

**Title: Fourier-Mukai transforms and spectral data of
harmonic tori into compact symmetric spaces**

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In this talk, we define an analogue of Fourier-Mukai transform. Applying it, we shall construct spectral data of harmonic maps of tori into compact symmetric spaces.

First we recall some definitions. Let $T = \mathbb{C}/(\mathbb{Z} \oplus \mathbb{Z}\tau)$ be a 1-dimensional complex torus and \hat{T} the dual torus of T . \hat{T} is also regarded as the moduli space $Pic^0(T)$ of line bundles of degree 0 on T . Let \mathcal{P} be the Poincare bundle on $\hat{T} \times T$. In particular, if $p \in \hat{T}$ corresponds to a line bundle $\mathcal{L} \in Pic^0(T)$, then $\mathcal{P}|_{\{p\} \times T}$ is isomorphic to \mathcal{L} . By using a modified Poincare bundle, we define an analogue of Fourier-Mukai transform.

We regard T as the 2-dimensional real torus with the conformal structure induced by τ . Let ϕ be a harmonic map from T to a compact symmetric space G/K . Let $\Phi_\lambda : \tilde{T} \rightarrow G^\mathbb{C}$ be an extended frame of ϕ ($\lambda \in \mathbb{P}^* = \mathbb{C} \setminus \{0\}$) where \tilde{T} is the universal cover $\mathbb{C} \rightarrow T$. We denote the Maurer-Cartan form of Φ_λ by $\alpha_\lambda = \Phi_\lambda^{-1} d\Phi_\lambda$. For each $\lambda \in \mathbb{P}^*$, we consider the vector bundle $E(\lambda)$ on T associated to the representation of fundamental group of T induced by α_λ .

By applying the above analogue of Fourier-Mukai transform, we shall construct a spectral data (X, π, \mathcal{L}) which corresponds to the family of vector bundles $\{E(\lambda) \mid \lambda \in \mathbb{P}^*\}$. Here, X is a complex curve in the product $\mathbb{P}^* \times H^1(T, \mathbb{C}^*)$ of \mathbb{P}^* and the moduli space $H^1(T, \mathbb{C}^*) = \mathbb{C}^* \times \mathbb{C}^*$ of flat \mathbb{C}^* -bundles on T . And π is a map from X to \mathbb{P}^* induced by the first projection $pr_1 : \mathbb{P}^* \times H^1(T, \mathbb{C}^*) \rightarrow \mathbb{P}^*$. And \mathcal{L} is a sheaf on X .

Moreover, we also get a map L_X from T to the category of sheaves on X . In some cases, L_X is considered as a linear flow on the Picard group $Pic(\bar{X})$ of a compactification \bar{X} of X .