Alexander duals to boundaries of polytopes

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Let $\mathbb{S}^r \subset \mathbb{R}^{r+1}$ be a unit sphere, and $\bigcup_{i\in[m]} H_i = \mathbb{S}^r$ — its covering by open hemispheres. The nerve of such covering will be called *a constellation complex*. Under some restrictions constellation complexes are exactly those simplicial complexes, which are Alexander dual to boundaries of polytopes. This fact follows from Gale duality. By using the correspondence between links in the complex and full subcomplexes of its dual we can do the following: (1) calculate bigraded Betti numbers of some polytopes using their dual constellation complexes; (2) relate bigraded Betti numbers of constellation complexes to f-numbers of polytopes; (3) do this sort of things for nonsimple polytopes.

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