Surfaces with singularities and Osserman-type Inequalities.

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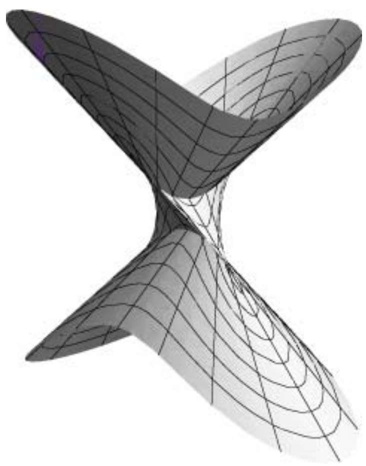
Abstract

Let M^2 be a 2-manifold, and $f : M^2 \to \mathbf{R}^3$ a complete minimal immersion of finite total curvature. Then it is well known that the degree of its Gauss map G satisfies the following inequality

$$\deg(G) \geq \chi(M^2) + \#(ends),$$

where the equality holds if and only if all ends of f are properly embedded. Similar inequalities hold for the following classes of surfaces:

(1) Surfaces of constant mean curvature 1 in hyperbolic 3-space H^3, (Yamada-U., Tsukuba J. Math. 21, 1997),
(2) Flat surfaces in H^3, (Kokubu-Yamada-U., Pacific J. Math. 216, 2004),
(3) Maximal surfaces (with certain kind of singularities) in Lorentz-Minkowski 3-space-time L^3, (Yamada-U., Hokkaido Math. J. 35, 2006),
(4) Surfaces (with certain kind of singularities) of constant mean curvature 1 in de Sitter 3-space-time S^3_1, (Fujimori-Kokubu-Rossman-Yang -Yamada-U., arXiv:0706.0973),
(5) Flat surfaces (with certain kind of singularities) with singularities in $\mathbf{R}^3$, (Murata-U., math/0605604).



flat surfaces with singularities with embedded ends

For all of such Osserman-type inequalities, the equalities hold if and only if all ends are properly embedded. A singular point $p \in M^2$ of a C^∞-map $f : M^2 \to \mathbf{R}^3$ is the point where f is not an immersion at p. The last four classes of surfaces admit singularities. This kind of Osserman-type inequalities are important philosophical criteria, when we must consider a class of C^∞-mappings admitting singularities.

Global study of surfaces with singularities gives several interesting view points in the differential geometry on surfaces: For example, a flat surface (as a wave front) in $\mathbf{R}^3$ whose ends are all embedded have at least four non-cuspidal edge points on it, which is a variant of classical four vertex theorem for convex planar curves. (In fact, in the above figure of a flat wave front, there are four swallowtails.)