

HARMONIC MAPS INTO GRASSMANNIANS AND ITS APPLICATIONS TO ISOPARAMETRIC FUNCTIONS AND MODULI PROBLEMS

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A nonlinear PDE for a harmonic map into a Grassmannian manifold relates to a linear PDE of Laplace type and we obtain a generalization of Theorem of Takahashi on a harmonic map into a sphere.

To construct a map into a Grassmannian, we use a vector bundle and a space of sections. The typical example is provided by the well-known Kodaira embedding of an algebraic manifold by a positive holomorphic line bundle and a space of holomorphic sections. Then, one holomorphic section gives a divisor. We also focus our attention on a specified section of a homogeneous vector bundle over a compact symmetric space and show that the zero set of the section is a totally geodesic submanifold. Moreover, it turns out that such a section gives an isoparametric function. We use the Radon transforms to obtain isoparametric functions on spheres. As a result, we get isoparametric functions on spheres whose level sets have two or four distinct principal curvatures. In particular, we obtain the isoparametric function on S^{15} defined by Ozeki-Takeuchi, which induces inhomogeneous isoparametric hypersurfaces.

Finally, we generalize a Theorem of do Carmo-Wallach (dCW) to describe moduli spaces of harmonic maps into Grassmannians with constant energy densities. In particular, we are interested in harmonic maps between complex projective spaces. A generalized dCW theorem gives another proof of a Theorem of Bando-Ohnita on the rigidity of minimal immersions from a complex projective line into a complex projective space. A similar method also gives the rigidity of Kähler isometries between complex projective spaces, which is a part of Calabi's theorem. We generalize a Toth's theorem on polynomial minimal immersions between complex projective spaces. In this case, we really obtain a finite dimensional moduli space and point out a similarity of the ADHM-construction of instantons.

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