

ENUMERATING PRIME LINK EXTERIORS WITH LENGTHS UP TO 10 BY A CANONICAL ORDER

AKIO KAWAUCHI and IKUO TAYAMA

Department of Mathematics, Osaka City University

Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan

kawauchi@sci.osaka-cu.ac.jp, CQX05126@nifty.com

ABSTRACT

In the previous paper we enumerated the prime links with lengths up to 10. In this paper we show a table of prime link exteriors with lengths up to 10 by using the enumeration of the prime links.

Keywords: Exterior, Lattice point, Length, Prime link

1. Introduction

It is a basic problem to enumerate all the prime links without overlaps. In [3] we made a table of prime links and in this paper we enumerate the prime link exteriors. This is an important step to enumerate the 3-manifolds. Here we consider unoriented links unless otherwise stated, so that we say that two links L and L' in S^3 are *equivalent* and we denote it by $L = L'$ if there is a homeomorphism $h : (S^3, L) \rightarrow (S^3, L')$.

A method of enumerating the set of links and the set of closed connected orientable 3-manifolds was suggested in [1]. The idea is to introduce a well-order on the set of links by embedding it into a canonical well-ordered set of (integral) lattice points. This well-order also induces a well-order on the set of closed connected orientable 3-manifolds. By using this method, the first 28 and 26 lattice points of lengths up to 7 corresponding to prime links and closed connected orientable 3-manifolds are respectively tabulated without any computer aid in [1]. We enlarged the table of the first 28 lattice points corresponding to prime links into that of the first 443 lattice points of lengths up to 10 corresponding to prime links in [3]. Our tentative goal of this study is to enumerate the lattice points of lengths up to 10 corresponding to 3-manifolds by hand to know which invariant is useful in hand calculations. In this

paper we show a table of the first 399 lattice points of lengths up to 10 corresponding to prime link exteriors, which is in a step of the goal.

In Section 2, we review the definition of the well-order described in [1]. In Section 3, we explain how to enumerate the lattice points corresponding to prime link exteriors. At the end, we show a table of the first 443 and 399 lattice points of lengths up to 10 corresponding to prime links and prime link exteriors respectively.

2 . Definition of a well-order on the set of links

Let \mathbf{Z} be the set of integers, and \mathbf{Z}^n the product of n copies of \mathbf{Z} . We put

$$\mathbf{X} = \prod_{n=1}^{\infty} \mathbf{Z}^n = \{(x_1, x_2, \dots, x_n) \mid x_i \in \mathbf{Z}, n = 1, 2, \dots\}.$$

We call elements of \mathbf{X} *lattice points*. For a lattice point $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbf{X}$, we put $\ell(\mathbf{x}) = n$ and call it the *length* of \mathbf{x} . Let $|\mathbf{x}|$ and $|\mathbf{x}|_N$ be the lattice points determined from \mathbf{x} by the following formulas:

$$\begin{aligned} |\mathbf{x}| &= (|x_1|, |x_2|, \dots, |x_n|) \text{ and} \\ |\mathbf{x}|_N &= (|x_{j_1}|, |x_{j_2}|, \dots, |x_{j_n}|) \text{ where } |x_{j_1}| \leq |x_{j_2}| \leq \dots \leq |x_{j_n}| \text{ and } \{j_1, j_2, \dots, j_n\} = \{1, 2, \dots, n\}. \end{aligned}$$

We define a well-order on \mathbf{X} as follows (See [1]):

Definition 2.1. We define a well-order on \mathbf{Z} by $0 < 1 < -1 < 2 < -2 < 3 < -3 \dots$, and for $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ we define $\mathbf{x} < \mathbf{y}$ if we have one of the following conditions (1)-(4):

- (1) $\ell(\mathbf{x}) < \ell(\mathbf{y})$.
- (2) $\ell(\mathbf{x}) = \ell(\mathbf{y})$ and $|\mathbf{x}|_N < |\mathbf{y}|_N$ by the lexicographic order (on the natural number order).
- (3) $|\mathbf{x}|_N = |\mathbf{y}|_N$ and $|\mathbf{x}| < |\mathbf{y}|$ by the lexicographic order (on the natural number order).
- (4) $|\mathbf{x}| = |\mathbf{y}|$ and $\mathbf{x} < \mathbf{y}$ by the lexicographic order on the well-order of \mathbf{Z} defined above.

For $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbf{X}$, we put

$$\min|\mathbf{x}| = \min_{1 \leq i \leq n} |x_i| \quad \text{and} \quad \max|\mathbf{x}| = \max_{1 \leq i \leq n} |x_i|.$$

Let $\beta(\mathbf{x})$ be the $(\max|\mathbf{x}| + 1)$ -string braid determined from \mathbf{x} by the identity

$$\beta(\mathbf{x}) = \sigma_{|x_1|}^{\text{sign}(x_1)} \sigma_{|x_2|}^{\text{sign}(x_2)} \dots \sigma_{|x_n|}^{\text{sign}(x_n)},$$

where we define $\sigma_{|0|}^{\text{sign}(0)} = 1$. We note that $\max|\mathbf{x}| + 1$ is the minimum string number of the braid indicated by the right-hand side of the identity. Let $\text{cl}\beta(\mathbf{x})$ be the closure of the braid $\beta(\mathbf{x})$. Let \mathbf{L} be the set of links. Then we have a map

$$\text{cl}\beta : \mathbf{X} \rightarrow \mathbf{L}$$

sending \mathbf{x} to $\text{cl}\beta(\mathbf{x})$. By Alexander's braiding theorem, the map $\text{cl}\beta$ is surjective. For $L \in \mathbf{L}$, we define a map

$$\sigma : \mathbf{L} \rightarrow \mathbf{X}$$

by $\sigma(L) = \min\{\mathbf{x} \in \mathbf{X} \mid \text{cl}\beta(\mathbf{x}) = L\}$. Then σ is a right inverse of the map $\text{cl}\beta$ and hence is injective. Now we have a well-order on \mathbf{L} by the following definition:

Definition 2.2. For $L, L' \in \mathbf{L}$, we define $L < L'$ if $\sigma(L) < \sigma(L')$.

For a link $L \in \mathbf{L}$, we call $\ell(\sigma(L))$ the *length* of L .

Let \mathbf{L}^p be the set of prime links, where we consider that the 2-component trivial link is not prime. We use the injection σ for our method of a tabulation of \mathbf{L}^p . For $k \in \mathbf{Z}$, let k^n and $-k^n$ be the lattice points determined by

$$k^n = \underbrace{(k, k, \dots, k)}_n \quad \text{and} \quad -k^n = (-k)^n,$$

respectively.

For $\mathbf{x} = (x_1, x_2, \dots, x_n)$, $\mathbf{y} = (y_1, y_2, \dots, y_m) \in \mathbf{X}$, let (\mathbf{x}, \mathbf{y}) be the lattice point determined by the following formula:

$$(\mathbf{x}, \mathbf{y}) = (x_1, \dots, x_n, y_1, \dots, y_m).$$

With these terminologies, we show a table of prime links with lengths up to 10 at the end of this paper.

3. A method of a tabulation of prime link exteriors

Since a knot is determined by its exterior by the Gordon-Luecke Theorem, we classify the exteriors of two or more component links.

Definition 3.1. For r -component links L and L' in S^3 , their Alexander polynomials $\Delta_L(t_1, \dots, t_r)$ and $\Delta_{L'}(t_1, \dots, t_r)$ are equivalent if there is an isomorphism $\varphi : \langle t_1, \dots, t_r \mid t_i t_j = t_j t_i (i, j = 1, \dots, r) \rangle \rightarrow \langle t_1, \dots, t_r \mid t_i t_j = t_j t_i (i, j = 1, \dots, r) \rangle$ satisfying

$$\Delta_{L'}(t_1, \dots, t_r) = \pm t_1^{a_1} \cdots t_r^{a_r} \Delta_L(\varphi(t_1), \dots, \varphi(t_r))$$

for some integers a_i , $i = 1, \dots, r$.

For a link L in S^3 , let $E(L) = \text{cl}(S^3 - N(L))$ be its exterior, where $N(L)$ is a regular neighborhood of L . Then we see the following lemma:

Lemma 3.2. For links L and L' in S^3 , if there is a homeomorphism $E(L) \rightarrow E(L')$, then their Alexander polynomials are equivalent.

By using the above lemma, we divide the prime links into several groups, each of which consists of the links with the equivalent Alexander polynomials. For two or more component prime links with lengths up to 10, there are 42 groups consisting of two or more elements:

- | | |
|---|--|
| (1) $4_1^2 < 7_7^2 < 9_{43}^2 < 9_{59}^2$ | (2) $6_1^2 < 9_{49}^2$ |
| (3) $5_1^2 < 7_8^2 < 8_{15}^2 < 9_{47}^2 < 10_{174}^2 < 10_{173}^2$ | (4) $6_3^2 < 8_{16}^2 < 9_{45}^2 < 10_{128}^2$ |
| (5) $7_3^2 < 9_{46}^2$ | (6) $9_{50}^2 < 10_{132}^2$ |
| (7) $7_5^2 < 9_{48}^2 < 10_{130}^2$ | (8) $7_2^2 < 9_{54}^2 < 10_{140}^2$ |
| (9) $7_4^2 < 9_{44}^2 < 10_{162}^2 < 10_{124}^2$ | (10) $10_{176}^2 < 10_{178}^2$ |
| (11) $9_{57}^2 < 10_{167}^2$ | (12) $7_6^2 < 9_{55}^2 < 9_{56}^2 < 10_{160}^2 < 10_{161}^2$ |
| (13) $9_{58}^2 < 10_{168}^2$ | (14) $8_{12}^2 < 8_{10}^2 < 10_{163}^2 < 10_{129}^2 < 10_{170}^2 < 10_{131}^2$ |
| (15) $8_{13}^2 < 10_{154}^2$ | (16) $8_{11}^2 < 10_{125}^2$ |
| (17) $9_{27}^2 < 9_{15}^2$ | (18) $9_{18}^2 < 9_{36}^2 < 10_{145}^2$ |
| (19) $9_{33}^2 < 9_{32}^2$ | (20) $9_{13}^2 < 10_{143}^2$ |
| (21) $9_{31}^2 < 10_A^2$ | (22) $10_{56}^2 < 10_{34}^2$ |
| (23) $10_{76}^2 < 10_{78}^2$ | (24) $10_{31}^2 < 10_{36}^2 < 10_{50}^2$ |
| (25) $10_{81}^2 < 10_{83}^2 < 10_{104}^2$ | (26) $10_{107}^2 < 10_{93}^2 < 10_{91}^2 < 10_{106}^2$ |
| (27) $10_{84}^2 < 10_{105}^2$ | (28) $10_{48}^2 < 10_{66}^2$ |
| (29) $6_3^3 < 8_7^3 < 10_{44}^3$ | (30) $6_1^3 < 8_8^3 < 9_{13}^3 < 9_{17}^3$ |
| (31) $6_2^3 < 8_9^3 < 9_{19}^3 < 9_{18}^3 < 10_{61}^3$ | (32) $7_1^3 < 9_{14}^3 < 10_{48}^3$ |
| (33) $8_3^3 < 10_{49}^3$ | (34) $8_1^3 < 10_{45}^3$ |
| (35) $10_{58}^3 < 10_{59}^3$ | (36) $8_5^3 < 10_{56}^3$ |
| (37) $8_6^3 < 10_{60}^3$ | (38) $8_3^4 < 10_A^4$ |
| (39) $8_2^4 < 10_{12}^4 < 10_B^4 < 10_C^4$ | (40) $10_{16}^4 < 10_{17}^4$ |
| (41) $8_1^4 < 10_D^4$ | (42) $10_{15}^4 < 10_{13}^4 < 10_{18}^4$ |

We divide each group into several subgroups with the homeomorphic exteriors. We have the following results, shown later.

For (2), (4), (5), (6), (7), (11), (13), (16), (29), (30), (32), (33), (34), (35), (38), (39), (40), (41) and (42), their exteriors are homeomorphic to each others. For the rest of the groups, we have the following homeomorphism types:

- (1) $E(4_1^2) \cong E(7_7^2) \cong E(9_{43}^2), E(9_{59}^2)$
- (3) $E(5_1^2) \cong E(7_8^2) \cong E(8_{15}^2) \cong E(9_{47}^2), E(10_{174}^2), E(10_{173}^2)$
- (8) $E(7_2^2), E(9_{54}^2), E(10_{140}^2)$
- (9) $E(7_4^2) \cong E(9_{44}^2) \cong E(10_{124}^2), E(10_{162}^2)$
- (10) $E(10_{176}^2), E(10_{178}^2)$
- (12) $E(7_6^2) \cong E(10_{160}^2) \cong E(10_{161}^2), E(9_{55}^2) \cong E(9_{56}^2)$
- (14) $E(8_{12}^2) \cong E(10_{131}^2), E(8_{10}^2) \cong E(10_{129}^2), E(10_{163}^2), E(10_{170}^2)$
- (15) $E(8_{13}^2), E(10_{154}^2)$
- (17) $E(9_{27}^2), E(9_{15}^2)$
- (18) $E(9_{18}^2), E(9_{36}^2), E(10_{145}^2)$
- (19) $E(9_{33}^2), E(9_{32}^2)$
- (20) $E(9_{13}^2), E(10_{143}^2)$
- (21) $E(9_{31}^2), E(10_A^2)$

- (22) $E(10_{56}^2), E(10_{34}^2)$
- (23) $E(10_{76}^2), E(10_{78}^2)$
- (24) $E(10_{31}^2), E(10_{36}^2), E(10_{50}^2)$
- (25) $E(10_{81}^2), E(10_{83}^2), E(10_{104}^2)$
- (26) $E(10_{107}^2), E(10_{93}^2), E(10_{91}^2), E(10_{106}^2)$
- (27) $E(10_{84}^2), E(10_{105}^2)$
- (28) $E(10_{48}^2), E(10_{66}^2)$
- (31) $E(6_2^3) \cong E(9_{18}^3), E(8_9^3) \cong E(9_{19}^3) \cong E(10_{61}^3)$
- (36) $E(8_5^3), E(10_{56}^3)$
- (37) $E(8_6^3), E(10_{60}^3)$

We show these results. For each group, we have $E(L) \cong E(L')$ by a composition of twist homeomorphisms along trivial components. Next we show that for each group, the classified exteriors are not homeomorphic to each others.

For (1), $E(4_1^2)$, $E(7_7^2)$ and $E(9_{43}^2)$ are Seifert manifolds since 4_1^2 is a torus link. On the other hand, 9_{59}^2 is decomposed into two nontrivial tangles and so $E(9_{59}^2)$ is not a Seifert manifold. We conclude that $E(4_1^2) \cong E(7_7^2) \cong E(9_{43}^2) \not\cong E(9_{59}^2)$.

For (3), let $5_1^2 = K_1 \cup K_2$ and $10_{174}^2 = K'_1 \cup K'_2$, where $K_1 = K_2 = K'_1 = O$ and $K'_2 = 6_2$. Suppose that there is a homeomorphism $h : E(5_1^2) \rightarrow E(10_{174}^2)$. We may assume $h(\partial N(K_1)) = \partial N(K'_1)$. Let $K'_{2,n}$ be a knot obtained by twisting K'_2 along K'_1 n times. Since $\text{lk}(K_1, K_2) = \text{lk}(K'_1, K'_2) = 0$, $E(K_2)$ should be homeomorphic to $E(K'_{2,n})$ for some integer n and then $K_2 = K'_{2,n}$. However this is impossible. So we have $E(5_1^2) \not\cong E(10_{174}^2)$. The same argument implies that $E(5_1^2) \not\cong E(10_{173}^2)$ and $E(10_{174}^2) \not\cong E(10_{173}^2)$. We conclude that there are 3 homeomorphism types of exteriors: $E(5_1^2) \cong E(7_8^2) \cong E(8_{15}^2) \cong E(9_{47}^2)$, $E(10_{174}^2)$ and $E(10_{173}^2)$.

For (9), (12), (14), (15), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26) and (27), we have the result in the same way as in (3).

For (8), suppose that there is a homeomorphism $h : E(7_2^2) \rightarrow E(9_{54}^2)$. From their Alexander polynomials $\Delta_{7_2^2}(t_1, t_2)$, $\Delta_{9_{54}^2}(t_1, t_2)$, we see $h_*(t_1) = t_i^{\pm 1}$ and $h_*(t_2) = t_j^{\pm 1}$ where $h_* : H_1(E(7_2^2)) \rightarrow H_1(E(9_{54}^2))$ is an isomorphism induced by h , t_1 and t_2 are meridians of the homology groups, and $\{i, j\} = \{1, 2\}$. Since the linking numbers for 7_2^2 and 9_{54}^2 are both non-zero, the homeomorphism h preserves the meridians of $E(7_2^2)$ and $E(9_{54}^2)$ and extends to a homeomorphism from S^3 to S^3 sending 7_2^2 to 9_{54}^2 , which is impossible. So we have $E(7_2^2) \not\cong E(9_{54}^2)$. By the same methods, we also have $E(7_2^2) \not\cong E(10_{140}^2)$ and $E(9_{54}^2) \not\cong E(10_{140}^2)$.

For (10), we compute the first homology groups of the double covering spaces. Those of $E(10_{176}^2)$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_7$, \mathbf{Z}^3 , \mathbf{Z}^3 , and those of $E(10_{178}^2)$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_9$, \mathbf{Z}^3 , \mathbf{Z}^3 , where we write $A^n = \underbrace{A \oplus \cdots \oplus A}_n$ for an abelian group A .

For (28), as in (10), we have the first homology groups of the double covering spaces of $E(10_{48}^2)$ are $\mathbf{Z}^2 \oplus \mathbf{Z}_{36}$, \mathbf{Z}^3 , \mathbf{Z}^3 and those of $E(10_{66}^2)$ are $\mathbf{Z}^2 \oplus \mathbf{Z}_3 \oplus \mathbf{Z}_{12}$, \mathbf{Z}^3 , \mathbf{Z}^3 .

For (31), the absolute values of the linking numbers of all pairs of the components for 6_2^3 are 0, 0, 0 and those for 8_9^3 are 0, 0, 2 and we have $E(6_2^3) \not\cong E(8_9^3)$.

For (36), the absolute values of the linking numbers of all pairs of the components for 8_5^3 are 0, 0, 1 and those for 10_{56}^3 are 0, 0, 3 and we have $E(8_5^3) \not\cong E(10_{56}^3)$.

For (37), we compute the first homology groups of the double covering spaces. Those of $E(8_6^3)$ are $\mathbf{Z}^3 \oplus (\mathbf{Z}_3)^2$, $\mathbf{Z}^4 \oplus \mathbf{Z}_3$, \mathbf{Z}^3 , \mathbf{Z}^3 , \mathbf{Z}^3 , \mathbf{Z}^4 , \mathbf{Z}^4 and those of $E(10_{60}^3)$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_9$, $\mathbf{Z}^4 \oplus \mathbf{Z}_3$, \mathbf{Z}^3 , \mathbf{Z}^3 , \mathbf{Z}^3 , \mathbf{Z}^4 , \mathbf{Z}^4 .

We obtain a table of the lattice points of lengths up to 10 corresponding to prime link exteriors, by omitting the lattice points corresponding to $7_7^2, 7_8^2, 8_7^3, 8_8^3, 8_{16}^2, 8_{15}^2, 9_{43}^2, 9_{44}^2, 9_{49}^2, 9_{13}^3, 9_{14}^3, 9_{19}^3, 9_{18}^3, 9_{17}^3, 10_{44}^3, 10_{45}^3, 10_{132}^2, 9_{45}^2, 10_{128}^2, 9_{56}^2, 9_{47}^2, 10_{160}^2, 10_{124}^2, 9_{48}^2, 10_{125}^2, 10_{12}^4, 9_{46}^2, 10_{129}^2, 10_A^4, 10_B^4, 10_C^4, 10_D^4, 10_{167}^2, 10_{13}^4, 10_{17}^4, 10_{130}^2, 10_{131}^2, 10_{168}^2, 10_{161}^2, 10_{18}^4, 10_{49}^3, 10_{48}^3, 10_{61}^3, 10_{59}^3$ from the table of the lattice points of lengths up to 10 corresponding to prime links because their exteriors have already appeared.

In the table shown at the end of this paper, we put \times in the above links on the column of E . Therefore, if we omit the lattice points with the mark \times , we have a table of the lattice points corresponding to prime link exteriors.

References

- [1] A. Kawauchi, *A tabulation of 3-manifolds via Dehn surgery*, Boletín de la Sociedad Matemática Mexicana, a special issue for FICOFEST (3) 10 (2004), 279–304 (<http://www.sci.osaka-cu.ac.jp/~kawauchi/index.htm>).
- [2] A. Kawauchi, *Characteristic genera of closed orientable 3-manifolds*, preprint (<http://www.sci.osaka-cu.ac.jp/~kawauchi/index.htm>).
- [3] A. Kawauchi and I. Tayama, *Enumerating prime links by a canonical order*, Journal of Knot Theory and Its Ramifications Vol. 15, No. 2 (2006), 217–237.
- [4] A. Kawauchi and I. Tayama, *Enumerating the exteriors of prime links by a canonical order*, in: Proc. Second East Asian School of Knots, Links, and Related Topics in Geometric Topology (Dalian, Aug. 2005), 269–277. (<http://www.sci.osaka-cu.ac.jp/~kawauchi/index.htm>).
- [5] A. Kawauchi and I. Tayama, *Enumerating 3-manifolds with lengths up to 9 by a canonical order*, International Conference on Topology and its Applications 2007-A Joint conference with 4th Japan Mexico Topology Conference', Topology and its applications.

x	L	E	x	L	E	x	L	E
0	0		$(1, -2, 1, -2, 1, -2, 1, -2)$	8 ₁₈		$(1^2, -2, 1, -2, 1, -2, 1, -2)$	9 ₄₂	
1 ²	2 ₁		$(1^3, 2, -1, -3, 2, -3)$	7 ₂₂		$(1^4, 2, -1, -3, 2, -3)$	8 ₆	
1 ³	3 ₁		$(1^3, -2, 1, 3, -2, 3)$	8 ₅		$(1^4, -2, 1, 3, -2, 3)$	9 ₁₁	
1 ⁴	4 ₁		$(1^2, 2, 1^2, -3, 2, -3)$	8 ₁₆	×	$(1^3, 2, 1^2, -3, 2, -3)$	9 ₄₃	
$(1, -2, 1, -2)$	4 ₁		$(1^2, 2, -1^2, -3, 2, -3)$	8 ₁₅	×	$(1^3, 2, -1^2, -3, 2, -3)$	9 ₄₄	
1 ⁵	5 ₁		$(1^2, -2, 1^2, 3, -2, 3)$	8 ₅		$(1^3, -2, 1^2, 3, -2, 3)$	9 ₃₆	
$(1^2, -2, 1, -2)$	5 ₁		$(1^2, -2, 1, -2, 3, -2, 3)$	8 ₂		$(1^3, -2, -1^2, 3, -2, 3)$	9 ₄₂	
1 ⁶	6 ₁		$(1^2, -2, 1, 3, -2^2, 3)$	8 ₁₂		$(1^3, 2, -1, 2, 3, -2, 3)$	7 ₂	
$(1^3, 2, -1, 2)$	5 ₂		$(1, -2, 1, -2, 1, 3, -2, 3)$	8 ₁₃		$(1^3, 2, -1, 2, -3, 2, -3)$	8 ₁₄	
$(1^3, -2, 1, -2)$	6		$(1^2, -2, 1, 3, -2, 3^2)$	8 ₂		$(1^3, -2, 1, -2, 3, -2, 3)$	9 ₂₆	
$(1^2, 2, 1^2, 2)$	6 ₃		$(1, -2, 1, -2, 3, -2^2, 3)$	8 ₁₀		$(1^3, -2, 1, -2, -3, 2, -3)$	8 ₄	
$(1^2, -2, 1^2, -2)$	6 ₁		$(1, -2, 1, 3, -2^3, 3)$	8 ₁₁		$(1^3, 2, -1, -3, 2^2, -3)$	8 ₃	
$(1^2, -2, 1, -2^2)$	6		$(1, 2^2, 1, 3, 2^2, 3)$	8 ₈		$(1^3, -2, 1, 3, -2^2, 3)$	9 ₆	
$(1, -2, 1, -2, 1, -2)$	6 ₃		$(1, 2^2, 1, 3, -2^2, 3)$	8 ₅		$(1^2, 2, 1^2, 2, -3, 2, -3)$	9 ₃	×
$(1, -2, 1, 3, -2, 3)$	6 ₃		$(1, -2^2, 1, 3, -2^2, 3)$	8 ₄		$(1^2, 2, -1^2, 2, -3, 2, -3)$	9 ₄	×
1 ⁷	7		$(1, -2, 3, -2, 1, -2, 3, -2)$	8 ₁₄		$(1^2, -2, 1^2, -2, 3, -2, 3)$	9 ₅	
$(1^4, 2, -1, 2)$	6 ₃		$(1, -2, 1, 3, -2, -4, 3, -4)$	8 ₁₂		$(1^2, 2, 1^2, -3, 2^2, -3)$	9 ₁₀	×
$(1^4, -2, 1, -2)$	7		1 ⁹	9 ₁		$(1^2, 2, -1^2, -3, 2^2, -3)$	9 ₈	×
$(1^3, 2, 1^2, 2)$	7	×	$(1^6, 2, -1, 2)$	8 ₂		$(1^2, -2, 1^2, 3, -2^2, 3)$	9 ₈	
$(1^3, 2, -1^2, 2)$	7	×	$(1^6, -2, 1, -2)$	9 ₁		$(1^2, 2, -1, 2, 1, 3, -2, 3)$	9 ₄₅	
$(1^3, -2, 1^2, -2)$	7		$(1^5, 2, 1^2, 2)$	9 ₂	×	$(1^2, -2, 1, -2, 1, 3, -2, 3)$	9 ₃₂	
$(1^3, -2, 1, -2^2)$	7		$(1^5, 2, -1^2, 2)$	9 ₃	×	$(1^2, -2, 1, 3, -2, 1, 3, -2)$	9 ₃₁	
$(1^2, -2, 1^2, -2^2)$	7		$(1^5, -2, 1^2, -2)$	9 ₁₃		$(1^3, 2, -1, -3, 2, -3^2)$	8 ₈	
$(1^2, -2, 1, -2, 1, -2)$	7		$(1^4, 2, 1^3, 2)$	9 ₁₀	×	$(1^3, -2, 1, 3, -2, 3^2)$	9 ₂₀	
$(1^2, 2, -1, -3, 2, -3)$	6 ₁		$(1^4, 2, -1^3, 2)$	9 ₂		$(1^2, -2, 1^2, 3, -2, 3^2)$	9 ₃	
$(1^2, -2, 1, 3, -2, 3)$	7 ₆		$(1^4, -2, 1^3, -2)$	9 ₁₉		$(1^2, 2, -1, 2^2, 3, -2, 3)$	7 ₄	
$(1, -2, 1, -2, 3, -2, 3)$	7 ₇		$(1^4, -2, -1^3, -2)$	9 ₂₀		$(1^2, 2, -1, 2^2, -3, 2, -3)$	8 ₁₁	
$(1, -2, 1, 3, -2^2, 3)$	7 ₃		$(1^5, 2, -1, 2^2)$	8 ₈		$(1^2, -2, 1, -2^2, 3, -2, 3)$	9 ₂₇	
1 ⁸	8 ₁		$(1^5, -2, 1, -2^2)$	9 ₃		$(1^2, -2, 1, -2^2, -3, 2, -3)$	8 ₁₃	
$(1^5, 2, -1, 2)$	7 ₃		$(1^4, 2, -1^2, 2^2)$	9 ₅		$(1^2, -2, 1, 3, 2^3, 3)$	8 ₁₅	
$(1^5, -2, 1, -2)$	8 ₂		$(1^4, -2, 1^2, -2^2)$	9 ₂		$(1^2, -2, 1, 3, -2^3, 3)$	9 ₂₄	
$(1^4, 2, 1^2, 2)$	8 ₃	×	$(1^4, 2, -1, 2, -1, 2)$	9 ₅		$(1^2, -2^2, 1, -2, 3, -2, 3)$	9 ₃₀	
$(1^4, 2, -1^2, 2)$	8 ₃	×	$(1^4, -2, 1, -2, 1, -2)$	9 ₁₁		$(1^2, 2^2, 1, 3, 2^2, 3)$	9 ₁₇	×
$(1^4, -2, 1^2, -2)$	8 ₁		$(1^3, 2, 1^3, 2^2)$	9 ₃		$(1^2, 2^2, 1, 3, -2^2, 3)$	9 ₁₆	
$(1^3, 2, 1^3, 2)$	8 ₁₉		$(1^3, 2, -1^3, 2^2)$	9 ₃		$(1^2, 2^2, 1, -3, 2^2, -3)$	9 ₁₅	
$(1^3, 2, -1^3, 2)$	8 ₂₀		$(1^3, -2, 1^3, 2^2)$	8 ₄		$(1^2, -2^2, 1, 3, -2^2, 3)$	9 ₄	
$(1^3, -2, 1^3, -2)$	8 ₅		$(1^3, -2, 1^3, -2^2)$	9 ₃		$(1, -2, 1, -2, 1, -2, 3, -2, 3)$	9 ₁₀	
$(1^4, 2, -1, 2^2)$	7 ₅		$(1^3, 2, -1^2, 2, -1, 2)$	9 ₇		$(1, -2, 1, -2, 1, 3, 2^2, 3)$	9 ₃₀	
$(1^4, -2, 1, -2^2)$	8 ₇		$(1^3, -2, 1^2, -2, 1, -2)$	9 ₅		$(1, -2, 1, -2, 1, 3, -2^2, 3)$	9 ₁₂	
$(1^3, 2, -1^2, 2^2)$	8 ₂₁		$(1^2, -2, 1^2, -2, 1^2, -2)$	9 ₁₀		$(1, -2, 1, -2, 1, -3, 2^2, -3)$	9 ₂₁	
$(1^3, -2, 1^2, -2^2)$	8 ₁₀		$(1^4, -2, 1, -2^3)$	9 ₂		$(1, -2, 1, -2^2, 1, 3, -2, 3)$	9 ₃₃	
$(1^3, 2, -1, 2, -1, 2)$	8 ₉		$(1^4, -2^2, 1, -2^2)$	9 ₂		$(1, 2, -1, 2, 3, -2, 1, -2, 3)$	9 ₄₆	
$(1^3, -2, 1, -2, 1, -2)$	8 ₅		$(1^3, -2, 1^2, -2^3)$	9 ₁		$(1, -2, 1, -2, 3, -2, 1, -2, 3)$	9 ₃₄	
$(1^2, -2, 1^2, -2, 1, -2)$	8 ₁₆		$(1^3, -2, 1, -2, 1, -2^2)$	9 ₄		$(1, -2, 1, -2, -3, -2, 1, -2, -3)$	9 ₄₇	
$(1^3, -2, 1, -2^3)$	8 ₉		$(1^3, -2, 1, -2^2, 1, -2)$	9 ₇		$(1^2, -2, 1, -2, 3, -2, 3^2)$	9 ₃₁	
$(1^3, -2^2, 1, -2^2)$	8 ₃		$(1^3, 2^2, 1^2, 2^2)$	9 ₇		$(1^2, -2, 1, 3, -2^2, 3^2)$	9 ₂₈	
$(1^2, -2, 1, -2, 1, -2^2)$	8 ₁₇		$(1^3, -2^2, 1^2, -2^2)$	9 ₉		$(1, -2, 1, 3, -2, 1, 3, -2, 3)$	9 ₄₀	
$(1^2, -2, 1, -2^2, 1, -2)$	8 ₃		$(1^2, -2, 1^2, -2, 1, -2^2)$	9 ₉		$(1^2, -2, 1, 3, -2, -4, 3, -4)$	9 ₁₁	
$(1^2, 2^2, 1^2, 2^2)$	8 ₅		$(1^2, 2, -1, 2, 1^2, 2^2)$	9 ₁		$(1, -2, 1, -2^3, 3, -2, 3)$	9 ₁₇	
$(1^2, -2^2, 1^2, -2^2)$	8 ₄		$(1^2, -2, 1, -2, 1^2, -2^2)$	9 ₁		$(1, -2, 1, -2, 3, -2^3, 3)$	9 ₂₂	

x	L	E	x	L	E	x	L	E
$(1, -2, 1, 3, -2^4, 3)$	9^3		$(1^3, -2, 1^2, -2, 1, -2^2)$	10^3		$(1^3, -2, 1, -2, 1, 3, -2, 3)$	10^2	
$(1, -2^2, 1, -2, 3, -2^2, 3)$	9^3		$(1^3, -2, 1^2, -2^2, 1, -2)$	10^4		$(1^3, 2, 1, 3, -2, 1, 3, 2)$	9^2	\times
$(1, -2^2, 3, -2, 1, -2, 3, -2)$	9^2		$(1^3, 2, -1, 2, 1^2, 2^2)$	10^6		$(1^3, 2, 1, -3, 2, 1, -3, 2)$	10^3	\times
$(1, -2, 1, -2, 3, -2, -4, 3, -4)$	9^2		$(1^3, 2, -1, 2, -1^2, 2^2)$	10^5		$(1^3, 2, -1, 3, -2, 1, 3, -2)$	9^2	\times
$(1, -2, 1, -2, -3, 2, 4, -3, 4)$	8^2		$(1^3, -2, 1, -2, 1^2, -2^2)$	10^6		$(1^3, 2, -1, 3, -2, -1, 3, 2)$	9^2	\times
$(1, -2, 1, 3, -2^2, -4, 3, -4)$	9^2		$(1^3, -2, 1, -2, 1, -2, 1, -2)$	10^6		$(1^3, 2, -1, 3, -2, -1, 3, 2)$	9^2	\times
1^{10}	10^1		$(1^3, 2^2, 1^3, 2^2)$	10^3		$(1^3, 2, -1, -3, 2, 1, -3, -2)$	10^2	\times
$(1^7, 2, -1, 2)$	9^3		$(1^3, -2^2, 1^3, -2^2)$	10^3		$(1^3, 2, -1, -3, 2, -1, -3, 2)$	10^2	\times
$(1^7, -2, 1, -2)$	10^2		$(1^3, -2^2, 1^3, -2^2)$	10^3		$(1^3, -2, 1, 3, -2, 1, 3, -2)$	10^2	\times
$(1^6, 2, 1^2, 2)$	10^3	\times	$(1^2, 2, -1^2, 2, 1^2, 2^2)$	10^3		$(1^3, -2, 1, -3, 2, 1, -3, -2)$	10^2	\times
$(1^6, 2, -1^2, 2)$	10^3	\times	$(1^2, -2, 1^2, -2, 1, -2, 1, -2)$	10^3		$(1^2, 2, -1^2, 2, 1, 3, -2, 3)$	10^2	\times
$(1^6, -2, 1^2, -2)$	10^1		$(1^2, -2, 1, -2, 1^2, -2, 1, -2)$	10^3		$(1^2, -2, 1^2, -2, 1, 3, -2, 3)$	10^2	\times
$(1^5, 2, 1^3, 2)$	10^1		$(1^5, 2, -1, -3, 2, -3)$	9^2		$(1^2, 2, 1^2, -3, -2, 1, -2, -3)$	10^2	\times
$(1^5, 2, -1^3, 2)$	10^1		$(1^5, -2, 1, 3, -2, 3)$	10^2		$(1^2, -2, 1^2, 3, 2, -1, 2, 3)$	10^2	\times
$(1^5, -2, 1^3, -2)$	10^4		$(1^4, 2, 1^2, -3, 2, -3)$	10^2		$(1^2, -2, 1^2, 3, -2, 1, -2, 3)$	10^2	\times
$(1^5, -2, -1^3, -2)$	10^1		$(1^4, 2, -1^2, -3, 2, -3)$	10^2		$(1^4, 2, -1, -3, 2, -3^2)$	9^2	\times
$(1^4, 2, 1^4, 2)$	10^3		$(1^4, -2, 1^2, 3, -2, 3)$	10^3		$(1^4, -2, 1, 3, -2, 3^2)$	10^2	\times
$(1^4, 2, -1^4, 2)$	10^3		$(1^4, -2, -1^2, 3, -2, 3)$	10^2	\times	$(1^3, -2, 1^2, 3, -2, 3^2)$	10^2	\times
$(1^4, -2, 1^4, -2)$	10^7		$(1^3, 2, 1^3, -3, 2, -3)$	10^4		$(1^3, -2, 1, -2^2, 3, -2, 3)$	10^2	\times
$(1^6, 2, -1, 2^2)$	9^6		$(1^3, 2, -1^3, -3, 2, -3)$	10^4		$(1^3, -2, 1, -2^2, -3, 2, -3)$	9^2	\times
$(1^6, -2, 1, -2^2)$	10^5		$(1^3, -2, 1^3, 3, -2, 3)$	10^4		$(1^3, 2, -1, 2, 3, 2^2, 3)$	9^2	\times
$(1^5, 2, -1^2, 2^2)$	10^1		$(1^4, -2, 1, -2^4)$	10^1		$(1^3, 2, -1, 2, -3, 2^2, -3)$	9^2	\times
$(1^5, -2, 1^2, -2^2)$	10^4		$(1^4, -2^2, 1, -2^3)$	10^4		$(1^3, -2, 1, -2, 3, 2^2, 3)$	10^2	\times
$(1^5, 2, -1, 2, -1, 2)$	10^3		$(1^3, -2, 1, -2, 1, -2^3)$	10^3		$(1^3, -2, 1, -2, 3, -2^2, 3)$	10^2	\times
$(1^5, -2, 1, -2, 1, -2)$	10^3		$(1^3, -2, 1, -2^2, 1, -2^2)$	10^3		$(1^3, -2, 1, -2, 3, -2^3, 3)$	10^2	\times
$(1^4, 2, 1^3, 2^2)$	10^1		$(1^3, -2, 1, -2^3, 1, -2)$	10^3		$(1^3, 2^2, -1, 2, -3, 2, -3)$	9^2	\times
$(1^4, 2, -1^3, 2^2)$	10^1		$(1^3, 2^2, 1^2, 2^3)$	10^1		$(1^3, -2^2, 1, -2, 3, -2, 3)$	10^2	\times
$(1^4, -2, 1^3, 2^2)$	9^9		$(1^3, -2^2, 1^2, -2^3)$	10^7		$(1^3, -2^2, 1, -2, 3, 2, -3)$	9^2	\times
$(1^4, -2, 1^3, -2^2)$	10^6		$(1^3, 2^2, -1, 2, -1, 2^2)$	10^1		$(1^3, 2^2, 1, 3, 2^2, 3)$	10^4	\times
$(1^4, -2, -1^3, -2^2)$	10^1		$(1^3, -2^2, 1, -2, 1, -2^2)$	10^1		$(1^3, 2^2, 1, 3, -2^2, 3)$	10^1	\times
$(1^4, 2, -1^2, 2, -1, 2)$	10^1		$(1^2, -2, 1^2, -2^2, 1, -2^2)$	10^9		$(1^3, 2^2, 1, -3, 2^2, -3)$	10^4	\times
$(1^4, -2, 1^2, -2, 1, -2)$	10^8		$(1^2, -2, 1, -2, 1, -2, 1, -2^2)$	10^3		$(1^3, 2^2, -1, -3, 2^2, -3)$	9^1	\times
$(1^3, 2, -1^3, 2, -1, 2)$	10^3		$(1^2, -2, 1, -2, 1, -2^2, 1, -2)$	10^1		$(1^3, -2^2, 1, 3, -2^2, 3)$	10^2	\times
$(1^3, -2, 1^3, 2, -1, 2)$	10^3		$(1^2, -2, 1, -2^2, 1^2, -2^2)$	10^1		$(1^3, 2^3, 1, 3, -2, 3)$	9^2	\times
$(1^3, -2, 1^3, -2, 1, -2)$	10^3		$(1, -2, 1, -2, 1, -2, 1, -2)$	10^1		$(1^3, 2, -3, 2, -1, 2, -3, 2)$	9^2	\times
$(1^3, 2, -1^2, 2, -1^2, 2)$	10^1		$(1^4, 2, -1, 2, -3, 2, -3)$	9^2		$(1^3, -2, 3, -2, 1, -2, 3, -2)$	10^2	\times
$(1^3, -2, 1^2, -2, 1^2, -2)$	10^1		$(1^4, -2, 1, -2, 3, -2, 3)$	10^2		$(1^2, 2, -1^2, 2^2, 3, -2, 3)$	9^2	\times
$(1^5, -2, 1, -2^3)$	10^9		$(1^4, -2, 1, -2, -3, 2, -3)$	9^2		$(1^2, 2, -1^2, 2^2, -3, 2, -3)$	10^2	\times
$(1^5, -2^2, 1, -2^2)$	10^3		$(1^4, 2, -1, -3, 2^2, -3)$	9^2		$(1^2, -2, 1^2, -2^2, 3, -2, 3)$	10^2	\times
$(1^4, -2, 1^2, -2^3)$	10^8		$(1^4, -2, 1, 3, -2^2, 3)$	10^2		$(1^2, -2, 1^2, -2^2, -3, 2, -3)$	9^2	\times
$(1^4, 2, -1, 2, -1, 2^2)$	10^1		$(1^3, 2, 1^2, 2, -3, 2, -3)$	10^2		$(1^2, 2, 1^2, 2, 3, 2^2, 3)$	10^4	\times
$(1^4, -2, 1, -2, 1, -2^2)$	10^8		$(1^3, 2, -1^2, 2, -3, 2, -3)$	10^2		$(1^2, 2, 1^2, 2, 3, -2^2, 3)$	10^4	\times
$(1^4, 2, -1, 2^2, -1, 2)$	10^3		$(1^3, -2, 1^2, -2, 3, -2, 3)$	10^2		$(1^2, 2, 1^2, 2, -3, 2^2, -3)$	10^4	\times
$(1^4, -2, 1, -2^2, 1, -2)$	10^3		$(1^3, -2, -1^2, -2, 3, -2, 3)$	10^2		$(1^2, 2, -1^2, 2, -3, 2^2, -3)$	10^4	\times
$(1^4, 2^2, 1^2, 2^2)$	10^3		$(1^3, 2, 1^2, -3, 2^2, -3)$	10^2		$(1^2, -2, 1^2, -2, 3, -2^2, 3)$	10^4	\times
$(1^4, 2^2, -1^2, 2^2)$	10^3		$(1^3, 2, -1^2, -3, 2^2, -3)$	10^2		$(1^2, 2, 1^2, -3, 2^3, -3)$	10^2	\times
$(1^4, -2^2, 1^2, -2^2)$	10^3		$(1^3, -2, 1^2, 3, -2^2, 3)$	10^2		$(1^2, 2, 1^2, -3, -2^3, -3)$	9^2	\times
$(1^3, -2, 1^3, 2^3)$	9^1		$(1^3, -2, -1^2, 3, -2^2, 3)$	10^2		$(1^2, -2, 1^2, 3, 2^3, 3)$	9^2	\times
$(1^3, -2, 1^3, -2^3)$	10^4		$(1^3, 2, -1, 2, 1, 3, -2, 3)$	10^2		$(1^2, -2, 1^2, 3, -2^3, 3)$	10^2	\times
$(1^3, 2, -1^2, 2, -1, 2^2)$	10^3		$(1^3, 2, -1, 2, -1, -3, 2, -3)$	10^2		$(1^2, -2, 1, -2, 1, -2, 3, -2, 3)$	10^2	\times

x	L	E	x	L	E	x	L	E
$(1^2, 2, -1, 2, 1, 3, -2^2, 3)$	10_{170}^2		$(1^3, -2, 1, 3, -2, 3^3)$	10_{17}^1		$(1^2, -2, 1, -2, 3, -2, -4, 3, -4)$	10_{42}^1	
$(1^2, -2, 1, -2, 1, 3, 2^2, 3)$	10_{172}^2		$(1^3, -2, 1, 3^2, -2, 3^2)$	10_{30}^1		$(1^2, -2, 1, -2, -3, 2, 4, -3, 4)$	9_8	
$(1^2, -2, 1, -2, 1, 3, -2^2, 3)$	10_{107}^1		$(1^2, -2, 1^2, 3^2, -2, 3^2)$	10_5^1		$(1^2, -2, 1, 3, 2^2, -4, 3, -4)$	9_{25}	
$(1^2, -2, 1, -2, 1, -3, 2^2, -3)$	10_{174}^1		$(1^3, 2, -1, -3, 2, 4, -3, 4)$	9_{15}		$(1^2, -2, 1, 3, -2^2, -4, 3, -4)$	10_{71}^1	
$(1^2, 2, -1, 2^2, 1, 3, -2, 3)$	10_{156}^2		$(1^3, -2, 1, 3, -2, -4, 3, -4)$	10_{29}^1		$(1, -2, 1, -2, 1, 3, -2, -4, 3, -4)$	10_{39}^2	
$(1^2, -2, 1, -2^2, 1, 3, -2, 3)$	10_{101}^1		$(1^2, 2, 1^2, -3, 2, 4, -3, 4)$	10_{39}^3	\times	$(1, -2, 1, -2, 1, -3, 2, 4, -3, 4)$	10_{61}^3	\times
$(1^2, 2, -1, 2, 3, 2, -1, 2, 3)$	10_{179}^1		$(1^2, 2, -1^2, -3, 2, 4, -3, 4)$	10_{48}^1	\times	$(1^2, 2, -1, -3, 2, -3, 4, -3, 4)$	9_{14}	
$(1^2, -2, 1, -2, -3, 2, -1, 2, -3)$	10_{175}^2		$(1^2, -2, 1^2, 3, -2, -4, 3, -4)$	10_5^1		$(1^2, 2, -1, -3, 2, -3, -4, 3, -4)$	8_3	
$(1^2, -2, 1, -2, 3, -2, 1, -2, 3)$	10_{118}^1		$(1^2, -2, 1, -2^3, 3, -2, 3)$	10_{26}^1		$(1^2, -2, 1, 3, -2, 3, 4, -3, 4)$	9_{12}	
$(1^2, -2, 1, -2, -3, -2, 1, -2, -3)$	10_{183}^1		$(1^2, 2, -1, 2^2, -3, 2^2, -3)$	9_{25}^1		$(1^2, -2, 1, 3, -2, 3, -4, 3, -4)$	10_{44}^1	
$(1^2, -2, 1, -2, 3, -2, 1, 3, -2)$	10_{103}^1		$(1^2, -2, 1, -2^2, 3, -2^2, 3)$	10_{36}^1		$(1^2, -2, 1, 3, -2, -4, 3^2, -4)$	10_{32}^1	
$(1^2, -2, 1, -2, -3, 2, 1, -3, -2)$	9_{26}^1		$(1^2, -2, 1, 3, -2^4, 3)$	10_{22}^1		$(1^2, -2, 1, 3, -2, -4, 3, -4^2)$	10_{43}^1	
$(1^2, -2, 1, 3, 2, -1, 2^2, 3)$	10_{142}^1		$(1^2, -2^2, 1, -2^2, 3, -2, 3)$	10_{43}^1		$(1, -2, 1, -2^4, 3, -2, 3)$	10_{25}^1	
$(1^2, -2, 1, 3, -2, 1, -2^2, 3)$	10_{108}^1		$(1^2, -2^2, 1, -2, 3, -2^2, 3)$	10_{79}^1		$(1, -2, 1, -2^3, 3, -2^2, 3)$	10_{34}^1	
$(1^2, -2, 1, 3, -2, 1, -2, 3, -2)$	10_{117}^1		$(1^2, 2^2, 1, 3, 2^3, 3)$	10_{149}^1		$(1, -2, 1, -2, 3, -2^4, 3)$	10_{39}^1	
$(1^2, -2, 1, 3, -2, 1, 3, -2^2)$	10_{67}^1		$(1^2, 2^2, 1, 3, -2^3, 3)$	10_{144}^1		$(1, -2, 1, 3, -2^5, 3)$	10_{55}^1	
$(1^2, 2, -1, 3, 2^2, 1, 3, -2)$	10_{15}^1		$(1^2, 2^2, 1, -3, 2^3, -3)$	10_{43}^1		$(1, -2^2, 1, -2, 3, -2^3, 3)$	10_{78}^1	
$(1^2, 2, -1, -3, 2^2, 1, -3, -2)$	10_{43}^1	\times	$(1^2, 2^2, 1, -3, -2^3, -3)$	10_{150}^1		$(1, -2^2, 1, 3, -2^4, 3)$	10_1^1	
$(1^2, -2, 1, 3, 2^2, 1, 3, -2)$	10_{19}^1		$(1^2, -2^2, 1, 3, 2^3, 3)$	10_{145}^1		$(1, 2^3, 1, -3, 2^3, -3)$	10_{146}^1	
$(1^2, -2, 1, 3, -2^2, 1, 3, -2)$	10_8^1		$(1^2, -2^2, 1, 3, -2^3, 3)$	10_{50}^1		$(1, -2^3, 1, 3, -2^3, 3)$	10_{52}^1	
$(1^2, -2, 1, -3, 2^2, 1, -3, -2)$	10_{20}^1		$(1^2, -2^3, 1, -2, 3, -2, 3)$	10_{48}^1		$(1, -2^3, 3, -2, 1, -2, 3, -2)$	10_{100}^1	
$(1^2, 2^2, 1^2, 2, -3, 2, -3)$	10_{165}^1		$(1, -2, 1, -2, 1, -2^2, 3, -2, 3)$	10_{83}^1		$(1, -2^2, 3, -2, 1, -2^2, 3, -2)$	10_{103}^1	
$(1^2, -2^2, 1^2, -2, 3, -2, 3)$	10_{72}^1		$(1, -2, 1, -2, 1, -2^2, -3, 2, -3)$	9_{32}^1		$(1, -2^2, 3, -2, 1, -2, 3, -2^2)$	10_{112}^1	
$(1^2, -2^2, 1^2, -2, -3, 2, -3)$	9_{17}^1		$(1, -2, 1, -2, 1, 3, -2^3, 3)$	10_{104}^1		$(1, -2, 1, -2^2, 3, -2, -4, 3, -4)$	10_{41}^1	
$(1^2, 2^2, 1^2, 3, 2^2, 3)$	10_{16}^1		$(1, -2, 1, -2, 1, -3, -3^3, -3)$	10_2^1		$(1, -2, 1, -2^2, -3, 2, 4, -3, 4)$	9_{19}	
$(1^2, 2^2, 1^2, 3, -2^2, 3)$	10_{14}^1		$(1, -2, 1, -2^2, 1, 3, 2^2, 3)$	10_7^1		$(1, -2, 1, -2, 3, 2^2, -4, 3, -4)$	10_{137}^1	
$(1^2, 2^2, 1^2, -3, 2^2, -3)$	10_{17}^1	\times	$(1, -2, 1, -2^2, 1, 3, -2^2, 3)$	10_{106}^1		$(1, -2, 1, -2, 3, -2^2, -4, 3, -4)$	10_{59}^1	
$(1^2, -2^2, 1^2, 3, -2^2, 3)$	10_4^1		$(1, -2, 1, -2^2, 1, -3, 2^2, -3)$	10_{173}^1		$(1, -2, 1, -2, -3, 2^2, 4, -3, 4)$	10_{136}^1	
$(1, -2, 1, -2, 1, -2, 1, 3, -2, 3)$	10_{114}^1		$(1, -2, 1, -2^3, 1, 3, -2, 3)$	10_{87}^1		$(1, -2, 1, -2, -3, -2^2, 4, -3, 4)$	10_{138}^1	
$(1, -2, 1, -2, 1, -2, 1, -3, 2, -3)$	10_{180}^1		$(1, 2, -1, 2^2, 3, -2, 1, -2, 3)$	10_{158}^1		$(1, -2, 1, 3, -2^3, -4, 3, -4)$	10_{70}^1	
$(1, 2, -1, 2, 1, 3, -2, 1, -2, 3)$	10_{181}^1		$(1, -2, 1, -2^2, 3, -2, 1, -2, 3)$	10_{115}^1		$(1, -2^2, 1, -2, 3, -2, -4, 3, -4)$	10_6^3	
$(1, 2, -1, 2, 1, -3, -2, 1, -2, -3)$	$10_{178}^2 = 10_{182}^2$		$(1, -2, 1, -2, 3, -2, 1, -2^2, 3)$	10_{116}^1		$(1, -2^2, 1, -2, -3, 2, 4, -3, 4)$	9_3^3	
$(1, -2, 1, -2, 1, 3, -2, 1, -2, 3)$	10_{120}^1		$(1, -2, 1, 3, -2^2, 1, -2^2, 3)$	10_{109}^1		$(1, 2^2, 1, 3, -2^2, -4, 3, -4)$	10_3^3	
$(1, -2, 1, -2, 1, -3, -2, 1, -2, -3)$	10_{184}^1		$(1^2, -2, 1, -2^2, 3, -2, 3^2)$	10_{27}^1		$(1, -2^2, 1, 3, -2^2, -4, 3, -4)$	10_{10}^1	
$(1^3, -2, 1, -2, 3, -2, 3^2)$	10_{19}^1		$(1^2, -2, 1, -2, 3, -2^2, 3^2)$	10_{45}^1		$(1, -2^2, 1, -3, -2^2, 4, -3, 4)$	10_4^3	
$(1^3, 2, -1, -3, 2^2, -3^2)$	9_{28}^1		$(1^2, -2, 1, -2, 3, -2, 3, -2, 3)$	10_{84}^1		$(1, -2, 3, -2, 1, -2, -4, 3, -2, -4)$	10_{37}^1	
$(1^3, -2, 1, 3, -2^2, 3^2)$	10_{24}^1		$(1^2, -2, 1, 3, 2^3, 3^2)$	9_{34}^1		$(1, -2, -3, -2, 1, -2, 4, -3, -2, 4)$	10_{59}^1	\times
$(1^3, 2, -1, -3, 2, -3, 2, -3)$	9_{33}^1		$(1^2, -2, 1, 3, -2^3, 3^2)$	10_{26}^1		$(1, -2, 1, -2, 3, -2, 3, -4, 3, -4)$	10_{45}^1	
$(1^3, -2, 1, 3, -2, 3, -2, 3)$	10_{81}^1		$(1^2, 2^2, 1, 3, 2^2, 3^2)$	10_{151}^1		$(1, -2, 1, -2, 3, -2, -4, 3^2, -4)$	10_3^3	
$(1^2, 2, 1^2, 2, -3, 2, -3^2)$	10_{130}^1	\times	$(1^2, 2^2, 1, 3, -2^2, 3^2)$	10_{48}^1		$(1, -2, 1, 3, -2^2, -4, 3^2, -4)$	10_{24}^1	
$(1^2, 2, -1^2, 2, -3, 2, -3^2)$	10_{131}^1	\times	$(1^2, 2^2, 1, -3, 2^2, -3^2)$	10_{47}^1		$(1, -2, 1, 3, -2, 3, -2, -4, 3, -4)$	10_{88}^1	
$(1^2, -2, 1^2, -2, 3, -2, 3^2)$	10_{37}^1		$(1^2, 2^2, 1, -3, -2^2, -3^2)$	10_{152}^1		$(1, -2, 1, 3, -2, -4, 3, -2, -4, 3)$	10_4^3	
$(1^2, 2, 1^2, -3, 2^2, -3^2)$	10_{168}^1	\times	$(1^2, -2^2, 1, 3, -2^2, 3^2)$	10_{54}^1		$(1, -2, 1, 3, -2, -4, 3, 5, -4, 5)$	10_{23}^1	
$(1^2, 2, -1^2, -3, 2^2, -3^2)$	10_{161}^1	\times	$(1^2, -2^2, 1, 3, -2, 3, -2, 3)$	10_{105}^1				
$(1^2, -2, 1^2, 3, -2^2, 3^2)$	10_{15}^1		$(1^2, -2, 3, -2, 1, 3, -2, 3^2, -2)$	10_{96}^1				
$(1^2, 2, 1^2, -3, 2, -3, 2, -3)$	10_{118}^1	\times	$(1, -2, 1, -2, 1, 3, -2, 3, -2, 3)$	10_9^1				
$(1^2, -2, 1^2, 3, -2, 3, -2, 3)$	10_4^1		$(1, -2, 1, -2, 1, -3, 2, -3, 2, -3)$	10_{21}^1				
$(1^2, -2, 1, -2, 1, 3, -2, 3^2)$	10_{86}^1		$(1, -2, 1, -2, 3, -2, 1, 3, -2, 3)$	10_{121}^1				
$(1^2, -2, 1, 3, -2, 1, 3, -2, 3)$	10_{119}^1		$(1^2, 2, -1, 2, 3, -2, -4, 3, -4)$	8_1				
$(1^2, -2, 1, 3, -2, 1, 3^2, -2)$	10_{61}^1		$(1^2, 2, -1, 2, -3, 2, 4, -3, 4)$	9_{21}				