

ABSTRACT

March 13 (Sunday)

9:30- Open and Tea

10:00-10:50 Mikiya Masuda (Osaka City University, Japan)
“Iterated circle bundles”

ABSTRACT: An iterated circle bundle over a point is a sequence of circle bundles starting with a point:

$$M_n \xrightarrow{p_n} M_{n-1} \xrightarrow{p_{n-1}} \cdots \xrightarrow{p_2} M_1 \xrightarrow{p_1} M_0 = \{\text{a point}\}.$$

Needless to say, M_1 is a circle and M_2 is a torus or a Klein bottle. This simple construction provides many interesting examples of aspherical manifolds. In this talk, I will discuss the topology and geometry of those manifolds. It turns out that they are infra-nilmanifolds (i.e. some finite covering space is a nilmanifold) and some of them are flat Riemannian manifolds. This talk is based on [1], [2] and a joint work in progress with Jong Bum Lee.

REFERENCES

- [1] Y. Kamishima and M. Masuda, *Cohomological rigidity of real Bott manifolds*, Algebraic & Geometric Topology 9 (2009) 2479-2502.
- [2] S. Choi, M. Masuda and S. Oum, *Classification of real Bott manifolds and acyclic digraphs*, arXiv:1006.4658.

11:10-12:00 Yng-Ing Lee (National Taiwan University, Taiwan)
“Special solutions to Lagrangian Mean Curvature Flow”

ABSTRACT: Mean curvature flow deforms a submanifold in the direction of its mean curvature vector. When the initial submanifold is Lagrangian in a Kahler-Einstein manifold, the solution will continue to be Lagrangian whenever it is smooth. It thus becomes a nice way to construct special Lagrangians. However, finite-time singularities may occur in general and cause the main difficulties. I will report some of my works with a few collaborators on special solutions to Lagrangian mean curvature flow that is closely related to the study of singularities. Most of the talk will concentrate on examples related to Schoen-Wolfson cones, which are the obstructions to the existence of special Lagrangians in two-dimension.

13:30-14:20 Kuo-Wei Lee (Academic Sinica, Taiwan)
“The mean curvature flow of compact submanifolds in higher codimension ”

ABSTRACT: In this talk, we will give two improvements of results of M.-T. Wang and M.-P. Tsui on mean curvature flow in higher codimension [1, 2]. Both the curvature condition and lower bound of $*\Omega$ (a geometric quantity) are weakened to obtain the long time existence and convergence of mean curvature flow. We also have some new applications on homotopy theory and on harmonic map theory. This is a joint work with Yng-Ing Lee.

REFERENCES

- [1] M.-P. Tsui; M.-T. Wang, Mean curvature flows and isotopy of maps between spheres. *Comm. Pure Appl. Math.* **57** (2004), no. 8, 1110–1126.
- [2] M.-T. Wang, Long-time existence and convergence of graphic mean curvature flow in arbitrary codimension. *Invent. Math.* **148** (2002), no. 3, 525–543.

14:30-15:20 Tai-Chia Lin (National Taiwan Univ., Taiwan)

“The Poisson-Nernst-Planck system for ion transport ”

ABSTRACT: Understanding ion transport is crucial in the study of many physical and biological problems, such as semiconductors, electro-kinetic fluids, transport of electrochemical systems and ion channels in cell membranes. One of the fundamental models for the ionic transport is the time dependent coupled diffusion-convection equations, the Poisson-Nernst-Planck (PNP) system. The PNP system consists of the electro-static Poisson and Nernst-Planck equations describing electro-diffusion and electrophoresis. In this lecture, I ’ ll introduce our recent results on the equilibrium of the PNP system and the linear stability problem.

15:40-16:30 Yoshie Sugiyama (Tsuda University, Japan)

“Measure valued solutions of the 2D Keller-Segel system ”

ABSTRACT: We deal with the two-dimensional Keller-Segel system describing chemotaxis in a bounded domain with smooth boundary under the nonnegative initial data. As for the Keller-Segel system, the L^1 norm is the scaling invariant one for the initial data, and so if the initial data is sufficiently small in L^1 , then the solution exists globally in time. On the other hand, if its L^1 norm is large, then the solution blows up in a finite time. The first purpose of my talk is to construct a time global solution as a measure valued function beyond the blow-up time even though the initial data is large in L^1 . The second purpose is to show the existence of two measure valued solutions of the different type depending on the approximation, while the classical solution is unique before the blow-up time.

16:40-17:30 Yohei Sato (OCAMI, Japan)

“The existence and non-existence of positive solutions for the nonlinear Schrödinger equations ”

ABSTRACT: In this talk, we consider the following Schrodinger equations:

$$-\Delta u + (1 + b(x))u = f(u) \quad \text{in } \mathbf{R}^N, \quad u \in H^1(\mathbf{R}^N),$$

where $b(x)$ satisfies $1 + b(x) \geq 0$, $\lim_{|x| \rightarrow \infty} b(x) = 0$ and $\limsup_{|x| \rightarrow \infty} e^{\beta|x|} b(x) \leq 0$ for some $\beta > 2$ and a typical example of our $f(u)$ is u^p .

By the concentration compactness arguments, we see that (*) has at least a positive solution for the case $b < b_0$, here b is a mountain pass value of the functional corresponding to (*) and b_0 is a mountain pass value corresponding to the limiting problem $-\Delta u + u = f(u)$ in \mathbf{R}^N , $u \in H^1(\mathbf{R}^N)$. In this talk, we also consider the case $b = b_0$. When $N \geq 2$ and $b = b_0$, we can also show the existence of the positive solution of (*) by the Bahri-Li’s minimax procedure. When $N = 1$ and $b = b_0$, depending on the $b(x)$, (*) has at least a positive solution or no non-trivial solutions.

March 14 (Monday)

- 9:30-10:20 Su-Jen Kan (Academic Sinica, Taiwan)
“Complete Ricci-flat metrics through a rescaled exhaustion ”
ABSTRACT: Typical existence result on Ricci-flat metrics is in manifolds of finite geometry, that is, on $F = \bar{F} - D$ where \bar{F} is a compact Kähler manifold and D is a smooth divisor. We view this existence problem from a different perspective. For a given complex manifold X , we take a suitable exhaustion $\{X_r\}_{r>0}$ admitting complete Kähler-Einstein metrics of negative Ricci. Taking a positive decreasing sequence $\{\lambda_r\}_{r>0}$, $\lim_{r \rightarrow \infty} \lambda_r = 0$, we rescale the metric so that g_r is the complete Kähler-Einstein metric in X_r of Ricci curvature $-\lambda_r$. The idea is to show the limiting metric $\lim_{r \rightarrow \infty} g_r$ does exist. If so, it is a Ricci-flat metric in X . Several examples: $X = \mathbb{C}^n$ and $X = TM$ where M is a compact rank-one symmetric space have been studied in this article.
The existence of complete Kähler-Einstein metrics of negative Ricci in bounded domains of holomorphy is well-known. Nevertheless, there is very few known for unbounded cases. In the last section we show the existence, through exhaustion, of such kind of metric in the unbounded domain $T^\pi H^n$.
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2000 Mathematics Subject Classification. 32Q20, 32Q25
- 10:30-11:20 Ryushi Goto (Osaka Univ., Japan)
“Holomorphic Poisson and generalized Kahler structures ”
ABSTRACT: First I will give a brief introduction of generalized complex and generalized Kaehler geometry. Deformations of generalized complex and generalized Kahler structures are discussed. Holomorphic Poisson structures gives rise to intriguing generalized Kaehler structures. Next I will construct Ricci-flat conical Kaehler metrics on crepant resolutions of Sasaki-Einstein cones in every Kaehler class. Applying the deformation method by Poisson structures in generalized geometry, I will deform ordinary Calabi-Yau structures to obtain generalized Calabi-Yau metrical structures on the crepant resolution.
- 11:30-12:20 Chiung-Ju Liu (National Taiwan University, Taiwan)
“Bergman kernel on singular Riemann surfaces ”
ABSTRACT: The Bergman kernel has asymptotic expansions on smooth polarized compact Kahler manifolds. However, some properties may not hold on singular manifolds. In this talk, we will study the information of the singularity in the expansion.
- 14:00-14:50 Kensuke Onda (Nagoya University & OCAMI)
“Sol-solitons on Lorentzian manifolds ”
ABSTRACT: J. Lauret introduced Sol-soliton and studied in the Riemannian framework. Sol-solitons have relevance to Ricci solitons, that is fixed point of the Ricci flow. We study sol-soliton theory and use it to give sol-solitons and Ricci solitons on Lorentzian manifolds.

- 15:00-15:40 Hitoshi Yamanaka (Osaka City University, Japan)
“Intersection of stable and unstable manifolds for invariant Morse functions ”
ABSTRACT: In Witten’s Morse theory, it is important to consider the negative gradient flows which connect two critical points whose Morse indices differ by 1. However there are too many examples of Morse functions which have no such pairs of critical points. In the case of an invariant Morse-Smale function, we investigate the intersection of a stable and an unstable manifold which corresponds to a pair of critical points whose Morse indices differ by 2. We also consider the group action on it.
- 16:00-16:40 Saki Okuhara (Tokyo Metropolitan University, Japan)
“Special Lagrangian 3-folds via harmonic maps ”
ABSTRACT: Special Lagrangian cones in \mathbb{C}^3 can be constructed from certain harmonic maps into a 6-symmetric space. We will review this process which was derived by Joyce and McIntosh, and use it to give a new construction of special Lagrangian cones in \mathbb{C}^3 .
- 16:50-17:30 Yohsuke Imagi (University of Kyoto, Japan)
“A Uniqueness Theorem for Gluing Special Lagrangian Submanifolds ”
ABSTRACT: Butscher, D. Lee, Y. Lee, and Joyce constructed a special Lagrangian submanifold by gluing a Lawlor neck into a transverse intersection point of two special Lagrangian submanifolds. I talk about a uniqueness theorem for the gluing of flat special Lagrangian tori of real dimension 3 in a flat complex torus of complex dimension 3.

March 15 (Tuesday)

- 9:30-10:20 Wu-yen Chuang (National Taiwan Univ., Taiwan)
“Wallcrossing in ADHM theory and Cohomology of the Hitchin system ”
ABSTRACT: I will introduce ADHM sheaf theory and present a conjectural recursive formula for the Poincare/Hodge polynomial of Hitchin system. The formula is derived from the wallcrossing of ADHM sheaf theory on curves. It establishes a connection between previous work of Hausel and Rodriguez-Villegas and refined local Donaldson-Thomas invariants.
- 10:30-11:20 Yasuyuki Nagatomo (Kyushu University, Japan)
“Harmonic maps into Grassmannians and its applications to isoparametric functions and moduli problems ”
ABSTRACT: A nonlinear PDE for a harmonic map into a Grassmannian manifold relates to a linear PDE of Laplace type and we obtain a generalization of Theorem of Takahashi on a harmonic map into a sphere.
To construct a map into a Grassmannian, we use a vector bundle and a space of sections. The typical example is provided by the well-known Kodaira embedding of an algebraic manifold by a positive holomorphic line bundle and a space of holomorphic sections. Then, one holomorphic section gives a divisor. We also focus our attention on a specified section of a homogeneous vector bundle over a compact symmetric spaces and show that the zero set of this section is a totally geodesic submanifold. Moreover, it turns out that such a section gives an isoparametric function. We use the Radon transformations to obtain isoparametric functions on spheres. As a result, we get isoparametric functions on spheres whose level sets have two or four distinct principal curvatures. In particular, we obtain the isoparametric function on S^{15} defined by Ozeki-Takeuchi, which induces inhomogeneous isoparametric hypersurfaces.
Finally, we generalize a Theorem of do Carmo-Wallach (dCW) to describe moduli spaces of harmonic maps into Grassmannians with constant energy densities. A generalized dCW theorem gives another proof of a Theorem of Bando-Ohnita on the rigidity of minimal immersions from a complex projective line into a complex projective space. A similar method also gives the rigidity of Kähler isometries between complex projective spaces, which is a part of Calabi’s theorem. We generalize a Toth’s theorem on polynomial minimal immersions between complex projective spaces. In this case, we really obtain a finite dimensional moduli space and point out a similarity of the ADHM-construction of instantons.
- 11:30-12:20 Ting-Hui Chang (Academic Sinica, Taiwan)
“On the existence of pseudoharmonic maps from pseudohermitian manifolds into Riemannian manifolds with nonpositive sectional curvature ”
ABSTRACT: In this talk, we first derive a CR Bochner identity for the pseudoharmonic map heat flow on pseudohermitian manifolds. Secondly, we are able to prove the existence of global smooth solution for the pseudoharmonic map heat flow from a closed pseudohermitian manifold into a Riemannian manifold with nonpositive sectional curvature. In particular, we prove the existence theorem of pseudoharmonic maps. This is served as the CR analogue of Eells-Sampson’s Theorem for the harmonic map heat flow.

13:40-14:30 Qing-Ming Cheng (Saga University, Japan)

“Estimates for eigenvalues on complete Riemannian manifolds ”

ABSTRACT: In this talk, we consider the Dirichlet eigenvalue problem of the Laplacian on a bounded domain in complete Riemannian manifolds. Estimates for eigenvalues will be discussed. We will talk about the conjecture of Polya for the lower bounds for eigenvalues on a bounded domain in Euclidean spaces and the generalized conjecture of Polya for the lower bounds of eigenvalues on a bounded domain in complete Riemannian manifolds. In order to obtain the lower bounds of eigenvalues, we need to have sharper universal inequalities for eigenvalues and the recursion formula of Cheng and Yang (Math. Ann. 2007). Furthermore, we will deal with the upper bounds for eigenvalues. In particular, for lower order eigenvalues, we will discuss the conjectures of Payne, Polya and Weinberger and their generalizations.