UNIMODALITY OF THE BETTI NUMBERS FOR HAMILTONIAN CIRCLE ACTION WITH ISOLATED FIXED POINTS

YUNHYUNG CHO

ABSTRACT. Let (M, ω) be a $2n$-dimensional compact symplectic manifold. We say that ω satisfies the *hard Lefschetz property* if

$$[\omega]^{n-k} : \begin{array}{ccc} H^k(M) & \longrightarrow & H^{n-k}(M) \\ \alpha & \mapsto & \alpha \wedge [\omega]^{n-k} \end{array}$$

is an isomorphism for every $k = 0, 1, \cdots, n$. In equivariant symplectic topology, one of the interesting question is as follows.

Question : If a smooth compact sympletic manifold (M, ω) admits a Hamiltonian circle action with only isolated fixed points, then does ω satisfies the hard Lefschetz property?

The above question was posed by Y. Karshon in [JHKLM]. If it is true, then we can easily show that the Betti numbers of M are unimodal. At the same time, S. Tolman posed the weaker version of the above question as follows.

Question (weaker version): If a smooth compact sympletic manifold (M, ω) admits a Hamiltonian circle action with only isolated fixed points, then are the Betti numbers of M unimodal?

In this talk, we will show that the answer for Tolman's question is true in eight dimensional cases. More precisely, let (M, ω) be an eight-dimensional closed symplectic manifold equipped with a Hamiltonian circle action with only isolated fixed points. Then we will show that the Betti numbers of M are unimodal, i.e. $b_0(M) \leq b_2(M) \leq b_4(M)$. This talk is based on joint work with M. K. Kim.

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School of Mathematics, Korea Institute for Advanced Study, 87 Hoegiro, Dongdaemun-gu, Seoul, 130-722, Republic of Korea

E-mail address: yhcho@kias.re.kr