From configuration to dynamics

Emergence of Lorentz signature in classical field theory

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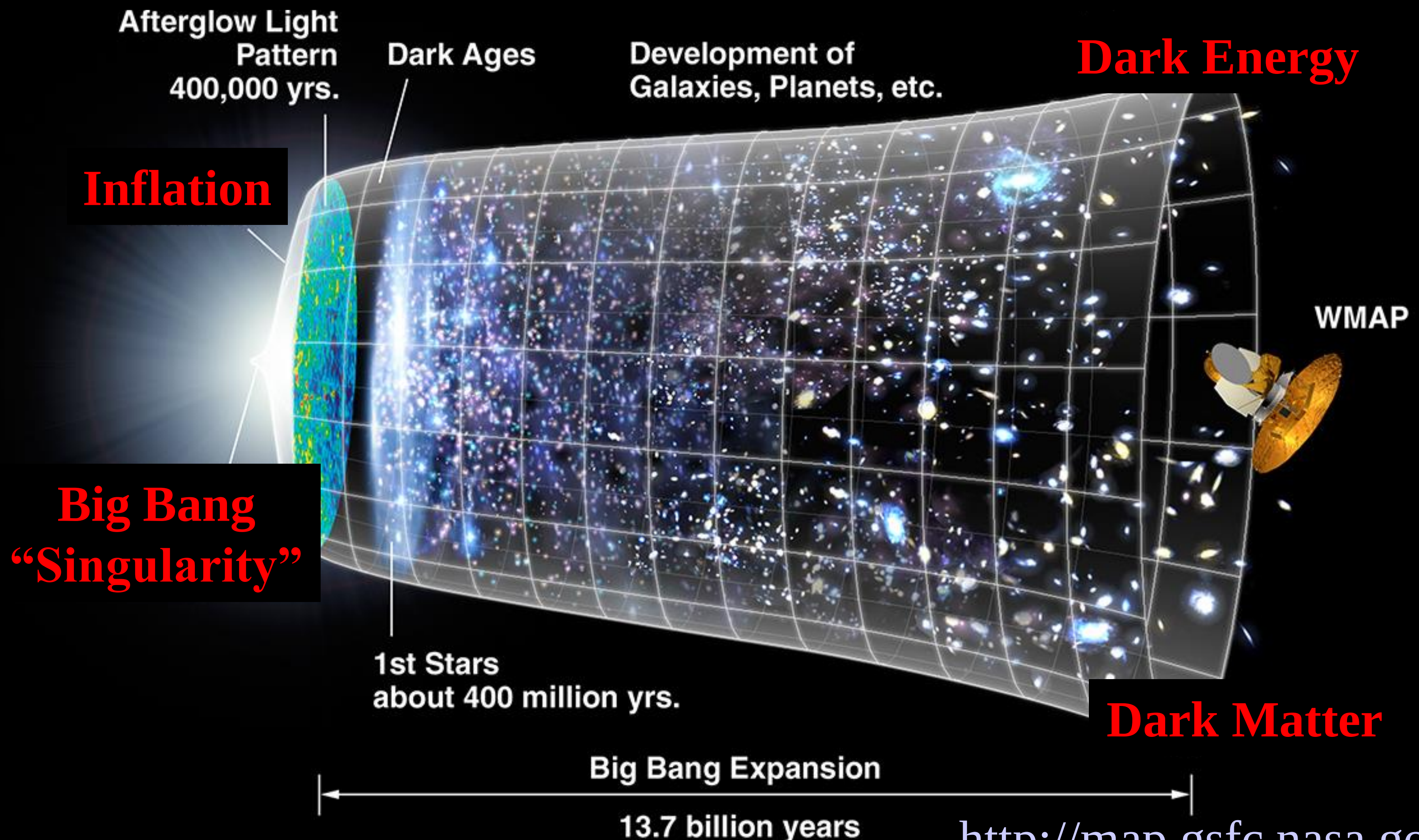
arXiv: 1301.1361 with Jean-Philippe Uzan
arxiv: 1303.1409
arXiv: 1403.0580 with John Kehayias & Jean-Philippe Uzan

INTRODUCTION

History of the universe



http://map.gsfc.nasa.gov/

History of the universe

- History = dynamics = sequence of configurations parameterized by time
- Beginning of the hot universe @ reheating
- Geometrical description of the universe breaks down @ initial singularity
- Space may be emergent
- How about time? Can time be emergent?

Time and dynamics

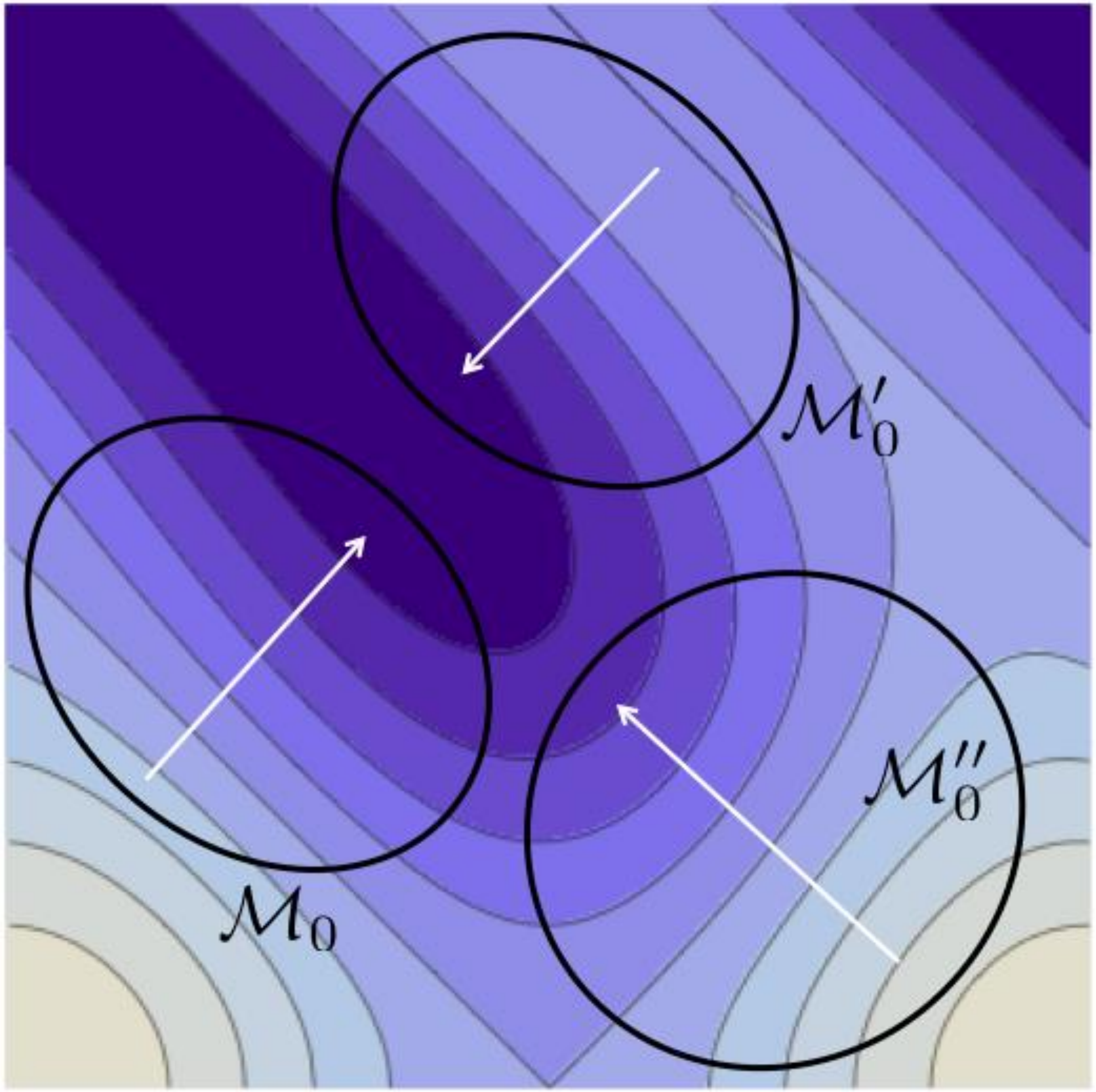
- In any diffeo-invariant theories of gravity, $H = \Sigma$ constraints = 0 (up to boundary terms) → no evolution of quantum state
- Dynamics should be encoded as correlations among various fields → one of the fields plays the role of time e.g. inflaton during inflation
- In this sense, concept of time and dynamics may be emergent

BASIC IDEA

Clock field

- Clock field = field playing the role of time
- It must carry at least one number
  → simplest: scalar field ϕ
- Time translational symmetry requires
  shift symmetry: $\phi \rightarrow \phi + c$
- Time reflection symmetry requires
  Z_2 symmetry: $\phi \rightarrow -\phi$

- Clock field does not have to be the same everywhere; multi-clock models also possible



$\mathcal{M}_0'$
$\mathcal{M}_0$
$\mathcal{M}_0''$

Effective metric

- Lorentz symmetry may be emergent (Chadha & Nielsen 1983)
- How about Lorentz signature?
- Let’s suppose that there is no concept of time @ fundamental level and start with 4D Riemannian (i.e. locally Euclidean) metric with (++++) signature.
- Physical fields couple to effective metric.
- Can effective metric have signature (-+++) ?

SIMPLE EXAMPLES

Scalar field χ in flat space

- Suppose that $\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0$
- Choose one of coordinates t so that $t \equiv \frac{\phi}{M^2}$
- Consider the Euclidean action

potential

$$S_\chi = \int d^4x \left[-\frac{1}{2}\delta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) + \frac{\alpha}{M^4}\left(\delta^{\mu\nu}\partial_\mu\phi\partial_\nu\chi\right)^2 \right]$$

Euclidean kinetic term — coupling to clock field

- This can be rewritten as

$$S_\chi = \int dt d^3x \left[-\frac{1}{2} g_{eff}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) \right]$$

$$g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if $\alpha > 1/2$!

Vector field A_μ in flat space

- Suppose that $\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0$
- Choose one of coordinates t so that $t \equiv \frac{\phi}{M^2}$
- Consider the Euclidean action

$$S_\chi = \int d^4x \left[-\frac{1}{4} \delta^{\mu\rho} \delta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{\alpha}{M^4} \delta^{\mu\rho} \delta^{\nu\sigma} \delta^{\alpha\beta} F_{\rho\alpha} F_{\sigma\beta} \partial_\mu \phi \partial_\nu \phi \right]$$

Euclidean kinetic term — coupling to clock field

- This can be rewritten as

$$S_\chi = \int dt d^3x \left[-\frac{1}{4} g_{eff}^{\mu\rho} g_{eff}^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

$$g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if $\alpha > 1/2$!

Abelian Higgs field ω in flat space

- Suppose that $\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0$
- Choose one of coordinates t so that $t \equiv \frac{\phi}{M^2}$
- Consider the Euclidean action

$$D_\mu \equiv \partial_\mu - iqA_\mu$$

potential

$$S_\chi = \int d^4x \left[-\frac{1}{2}\delta^{\mu\nu}(D_\mu\omega)^*(D_\nu\omega) - U(|\omega|^2) + \frac{\alpha}{M^4}\left|\delta^{\mu\nu}\partial_\mu\phi D_\nu\omega\right|^2 \right]$$

Euclidean kinetic term

coupling to clock field

- This can be rewritten as

$$S_\chi = \int dt d^3x \left[-\frac{1}{2} g_{eff}^{\mu\nu}(D_\mu\omega)^*(D_\nu\omega) - U(|\omega|^2) \right]$$

$$g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if α > 1/2!

GRAVITY

Model of clock field and gravity

- Shift symmetry: $\phi \rightarrow \phi + c$
- Z_2 symmetry: $\phi \rightarrow -\phi$
- Minimal # of d.o.f. → 2nd-order EOM →
  Riemannian version of covariant Galileon

$$S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} - g_5 G_{\mathrm{E}}^{\mu\nu} \partial_\mu\phi\partial_\nu\phi + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2\phi)^2 - (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2 \right] \Big\}$$

$$X_{\mathrm{E}} \equiv g_{\mathrm{E}}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2 \equiv g_{\mathrm{E}}^{\nu\rho} g_{\mathrm{E}}^{\sigma\mu} (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)(\nabla_\rho^{\mathrm{E}}\nabla_\sigma^{\mathrm{E}}\phi)$$

- Redefinition of $G_4(X_E)$ → $g_5 = 0$

$$S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2\phi)^2 - (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2 \right] \Big\}$$

Simple case

- Consider the Riemannian action

couplings to clock field

$$I_g = -M^2 \int dx^4 \sqrt{g_E} \left[\frac{\kappa_g}{2} R_E + \frac{\alpha_g}{M^4} G_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

Euclidean Einstein-Hilbert

- Adopt ADM in "unitary gauge" $t \equiv \dfrac{\phi}{M^2}$

$$g^E_{\mu\nu} dx^\mu dx^\nu = N_E^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Use formulas

$$\frac{1}{2} \int dx^4 \sqrt{g_E} R_E = \frac{1}{2} \int dt dx^3 N_E \sqrt{\gamma} (-K_E^{ij} K_{ij}^E + K_E^2 + R^{(3)})$$

$$\frac{1}{M^4} G_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = \frac{1}{2N_E^2} (-K_E^{ij} K_{ij}^E + K_E^2 - R^{(3)})$$

- The action is rewritten as

$$I_g = \frac{M^2}{2} \int dt dx^3 N_E \sqrt{\gamma} \left[\left(\frac{\alpha_g}{N_E^2} + \kappa_g \right) (K_E^{ij} K_{ij}^E - K_E^2) + \left(\frac{\alpha_g}{N_E^2} - \kappa_g \right) R^{(3)} \right]$$

- This describes Lorentzian gravity if $\dfrac{\alpha_g}{N_E^2} > |\kappa_g|$ (for tensor sector)

Correspondence

$$S_g = \int dx^4 \sqrt{g_{\rm E}} \left\{ G_4(X_{\rm E}) R_{\rm E} + \mathcal{K}(X_{\rm E}) - 2G_4'(X_{\rm E}) \left[(\nabla^2_{\rm E}\phi)^2 - (\nabla^{\rm E}_\mu \nabla^{\rm E}_\nu \phi)^2 \right] \right\}$$

$$g_{\mu\nu} = g^{\rm E}_{\mu\nu} - \frac{\partial_\mu\phi\partial_\nu\phi}{X_c}$$

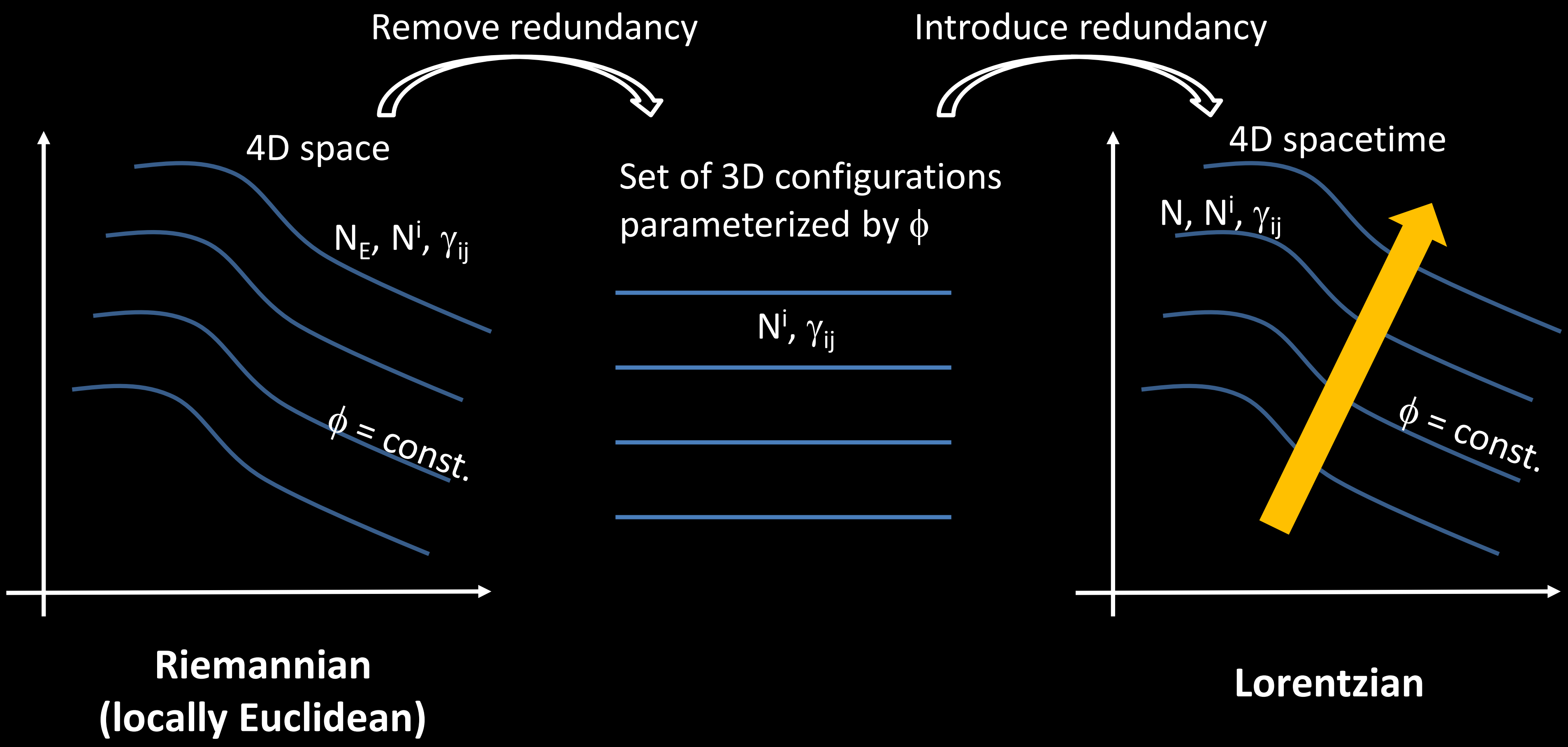
$$g^{\mu\nu} = g^{\mu\nu}_{\rm E} + \frac{g^{\mu\rho}_{\rm E} g^{\nu\sigma}_{\rm E} \partial_\rho\phi\partial_\sigma\phi}{X_c - X_{\rm E}}$$

$$\frac{1}{X} = \frac{1}{X_c} - \frac{1}{X_{\rm E}} \qquad X_c = \frac{M^4}{N_c^2}$$

$$\frac{f(X)}{\sqrt{X}} = \frac{G_4(X_{\rm E})}{\sqrt{X_{\rm E}}} \qquad \frac{P(X)}{\sqrt{X}} = \frac{\mathcal{K}(X_{\rm E})}{\sqrt{X_{\rm E}}}$$

$$S_g = \int dx^4 \sqrt{-g} \left\{ f(X) R + 2f'(X) \left[(\nabla^2\phi)^2 - (\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi) \right] + P(X) \right\}$$

Riemannian diffeo. vs Lorentzian diffeo.



Riemannian action in unitary gauge

- Riemannian metric in unitary gauge

$$g^{\rm E}_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu = N_{\rm E}^2 {\rm d}t^2 + \gamma_{ij}({\rm d}x^i + N^i {\rm d}t)({\rm d}x^j + N^j {\rm d}t)$$

$$t \equiv \frac{\phi}{M^2} \qquad N_{\rm E} \equiv \frac{1}{\sqrt{g_{\rm E}^{tt}}} = \frac{M^2}{\sqrt{X_{\rm E}}} \qquad N^i \equiv \gamma^{ij} g^{\rm E}_{tj} \qquad \gamma_{ij} \equiv g^{\rm E}_{ij}$$

- Extrinsic curvature $K^{\rm E}_{ij} \equiv \frac{1}{2N_{\rm E}}(\partial_t \gamma_{ij} - D_i N_j - D_j N_i)$
- Useful formulas

$$\begin{aligned}\sqrt{g_{\rm E}} R_{\rm E} &= N_{\rm E}\sqrt{\gamma}(-K_{\rm E}^{ij}K^{\rm E}_{ij} + K_{\rm E}^2 + R^{(3)}) \\ &\quad -2\partial_i(\sqrt{\gamma}\gamma^{ij}\partial_j N_{\rm E}) - 2\partial_t(\sqrt{\gamma}K_{\rm E}) + 2\partial_i(\sqrt{\gamma}N^i K_{\rm E})\end{aligned}$$

$$\begin{aligned}\phi^{\rm E}_{;ij} &\equiv \nabla^{\rm E}_i \nabla^{\rm E}_j \phi = \sqrt{X_{\rm E}} K^{\rm E}_{ij} \\ \phi^{\rm E}_{;\perp i} &\equiv \phi^{\rm E}_{;i\perp} \equiv n^\mu_{\rm E} \nabla^{\rm E}_\mu \nabla^{\rm E}_i \phi = \frac{1}{2}\sqrt{X_{\rm E}}\partial_i \ln X_{\rm E} \\ \phi^{\rm E}_{;\perp\perp} &\equiv n^\mu_{\rm E} n^\nu_{\rm E} \nabla^{\rm E}_\mu \nabla^{\rm E}_\nu \phi = \frac{1}{2}\sqrt{X_{\rm E}}\partial^{\rm E}_\perp \ln X_{\rm E}\end{aligned}$$

$$\partial^{\rm E}_\perp \equiv n^\mu_{\rm E}\partial_\mu \equiv \frac{1}{N_{\rm E}}(\partial_t - N^i \partial_i)$$

- Riemannian action is rewritten as

$$S_g = \int {\rm d}t{\rm d}x^3 N_{\rm E}\sqrt{\gamma}\left\{(2G_4' X_{\rm E} - G_4)(K_{\rm E}^{ij}K^{\rm E}_{ij} - K_{\rm E}^2) + G_4 R^{(3)} + \mathcal{K}(X_{\rm E})\right\}$$

Apparent Lorentzian structure

- Lorentzian metric

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{-N^2\mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(dx^j + N^j\mathrm{d}t)$$

$$N\mathrm{d}N = -N_{\mathrm{E}}\,\mathrm{d}N_{\mathrm{E}} \qquad \boxed{N = \sqrt{N_c^2 - N_{\mathrm{E}}^2}}$$

- Extrinsic curvature $K_{ij} \equiv \frac{1}{2N}(\partial_t\gamma_{ij} - D_iN_j - D_jN_i)$
- Apparent Lorentzian structure

$$\boxed{\begin{aligned} S_g = \int \mathrm{d}t\mathrm{d}x^3 N\sqrt{\gamma}\,\{[f(X) - 2Xf'(X)]\,(K^{ij}K_{ij} - K^2) \\ +f(X)R^{(3)} + P(X)\Big\} \end{aligned}}$$

$$f(X) \equiv \frac{N_{\mathrm{E}}}{N}G_4(X_E) \qquad f'(X) \equiv \frac{\mathrm{d}f(X)}{\mathrm{d}X} \qquad P(X) \equiv \frac{N_{\mathrm{E}}}{N}\mathcal{K}(X_{\mathrm{E}})$$

$$X \equiv \frac{M^4}{N^2}$$

Undo unitary gauge

- Useful formulas

$$\sqrt{-g}R = N\sqrt{\gamma}\left[K^{ij}K_{ij} - K^2 + R^{(3)}\right] - \Delta$$

$$\Delta = 2\partial_i\left(\sqrt{\gamma}\gamma^{ij}\partial_j N\right) - 2\partial_t\left(\sqrt{\gamma}K\right) + 2\partial_i\left(\sqrt{\gamma}N^i K\right)$$

$$\phi_{;ij} \equiv \nabla_i\nabla_j\phi = -\sqrt{X}K_{ij}$$

$$\phi_{;\perp i} \equiv \phi_{;i\perp} \equiv n^\mu\nabla_\mu\nabla_i\phi = \frac{1}{2}\sqrt{X}\partial_i \ln X$$

$$\phi_{;\perp\perp} \equiv n^\mu n^\nu\nabla_\mu\nabla_\nu\phi = \frac{1}{2}\sqrt{X}\partial_\perp \ln X$$

$$\partial_\perp = n^\mu\partial_\mu = \frac{1}{N}(\partial_t - N^i\partial_i)$$

- Lorentzian action

$$S_g = \int dx^4\sqrt{-g}\left\{f(X)R + 2f'(X)\left[(\nabla^2\phi)^2 - (\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\right] + P(X)\right\}$$

- "Covariantization"

$$X \equiv \frac{M^4}{N^2} \Rightarrow X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Correspondence

$$S_g = \int dx^4 \sqrt{g_{\mathrm{E}}} \left\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2 \phi)^2 - (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \right] \right\}$$

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{E}} - \frac{\partial_\mu \phi \partial_\nu \phi}{X_c}$$

$$g^{\mu\nu} = g_{\mathrm{E}}^{\mu\nu} + \frac{g_{\mathrm{E}}^{\mu\rho} g_{\mathrm{E}}^{\nu\sigma} \partial_\rho \phi \partial_\sigma \phi}{X_c - X_{\mathrm{E}}}$$

$$\frac{1}{X} = \frac{1}{X_c} - \frac{1}{X_{\mathrm{E}}} \qquad X_c = \frac{M^4}{N_c^2}$$

$$\frac{f(X)}{\sqrt{X}} = \frac{G_4(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}} \qquad \frac{P(X)}{\sqrt{X}} = \frac{\mathcal{K}(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}}$$

$$S_g = \int dx^4 \sqrt{-g} \left\{ f(X) R + 2f'(X) \left[(\nabla^2 \phi)^2 - (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi) \right] + P(X) \right\}$$

CANDIDATE UV THEORY

Power-counting renormalizable theory of clock field & gravity

arxiv: 1303.1409

- Riemannian (i.e. locally Euclidean) theory
- Shift symmetry: $\phi \rightarrow \phi + c$
- Z_2 symmetry: $\phi \rightarrow -\phi$
- 4D parity invariance: $x^\mu \rightarrow -x^\mu$
- Power-counting renormalizable
- Reduces to a special case of shift- and Z_2-symmetric covariant Galileon in IR

$$
\begin{aligned}
I_{\rm IR} \ = \ & \int dx^4 \sqrt{g_{\rm E}}\, [2Z\Lambda_{\rm E} - ZR_{\rm E} \\
& + X_{\rm E}^2 - 2X_\star X_{\rm E} - 2\gamma(\nabla_{\rm E}^2\phi)^2 + 2\gamma(\nabla^{\rm E}_\mu\nabla^{\rm E}_\nu\phi)^2 + \gamma X_{\rm E} R_{\rm E}]
\end{aligned}
$$

UV action (4th derivatives)

$$I_4 = \int dx^4 \sqrt{g_{\rm E}} \Big[c_1 R_{\rm E}^2 + c_2 R_{\rm E}^{\mu\nu} R^{\rm E}_{\mu\nu} + c_3 R_{\rm E}^{\mu\nu\rho\sigma} R^{\rm E}_{\mu\nu\rho\sigma} + c_4 X_{\rm E} R_{\rm E}$$
$$+ c_5 R_{\rm E}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + c_6 X_{\rm E}^2 + c_7 (\nabla_{\rm E}^2 \phi)^2 + c_8 (\nabla_\mu^{\rm E} \nabla_\nu^{\rm E} \phi)^2\Big]$$

Relevant deformations (2nd derivatives & 0th derivatives)

$$I_2 = \int dx^4 \sqrt{g_{\rm E}} \left[c_9 R_{\rm E} + c_{10} X_{\rm E}\right] \qquad I_0 = c_{11} \int dx^4 \sqrt{g_{\rm E}}$$

Integration by parts & rescaling of ϕ

$$\Rightarrow \quad c_5 = 0 \qquad c_6 = 1$$

Total action

$$I = \int dx^4 \sqrt{g_{\rm E}} \bigg[2Z\Lambda_{\rm E} - Z R_{\rm E} + \frac{1}{2\lambda} C_{\rm E}^2 - \frac{\omega}{3\lambda} R_{\rm E}^2 + \frac{\theta}{\lambda} E_{\rm E}$$
$$+ X_{\rm E}^2 - 2 X_\star X_{\rm E} + \alpha (\nabla_{\rm E}^2 \phi)^2 + \beta (\nabla_\mu^{\rm E} \nabla_\nu^{\rm E} \phi)^2 + \gamma X_{\rm E} R_{\rm E}\bigg]$$

$$C_{\rm E}^2 \equiv R_{\rm E}^{\mu\nu\rho\sigma} R^{\rm E}_{\mu\nu\rho\sigma} - 2 R_{\rm E}^{\mu\nu} R^{\rm E}_{\mu\nu} + R_{\rm E}^2/3$$

$$E_{\rm E} \equiv R_{\rm E}^{\mu\nu\rho\sigma} R^{\rm E}_{\mu\nu\rho\sigma} - 4 R_{\rm E}^{\mu\nu} R^{\rm E}_{\mu\nu} + R_{\rm E}^2$$

COSMOLOGICAL SOLUTION

Cosmological solution

- Flat (K=0) FLRW

$$N = 1 \quad N_i = 0 \quad \gamma_{ij} = a(t)^2\delta_{ij} \quad \phi = \phi_0(t)$$

- EOM for ϕ = shift charge conservation

$$\dot{J}_\phi + 3HJ_\phi = 0 \quad \Rightarrow \quad \boxed{J_\phi \propto 1/a^3}$$

$$J_\phi \equiv \left[P_0' + 6H^2(2X_0 f_0'' + f_0')\right]\dot{\phi}_0$$

- Metric EOM

$$3M_{\rm eff}^2 H^2 = 2J_\phi\dot{\phi}_0 - P_0 \qquad M_{\rm eff}^2 \equiv 2(f_0 - 2X_0 f_0')$$

- P’(X) and f’(X) near a local minimum of P(X)

$$P'(X) = p_2\delta + \mathcal{O}(\delta^2) \quad f'(X) = \frac{f_1 + f_2\delta}{M^2} + \mathcal{O}(\delta^2) \quad \delta \equiv \frac{X}{M^4} - q$$

$$J_\phi \propto 1/a^3 \Rightarrow \delta + \mathcal{O}(H^2/M^2) \propto 1/a^3 \to 0$$

$\Rightarrow$ $3M_{\rm eff}^2H^2$ = DM ($\propto 1/a^3$) + DE ($\sim$const)

Stability of tensor perturbation

- Tensor-type perturbation

$$N = 1 \quad N_i = 0 \quad \gamma_{ij} = a(t)^2 \left[\mathrm{e}^h \right]_{ij}$$

$$\phi = \phi_0(t) \qquad \partial_i h^i_k = 0 = \delta^{ij} h_{ij}$$

- Quadratic action in Fourier space

$$\delta S^{(2)}_{\mathrm{T},\boldsymbol{k}} = \frac{1}{8} \int \mathrm{d}t a^3 \left[M^2_{\mathrm{eff}} \dot{h}^2_{\boldsymbol{k}} - 2 f_0 \frac{\boldsymbol{k}^2}{a^2} h^2_{\boldsymbol{k}} \right]$$

$$M^2_{\mathrm{eff}} \equiv 2(f_0 - 2X_0 f_0')$$

- Stability condition

$$M^2_{\mathrm{eff}} > 0 \qquad f_0 > 0$$

Stability of scalar perturbation

- Scalar-type perturbation in unitary gauge

$$N = 1 + \alpha \quad N_i = \partial_i \beta \quad \gamma_{ij} = a(t)^2 \mathrm{e}^{2\zeta} \delta_{ij}$$

$$\phi = \phi_0(t)$$

- Quadratic action after eliminating α and β

$$\delta S^{(2)}_{\mathrm{S},\boldsymbol{k}} = \frac{1}{2}\int \mathrm{d}t a^3 \left[\mathcal{A} \dot{\zeta}^2_{\boldsymbol{k}} - \mathcal{B} \frac{\boldsymbol{k}^2}{a^2} \zeta^2_{\boldsymbol{k}} \right]$$

$$\mathcal{A} = \frac{M^2_{\mathrm{eff}}}{H^2 \mathcal{G}^2} \left(6 + M^2_{\mathrm{eff}} \mathcal{F}\right) \qquad \mathcal{B} = \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a M^4_{\mathrm{eff}}}{H \mathcal{G}^2}\right) + 4 f_0$$

$$\mathcal{F} = P''_0 X^2_0 + \frac{1}{2} J_\phi \dot{\phi}_0 + 3H^2 \left[4 f'''_0 X^3_0 + 14 f''_0 X^2_0 + 6 f'_0 X_0 - f_0\right]$$

$$\mathcal{G} = 4 f''_0 X^2_0 + 4 f'_0 X_0 - f_0 \qquad M^2_{\mathrm{eff}} \equiv 2(f_0 - 2X_0 f'_0)$$

- Stability condition

$$\mathcal{A} > 0 \qquad \mathcal{B} > 0$$

PHENOMENOLOGY

Free functions/parameters

- Gravity sector: $G_4(X_E)$, $K(X_E)$ [or $f(X)$, $P(X)$]

$$S_g = \int dx^4 \sqrt{g_E}\Big\{ \boxed{G_4(X_E)} R_E - g_5 G_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \boxed{\mathcal{K}(X_E)} - 2\boxed{G_4'(X_E)}\left[(\nabla_E^2\phi)^2 - (\nabla_\mu^E\nabla_\nu^E\phi)^2\right]\Big\}$$

- Matter sector: $(\kappa_\chi, \alpha_\chi)$, (κ_A, α_A), $(\kappa_\omega, \alpha_\omega)$

$$S_\chi = \int dx^4 \sqrt{g_E}\left[-\frac{\boxed{\kappa_\chi}}{2} g_E^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \tilde{V}(\chi) + \frac{\boxed{\alpha_\chi}}{2M^4}(g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\chi)^2\right]$$

$$S_A = \frac{1}{4}\int dx^4 \sqrt{g_E}\left[-\boxed{\kappa_A} F_E^{\mu\nu}F_{\mu\nu} + 2\frac{\boxed{\alpha_A}}{M^4} F_E^{\mu\rho}F_{E\rho}^{\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

$$S_\psi = \int dx^4 \sqrt{g_E}\Big\{-\frac{\boxed{\kappa_\psi}}{2} g_E^{\mu\nu}(\partial_\mu + iqA_\mu)\psi^*(\partial_\nu - iqA_\nu)\psi + \frac{\boxed{\alpha_\psi}}{2M^4}|g_E^{\mu\nu}\partial_\mu\phi(\partial_\nu - iqA_\nu)\psi|^2 - \tilde{U}(|\psi|^2)\Big\}$$

- Clock field configuration: $\phi(x)$

$$X_E \equiv g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Constraints

- Stability of clock field/gravity sector

$$M_{\rm eff}^2 > 0 \qquad f_0 > 0 \qquad \mathcal{A} > 0 \qquad \mathcal{B} > 0$$

- Amount of DE/DM

$$P_0 \sim -3\Omega_{\Lambda 0} M_{\rm eff}^2 H_0^2 \sim -2.1 M_{\rm eff}^2 H_0^2$$

$$\frac{2}{3}\frac{J_{\phi_0}}{M_{\rm eff}^2}\sqrt{q}\frac{M^2}{H_0^2} \leq \Omega_{\rm m0} \sim 0.3$$

- Stability of matter sector

$$\frac{\alpha_\chi}{N_{\rm E}^2} > \kappa_\chi > 0 \qquad \frac{\alpha_A}{N_{\rm E}^2} > \kappa_A > 0$$

- Coincidence of speed limits in matter sector

$$\frac{\kappa_A}{\alpha_A} = \frac{\kappa_\chi}{\alpha_\chi}$$ independently from clock field configuration

- Avoidance of gravi-Cerenkov radiation

$$\frac{c_\gamma - c_{\rm GW}}{c_\gamma} < 2 \times 10^{-15} \qquad c_\gamma^2 = \left[\frac{\alpha_A X_{\rm E}}{\kappa_A M^4} - 1\right]^{-1} \qquad c_{\rm GW}^2 = \left[\frac{2G_4' X_{\rm E}}{G_4} - 1\right]^{-1}$$

SUMMARY & DISCUSSIONS

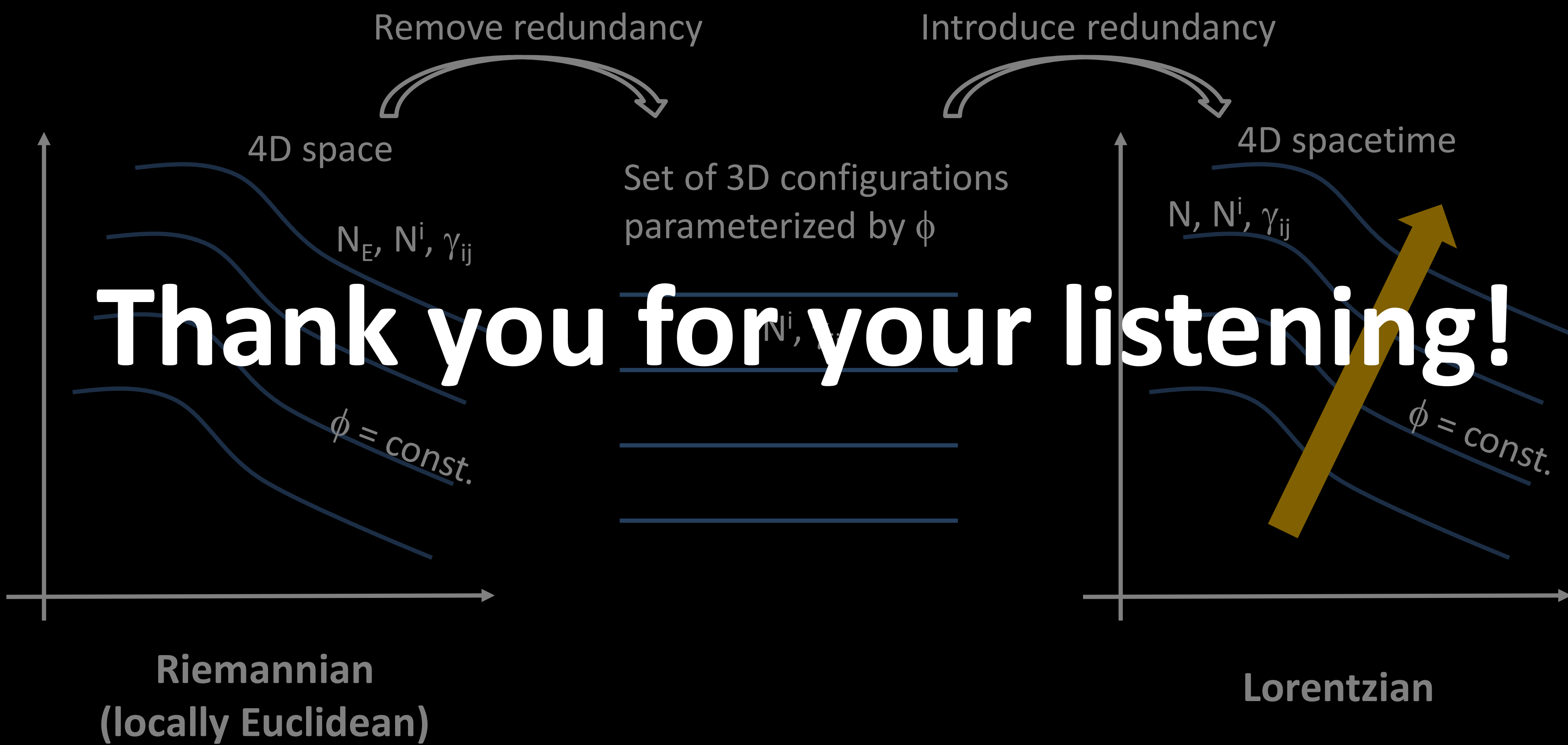
Summary

- Lorentzian dynamics can emerge as an effective property of a fundamentally Riemannian theory.
- This requires introduction of a field playing the role of time, a clock field.
- This idea was applied to scalar, vector, (Dirac, Weyl, Majorana) spinor fields and gravity as explicit examples.
- In our simple realization, the clock field/gravity sector is described by the Riemannian version of a shift- and Z_2-symmetric covariant Galileon.
- We obtained the dictionary for the mapping from Riemannian Galileon to Lorentzian Galileon.
- We proposed a power-counting renormalizable Riemanian theory as a candidate UV theory.
- We found a FLRW solution and analyzed stability of scalar- and tensor- perturbations.

Future works

- Development of quantum theory
- CPT-invariant construction of spinor fields
- Understanding of black holes and singularities
- Possibility of embedding Lorentzian dS/CFT inside Euclidean AdS/CFT
- Emergence of Lorentz symmetry at low energy
- …
- Multi-clock models
- Time emergence & compactification → landscape with various signatures & dimensions?

Riemannian diffeo. vs Lorentzian diffeo.



Thank you for your listening!