ABSTRACT

March 9 (Mon)

10:00-11:00 **Yoshihiro Ohnita** (Osaka City University, Japan)

"Differential geometry of Lagrangian submanifolds and Hamiltonian variational problems."

ABSTRACT: In this talk I will explain Hamiltonian minimality and Hamiltonian stability problem for Lagrangian submanifolds in specific Kähler manifolds and I will mention recent results in my joint works with Hui Ma on minimal Lagrangian submanifolds in complex hyperquadrics obtained as the Gauss images of isoparametric hypersurfaces in the standard unit sphere.

11:10-12:10 **Yng-Ing Lee** (National Taiwan University, Taiwan)

"On the existence of Hamiltonian stationary Lagrangian submanifolds in symplectic manifolds"

ABSTRACT: In this talk, I will report my recent joint work with D. Joyce and R. Schoen. Let (M, ω) be a compact symplectic $2n$-manifold, and g be a Riemannian metric on M compatible with ω. For instance, g could be Kähler, with Kähler form ω. Consider compact Lagrangian submanifolds L of M. We call L Hamiltonian stationary, or H-minimal, if it is a critical point of the volume functional under Hamiltonian deformations. Our main result is that if L is a compact, Hamiltonian stationary Lagrangian in $\mathbb{C}^n$ which is Hamiltonian rigid, then for any (M, ω, g) as above there exist compact Hamiltonian stationary Lagrangians L' in M contained in a small ball about some point p and locally modelled on tL for small $t > 0$, identifying M near p with $\mathbb{C}^n$ near 0. If L is Hamiltonian stable, we can take L' to be Hamiltonian stable. Applying this to known examples in $\mathbb{C}^n$ shows that there exist families of Hamiltonian stable, Hamiltonian stationary Lagrangians diffeomorphic to T^n, and to $S^1 \times S^{n-1}/Z_2$, and with other topologies, in every compact symplectic $2n$-manifold (M, ω) with compatible metric g.

13:30-14:30 **River Chiang** (National Cheng Kung University, Taiwan)

"On the construction of certain 6-dimensional Hamiltonian $SO(3)$ manifolds

ABSTRACT: In this talk, I will discuss a construction of 6-dimensional Hamiltonian $SO(3)$ manifolds. In 2005, I gave the invariants to distinguish these manifolds up to equivariant symplectomorphisms, which constitute the uniqueness part of the classification. The construction in this talk is one step toward the existence part.

14:50-15:50 **Futoshi Takahashi** (Osaka City University, Japan)

"On an eigenvalue problem related to the critical Sobolev exponent: variable coefficient case"

ABSTRACT: Let $\Omega \subset \mathbb{R}^N$ $(N \geq 3)$ be a smooth bounded domain, $p = (N+2)/(N-2)$, $c_0 = N(N-2)$, $\varepsilon > 0$ and $p_\varepsilon = p - \varepsilon$. $K \in C^2(\overline{\Omega})$, $K > 0$ is a given function.
In this talk, we are concerned with some spectral properties of least energy solutions u_ε to the problem

$$(P_{\varepsilon,K}) \begin{cases} -\Delta u = c_0 K(x) u^{p_\varepsilon} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $\varepsilon > 0$ is small.
Let us consider the linearized eigenvalue problem around the least energy solution u_ε to $(P_{\varepsilon,K})$:

$$(E_{\varepsilon,K}) \begin{cases} -\Delta v_{i,\varepsilon} = \lambda_{i,\varepsilon}(c_0 p_\varepsilon K(x) u_\varepsilon^{p_\varepsilon - 1}) v_{i,\varepsilon} & \text{in } \Omega, \\ v_{i,\varepsilon} = 0 & \text{on } \partial\Omega, \\ \|v_{i,\varepsilon}\|_{L^\infty(\Omega)} = 1 \end{cases}$$

for $i \in \mathbb{N}$. Under some assumptions of K (including the nondegeneracy of its maximum point as a critical point of K), we prove precise asymptotic estimates for the first $(N+2)$ eigenvalues and eigenfunctions of $(E_{\varepsilon,K})$ as $\varepsilon \longrightarrow 0$. These results correspond to the ones recently obtained by Grossi and Paceella (Math.Z., 2005) for the "one-point blow up solutions" to $(P_{\varepsilon,K})$ with $K \equiv 1$.

16:00-17:00 **Chang-Shou Lin** (National Taiwan University, Taiwan)

"Green function and Mean field equations on torus"

ABSTRACT: The Liouville equation is an integrable system. Locally, solutions can be written explicitly by Liouville theorem. So, it is interesting to study solution structure globally. In my talk, I will show you how applying Elliptic function theory and PDE technique together to study the equation. As application of our theory, we prove the Green function of torus has at most five critical points.

March 10 (Tue)

10:00-11:00 **Jost Hinrich Eschenburg** (University of Augsburg, Germany)

"Constant mean curvature surfaces and monodromy of Fuchsian equations"

Abstract: We will discuss classical theory (going back to H.A. Schwarz) of certain Fuchsian equations, i.e. second order linear ODEs

$$y'' + py' + qy = 0$$

where p, q are real rational functions with only regular singularities lying on the real line. In particular we investigate in which cases the monodromy group is (up to conjugation) contained in the isometry group of either the 2-sphere or euclidean or hyperbolic plane. As an application we study punctured spheres of constant mean curvature in euclidean 3-space where all punctures lie on a common circle.

11:10-12:10 **Derchyi Wu** (Academia Scinica, Taiwan)

"The Cauchy Problem of the Ward Equation"

Abstract: Taking a dimension reduction and a gauge fixing of the self-dual Yang-Mills equation in the space-time with signature $(2, 2)$, one derives a $2+1$ dimensional $SU(N)$ chiral field equation with an additional torsion term.

$$\begin{aligned} &-\left(J^{-1}J_t\right)_t + \left(J^{-1}J_x\right)_x + \left(J^{-1}J_y\right)_y + \nu_0 \left\{\left(J^{-1}J_y\right)_x - \left(J^{-1}J_x\right)_y\right\} \\ &+\nu_1 \left\{\left(J^{-1}J_t\right)_y - \left(J^{-1}J_y\right)_t\right\} + \nu_2 \left\{\left(J^{-1}J_x\right)_t - \left(J^{-1}J_t\right)_x\right\} = 0. \end{aligned}$$

Where J lies in $SU(N)$ and $\nu = (\nu_0, \nu_1, \nu_2)$ is a constant unit vector. Letting $\nu = (1, 0, 0)$ (time-like) and $\nu = (0, 1, 0)$ (space-like), we obtain two integrable systems, the 3-dimensional relativistic-invariant system and the Ward equation.

One important class of solutions for integrable systems are solitons of which the associated eigenfunctions $\psi(x, y, t, \lambda)$ are λ-rational functions. The construction of simple solitons, and the study of their scattering properties was done by [4] for the 3-dimensional relativistic-invariant system and by many mathematicians for the Ward equation, see [1] for references.

Besides, mathematicians study the inverse scattering problem and solve the Cauchy problem of the 3-dimensional relativistic-invariant system [4], [5] and of the Ward equation [6], [3], [2] if the initial potential is sufficiently small. Under the small data condition, the associated eigenfunction $\psi(x, y, t, \lambda)$ is λ-holomorphic outside a contour in the complex plane. Therefore this class of solutions does not include solitons in previous study.

Our main contribution is solving the inverse scattering problem and the Cauchy problem of the Ward equation without small data constraints.

References

[1] B. Dai and C. L. Terng: Backlund transformations Ward solitons, and unitons. *J. Differential Geom.*, **75** (2007), no. 1, 57–108.

[2] B. Dai and C. L. Terng and K. Uhlenbeck: On the space-time Monopole equation, Surveys in differential geometry. Vol. X, 1–30, Surv. Differ. Geom., 10, Int. Press, Somerville, MA, 2006.

[3] A. S. Fokas and T. A. Ioannidou: The inverse spectral theory for the Ward equation and for the $2 + 1$ chiral model, *Comm. Appl. Analysis*, **5** (2001), 235-246.

[4] S. V. Manakov and V. E. Zakharov: Three-dimensional model of relativistic-invariant theory, integrable by the inverse scattering transform, *Lett. Math. Phys.*, **5**, 247-253.

[5] J. Villarroel: Scattering data for the self-duality equations, *Inverse Problems*, **5** (1989), 1157-1162.

[6] J. Villarroel: The inverse problem for Ward's system, *Stud. Appl. Math.*, **83** (1990), 211-222.

13:30-14:30 **Manabu Akaho** (Tokyo Metropolitan University, Japan)

"Lagrangian mean curvature flow and symplectic area"

ABSTRACT: I'll explain a very easy observation of Lagrangian mean curvature flow in an Einstein-Kähler manifold and the symplectic area of smooth maps from a Riemann surface with boundary on the flow.

14:50-15:50 **Hajime Ono** (Science University of Tokyo, Japan)

"Variation of Reeb vector fields and its applications"

ABSTRACT: In this talk, we give some applications of the variation of Reeb vector fields of Sasaki manifolds: Given a Fano manifold there are obstructions for asymptotic Chow semistability described as integral invariants. One of them is the Futaki invariant which is an obstruction for the existence of Kähler-Einstein metrics. We show that these obstructions are obtained as derivatives of the Hilbert series. Especially, in toric case, we can compute the Hilbert series and its derivative using the combinatorial data of the image of the moment map. This observation should be regarded as an extension of the volume minimization of Martelli, Sparks and Yau.

16:00-17:00 **Shu-Cheng Chang** (National Taiwan University, Taiwan)

"The CR Bochner formulae and its applications"

ABSTRACT: (i) In first half, we will prove the CR analogue of Obata's theorem on a closed pseudohermitian manifold with vanishing pseudohermitian torsion. The key step is a discovery of CR analogue of Bochner formula which involving the CR Paneitz operator and nonnegativity of CR Paneitz operator. This is a joint work with H.-L. Chiu which to appear in Math. Ann. and JGA.

(ii) In second half, we obtain a new Li-Yau-Hamilton inequality for the CR Yamabe flow. It follows that the CR Yamabe flow exists for all time and converges smoothly on a spherical closed CR 3-manifold with positive Yamabe constant and vanishing torsion. This is a joint work with H.-L. Chiu and C.-T. Wu which to appear in Transactions of AMS.

Concentrated Lectures
March 12 (Thu)- March 13 (Fri)

10:00-12:00 **Jost Hinrich Eschenburg**

"Pluriharmonic Maps and Submanifolds"

ABSTRACT: A (simply connected) Kähler manifold M allows for a family of parallel rotations R_θ on its tangent bundle, namely multiplication by the complex scalars $e^{i\theta}$. Given a smooth map $f : M \longrightarrow S$ into some symmetric space $S = G/K$, one may ask when $df \circ \mathsf{R}_\theta$ is the differential of another map f_θ. When S has semi-definite curvature operator (e.g. when S is compact or dual to a compact symmetric space), this holds if and only if f is pluriharmonic, i.e. the $(1,1)$ part of its hessian vanishes $(\nabla df)^{(1,1)} = \nabla'' d' f = 0$ (without the semi-definitness, the "if" statement is unknown) and $(f_\theta)_{\theta\in[0,2\pi]}$ is called *associated family* of f. Identifying $df \circ \mathsf{R}_\theta$ with df_θ needs an identification of the two tangent spaces $T_{f(x)}S$ and $T_{f_{\theta(x)}}S$ by applying an element $\Phi_\theta(x) \in G$. This defines a smooth map $\Phi : \mathbb{S}^1 \times M \to G$, the so called *extended solution* (introduced by Uhlenbeck). Further, if a *frame* F for f is given, i.e. a smooth map $F : M \to G$ with $f = \pi \circ F$ for the projection $\pi : G \to G/K$, then $F_\theta = \Phi_\theta F$ is a frame for f_θ, and the family of maps $(F_{\theta)}$ is called an *extended frame* of f. Extended solutions and extended frames are two different descriptions of pluriharmonic maps. We shall discuss the advantages of both notions in various applications.

Extended frames take values in the loop group ΛG. There is some freedom in the choice of F which is fixed by passing to the quotient space $\Lambda G/K$. This can be viewed as *universal twistor space*: Every pluriharmonic map f is the projection of a holomorphic and "superhorizonta" map $\hat{f} : M \to \Lambda G/K$. This infinite dimensional space carries a homogeneous holomorphic structure acted on by the complexified loop group ΛG^c, and this action also preserves the superhorizontal distribution. Applied to $\hat{f}$, these transformations give new pluriharmonic maps which are called dressing transformations of f.

An important special case happens when the associated family is constant, $f_\theta = f$ (*isotropic* case). Then $\hat{f}$ takes values in a finite dimensional complex homogeneous subspace of $\Lambda G/K$, a *twistor space*. We will discuss this notion in detail. There are two special cases where all isotropic pluriharmonic maps can be written down explicitly: when S is a complex Grassmannian or a quaternionic symmetric space (Wolf space). For all other compact symmetric spaces S, superhorizontality is a complicated nonholonomic condition, and the general solution seems to be unknown. The general (non-isotropic) case can be solved by the DPW method (Dorfmeister-Pedit-Wu). However, its application is essentially more difficult than in the surface case ($\dim_{\mathbb{C}} M = 1$) since in higher dimensions it requires solving the *complex curved flat condition*: Finding all closed $\mathfrak{p}^c$-valued one-forms η with $[\eta, \eta] = 0$, where $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ is the Cartan decomposition of $\mathfrak{g} = \mathrm{Lie}(G)$ corresponding to the symmetric space S and $\mathfrak{p}^c = \mathfrak{p} \otimes \mathbb{C}$. The generalsolution of this problem seems to be unknown.

Extended solutions take values in the set of based loops $\Omega G = \{\omega : \mathbb{S}^1 \to G;\ \omega(1) = e\}$. One striking application of extended solutions is a reduction of a large class of pluriharmonic maps to the isotropic case: pluriharmonic maps of finite uniton number, where Φ is a rational function of $\lambda = e^{-i\theta}$, i.e. its Fourier series in λ is finite. By a theorem of Uhlenbeck and Ohnita-Valli, this holds always if M is compact. Burstall and Guest have shown that Φ can be deformed to an isotropic extended solution Φ^∞ (where the loops $\Phi^\infty(x) : \mathbb{S}^1 \to G$ are group homomorphisms) using the energy flow on ΩG which acts by dressing. In the original paper S was an inner symmetric space, but there is a version for outer symmetric spaces as well. In both cases one obtaines a classification of pluriharmonic maps f by certain isotropic pluriharmonic maps f^∞ in the closure of the dressing orbit of f.

Pluriharmonic maps are useful for submanifold theory in various ways. By a classical theorem of Ruh and Vilms, a surface isometrically immersed in euclidean 3-space has constant mean curvature (CMC) iff its ($\mathbb{S}^2$-valued) Gauss map is harmonic. What are the Kähler submanifolds in euclidean n-space whose (Grassmann-valued) Gauss map is pluriharmonic? They turn out to have parallel *pluri-mean curvature*, "PPMC", i.e. $\nabla\alpha(1,1) = 0$, where the pluri-mean curvature $\alpha(1,1)$ is the $(1,1)$ component of the second fundamental form α. Examples are rare. The only examples known so far are CMC surfaces in $\mathbb{R}^3$ or S^3, pluriminimal submanifold (where $\alpha^{(1,1)} = 0$), and extrinsic hermitian symmetric spaces (where $\nabla\alpha = 0$). It can be shown that further examples must have high codimension.

CMC surfaces in euclidean 3-space yet enjoy another property: They can be computed from their (harmonic) Gauss map, using the associated family. This was discovered by Bonnet and restated differently by Bobenko, using a result of Sym, and it is often called Sym-Bobenko formula. Can one obtain PPMC submanifolds from their (pluriharmonic) Gauss map by a generalized Sym-Bobenko formula? Unfortunately this is not true. However, there is a generalization of this formula where the sphere $\mathbb{S}^2$ is replaced by any hermitian symmetric space of compact type, but instead of PPMC it leads to a new class of Kähler submanifolds which share many properties of CMC surfaces; one might call them "pluri-CMC".

15:00-17:00 Quo-Shin Chi

"The Isoparametric Story"

ABSTRACT: I will talk about the almost complete classification of isoparametric hypersurfaces in spheres, and, if time allows, end with a new look at two of the four unclassified cases.

Differential Geometry Seminar

(in Japanese)

March 11 (Wed)

10:40-12:40 Toyoko Kashiwada

"Conformal Killing Tensors and Tensor Analysis"

ABSTRACT: A Killing tensor is a p-form u which satisfies the relation

$$\nabla_{i_0} u_{i_1 \cdots i_p} + \nabla_{i_1} u_{i_0 \cdots i_p} = 0.$$

The notion of a Killing tensor has produced fruitful results.
As a development of a Killing tensor, a conformal Killing tensor of order p is defined as a p-form u satisfying

$$\begin{aligned}&\nabla_{i_0} u_{i_1 \cdots i_p} + \nabla_{i_1} u_{i_0 \cdots i_p} \\ &\qquad = 2\rho_{i_2 \cdots i_p} g_{i_0 i_1} - \textstyle\sum_{k=2} (-1)^k (\rho_{i_1 \cdots \hat{i_k} \cdots i_p} g_{i_0 i_k} + \rho_{i_0 i_2 \cdots \hat{i_k} \cdots i_p} g_{i_1 i_k})\end{aligned}$$

with a $(p-1)$-form ρ.
In this talk, we mention the context of the introduction of conformal Killing tensors, proofs of several results, problems in that connection. In the process, advantage of tensor analysis will be shown.

14:40-16:10 Yukinori Yasui (Dept. of Physics, Osaka City University)

"Einstein Manifold with Conformal Killing-Yano tensor"

ABSTRACT: We talk about Einstein manifolds admitting a conformal Killing-Yano(CKY) tensor. The black hole spacetime is taken into consideration for a concrete example. We give a local classification of the spacetimes with a closed rank-2 CKY tensor. By compactifying the spacetimes Einstein metrics are constructed on the sphere bundles over Kähler manifolds.

March 12 (Wed)

17:30-19:00 Yu Kawakami (Kyushu University & OCAMI)

"Non-existence of minimal surfaces with embedded planar ends"

ABSTRACT: We will talk about the algebraic characterization of embedded planar ends of minimal surfaces and its applications by Kusner and Schmitt.