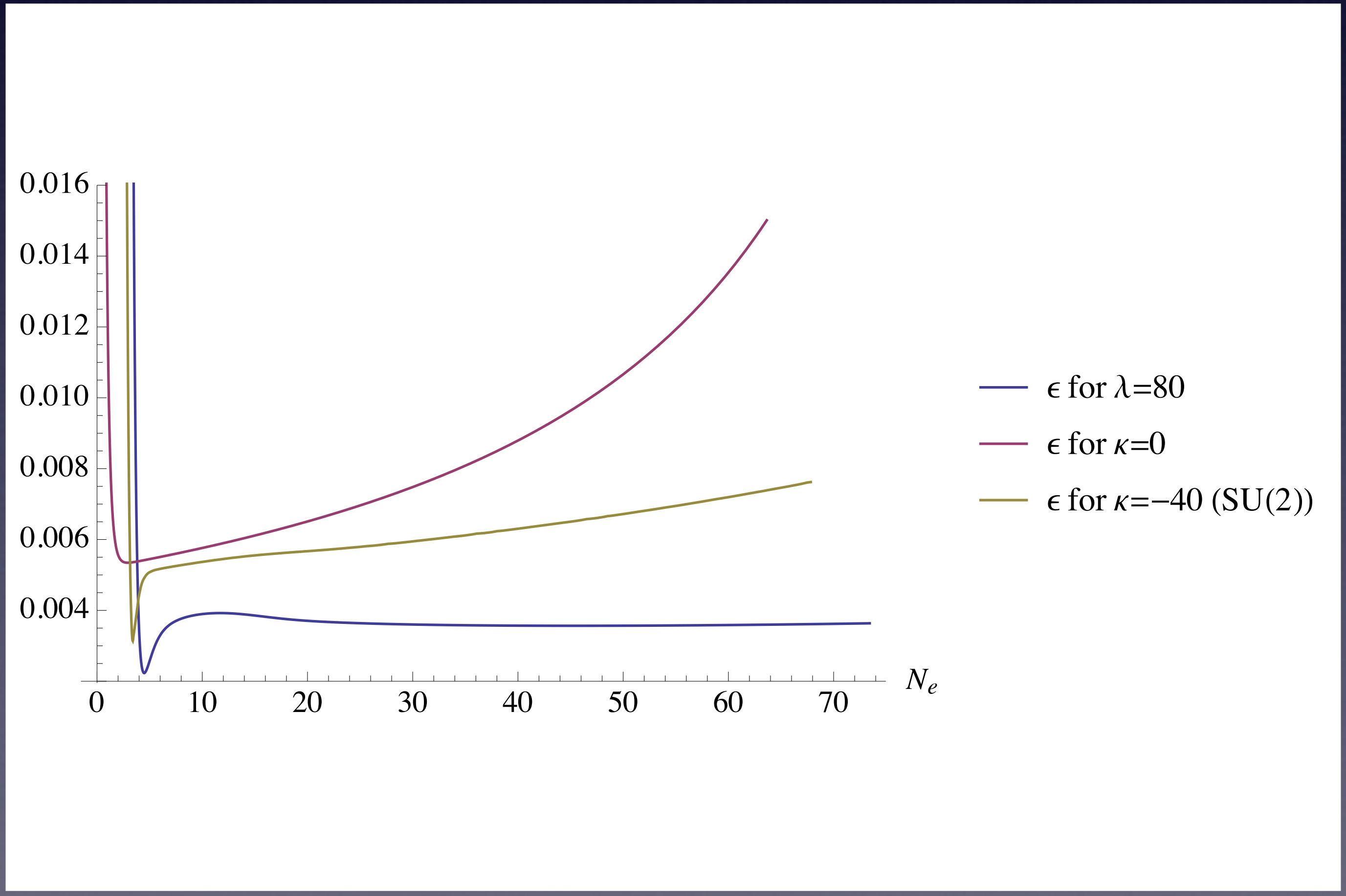


Anisotropy summary

- Always bounded by slow-roll

$$\frac{\beta'_{\pm}}{H} \lesssim O(1)\frac{\rho_{gauge}}{H^2} \lesssim \epsilon$$

- No anisotropic inflation from Chern-Simons

- Isotropy in SU(2) gauge-kinetic requires $\eta \ll \epsilon$

-> Anisotropic phase lasts at least O(10) e-folds

What are observables?

- New parameters: anisotropy and gauge energy density

$$\epsilon = \frac{3\beta_+^2}{H^2} + \frac{2\rho_g}{3H^2} + \frac{\varphi'^2}{2H^2}$$

- Anisotropy -> dependent and subdominant

$$\frac{\beta_+^2}{H^2} \sim O(1)\frac{\rho_g^2}{H^4} \ll \epsilon$$

- Two-parameter system

$$\epsilon \sim (1+I)\,\epsilon_\varphi \ , \quad \epsilon_\varphi = \frac{\varphi'^2}{2H^2} \ , \quad I = \frac{4\rho_g}{3\varphi'^2}$$

What are observables?

- Signature (if any) can only come from CMB
- Perturbation in the presence of background gauge fields

1. Statistical anisotropy of energy density
2. Modification to tensor mode spectrum

Statistical anisotropy

- Linearise around the U(1) gauge-kinetic background
- Want to compute density perturbation $\delta\rho$
- Target variables corresponding to scalar mode on FLRW
- WLOG assume $\vec{k} = (k_x, k_y, 0)$

Statistical anisotropy

- Gravitational sector irrelevant in flat gauge (same as FLRW)

- Matter sector characterised by two modes

1. Inflaton: $\varphi \to \varphi + \delta\varphi$

2. Longitudinal gauge field:

$$A_\mu = (0, A(t), 0, 0) + (\partial_x \delta A_0, \delta A_x, 0, 0)$$

Statistical anisotropy

- Inflaton-gauge coupling not suppressed

$$\delta\varphi'' + 3H\delta\varphi' + \frac{k^2}{a^2}\delta\varphi$$

$$+ 12IH^2(1-\sin^2\theta)\delta\varphi + \frac{2\sqrt{6I}H\sin\theta}{a^3}\delta A'_x = 0$$

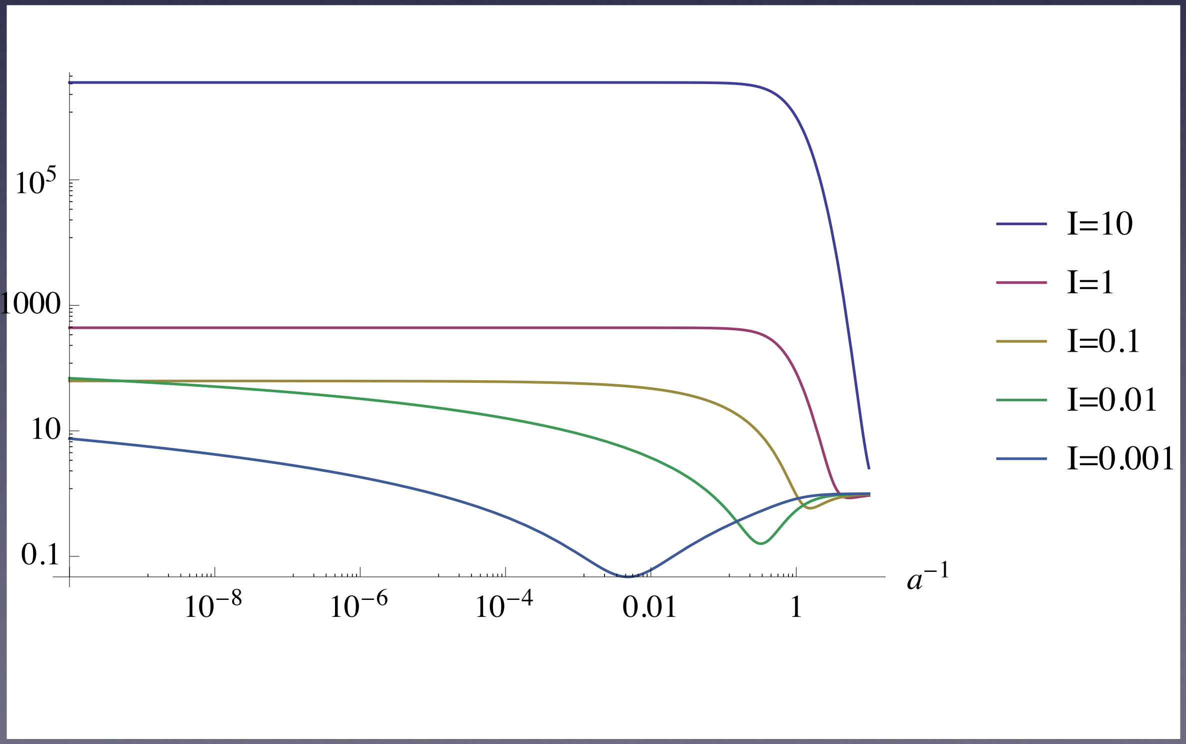
$$\delta\tilde{A}''_x - 3H\delta\tilde{A}'_x + \frac{k^2}{a^2}\delta\tilde{A}_x - 2\sqrt{6I}a^3 H\sin\theta\delta\varphi' = 0$$

$$\sin\theta = k_x/\sqrt{k_x^2+k_y^2}\,, \quad \delta\tilde{A}_x = e^{-2\beta_+}\sin\theta\delta A_x$$

- Interaction depends on the angle -> anisotropy
- Gauge energy density I may be observable

Statistical anisotropy

- Superhorizon evolution -> large effect
- Tight constraint on the parameter I
- Plot of $|\delta\rho_k|^2\big|_{\theta=\frac{\pi}{2}} / |\delta\rho_k|^2\big|_{\theta=0} - 1$



Gravitational waves

- SU(2) gauge field in isotropic background can source tensor mode

$$\delta\left(F_{\mu\alpha}F_{\nu}{}^{\alpha}\right) \supset A'\delta_i^a \delta A_j^{a\prime}\ ,\ f A^2 \epsilon^{abc}\delta_i^b \delta_k^c \partial_k \delta A_j^a \ \text{etc.}$$

- "Tensor" perturbation of gauge field

$$\omega_{ij} = \left(\delta A_j^i + \delta A_i^j\right)_{TT}$$

- Chiral mixing through the structure constant

Gravitational waves

- Equations take similar form as density

$$h''_{ij} + 3Hh'_{ij} - \frac{\nabla^2}{a^2}h_{ij} - 2f^2\left(A'^2 - \frac{g^2A^4}{a^2}\right)h_{ij}$$
$$+ 2f^2A'\omega'_{ij} - \frac{2g^2f^2A^3}{a^2}\omega_{ij} + \frac{2gf^2A^2}{a^2}\epsilon_{(i|kl}\partial_k\omega_{l|j)} = 0$$

- Tensor-gauge coupling -> superhorizon growth
- Chiral mixing gives transient effect
- Need scalar mode too for phenomenology

End

Thanks!

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\Phi & a^2\partial_x\beta_1 & a^2\partial_y\beta_2 & 0 \\ a^2\partial_x\beta_1 & 2a^2e^{4\beta_+}G & 0 & 0 \\ a^2\partial_y\beta_2 & 0 & 2a^2e^{-2\beta_+}G & 0 \\ 0 & 0 & 0 & -2a^2e^{-2\beta_+}G \end{pmatrix}$$