

# ABSTRACT

## The 10th Pacific Rim Geometry Conference 2011 Osaka-Fukuoka, Part I

December 1 (Thursday)

9:50-10:50 **Shu-Cheng Chang** (National Taiwan University, Taiwan, ROC)

“Li-Yau gradient estimate and entropy formulae for the CR heat equation in a closed pseudohermitian 3-manifold”

ABSTRACT: In this paper, we derive two sub-gradient estimates of the CR heat equation in a closed pseudohermitian 3-manifold which are served as the CR version of Li-Yau gradient estimate. With its applications, we first get a subgradient estimate of logarithm of the positive solution of CR heat equation. Secondly, we have the Harnack inequality and upper bound estimate for the heat kernel. Finally, we obtain Perelman-type entropy formulae for the CR heat equation.

11:00-12:00 **Carlos Olmos** (National University of Cordoba, Argentina)

“Killing fields, holonomy and the index of symmetry”

ABSTRACT: This talk is mainly based on a work, still in preparation, with Silvio Reggiani. We would like to draw the attention to some concept that we call the index of symmetry  $0 \leq i_s(M) \leq n$  of a Riemannian manifold  $M^n$ . The index of symmetry can be defined as the dimension of the tangent subspace where any natural Riemannian tensor is parallel (or, equivalently, the dimension of the space of Killing fields that are parallel at a given point). One has that  $M$  is symmetric if and only if  $i_s(M) = n$ . We are, of course, interested on non-symmetric spaces with positive index of symmetry. In this case one can prove that  $i_s(M) \leq n - 2$  (in other words, the co-index of symmetry is at least 2, for a non-symmetric space). We have some general results and many questions.

Many examples of spaces with non-trivial index of symmetry arise from naturally reductive spaces (we will also refer to a previous joint work with Reggiani related to naturally reductive spaces and holonomy, Crelle's 2011).

Also the unit tangent bundle over the sphere  $S^n$  of curvature 2 has  $i_s(S^n) = n - 1$ . We prove the following result

**Theorem** *Let  $M^n$  be a compact locally irreducible homogeneous Riemannian manifold which is not locally symmetric. Let  $k := n - i_s(M)$  be its co-index of symmetry. Then there is a subgroup of isometries  $G \subset I(M)$ , which acts transitively on  $M$  and such that  $\dim(G) \leq \frac{1}{2}k(k + 1)$ . Moreover, if the equality holds, then, up to a cover,  $G = Spin(k + 1)$  and  $G$  has non trivial isotropy, if  $k \geq 3$ .*

This allows us to classify the homogeneous spaces with low co-index of symmetry. For instance the spaces with co-index of symmetry 2 correspond to two distinguished families of one-parameter left invariant metrics on  $Spin(3)$ .

It is an interesting fact that there is a nice equivariant “Gauss map” from a homogeneous space  $M$  with non-trivial index of symmetry, into an appropriate Grassmannian.

The subjects of this talk may be regarded as an effort to explore Riemannian manifolds that are symmetric up to some defect (in the hope of finding distinguished non-symmetric homogeneous manifolds). In some sense, our philosophy is in the direction of the concept of co-polarity by Claudio Gorodski, that measures how a representation, orbit like, differ from a symmetric (isotropy) representation .

## Parallel Session (A)

- 13:30-14:20 **Yaroslav V. Bazaikin** (Sobolev Institute of Mathematics, Novosibirsk, Russia)  
“On  $G_2$ -holonomy metrics based on  $S^3 \times S^3$ ”  
ABSTRACT: We discuss one-parameter family of complete  $G_2$ -holonomy Riemannian metrics obtained by deformation of standard cone metric over  $S^3 \times S^3$ .
- 14:30-15:20 **Nikolai Erokhovets** (Moscow State University, Russia)  
“Towards the Theory of Buchstaber Invariant”  
ABSTRACT: Toric topology associates to each simple  $n$ -polytope  $P$  with  $m$  faces the smooth moment-angle manifold  $\mathcal{Z}_P$  with the canonical  $T^m$  action such that  $\mathcal{Z}_P/T^m = P$ . This gives a way to study the combinatorics of  $P$  in terms of the topology of  $\mathcal{Z}_P$  and vice versa. This idea is realized by the *Buchstaber invariant*  $s(P)$  – the combinatorial invariant of simple polytope  $P$  equal to the maximal dimension of torus subgroups  $H \simeq T^k \subseteq T^m$  acting freely on  $\mathcal{Z}_P$  – which is in some sense a measure of symmetry of  $\mathcal{Z}_P$ . It can be shown that  $1 \leq s(P) \leq m - n$ . In 2002 Victor M. Buchstaber stated a problem to find an effective method to calculate  $s(P)$  in terms of the combinatorics of  $P$ . The Buchstaber invariant has been studied since 2001. Nowadays there are quite enough general results about  $s(P)$  to look towards the theory of Buchstaber invariant. For example, we will show that  $s(P) = 1$  iff  $P = \Delta^n$ ; for any  $k \geq 2$  there exists a polytope  $P$  with  $m - n = k$  and  $s(P) = 2$ ;  $s(P)$  can not be calculated if only the  $f$ -vector and the chromatic number  $\gamma(P)$  are known. We will also show the behavior of the Buchstaber invariant under constructions of the polytope theory and its connection with classical and modern combinatorial invariants of polytopes.
- 16:00-16:30 **Imsoon Jeong\***, **Seonhui Kim** and **Young Jin Suh** (Kyungpook National University, Korea) (\*: speaker)  
“Real hypersurfaces in complex two-plane Grassmannians with  $\xi$ -parallel structure Jacobi operator”  
ABSTRACT: In this talk we give a characterization of Hopf hypersurfaces of Type (A) in complex two-plane Grassmannians  $G_2(\mathbb{C}^{m+2})$ , that is, a tube over a totally geodesic  $G_2(\mathbb{C}^{m+1})$  in  $G_2(\mathbb{C}^{m+2})$  with  $\xi$ -parallel structure Jacobi operator.
- 16:30-17:00 **Imsoon Jeong**, **Hyunjin Lee\*** and **Young Jin Suh** (Kyungpook National University, Korea) (\*: speaker)  
“Characterizations of Hopf hypersurfaces in complex two-plane Grassmannians related to generalized Tanaka-Webster connection”  
ABSTRACT: In this talk, we introduce the notion of generalized Tanaka-Webster connection (in short,  $g$ -Tanaka-Webster connection) for hypersurfaces in complex two-plane Grassmannians  $G_2(\mathbb{C}^{m+2})$ . Moreover, we consider various parallelisms of shape operator with respect to  $g$ -Tanaka-Webster connection, namely,  $g$ -Tanaka-Webster parallel,  $g$ -Tanaka-Webster  $\xi$ -parallel,  $g$ -Tanaka-Webster  $\xi$ -parallel, and  $g$ -Tanaka-Webster  $\mathbb{D}^\perp$ -parallel. By using there concepts, we give some characterizations of Hopf hypersurfaces in  $G_2(\mathbb{C}^{m+2})$ .

## Parallel Session (B)

13:30-14:20 **Makiko Sumi Tanaka** (Tokyo University of Science, Japan)

“Antipodal sets of compact Riemannian symmetric spaces and their applications”

ABSTRACT: This talk is based on a joint work with Hiroyuki Tasaki. We investigate fundamental properties of antipodal sets of symmetric  $R$ -spaces. We also investigate the intersection of two real forms in a Hermitian symmetric space of compact type and obtained that the intersection is an antipodal set, and moreover, we obtained that its cardinality is equal to the 2-number of the real form if two real forms are congruent. As a consequence we obtained that every real form of a Hermitian symmetric space of compact type is a globally tight Lagrangian submanifold.

14:30-15:20 **Adela Mihai** (University of Bucharest, Romania)

“Normal Complex Contact Metric Manifolds”

ABSTRACT: In this lecture the complex contact manifolds from a Riemannian geometric point of view, comparing the ideas with those of real contact metric geometry, are discussed. One important notion is that of a normal complex contact metric structure.

In the first part, I will present the recent work (D. E. Blair, A. Mihai) on locally symmetric normal complex contact metric manifolds along with the role played by reflections in the integral submanifolds of the vertical subbundle. Also, the properties of homogeneity and local symmetry of complex  $(k, \mu)$ -spaces are shown. The second part consists in recent definitions and studies of submanifolds of complex contact metric manifolds.

### References

D. E. Blair, A. Mihai, *Symmetry in complex contact geometry*, Rocky Mount. J. Math., to appear.

D. E. Blair, A. Mihai, *Homogeneity and local symmetry of complex  $(k, \mu)$ -spaces*, Israel J. Math., DOI: 10.1007/s11856-011-0089-2.

16:00-16:30 **Xianfeng Wang** (Nankai University, P. R. China)

“Lagrangian submanifolds in complex projective space with parallel second fundamental form”

ABSTRACT: From the Riemannian geometric point of view, one of the most fundamental problems in the study of Lagrangian submanifolds is the classification of Lagrangian submanifolds with parallel second fundamental form. In 1980's, H. Naitoh classified the Lagrangian submanifolds with parallel second fundamental form and without Euclidean factor in complex projective space, by using the theory of Lie groups and symmetric spaces. He showed that such a submanifold is always locally symmetric and is one of the symmetric spaces:  $\mathbf{SO}(k+1)/\mathbf{SO}(k)$  ( $k \geq 2$ ),  $\mathbf{SU}(k)/\mathbf{SO}(k)$  ( $k \geq 3$ ),  $\mathbf{SU}(k)$  ( $k \geq 3$ ),  $\mathbf{SU}(2k)/\mathbf{Sp}(k)$  ( $k \geq 3$ ),  $\mathbf{E}_6/\mathbf{F}_4$ .

In this paper, we completely classify the Lagrangian submanifolds in complex projective space with parallel second fundamental form by an elementary geometrical method. We prove that such a Lagrangian submanifold is either totally geodesic or the Calabi product of a point with a lower dimensional Lagrangian submanifold with parallel second fundamental form, or the Calabi product of two lower dimensional Lagrangian submanifolds with parallel second fundamental form, or one of the standard symmetric spaces:  $\mathbf{SO}(k+1)/\mathbf{SO}(k)$  ( $k \geq 2$ ),  $\mathbf{SU}(k)/\mathbf{SO}(k)$  ( $k \geq 3$ ),  $\mathbf{SU}(k)$  ( $k \geq 3$ ),  $\mathbf{SU}(2k)/\mathbf{Sp}(k)$  ( $k \geq 3$ ),  $\mathbf{E}_6/\mathbf{F}_4$ .

This is joint work with Professor Franki Dillen, Professor Haizhong Li and Professor Luc Vrancken.

16:30-17:00 **Seonhui Kim\***, **Hyunjin Lee** and **Young Jin Suh** (Kyungpook National University, Korea) (\*: speaker)

“A new condition of real hypersurfaces in complex two-plane Grassmannians”

ABSTRACT: We give a characterization of Hopf hypersurfaces of Type (A), that is, a tube over a totally geodesic  $G_2(\mathbb{C}^{m+1})$  in complex two plane Grassmannians  $G_2(\mathbb{C}^{m+2})$  in terms of commuting condition  $\varphi\varphi_1A = A\varphi_1\varphi$  between the shape operator  $A$  and the structure tensors  $\varphi, \varphi_1$  for real hypersurfaces in  $G_2(\mathbb{C}^{m+2})$ .

## December 2 (Friday)

9:50-10:50 **Victor Buchstaber** (Steklov Mathematical Institute & Moscow State University, Russia)

“Symplectic nilmanifolds and applications”

ABSTRACT: The talk will be devoted to the remarkable sequence of bundles  $M^n \rightarrow M^{n-1}$ ,  $n \geq 1$ , with fiber the circle. Each  $M^n$  is a smooth nilmanifold with a 2-form for  $n \geq 2$ , which gives a symplectic structure on  $M^{2k}$ , and a contact structure on  $M^{2k+1}$ .

This sequence plays an important role in different areas of mathematics. We will discuss the differential-geometric and algebro-topologic results and unsolved questions, concerning this sequence.

11:00-12:00 **Yael Karshon** (Toronto University, Canada)

“Counting toric actions”

ABSTRACT: In how many different ways can a two-torus act on a given simply connected symplectic four-manifold? If the second Betti number is one or two, the answer has been known for a while. For a higher Betti number, our (“soft”) proof that there are only finitely many inequivalent torus actions did not enable us to count these actions.

I will report on recent work, in which we reduce this counting question to combinatorics by expressing the manifold as a symplectic blowup in a way that is compatible with all the torus actions simultaneously. For this we use the theory of pseudoholomorphic curves. This work is joint with Liat Kessler and Martin Pinsonnault.

13:30-14:20 **Dmitry Gugnin** (Moscow State University, Russia)

“Smith-Dold Branched Coverings and Cup-Length”

ABSTRACT: Smith-Dold branched coverings are finite-fold branched coverings of Hausdorff spaces of a special type. They were defined by L.Smith in 1983 as a generalization of unbranched finite-fold coverings on which can be extended a (co)homology transfer. A.Dold gave a characterization of such a coverings in terms of actions of finite groups on topological spaces. Subsequently branched coverings of such a type were called Smith-Dold branched coverings.

There are at least 3 important for topology classes of maps which are  $n$ -fold Smith-Dold branched coverings:

- (1) unbranched  $n$ -fold coverings of Hausdorff spaces.
- (2) the projection map  $f : X \rightarrow X/G$  on the quotient space of  $X$  by an action of a group  $G$  of order  $n$ .
- (3) usual  $n$ -fold branched coverings of PL (smooth) manifolds.

In item (3) (the PL case) branched coverings of manifolds mean open-closed PL finite-fold maps of connected PL manifolds.

Using the cohomology transfer due to L. Smith it is easy to show that for any  $n$ -fold Smith-Dold branched covering  $f : X \rightarrow Y$  the induced homomorphism of rational and  $Z_p$ ,  $p > n$ , cohomology rings is a monomorphism. Therefore if a space  $X$  has at least one Betti number (rational or  $Z_p$ ,  $p > n$ )  $b_q(X)$  which is less than the corresponding Betti number  $b_q(Y)$  of a space  $Y$ , then there is no  $n$ -fold branched covering  $f : X \rightarrow Y$ . So there is the following easy principle:

(Easy P.) A simple space cannot cover (with an arbitrarily large  $n$ ) a complicated space.

One of the most common numerical invariants of a cohomology ring of a space is a cup-length. Cup-length  $l_R(X)$  of a space  $X$  is the maximal number of homogeneous elements of positive degrees of a cohomology ring  $H^*(X; R)$  which when multiplied give a nonzero product ( $R$  is an arbitrary commutative coefficient ring). Cup-length is the lower bound for Lusternik-Schnirelmann category of a space. Denote by  $l(X)$  the rational cup-length of a space  $X$ , and by  $l_p(X) = \dots = Z_p$  cup-length of  $X$ . I. Berstein and A. L. Edmonds in 1978 proved the following

**Theorem.** For any  $n$ -fold branched covering  $f : X^m \rightarrow Y^m$  of connected PL (Top) orientable closed manifolds the following inequality holds:  $l(Y^m) \geq l(X^m)/n$ . This result of I. Berstein and A. L. Edmonds made a good start for the following “hard” principle.

(“Hard”P.) A too complicated space cannot cover a too simple space with a relatively small number of sheets  $n$ .

The main result to be presented on the talk is the following general inequality slightly more weaker than Berstein-Edmonds inequality.

**Theorem 1.** For any  $n$ -fold Smith-Dold branched covering  $f : X \rightarrow Y$  of locally contractible paracompact spaces the following inequality holds:

$$l(Y) + 1 \geq (l(X) + 1)/n; \quad l_p(Y) + 1 \geq (l_p(X) + 1)/n, \quad \forall p > n.$$

The second result extends the original Berstein-Edmonds inequality:

**Theorem 2.** For any  $n$ -fold Smith-Dold branched covering  $f : X^m \rightarrow Y^m$ , with the base — connected top. orientable closed manifold, and the ENR (euclidean neighborhood retract) total space, the following inequality holds:

$$l(Y^m) \geq l(X^m)/n; \quad l_p(Y^m) \geq l_p(X^m)/n, \quad \forall p > n.$$

Our techniques is quite different from the one of I. Berstein and A. L. Edmonds, and is based on our extension of V. M. Buchstaber and E. G. Rees theory of Frobenius  $n$ -homomorphisms to graded algebras. Moreover, by the Berstein-Edmonds technics it cannot be proven even Theorem 2, because their approach needs the Poincare Duality of the total space, which does not hold for arbitrary ENR’s (arbitrary pseudomanifolds).

14:30-15:20 **Megumi Harada** (McMaster University, Canada)

“An invitation to (Newton-)Okounkov bodies”

ABSTRACT: This is not a standard research talk. It is not even a standard expository talk. Instead, it is an invitation to, and an advertisement of, a new and rapidly developing research area at the intersection of algebraic geometry, symplectic geometry, representation theory, and combinatorics. My main goal is to set the stage, to illustrate (some of) the connections between Okounkov bodies and the above-mentioned research areas, and (time permitting) to outline a small sample of the many open questions in the field.

The celebrated Bernstein-Kushnirenko theorem from Newton polyhedra theory relates the number of solutions of a system of polynomial equations with the volumes of their corresponding Newton polytopes. This motivated developments in the theory of toric varieties, which connects the combinatorics of a convex integral polytope  $\Delta$  with the (equivariant) geometry of the associated toric variety  $X(\Delta)$ . In the more general setting of symplectic manifolds and Hamiltonian actions, the Atiyah/Guillemin-Sternberg and Kirwan convexity theorems link equivariant symplectic and algebraic geometry to the combinatorics of moment map polytopes.

In the case of a toric variety  $X(\Delta)$ , the moment map polytope  $\Delta$  fully encodes the geometry of  $X(\Delta)$ , but this fails in general. In ground-breaking work, Okounkov constructs, for an (irreducible) projective variety  $X \subseteq P(V)$  equipped with an action of a reductive algebraic group  $G$ , a convex body  $\tilde{\Delta}$  and a natural projection from  $\tilde{\Delta}$  to the moment map polytope  $\Delta$  of  $X$ . The volumes of the fibers of this projection encode the so-called Duistermaat-Heckman measure, and in particular, one recovers the degree of  $X$  (i.e. the symplectic volume) from  $\tilde{\Delta}$ . Recently, Kaveh-Khovanskii and Lazarsfeld-Mustata have vastly generalized Okounkov's ideas; specifically, given the data of a variety  $X$  and a (big) divisor  $D$  on  $X$ , they construct a convex body  $\tilde{\Delta}(X, D)$  with  $\dim_{\mathbf{R}}(\tilde{\Delta}(X, D)) = \dim_{\mathbf{C}}(X)$  (called a *Newton-Okounkov body* or *Okounkov body*) even without presence of any group action. Thus the constructions of Kaveh-Khovanskii and Lazarsfeld-Mustata show that there are combinatorial objects of 'maximal' dimension associated to  $X$  in great generality. As a first application, Kaveh-Khovanskii use this to prove a far-reaching generalization of the Bernstein-Kushnirenko theorem to arbitrary varieties which relates the self-intersection number of a divisor with the volume of the corresponding Newton-Okounkov body. This theory is still in its infancy and the subject is wide open. The fundamental question is: What (asymptotic) geometric data of  $(X, D)$  do the combinatorics of Okounkov bodies encode, and how?

15:40-16:30 **Qun Chen** (Wuhan University, P. R. China)

“The maximum principle and the Dirichlet problem for Dirac-harmonic maps”

ABSTRACT: In this talk, we will introduce a recent work joint with J. Jost and G. F. Wang on Dirac-harmonic maps, which satisfy a system of equations consisting of a second order elliptic system and a Dirac equation on Riemannian spin manifolds. We first give a maximum principle for Dirac-harmonic maps from a Riemannian spin manifold with boundary into a regular ball in any Riemannian manifold. Then we establish a general existence theorem for boundary value problems of Dirac-harmonic maps.

16:40-17:30 **Yuxin Dong** (Fudan University, P. R. China)

“Monotonicity Formulae and Holomorphicity of Harmonic Maps between Kähler manifolds”

ABSTRACT: In this work, we introduce the stress-energy tensors of the partial energies  $E'(f)$  and  $E''(f)$  of maps between Kaehler manifolds. Assuming the domain manifolds poss some special exhaustion functions, we use these stress-energy tensors to establish some monotonicity formulae of the partial energies of pluriharmonic maps into any Kaehler manifolds and harmonic maps into Kaehler manifolds with strongly semi-negative curvature respectively. These monotonicity inequalities enable us to derive some holomorphicity and Liouville type results for these pluriharmonic maps and harmonic maps. We also use the stress-energy tensors to investigate the holomorphic extension problem of CR maps.

## December 3 (Saturday)

9:50-10:50 **Kaoru Ono** (Hokkaido University, Japan)

“Lagrangian Floer theory on compact toric manifolds”

ABSTRACT: I will present a review on Lagrangian Floer theory on compact toric manifolds based on my joint works with K. Fukaya, Y.-G. Oh, H. Ohta. If time allows, I will also discuss applications to Calabi quasi-morphisms on the universal covering group of Hamiltonian diffeomorphisms, symplectic partial quasi-states, which are discovered by Entov and Polterovich, etc.

11:00-12:00 **Yong-Geun Oh** (University of Wisconsin-Madison, USA)

“Localization of Floer homology in  $C^0$  hamiltonian topology”

ABSTRACT: Localization of Floer homology was first introduced by Floer for a  $C^2$ -small Hamiltonian  $H$ , which was the employed by the present speaker in the study of topology of Lagrangian submanifolds. In this talk, we will explain how this localization process can be carried out for a Hamiltonian that is small both in  $C^0$  topology of Hamiltonian flows and in Hofer topology of Hamiltonians, i.e., small in the sense of hamiltonian topology that is introduced by Mueller and the speaker. We will also explain how this localization gives rise to a comparison result between a ‘global’ spectral invariant and a ‘local’ spectral invariant. This comparison result plays a crucial role in the author’s recent study of nonsimpleness question of area-preserving homeomorphism group of the two disc.

13:30-14:20 **Hong Van Le** (Institute of Mathematics of ASCR, Czech Republic)

“Twisted symplectic manifolds and the associated cohomologies”

ABSTRACT: This is my joint work with Jiri Vanzura and Alexandre Vinogradov. We introduce the notion of a twisted symplectic manifold  $(M^{2n}, \mathcal{L}, \nabla, \omega)$ . Locally conformally symplectic manifolds are particular cases of twisted symplectic manifolds when  $\mathcal{L}$  is the trivial line bundle. We associate with a twisted symplectic manifold  $(M^{2m}, \mathcal{L}, \nabla, \omega)$  cohomology groups using the Lepage-Lefschetz decomposition. We study the relation between these new invariants, using and extending the technique of spectral sequences developed by Di Pietro and Vinogradov for symplectic manifolds. We discuss related results by many peoples, e.g. Bouche, Lychagin, Rumin, Tseng-Yau, in light of our spectral sequences. We calculate the associated cohomologies of a  $(2n+2)$ -dimensional locally conformally symplectic nilmanifold as well as those of a solvable 4-manifold. Using our theory we show an explicit example of a coorientation preserving contactomorphism of a connected contact 3-manifold, which is not isotopic to the identity through contactomorphisms.

14:30-15:20 **Hiroshi Iriyeh** (Tokyo Denki University, Japan)

“Floer homology and Hamiltonian volume minimizing properties of real forms of complex hyperquadric”

ABSTRACT: In this talk we first calculate the Lagrangian Floer homology  $HF(L_0, L_1 : \mathbb{Z}_2)$  of a pair of real forms  $(L_0, L_1)$  in the complex hyperquadric  $Q_n(\mathbb{C})$  in the case where  $L_0$  is not necessarily congruent to  $L_1$ . This yields a generalization of the Arnold-Givental inequality. Then we obtain a volume estimate for all real forms of  $Q_n(\mathbb{C})$  under Hamiltonian isotopies, combining the inequality and a Crofton type formula obtained by Lê Hồng Vân. In particular, we prove that the totally geodesic Lagrangian sphere in the complex hyperquadric is globally volume minimizing under Hamiltonian deformations. This talk is based on a joint work with Takashi Sakai and Hiroyuki Tasaki.

## Parallel Session (A)

15:50-16:20 **Toru Kajigaya**(Tohoku University, Japan)

“Legendrian minimal submanifolds in Sasakian manifolds and its stability”

ABSTRACT: Y. G. Oh introduced the notion of Hamiltonian-minimal Lagrangian submanifolds in Kähler manifolds, and his main concern is stability. An odd-dimensional version of Kähler manifolds would be Sasakian manifolds in the contact geometry. Corresponding to Hamiltonian minimal Lagrangian submanifolds, we introduce the notion of Legendrian-minimal Legendrian submanifolds in Sasakian manifolds, and investigate the stability.

16:30-17:20 **Hui Ma** (Tsinghua University, P. R. China)

“On Lagrangian submanifolds in complex hyperquadrics and Hamiltonian volume variational problem”

ABSTRACT: We will give a short survey on Hamiltonian volume variational problem related to Lagrangian submanifolds in Kähler manifolds. Then we will mainly discuss the properties of compact minimal Lagrangian submanifolds embedded in a complex hyperquadric obtained as the Gauss images of isoparametric hypersurfaces in a sphere. Our main results are as follows:

- (a) The Gauss image is a monotone and cyclic Lagrangian submanifold in a complex hyperquadric with minimal Maslov number  $2n/g$ , where  $g$  denotes the number of distinct principal curvatures of the isoparametric hypersurface.
- (b) The classification of homogeneous Lagrangian submanifolds.
- (c) The determination of the (strictly) Hamiltonian stability of the Gauss images of all compact homogeneous isoparametric hypersurfaces in spheres, by harmonic analysis on homogeneous spaces and fibrations on homogeneous isoparametric hypersurfaces.

This talk is mainly based on the joint work with Professor Yoshihiro Ohnita.

17:30-18:20 **Taras Panov**(Moscow State University, Russia)

“Intersections of quadrics and H-minimal Lagrangian submanifolds”

ABSTRACT: We study the topology of Hamiltonian-minimal Lagrangian submanifolds  $N$  in  $\mathbb{C}^m$  constructed from intersections of real quadrics in the work of the first author. This construction is linked via an embedding criterion to the well-known Delzant construction of Hamiltonian toric manifolds.

We establish the following topological properties of  $N$ : every  $N$  embeds as a submanifold in the corresponding moment-angle manifold  $Z$ , and every  $N$  is the total space of two different fibrations, one over a torus with fibre a real moment-angle manifold  $R$ , and another over a quotient of  $R$  by a finite group (known as a small cover) with fibre a torus. These properties are used to produce new examples of H-minimal Lagrangian submanifolds with quite complicated topology. The interpretation of our construction in terms of symplectic reduction leads to its generalisation providing new examples of H-minimal submanifolds in toric varieties.

The talk is based on a joint work with Andrey Mironov.

Reference: Andrey Mironov and Taras Panov. Intersections of quadrics, moment-angle manifolds, and Hamiltonian-minimal Lagrangian embeddings. Preprint(2011); arXiv:1103.4970.

## Parallel Session (B)

15:50-16:20 **Peng Wang** (Tongji University, P. R. China)

“Willmore two-spheres in  $S^{n+2}$  via Loop group theory”

ABSTRACT: We consider the harmonic conformal Gauss maps of Willmore surfaces by use of loop group methods. First, we derive a generic description of the normalized potential of a Willmore harmonic map into  $SO(1, n+3)/SO(1, 3) \times SO(n)$ . Then we consider such harmonic maps of finite uniton, via the DPW version of the theory of Burstall-Guest on harmonic maps of finite uniton. Then we give a classification of the normalized potential of Willmore harmonic maps of finite uniton. As an application, we derive a totally isotropic Willmore sphere in  $S^6$ , which is not S-Willmore.

16:30-17:20 **Xiang Ma** (Peking University, P. R. China)

“The global geometry of stationary surfaces in 4-dimensional Lorentz space”

ABSTRACT: We study the global geometry of complete stationary surfaces (i.e. zero mean curvature and space-like surfaces) in 4-dimensional Lorentz space based on a Weierstrass type representation of them. We find a series of examples with finite total curvature whose Gauss maps could not be extended to one end. We also generalize the construction of catenoid,  $k$ -noids and Enneper surface, all of them being embedded. These phenomena differ greatly with the classical minimal surfaces in 3-dimensional Euclidean space. We will also report our work on Gauss-Bonnet type theorems and the exceptional value problem about the Gauss maps. (This is a joint work with Zhiyu Liu, Changping Wang and Peng Wang.)

17:30-18:20 **Emma Carberry** (University of Sydney, Australia)

“Harmonic maps, Toda frames and extended Dynkin diagrams”

ABSTRACT: I shall discuss harmonic maps from surfaces into homogeneous spaces  $G/T$  where  $G$  is any simple real Lie group (not necessarily compact) and  $T$  is a Cartan subgroup. All immersions of a genus one surface into  $G/T$  possessing a Toda frame can be constructed by integrating a pair of commuting vector fields on a finite dimensional Lie algebra. I will provide necessary and sufficient conditions for the existence of a Toda frame and describe those  $G/T$  to which the theory applies in terms of involutions of extended Dynkin diagrams. Applications will be given to harmonic maps into de Sitter spaces and to Willmore tori in  $S^3$ . This is joint work with Katharine Turner (University of Chicago).

## December 4 (Sunday)

9:50-10:50 **Xiaobo Liu** (University of Notre Dame, USA and Peking University, P. R. China)

“Universal Equations for Gromov-Witten Invariants”

ABSTRACT: There is a class of differential equations which holds for generating functions of Gromov-Witten invariants of all compact symplectic manifolds. Such equations are called universal equations. Universal equations can be used to compute Gromov-Witten invariants. They also play important roles in the study of the Virasoro conjecture. It is well known that relations in tautological ring of moduli spaces of stable curves can produce universal equations, not only for Gromov-Witten invariants, but also for any cohomological field theory which satisfies the splitting principle. A typical example of such an equation is the WDVV equation, which is a genus-0 equation and gives the associativity of the quantum cohomology. Finding such relations in higher genera is a very difficult problem. Mumford, Getzler, Belorousski-Pandharipande have found some universal equations of genus-1 and genus-2. Together with Takashi Kimura, we obtained two genus-3 universal equations. I will also talk about some topological recursion relations for all genera which was proved in a joint paper with R. Pandharipande. Some of these relations can be used to prove a conjecture of Kefeng Liu and Hao Xu.

11:00-12:00 **Hiroshi Iritani** (Kyoto University, Japan)

“Quantum cohomology and periods”

ABSTRACT: Mirror symmetry predicts that quantum cohomology of a given manifold can be calculated by periods of the mirror manifold. In this talk, I will explain that a vector bundle on the original manifold should correspond to an integration cycle of mirror periods. In this correspondence, we need a transcendental characteristic class, called the Gamma class. Conjecturally, vector bundles and Gamma class should define a pure and polarized Hodge structure on the quantum cohomology globally. I will also explain its application to the functoriality of quantum cohomology.

13:30-14:20 **Siu-Cheong Lau** (IPMU, Japan)

“SYZ and mirror maps for semi-Fano toric manifolds”

ABSTRACT: In this talk I will explain my recent joint work with K. W. Chan, N. C. Leung and H. H. Tseng on mirror maps via the SYZ approach. We derive an open analog of closed-string mirror symmetry for a class of toric manifolds, which leads to a computational method of open Gromov-Witten invariants in the toric cases via mirror symmetry.

14:30-15:20 **Jianxun Hu** (Sun Yat-sen University, P. R. China)

“Degeneration formulae and its applications to local GW and DT invariants”

ABSTRACT: Degeneration formula is one of the most important techniques in the Gromov-Witten and Donaldson-Thomas theory. In this talk, I will first introduce the degeneration formulae and then talk about how to use the degeneration technique to study the change of Gromov-Witten and Donaldson-Thomas invariants of local surfaces under blowing up along points.

15:40-16:30 **Chin-Lung Wang**(National Taiwan University, Taiwan, ROC)

“Quantum Leray-Hirsch”

ABSTRACT: Let  $X$  be a split toric bundle over a smooth base  $S$ . I will explain how to construct the Dubrovin connection on  $X$  in terms of the Dubrovin connection on  $S$  and the Picard-Fuchs system associated to the toric fiber. The construction is natural in the sense that under an ordinary flop over  $S$  we get analytic continuations of quantum cohomology. This is a joint work with Y. P. Lee and H. W. Lin.

16:40-17:30 **Mohammad Ghomi** (Georgia Institute of Technology, USA)

“Tangent lines, inflection points, and vertices of closed curves”

ABSTRACT: We show that every smooth closed curve  $C$  immersed in Euclidean 3-space satisfies the sharp inequality  $2(P + I) + V > 5$  which relates the numbers  $P$  of pairs of parallel tangent lines,  $I$  of inflections (or points of vanishing curvature), and  $V$  of vertices (or points of vanishing torsion) of  $C$ . The proof, which employs curve shortening flow, is based on a corresponding inequality for the numbers of double points, singularities, and inflections of closed contractible curves in the real projective plane which intersect every closed geodesic. In the process we will also obtain some generalizations of classical theorems due to Mobius, Fenchel, and Segre (including Arnold’s “tennis ball theorem”).

17:40-18:20 **Qingchun Ji** (Fudan University, P. R. China)

“Division theorems for exact sequences”

ABSTRACT: I will talk about Skoda-type division theorems for exact sequences of holomorphic vector bundles, and give applications to the Koszul complex. I will also discuss how to use Skoda triples to establish global division theorems.

## December 5 (Monday)

- 9:50-10:50    **Zizhou Tang** (Beijing Normal University, P. R. China)  
“Gromov-Lowson-Schoen-Yau theory and isoparametric hypersurfaces ”  
ABSTRACT: Motivated by the Gromov-Lawson-Schoen-Yau surgery theory on metrics of positive scalar curvature, we construct a double manifold associated with a minimal isoparametric hypersurface in the unit sphere. The resulting double manifold carries a metric of positive scalar curvature and an isoparametric foliation as well.
- 11:00-12:00    **Mu-Tao Wang** (Columbia University, USA)  
“Mean curvature flows and isotopy problems”  
ABSTRACT: I shall discuss how mean curvature flows give canonical deformation of maps between Riemannian manifolds. Applications include estimations of null-homotopy constants of maps between spheres and smooth retractions of symplectomorphism groups of closed Riemann surfaces and complex projective spaces.
- 13:30-14:20    **Fuminori Nakata** (Tokyo University of Science, Japan)  
“Integral transforms and the twistor theory for indefinite metrics”  
ABSTRACT: Twistor theory for indefinite metrics, originated with LeBrun and Mason, is progressing steadily. By this theory, one can establish one-to-one correspondence between certain indefinite geometries and families of holomorphic disks on complex manifolds.  
While the general theory for this type of twistor correspondence is studied, several explicit examples are constructed. These examples are described by making a use of Radon type integral transforms, and give a new insight to the theory of hyperbolic PDE's.  
In this talk, an introduction to the LeBrun-Mason type twistor theory is given with showing examples and applications to hyperbolic PDE's.
- 14:30-15:20    **Shohei Honda** (Kyushu University, Japan)  
“Convergence of Lipschitz functions and a weak second differentiable structure on limit spaces”  
ABSTRACT: In this talk, we will give a new notion for convergence of Lipschitz functions with respect to the Gromov-Hausdorff topology and several properties of the convergence. As an application, we will show that all limit spaces of Riemannian manifolds with lower Ricci curvature bounds have second differentiable structure in some weak sense.

### REFERENCES

- [1] S. HONDA, *Ricci curvature and convergence of Lipschitz functions*, Commun. Anal. Geom. 19 (2011), 79-158.
- [2] S. HONDA, *A weak second differentiable structure on rectifiable metric measure spaces*, preprint.

# ABSTRACT

## The 10th Pacific Rim Geometry Conference 2011 Osaka-Fukuoka, Part II

December 7 (Wednesday)

- 9:50-10:50    **Henry Wente** (The University of Toledo, USA)  
“Exotic capillary tubes”  
ABSTRACT: In contrast to the standard capillary tube, an exotic capillary tube is a rotationally symmetric tube of variable cross-section which if positioned correctly in a vessel of fluid possesses a continuum of equilibrium configurations. The controlling variables are the capillary constant  $k = \rho g / \sigma$  and the contact angle  $\gamma$ . Lowering the tube slightly from its natural position causes the tube to completely fill up while raising the tube slightly forces the tube to drain out. Other surprising consequences follow. Potential commercial applications will also be discussed.  
Note: My paper discussing this topic recently appeared in the “Journal of Mathematical Fluid Mechanics” 13 (2011) 355-370.
- 11:00-12:00    **Jaigyoung Choe** (KIAS, Korea)  
“A sharp isoperimetric inequality for minimal surfaces”  
ABSTRACT: It is proved that a minimal surface with no genus and three boundary components in  $\mathbb{R}^3$  satisfies the classical isoperimetric inequality  $4\pi A \leq L^2$ . (Joint work with Richard Schoen)
- 13:30-14:30    **Miyuki Koiso** (Kyushu University/PREST, JST, Japan)  
“Pitchfork bifurcation for hypersurfaces with constant mean curvature”  
ABSTRACT: We consider hypersurfaces with constant mean curvature with given boundary conditions. Choosing the mean curvature  $H$  or the volume  $V$  enclosed by the hypersurface as parameter, we construct conditions under which a pitchfork bifurcation occurs. We apply our results to isoperimetric problems in the Riemannian products of  $S^1$  and simply connected space forms introduced by Pedrosa and Ritoré (1999).
- 14:40-15:20    **Keomkyo Seo** (Sookmyung Women’s University, Korea)  
“ $L^2$  harmonic 1-forms on a complete minimal submanifold in hyperbolic space ”  
ABSTRACT: We study the nonexistence of  $L^2$  harmonic 1-forms and topological property on minimal submanifolds in hyperbolic space. We also estimate the first eigenvalue for the Laplacian operator on minimal submanifolds in hyperbolic space.

15:50-16:20 **Rung-Tzung Huang** (National Central University, Taiwan)

“The comparison theorem of the refined analytic torsions on manifolds with boundary for an acyclic Hermitian connection”

ABSTRACT: The refined analytic torsion was introduced by M. Braverman and T. Kappeler as a canonical refinement of analytic torsion on odd dimensional closed Riemannian manifolds. It is defined by using the graded zeta-determinant of the odd signature operator. The refined analytic torsion on compact Riemannian manifolds with boundary has been discussed by B. Vertman and by Y. Lee and myself, but these two constructions are completely different. In this talk we will discuss the comparison theorem of these two constructions when the odd signature operator comes from an acyclic Hermitian flat connection.

16:20-17:00 **Toshiaki Omori** (Tohoku U., Japan)

“On existence of harmonic maps via exponentially harmonic maps”

ABSTRACT: An exponentially harmonic map  $u : (M, g) \rightarrow (N, h)$  is a map between compact Riemannian manifolds which extremize the functional

$$\mathbb{E}(u) := \int_M e^{|\nabla u|^2} d\text{vol}_g.$$

A remarkable fact on them is that, unlike harmonic maps, exponentially harmonic maps are known to always exist in a given homotopy class  $\mathcal{H} \in [M, N]$  and to be necessarily smooth. In the present talk, I would like to introduce an approximation of a harmonic map via a sequence of exponentially harmonic maps. Also, if possible, I would like to mention a time-evolution equation for exponentially harmonic maps, which is also expected to well approximate harmonic maps.

17:00-17:30 **Sung-Hong Min** (KIAS, Korea)

“Embeddedness of proper minimal submanifolds in homogeneous spaces”

ABSTRACT: In this presentation, we briefly show the three embeddedness results as follows.

- (1) Let  $\Gamma_{2m+1}$  be a polygon with  $2m + 1$  vertices in  $\mathbb{R}^n$ , where  $m$  is an integer  $\geq 2$ . Then the total curvature of  $\Gamma_{2m+1} < 2m\pi$ . In particular, the total curvature of  $\Gamma_5 < 4\pi$  and thus any minimal surface  $\Sigma \subset \mathbb{R}^n$  bounded by  $\Gamma_5$  is embedded. Let  $\Gamma_5$  be a piecewise geodesic Jordan curve with 5 vertices in  $\mathbb{H}^n$ . Then any minimal surface  $\Sigma \subset \mathbb{H}^n$  bounded by  $\Gamma_5$  is embedded. If  $\Gamma_5$  is in a geodesic ball of radius  $\frac{\pi}{4}$  in  $\mathbb{S}_+^n$ , then  $\Sigma \subset \mathbb{S}_+^n$  is also embedded. As a consequence,  $\Gamma_5$  is an unknot in  $\mathbb{R}^3$ ,  $\mathbb{H}^3$  and  $\mathbb{S}_+^3$ .
- (2) Let  $\Sigma$  be an  $m$ -dimensional proper minimal submanifold in  $\mathbb{H}^n$  with the ideal boundary  $\partial_\infty \Sigma = \Gamma$  in the infinite sphere  $\mathbb{S}^{n-1} = \partial_\infty \mathbb{H}^n$ . If the Möbius volume  $\widetilde{\text{vol}}(\Gamma)$  of  $\Gamma$  satisfies  $\widetilde{\text{vol}}(\Gamma) < 2\text{vol}(\mathbb{S}^{m-1})$ , then  $\Sigma$  is embedded. If  $\widetilde{\text{vol}}(\Gamma) = 2\text{vol}(\mathbb{S}^{m-1})$ , then  $\Sigma$  is embedded unless it is a cone.
- (3) Let  $\Sigma$  be a proper minimal surface in  $\mathbb{H}^2 \times \mathbb{R}$ . If  $\Sigma$  is vertically regular at infinity and has two ends, then  $\Sigma$  is embedded.

## December 8 (Thursday)

- 9:50-10:50 **Paolo Piccione** (University of Sao Paulo, Brazil)  
“Equivariant bifurcation in geometric variational problem”  
ABSTRACT: I will first discuss some abstract equivariant bifurcation results for variational problems. Then I will present some applications, including bifurcation of constant mean curvature embeddings, bifurcation of solutions of the Yamabe problems in product manifolds and in some special Riemannian submersions, and bifurcation of solutions of the  $\sigma_2$ -Yamabe problem in product of Einstein manifolds.
- 11:00-12:00 **Yoonweon Lee** (Inha University, Korea)  
“Gluing formula of the refined analytic torsion”  
ABSTRACT: The refined analytic torsion was introduced by Braverman and Kappeler on an odd dimensional closed Riemannian manifold in 2000’s as an analytic analogue of the Turaev torsion. It is defined by using the spectrum of the odd signature operator and is described as an element of the determinant line for cohomologies. Specially, when the odd signature operator is defined by an acyclic Hermitian connection, the refined analytic torsion is a complex number whose modulus part is a classical Ray-Singer analytic torsion and the phase part is the rho invariant, the difference of two eta invariants. In earlier work we introduced a well-posed boundary condition for the odd signature operator to define the refined analytic torsion on a compact manifold with boundary. In this talk we discuss the gluing formula for the refined analytic torsion on a closed Riemannian manifold with respect to this boundary condition in case that the odd signature operator is defined by an acyclic Hermitian connection. Basic tools are BFK-gluing formula for zeta-determinants and the gluing formula of eta invariant given by Brüning, Lesch and Kirk.
- 13:30-14:30 **Entao Zhao** (Zhejiang U., P.R.China)  
“The mean curvature flow in higher codimensions”  
ABSTRACT: The mean curvature flow is the negative gradient flow of the volume functional of the submanifolds. Many convergence theorems have been proved for the mean curvature flow of hypersurfaces or of submanifolds with low dimension or admitting some special structure. Recently, B. Andrews and C. Baker proved some beautiful convergence theorems for the mean curvature flow of submanifolds in an Euclidean space or a sphere.  
In this talk, I will discuss several new convergence theorems for mean curvature flow in higher codimensions. The initial submanifold is assumed to satisfy suitable pointwise or integral curvature pinching conditions. The talk is based on the joint works with Kefeng Liu, Hongwei Xu and Fei Ye.
- 14:40-15:20 **Yu Kawakami** (Yamaguchi U., Japan)  
“A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space”  
ABSTRACT: We provide an effective ramification theorem for the ratio of canonical forms of weakly complete flat fronts in the hyperbolic three-space. As an application, we give a simple proof of the classification of complete nonsingular flat surfaces in the hyperbolic three-space.

### Ph. D. Students Session:

15:50-16:15 **Renato Bettiol** (University of Notre Dame, USA)

“Bifurcation and local rigidity of homogeneous solutions to the Yamabe problem on spheres”

ABSTRACT: We study local rigidity and bifurcation of the Yamabe problem on 1-parameter families of homogeneous metrics on spheres. More precisely, we use variational techniques to study existence and non-existence of constant scalar curvature metrics conformal and arbitrarily close to homogeneous metrics. The abstract tools used are an implicit function theorem and a bifurcation criterion relying on jumps of the Morse index, which also have applications to other geometric variational problems. In the case of the Yamabe problem, this means that finding bifurcation instants amounts to computing the spectrum of the Laplacian and scalar curvature of each metric in the family. Applying this to 1-parameter families of  $U(n+1)$ ,  $Sp(n+1)$  and  $Spin(9)$ -homogeneous metrics, we prove local rigidity in the first and existence of infinitely many bifurcation instants in the last two families. As corollaries, we obtain some global uniqueness and multiplicity results on such families.

[Joint work with P. Piccione.]

16:15-16:40 **Nguyen Thac Dung** (National Tsinghua U., Taiwan)

“Complete Smooth Metric Measure Spaces with Spectrum Bounded from Below ”

ABSTRACT: Join work with Prof. Chiung Jue Sung (National Tsinghua University). We consider smooth metric measure spaces  $(M, g, e^{-f} dv)$  with weighted Laplacian  $\Delta_f$ . Assuming  $\lambda_1(\Delta_f)$  is bounded from below in term of  $|\text{grad} f|$  and its Bakry-Émery curvature bounded from below by  $\lambda_1(\Delta_f)$ , we prove the rigidity of  $M$ . This result generalizes the work of Li and Wang (see [2]) on complete non-compact Riemannian manifolds and extends the work of Munteanu and Wang (see citeM-W2) on the smooth metric measure spaces with its Bakry-Émery curvature bounded from below. At the same time, we address the space of  $f$ -harmonic functions with finite  $f$ -energy. A structure property of this space is given if the Bakry-Émery bounded from below. References.

#### REFERENCES

- [1] N. T. Dung and C. J. Sung, *Some Remarks on Manifolds with positive spectrum*, preprint.
- [2] P. Li and J. Wang, *Complete manifolds with positive spectrum*, Jour. Diff. Geom. **58** (2001) 501-534.
- [3] O. Munteanu and J. Wang, *Smooth metric measure spaces with non-negative curvature*, Comm. Anal. Geom. **19** (2011), no. 3, 451-486.
- [4] O. Munteanu and J. Wang, *Analysis of weighted Laplacian and application to Ricci solitons*, Preprint.

16:40-17:05 **Abdullah Kizilay** (Tohoku University, Japan)

“Viscosity solutions on a Riemannian manifold”

ABSTRACT: In this talk, we consider viscosity solutions to second order partial differential equations, in particular Cauchy-Dirichlet problem [CDP] of the form;  $u_t + F(t, x, u, Du, D^2u) = 0$  with a boundary condition and an initial condition at time  $t = 0$ . We work on Riemannian manifold and present the structures of semi-jets, comparison principle and Perron’s method. Existence, uniqueness and stability results for the Cauchy-Dirichlet problem on a Riemannian manifold are studied.

17:10-17:35 **Kotaro Kawai** (Tohoku University, Japan)

“Construction of special Lagrangian submanifolds”

ABSTRACT: The notion of special Lagrangian submanifolds was introduced by Harvey and Lawson in 1982. They are important in mirror symmetry due to the SYZ conjecture. In this short talk, I will construct special Lagrangian submanifolds in non-flat spaces explicitly. From the construction, these spaces are fibered by special Lagrangian submanifolds.

## December 9 (Friday)

9:50-10:50 **Boris Botvinnik** (University of Oregon, USA)

“Surgery, concordance and isotopy of metrics with positive scalar curvature”

ABSTRACT: Two positive scalar curvature metrics  $g_0, g_1$  on a manifold  $M$  are psc-isotopic if they are homotopic through metrics of positive scalar curvature. It is well known that if two metrics  $g_0, g_1$  of positive scalar curvature on a closed compact manifold  $M$  are psc-isotopic, then they are psc-concordant, i.e. there exists a metric  $\bar{g}$  of positive scalar curvature on the cylinder  $M \times I$  which has zero mean curvature along the boundary and extends the metrics  $g_0$  on  $M \times \{0\}$  and  $g_1$  on  $M \times \{1\}$ . In my lecture, I will discuss the problem whether a psc-concordance implies psc-isotopy. There is a combination of relevant methods to be used here: surgery tools related to Gromov-Lawson construction, classic results on isotopy and pseudo-isotopy of diffeomorphisms, standard geometric analysis related to the conformal Laplacian, and the Ricci flow.

11:00-12:00 **Harish Seshadri** (Indian Institute of Science, Bangalore, India)

“On Wilking’s criterion for the Ricci flow”

ABSTRACT: Recently Wilking gave a simple criterion for generating Ricci flow invariant nonnegative curvature conditions which recovers most of the known such conditions: In dimension  $n$  let  $S$  be an  $Ad_{SO(n, \mathbb{C})}$ -invariant subset of  $so(n, \mathbb{C})$ . Then the cone  $C(S)$  of curvature operators which are positive on  $S$  is invariant under Ricci flow.

In this talk we show that the class of  $Ad_{SO(n, \mathbb{C})}$ -invariant subset of  $so(n, \mathbb{C})$  is a disjoint union of two subclasses with the following properties:

- (i) Let  $S$  belong to the first class. Then the connected sum of manifolds whose curvature operators lie in  $C(S)$  also admits a metric with curvature operator in  $C(S)$ .
- (ii) If  $S$  is in the second class then the normalized Ricci flow of a manifold whose curvature operator lies in  $C(S)$  converges to a metric of constant positive sectional curvature.

13:30-14:30 **Kazuo Akutagawa** (Tohoku University, Japan)

“3-manifolds with positive flat conformal structure”

ABSTRACT: In this talk, we consider a closed 3-manifold  $M$  with flat conformal structure  $C$ . We will show that, *if the Yamabe constant of  $(M, C)$  is positive, then  $(M, C)$  is Kleinian.* This result is probably a *folk theorem*, by using Witten’s approach to positive mass theorems. Here, we will give another explicit proof of it.

14:40-15:30 **Bennett Palmer** (Idaho State University, USA)

“Anisotropic variational problems for surfaces”

ABSTRACT: We will discuss recent progress in the study of surfaces which are critical of an anisotropic surface energy. This will include results for free boundary problems and the anisotropic mean curvature flow.

15:30-16:00 **Naoya Ando** (Kumamoto University, Japan)

“Hopf’s theorem for surfaces with constant mean curvature and its generalizations”

ABSTRACT: It is well-known that if a surface with constant mean curvature in  $\mathbb{E}^3$  is homeomorphic to a sphere, then the surface is a round sphere. The same conclusion holds in the case where the surface is special Weingarten. Koiso-Palmer obtained an analogous result for surfaces with constant anisotropic mean curvature. The purpose of this talk is to introduce the outline of a proof by the speaker of Koiso-Palmer’s theorem.

## Poster Session and Presentation

Part I & II **Tatsuyoshi Hamada** (Fukuoka University, Japan)

“MathLibre: an open source project for enjoying mathematics with computer ”

ABSTRACT: MathLibre is a new open source project offering many documents and mathematical software packages. MathLibre is the direct descendant project of “KNOPPIX/Math”. Once you run the live system, you can enjoy a wonderful world of mathematical software without needing to install anything yourself. We will demonstrate how to boot and use this system.

Part I **Makoto Narita** (Okinawa National College of Technology, Japan)

“On gravitational collapse of five dimensional triaxial Bianchi-IX spacetimes with matter”

ABSTRACT: We prove that five dimensional spacetimes with matter developing from suitable asymptotically flat triaxial Bianchi-IX symmetric initial data and containing a trapped or marginally trapped three-surface necessarily possess a complete future null infinity. The past region of the null infinity is bounded to the future by a regular null hypersurface (event horizon), whose cross-sectional volume satisfies a Penrose-like inequality, relating it to the gravitational (final Bondi) mass.

Part I **Masashi Yasumoto** (Kobe University, Japan)

“Construction of discrete constant mean curvature surfaces”

ABSTRACT: In the case for smooth surfaces, any constant mean curvature surface in  $\mathbb{R}^3$  can be obtained by solving a certain differential equation, using a loop group splitting, and inserting one component of the splitting into a Sym-Bobenko formula. This recipe is called the DPW method. Also in the case for discrete surfaces, there exists a discrete analogue of the DPW method, which we briefly explain in this poster.

Part I **Yuriko Umemoto** (Osaka City University, Japan)

“On the growth functions of hyperbolic Coxeter groups”

ABSTRACT: We will talk about the growth functions of the Coxeter groups, which are known to be rational functions. In particular we will study the distributions of poles of the growth functions of simplex hyperbolic Coxeter groups.

Part I **Hassanien Samah Gaber Mohamed** (Kobe University, Japan)

“Inextensible flow of spacelike and timelike curves in de Sitter space  $S^{2,1}$ ”

ABSTRACT: In this poster we will study the motion of spacelike and timelike curves in de Sitter space  $S^{2,1}$ . The evolution equations for curvature and torsion are given as a system of partial differential equations. In addition, we will study inextensible flow of spacelike and timelike curves in de Sitter space  $S^{2,1}$ , and we will get necessary and sufficient conditions for the flows of spacelike and timelike curves to be inextensible.

Part I & II **Hisayoshi Muroya** (OCAMI, Japan)

“ $n$ -end catenoids of genus one”

ABSTRACT: An  $n$ -end catenoid is a complete minimal surface in the three-dimensional Euclidean space with finite total curvature and  $n$  catenoidal ends. We give a necessary and sufficient condition for the existence of an  $n$ -end catenoid of genus one. By using the condition, we construct several new examples.

Part I & II **Xianfeng Wang** (Nankai University, P. R. China)

“Second eigenvalue of a Jacobi operator of hypersurfaces with constant scalar curvature”

ABSTRACT: Let  $x : M \rightarrow \mathbb{S}^{n+1}(1)$  be an  $n$ -dimensional compact hypersurface with constant scalar curvature  $n(n-1)r$ ,  $r \geq 1$ , in a unit sphere  $\mathbb{S}^{n+1}(1)$ ,  $n \geq 5$  and  $J_s$  be the Jacobi operator of  $M$ . In this case, the Jacobi operator  $J_s$  is given by  $J_s = -\square - \{n(n-1)H + nHS - f_3\}$ , which is associated with the variational characterization of the hypersurfaces with constant scalar curvature in  $\mathbb{S}^{n+1}(1)$ , where  $f_3 = \sum_{j=1}^n k_j^3$ . The

spectral behavior of  $J_s$  is directly related to the instability of hypersurfaces with constant scalar curvature. In 2004, L. J. Alías, A. Brasil and L. A. M. Sousa studied the first eigenvalue of  $J_s$  of the hypersurface with constant scalar curvature  $n(n-1)$  in  $\mathbb{S}^{n+1}(1)$ ,  $n \geq 3$ . In 2008, Q.-M. Cheng studied the first eigenvalue of the Jacobi operator  $J_s$  of the hypersurface with constant scalar curvature  $n(n-1)r$ ,  $r > 1$  in  $\mathbb{S}^{n+1}(1)$ . In this paper, we study the second eigenvalue of the Jacobi operator  $J_s$  of  $M$  and give an optimal upper bound for the second eigenvalue of  $J_s$ .

This is joint work with Professor Haizhong Li.

Part I & II **Ayaka Shimizu** (OCAMI, Japan)

“Region Select—a game using knot theory”

ABSTRACT: We introduce Region Select which is a game using knot theory. In this game we consider a knot projection on a display whose crossings have “lamps” which can be turned on or off by clicking on the region bordering it. The goal of this game is to light up all of the lamps by clicking on regions. In this poster, we show that we can complete the game for any knot projection with lamps by considering “region crossing change” which is a local move on knot diagrams. This is a joint work with Akio Kawauchi and Kengo Kishimoto.

AND OTHERS