Abstract

We consider conformal cmc immersions of $\mathbb{C}$ to $\mathbb{S}^3$, which are periodic with respect to a one-dimensional lattice in $\mathbb{C}$, and whose Hopf differential is constant on $\mathbb{C}$. These immersions are up to isometry determined by a periodic solution of the sinh-Gordon equation. We consider only those immersions, whose solutions of the sinh-Gordon equation are of finite type. For this purpose we parameterize the moduli space of spectral curves, corresponding to such periodic solutions of the sinh-Gordon equation, and describe the subset of solutions corresponding to periodic cmc immersions.

We call an immersion of a surface into $\mathbb{S}^3$ 1-sided Alexandrov embedded, iff it extends to an immersion of a three manifold with boundary equal to the surface and non-negative mean curvature with respect to the inner normal. Furthermore, we assume that the three manifold is complete with respect to the induced metric. With a maximum principle at infinity of Rosenberg we show that the 1-sided Alexandrov embeddedness is preserved under continuous deformations. With the help of the parametrisation of the moduli space we are able to deform all 1-sided Alexandrov embedded cmc cylinders of finite type into flat 1-sided Alexandrov embedded cylinders. Finally we classify the 1-sided Alexandrov embedded flat cylinders and their connected component, containing only Delaunay surfaces.