Abstract

Anna Abczynski (Hausdorff Institute, Germany)
Title: On the polarisation of manifolds
Abstract: In this talk I will explain how modified surgery theory can be used to study the polarised structure. The polarised structure set of some manifold $M$ is the set of diffeomorphism classes of manifolds whose cohomology ring is isomorphic to the one of $M$, preserving Pontrjagin and Stiefel-Whitney classes. I’ll give an upper bound for the order of the polarised structure set of a complex 4-dimensional Spin Bott manifold.

Hiraku Abe (Tokyo Metropolitan Univ., Japan)
Title: On the rational $T$-equivariant cohomology of the weighted Grassmannians
Abstract: The weighted Grassmannian $wGr(d,n)$ is defined by the Plucker relation in a certain weighted projective space $w\mathbb{P}(\wedge^d \mathbb{C}^n)$. The GKM theory provides us a way to describe the rational $T$-equivariant cohomology $H^*_T(wGr(d,n) : \mathbb{Q})$ as a certain combinatorially defined subalgebra of the $T$-equivariant cohomology of the $T$-fixed point set. Among equivariant classes, the canonical classes form an $H^*_T(pt : \mathbb{Q})$-module basis of $H^*_T(wGr(d,n) : \mathbb{Q})$. Following the argument given by Knutson-Tao, we will see an inductive formula for the structure constants with respect to the canonical classes.

Ivan Arzhantsev (Moscow State Univ., Russia)
Title: The automorphism group of a variety with torus action of complexity one
Abstract: In his seminal article, Michel Demazure gave a combinatorial description of the automorphism group of a complete smooth toric variety as a linear algebraic group. The central concept is a root system associated with a complete fan. This approach was investigated and generalized by Winfried Bruns, David Cox, Joseph Gubeladze, Benjamin Nill, and others. Recently it was realized in the framework of symplectic geometry by Mikiya Masuda.

We describe the automorphism group of a complete algebraic variety $X$ with torus action of complexity one. The result is based on a presentation of the Cox ring $\mathbb{R}(X)$ in terms of trinomials and on an interpretation of Demazure’s roots as homogeneous locally nilpotent derivations of $\mathbb{R}(X)$. This is a joint work with Juergen Hausen and Alvaro Liendo.

Anton Ayzenberg (Moscow State Univ., Russia)
Title: Nerve-complexes of nonsimple polytopes
Abstract: Let $P$ be an $n$-dimensional polytope with $m$ facets defined as an intersection of halfspaces $P = \{x \in \mathbb{R}^n | \langle a_i, x \rangle + b_i \geq 0\}$.
Let $j_P : P \to \mathbb{R}^m$ be an affine inclusion defined by $j_P(x) = (y_1, \ldots, y_m)$, where $y_i = \langle a_i, x \rangle + b_i$.
In this case $j_P(P) \subseteq \mathbb{R}^m$ and the moment-angle space $Z_P$ is defined as a pullback in the diagram:

\[
\begin{array}{ccc}
\mathbb{C}^m & \overset{p}{\longrightarrow} & \mathbb{R}^m \\
\downarrow & & \downarrow \\
\mathbb{P}^C & \overset{j_P}{\longrightarrow} & \mathbb{R}_+^m
\end{array}
\]

The vertical map on the right is the standard moment map given by $p : (z_1, \ldots, z_m) \mapsto (|z_1|^2, \ldots, |z_m|^2)$. As in the case of simple polytopes, $\dim Z_P = m + n$ and the space $Z_P$ carries the canonical action of torus $T^m$ with $P$ being the orbit space.

The theory of moment-angle spaces is well developed for simple polytopes. In this particular case $Z_P$ is a manifold and $Z_P$ is equivariantly homeomorphic to the space $(D^2, S^1)^{n|m}$ (theorem of Buchstaber and Panov).
The space $S$ is called a spherical nerve-complex of rank $n$ if the following conditions hold:

1. Let $P$ be an $n$-dimensional polytope. Then $S$ is a graded poset of rank $n$ (it means that all its maximal under inclusion chains have the same length $n$).

2. $S$ is a simplicial complex $K_P$, which was introduced to study $S$, is an object of an independent interest. We define new combinatorial invariants of polytopes in terms of this complex. An important question is: what are the properties characterizing simplicial complex $K_P$? This question led us to the notion of a spherical nerve-complex.

3. Suppose $S \in F(K)$. Then $\text{link}_K S$ is homotopy equivalent to a sphere $S^{n-rk(S)-1}$. Here, by definition, $\text{link} \emptyset = K$ and $S^{-1} = \emptyset$.

Theorem. Let $P$ be an $n$-dimensional polytope. Then $K_P$ is the spherical nerve complex of rank $n$.

This gives a partial characterization of the complexes $K_P$ for convex polytopes $P$.

A spherical nerve complex is a generalization of a combinatorial sphere. The definition of a nerve complex allows to get combinatorial formulas for the $f$-vector of a polytope.

Let $P$ be a convex polytope and $F$ — its face. Let $m(F)$ be the number of facets containing $F$. Consider the polynomial

$$F_P(\alpha, t) = \sum_{F \in P} \alpha^{\dim F} t^{m(F)},$$

where the sum is taken over all faces of $P$ including $P$ itself. For a simplicial complex $K$ the $f$-polynomial is defined by $f_K(t) = \sum_{\sigma \in K} t^{\dim \sigma}$. Then for a nerve complex $K_P$ of a polytope $P$ we prove the formula:

$$f_K(t) = F_P(-1, t + 1).$$

For simplicial polytopes this gives famous Dehn-Sommerville relations and $F_P(\alpha, t)$ coincides with two-parametric $F$-polynomial studied in [3]. Some aspects of the theory of the ring of simple polytopes allow generalization for nonsimple polytopes.

We also provide a formula, connecting the polynomial $F_P(\alpha, t)$ with the bigraded Betti numbers of the space $Z_P = (D^2, S^1)^{K_P}$.

References


We prove that it is easy to see that the polytope with $M$ is discussed. The idea is based on special blow-up construction of quasi-toric orbifold $M^{2n}$ with positive Ricci curvature: if we have singularity set $S^{2m} \subset M^{2n}$ with some neighborhood which is diffeomorphic to $S^{2m} \times (\mathbb{C}^{m-m}/\mathbb{Z}_{n-m})$ then we can produce blow-up construction preserving positivity of Ricci tensor. Combining this construction with Riemannian submersion technique we obtain new topological types of moment-angle manifolds with positively Ricci-curved Riemannian metrics. All these manifolds correspond to polyhedrons of the following form: truncated products of simplexes of arbitrary dimensions.

Yaroslav Bazaikin (Sobolev Mathematical Institute, Russia)

Title: Riemannian metrics with positive Ricci curvature on moment-angle manifolds.

Abstract: The construction of positively Ricci-curved Riemannian metrics on some moment-angle manifolds is discussed. The idea is based on special blow-up construction of quasi-toric orbifold $M^{2n}$ with positive Ricci curvature: if we have singularity set $S^{2m} \subset M^{2n}$ with some neighborhood which is diffeomorphic to $S^{2m} \times (\mathbb{C}^{m-m}/\mathbb{Z}_{n-m})$ then we can produce blow-up construction preserving positivity of Ricci tensor. Combining this construction with Riemannian submersion technique we obtain new topological types of moment-angle manifolds with positively Ricci-curved Riemannian metrics. All these manifolds correspond to polyhedrons of the following form: truncated products of simplexes of arbitrary dimensions.

Suyoung Choi (Ajou Univ., Korea)

Title: Invariance of Pontrjagin classes of torus manifolds.

Abstract: A torus manifold of dimension $2n$ is a closed smooth manifold having a half-dimensional effective torus $T^n$ action with the non-empty fixed point set. A typical example of torus manifolds is $\mathbb{C}P^n$ with the standard torus action. Petrie has shown that all homotopy equivalence between a homotopy projective space $M$ and $\mathbb{C}P^n$ preserve their Pontrjagin classes if $M$ is a torus manifold, although Pontrjagin classes are not invariant under homotopy equivalences. In this talk, we discuss about the invariance of Pontrjagin classes of torus manifolds. In general, the Pontrjagin classes of torus manifolds is not preserved by homotopy equivalences. However, we show that, in the case where the cohomology of $M$ is isomorphic to that of a Bott manifold or a product of projective spaces, their Pontrjagin classes are preserved by homotopy equivalences, which generalizes the Petrie's theorem immediately.

Haibao Duan (Chinese Academy of Sciences, China)

Title: Schubert calculus and Cohomology of Lie groups

Abstract: Schubert calculus is an old and important problem. Its main focus has been the determination of the intersection product on flag manifolds with respect to the Schubert basis. Based on recent result of this classical calculus, we present a unified construction of the integral cohomology of all 1-connected simple Lie groups.

Nickolai Erokhovets (Moscow State Univ., Russia)

Title: Toric topology of polytopes with few facets.

Abstract: Central objects of toric topology are a simple polytope $P^m$ with $m$ facets and an associated $(m + n)$-dimensional moment-angle manifold $\mathbb{Z}_P$. We study the case of polytopes with few facets from the point of view of bigraded Betti numbers $\beta^{i,2j}(\mathbb{Z}_P)$ and the Buchstaber invariant $s(P)$ which is the maximal dimension of a torus $T^n \subset T^m$ that acts freely on $\mathbb{Z}_P$.

1. We prove that $s(P) = 1$ if and only if $P = \Delta^n$, that is $m = n + 1$. Then $\mathbb{Z}_P = S^{2n+1}$.

2. It is easy to see that the polytope with $m = n + 2$ is projectively equivalent to the product $\Delta^i \times \Delta^j$, so $\mathbb{Z}_P$ is homeomorphic to $S^{2i+1} \times S^{2j+1}$ and $s(P) = 2$.

3. The most interesting case is $m = n + 3$. Due to M. Perles in this case the combinatorial polytope $P$ can be represented in terms of a regular $(2k - 1)$-gon $M_{2k-1}$, $k \geq 3$, and a surjective map from the set of facets $\{F_1, \ldots, F_m\}$ to the set of vertices of $M_{2k-1}$ such that the facets $F_{i_1}, \ldots, F_{i_k}$ intersect is a vertex if and only if the triangle formed by the vertices corresponding to the rest three facets contains the center of $M_{2k-1}$. For a given polytope $P$ the number $k$ is defined in a unique way. Let us order the vertices of the polygon $M_{2k-1}$ clockwise, and let $a_i$ be the number of the preimages of the $i$-th vertex. Let $\varphi_i = a_i + \cdots + a_{i+k-2}$, $\psi_j = a_1 + \cdots + a_{j+k-1}$, where indices are taken modulo $2k - 1$.

It is easy to see that for the triangle with the numbers $a_1, a_2, a_3$ we have $\mathbb{Z}_P = S^{2\varphi_1-1} \times S^{2\varphi_2-1} \times S^{2\varphi_3-1}$.

According to the results by Lopez de Medrano [LM] for $k \geq 3$ the manifold $\mathbb{Z}_P$ is homeomorphic to $\bigotimes_{i=1}^{2k-1} S^{2\varphi_i-1} \times S^{2\psi_{i+k-1}-2}$.

Our main results are the following:
(a) If \( k \leq 4 \), then \( s(P) = 3 \), else \( s(P) = 2 \). We give the series of pairs of polytopes \((P, Q)\) such that \( P \) and \( Q \) have equal \( f \)-vectors and chromatic numbers, but \( s(P) = 3 \) and \( s(Q) = 2 \). The simplest pair is \((2, 1, 1, 1, 1, 1, 1, 1), (2, 1, 1, 2, 1, 2, 1, 1)\).

(b) **Theorem.** For the polytope \( P \) with \( m = n + 3 \) facets the bigraded cohomology ring \( H^{\bullet, \bullet}(\mathbb{Z}_P) \) is isomorphic to the free abelian group \( \mathbb{Z} \oplus \mathbb{Z}^{2k-1} \oplus \mathbb{Z}^{2k-1} \oplus \mathbb{Z} \) with the generators

\[
1, \text{ bideg } 1 = (0, 0); \\
X_i, \text{ bideg } X_i = (-1, 2\varphi), \ i = 1, \ldots, 2k - 1; \\
Y_j, \text{ bideg } Y_j = (-2, 2\psi), \ j = 1, \ldots, 2k - 1; \\
Z, \text{ bideg } Z = (-3, 2(n + 3)).
\]

For \( k \geq 3 \) we have: \( X_i \cdot X_j = 0; \ X_i \cdot Y_j = \delta_{i+1,j}, Z; \ Y_i \cdot Y_j = 0 \), and for \( k = 2 \) we have:

\[
X_i^2 = 0; \ X_iX_{i+1} = -X_{i+1}X_i = Y_i; \ X_1X_2X_3 = Z.
\]

In particular \( 2k - 1 = \sum_i \mu^{-1,2i}(\mathbb{Z}_P) \), so in the case of \( m = n + 3 \) the \( s \)-number can be calculated in terms of bigraded Betti numbers. In general case it is an open problem.

We also present examples of combinatorially different polytopes with isomorphic bigraded cohomology rings \( H^{\bullet, \bullet}(\mathbb{Z}_P) \) and examples of rigid (see [CPS]) polytopes.

**References**


**Yukiko Fukukawa** (Osaka City Univ., Japan)

**Title:** The cohomology ring of the GKM graph of a flag manifold

**Abstract:** If a closed smooth manifold with an action of a torus satisfies certain conditions, then its equivariant cohomology ring is determined by the fixed point sets of codimension one subtori of the torus. In addition, the labeled graph corresponding to the fixed point sets of the codimension one subtori and the cohomology ring of the labeled graph are defined. It is known that the equivariant cohomology ring of a flag manifold is isomorphic to the cohomology ring of the labeled graph associated with the flag manifold. I determined the ring structure of the cohomology of the labeled graph of a flag manifold combinatorially, namely without using this fact. In my talk, I would like to draw the labeled graph concretely and introduce the result for a flag manifold of type \( G_2 \).

**Akihiro Higashitani** (Osaka Univ., Japan)

**Title:** Ehrhart polynomials of multi-polygons

**Abstract:** Around 1950’s, Ehrhart theory on lattice polytopes was established by Ehrhart, and that has been generalized recently to multi-polytopes by Hattori and Masuda. In this talk, following the complete classification of the Ehrhart polynomials of lattice polygons, we give the complete classification of the Ehrhart polynomials of multi-polygons. (This talk is based on the joint work with M. Masuda.)

**Hirokaki Ishida** (Osaka City Univ. Advanced Mathematical Institute , Japan)

**Title:** Complex torus manifolds

**Abstract:** A closed connected oriented manifold of dimension \( 2n \) with an effective \( (S^1)^n \)-action having a fixed point is called a torus manifold. A typical example of torus manifold is a complete non-singular toric variety (we call it a toric manifold in this talk). In this talk, we will see that a torus manifold with an \( (S^1)^n \)-invariant complex structure is equivariantly biholomorphic to a toric manifold. This is joint work with Yael Karshon.
Shizuo Kaji (Yamaguchi Univ., Japan)
Title: Constructing a Schubert-like basis for GKM G-manifolds
Abstract: Among the remarkable properties of the cohomology of a flag variety is the existence of the Schubert basis, which relates the topology of the flag variety with the combinatorics of the Weyl group. Our aim is to find such a good basis for a little wider class of manifolds. More precisely, we work with the manifolds with nice Lie group action and their torus equivariant cohomology, where we can make use of the powerful machinery of localization.

Shintaro Kuroki (Osaka City Univ. Advanced Mathematical Institute, Japan)
Title: Infrasolv real moment-angle manifolds
Abstract: A real moment-angle manifold $\mathbb{R}Z_P$ is a real analogue of the moment angle manifold $Z_P$ over simple convex polytope $P$. If a small cover $M$ can be defined over $P$, the real moment-angle manifold $\mathbb{R}Z_P$ over $P$ is the finite covering of $M$. The universal covers over small covers $M$ and real moment-angle manifolds $\mathbb{R}Z_P$ are often contractible, i.e., $M$ and $\mathbb{R}Z_P$ are often aspherical manifolds. In this talk, we prove the following theorem: if the real moment-angle manifold $\mathbb{R}Z_P$ over $P$ is an aspherical manifold with virtually solvable fundamental group, then $\mathbb{R}Z_P$ is homeomorphic to the $n$-dimensional torus $T$, where a virtually solvable group is a group which has a solvable subgroup of finite index. The typical examples of aspherical manifolds with virtually solvable fundamental groups are infrasolv manifolds. Therefore, by using our theorem and the smooth rigidity theorem of infrasolv manifolds proved by Baues, we have that the moment angle manifold $\mathbb{R}Z_P$ (resp. small cover $M$) is an infrasolv manifold if and only if $\mathbb{R}Z_P$ (resp. $M$) is diffeomorphic to the torus (resp. real Bott manifold). This is a joint work with Mikiya Masuda and Li Yu.

Ivan Limonchenko (Moscow State Univ., Russia)
Title: Bigraded Betti numbers of some simple polytopes
Abstract: The bigraded Betti numbers $\beta^{-i,2j}(P)$ of a simple polytope $P$ are the dimensions of the bigraded components of the Tor groups of the face ring $k[P]$. The numbers $\beta^{-i,2j}(P)$ reflect the combinatorial structure of $P$ as well as the topology of the corresponding moment-angle manifold $Z_P$, and therefore they find numerous applications in combinatorial commutative algebra and toric topology. Here we calculate some bigraded Betti numbers of the type $\beta^{-i,2j}(P)$ for associahedra, and relate the calculation of the bigraded Betti numbers for truncation polytopes to the topology of their moment-angle manifolds. These two series of simple polytopes provide conjectural extrema for the values of $\beta^{-i,2j}(P)$ among all simple polytopes $P$ with the fixed dimension and number of facets.

References
Tomoo Matsumura (KAIST, Korea)
Title: Moment Angle Complex and Integral Cohomology of Toric orbifolds (joint work with Shisen Luo, Frank Moore)
Abstract: The moment-angle complex $Z_K$ was introduced by Buchstaber and Panov as a disc-circle decomposition of the Davis-Januszkiewicz universal space associated to a simplicial complex $K$ where they defined a quasi-toric manifold as a partial quotient of the moment angle complexes associated to a simple polytope. In this talk, I will explain that the equivariant cohomology of moment angle complexes with respect to the action of a subgroup of torus can be described in terms of Tor module of the Stanley-Reisner rings, and based on that, I will discuss about when the cohomology of a toric orbifold is the quotient of the Stanley-Reisner ring by linear terms. If time allows, I will also talk about the cohomological properties of the connected sum of simplicial complexes, which is motivated by the symplectic cut of toric orbifolds.

Mayumi Nakayama (Tokyo Metropolitan Univ., Japan)
Title: NilBott Tower of aspherical manifolds and Torus actions
Abstract: We shall introduce a notion of nilBott tower which is an iterated fiber space with fiber a nilmanifold. The top space is called a nilBott aspherical manifold. This generalizes real Bott tower as fibration. In this talk we discuss the following things.
(1) The structure of NilBott tower, Holomorphic NilBott tower.
(2) Homological torus actions on Kaehler Bott manifolds.
(3) Application to the Halperin-Carlsson conjecture.
Some of the results obtained here are based on the joint work with Y. Kamishima.

Seonjeong Park (KAIST)
Title: Projective bundles over 4-dimensional toric manifolds
Abstract: In this talk, we consider a quasitoric manifold whose cohomology ring is isomorphic to that of the projectivization of the Whitney sum of complex line bundles over a 4-dimensional toric manifold. The orbit space of such a quasitoric manifold is the product of a polygon with an simplex, and those quasitoric manifolds are equivalent (or homeomorphic) to projective bundles over 4-dimensional toric manifolds. Moreover, some of such quasitoric manifolds are distinguished by their cohomology rings up to homeomorphism.

Soumen Sarkar (ISI, India)
Title: A class of torus manifolds with nonconvex orbit space
Abstract: We study a class of smooth torus manifolds whose orbit space has the structure of a simple polytope with holes. We prove that these manifolds have stable almost complex structure and give combinatorial formula for some of their Hirzebruch genera. They have (invariant) almost complex structure if they admit positive omniorientation. In dimension four, we calculate their homology groups, construct a symplectic structure on a large class of these manifolds, and give a family which is symplectic but not complex.

Jerome Tambour (KAIST, Korea)
Title: Geometry of moment-angle complexes
Abstract: In order to understand complex geometry, it can be useful to construct “pathological” examples. Nonkaehler or nonprojective compact complexe manifolds are such examples. Hopf (in 1948) and Calabi and Eckmann (1953) constructed complex structures on product of spheres with odd dimension which are not kaehler. In this talk, we will present a generalization due to Bosio of these manifolds. These manifolds are called LVMB manifolds and their interest lies in the very combinatorial nature. In particular, we will show that LVMB manifolds are homeomorphic to quotients of important objects in toric topology, namely moment-angle complexes (MAC). We will also show the relationship between LVMB manifolds and toric (algebraic) varieties.
Yuri Ustinovsky (Moscow State Univ., Russia)
Title: Characteristic numbers of quasitoric manifolds from the combinatorial point of view.
Abstract: We will discuss an application of localization formulas to the problem of computation of characteristic numbers of toric varieties and quasitoric manifolds. We describe relations of these computations with combinatorial operations on simplicial spheres (e.g. stellar subdivisions, bistellar moves).

Wei Wang (Fudan Univ., China)
Title: On the topological decomposition of non-singular hypersurfaces in projective toric manifolds
Abstract: In this talk, we want to discuss the topology of a non-singular hypersurface $Y_{2n}$ in projective toric manifold. When $n$ is odd, our main results are a decomposition of $Y_{2n}$ as a connected sum of $k$ copies of $S^n \times S^n$ with a differential manifold $M$ such that $b_n(M) \leq 2$. We will also discuss the case when $n$ is even and some partial results will be mentioned.

Takahiko Yoshida (Tokyo Denki Univ., Japan)
Title: Equivariant local index
Abstract: This talk is based on a joint work with Hajime Fujita and Mikio Furuta. In our joint work we developed an index theory for Dirac-type operators on possibly noncompact Riemannian manifolds and applied the theory to the geometric quantization, in particular, the relationship between the Riemann-Roch index and Bohr-Sommerfeld fibers. The purpose of this talk is to explain its equivariant version in a simple symplectic case. In particular, as an application, we give a proof of the quantization conjecture for a circle action.

Li Yu (Osaka City Univ. & Nanjing Univ., Japan & China)
Title: On transition functions of topological toric manifolds
Abstract: We show that any topological toric manifold can be covered by finitely many open charts so that all the transition functions between these charts are Laurent monomials of $z_j$’s and $\bar{z}_j$’s. In addition, we will describe toric manifolds and some special class of topological toric manifolds in terms of transition functions of charts up to (weakly) equivariant diffeomorphism.