PARALLELISM OF NORMAL JACOBI OPERATOR FOR REAL HYPERSURFACES IN COMPLEX TWO-PLANE GRASSMANNIANS

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Abstract. In this talk, we introduce a notion of normal Jacobi operator $\bar{R}_N$ for hypersurfaces M in a complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ in such a way that

$$\bar{R}_N X = \bar{R}(X, N)N \in \text{End } (T_x M), \quad x \in M$$

for any tangent vector field X on M, where $\bar{R}$ and N respectively denote the Riemannian curvature tensor and a unit normal vector field of M in $G_2(\mathbb{C}^{m+2})$. The ambient space $G_2(\mathbb{C}^{m+2})$ has a remarkable geometric structure. It was known that $G_2(\mathbb{C}^{m+2})$ is the unique compact irreducible Riemannian symmetric space equipped with both a Kähler structure J and a quaternionic Kähler structure $\mathfrak{J}$. And the structure vecror field ξ, $\xi = -JN$, of a real hypersurface M in $G_2(\mathbb{C}^{m+2})$ is said to be a *Reeb vector field*. The almost contact structure vector fields $\{\xi_1, \xi_2, \xi_3\}$ are defined by $\xi_i = -J_i N$, $i = 1, 2, 3$, where $\{J_1, J_2, J_3\}$ denote a canonical local basis of quaternionic Kähler structure $\mathfrak{J}$ on $G_2(\mathbb{C}^{m+2})$. If the distributions $\mathfrak{D}$ and $\mathfrak{D}^\perp = \text{Span}\{\xi_1, \xi_2, \xi_3\}$ are invariant by the shape operator A of M, that is, $g(A\mathfrak{D}, \mathfrak{D}^\perp) = 0$, where $T_x M = \mathfrak{D} \oplus \mathfrak{D}^\perp$, $x \in M$, then we call M is $\mathfrak{D}^\perp$-invariant. The normal Jacobi operator $\bar{R}_N$ is said to be *Reeb parallel* on M if the covariant derivative of the normal Jacobi operator $\bar{R}_N$ along the direction of the Reeb vector ξ identically vanishes, that is, $\nabla_\xi \bar{R}_N = 0$.

Related to such a Reeb parallel normal Jacobi operator $\bar{R}_N$, we give a complete classification of $\mathfrak{D}^\perp$-invariant real hypersurfaces in complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ with Reeb parallel normal Jacobi operator.

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