BUCHSTABER INVARIANT, 2-SURFACES AND MATROIDS

NIKOLAY EROKHOVETS

With each simplicial $(n-1)$-complex K on m vertices toric topology associates an $(m+n)$-dimensional moment-angle complex $\mathcal{Z}_K$ with a canonical action of a torus T^m. The equivariant topology of $\mathcal{Z}_K$ depends only on the combinatorics of K, which gives a tool to study the combinatorics of simplicial complexes in terms of the algebraic topology of moment-angle complexes and vice versa. A *Buchstaber invariant* $s(K)$ is equal to the maximal dimension of torus subgroups $H \subset T^m$, $H \simeq T^k$, that act freely on $\mathcal{Z}_K$. In 2002 V.M. Buchstaber stated the problem *to find an effective description of* $s(K)$ *in terms of the combinatorics of* K. There is an n-dimensional analog $\mathbb{R}\mathcal{Z}_K$ for the $\mathbb{Z}_2^m$-action. The corresponding number $s_{\mathbb{R}}(K)$ is called a *real Buchstaber invariant*. We have $s(K) \leqslant s_{\mathbb{R}}(K) \leqslant m-n$.

The talk is based on the following two theorems and their consequences.

Theorem 1. *Let* $\dim K = 2$. *Then*

$$m-1-\lceil \log_2 \gamma(K) \rceil \leqslant s(K) \leqslant m-1-\left\lceil \log_2 \max\left\{\frac{\gamma(K)+1}{2}, \gamma(\mathrm{lk}_K v)+1\right\}_{v\in[m]}\right\rceil,$$

where $\gamma(K)$ *is the chromatic number of the* 1*-skeleton of* K. *In particular* $s(K) = m-l-1$ *if* $\gamma(K) = 2^l$.

This is a generalization of the result by A.Ayzenberg that states that $s(K) \leqslant m - \lceil \log_2(\gamma(K)+1) \rceil$ for all K, and $s(K) = m - \lceil \log_2(\gamma(K)+1) \rceil$ if $\dim K = 1$.

Corollary (Ayzenberg, Masuda and Fukukawa, 09)**.** *For the* 2*-skeleton of an* $(m-1)$*-simplex we have* $s(\Delta^{m-1}_{(2)}) = m-1-\lceil \log_2 m \rceil$

Corollary. *Let* $\mathcal{H}(K) = \frac{7+\sqrt{49-24\chi(K)}}{2}$. *Then for a triangulation* K *of a* 2*-dimensional surface we have*

$$s(K) \geqslant m-1-\lceil \log_2 [\mathcal{H}(K)] \rceil \geqslant \lceil \mathcal{H}(K) \rceil - 1 - \lceil \log_2 [\mathcal{H}(K)] \rceil.$$

Theorem 2. Let $m \geqslant n+2$. Then $s_{\mathbb{R}}(K) \geqslant r$ if and only if there is an $(r-1)$-neighborly subcomplex $\widehat{M} \subset \widehat{K}$ such that $(\sigma_1 \cap \sigma_2) \cup \{i,j\} \in \widehat{M}$ for any two distinct maximal simplices $\sigma_1, \sigma_2 \in \widehat{M}$ and possibly coinciding vertices $i, j \notin \sigma_1 \cup \sigma_2$.

This result follows from known results on binary matroids and a connection of the Buchstaber number to matroid theory discovered by the author.

References

[1] N.Yu. Erokhovets, *Theory of the Buchstaber invariant of simplicial complexes and convex polytopes*, accepted to Proceedings of the Steklov Institute of Mathematics.

Lomonosov Moscow State University, Moscow, Russia
E-mail address: erochovetsn@hotmail.com