

# Totally complex submanifolds of a complex Grassmann manifold of 2-planes

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A complex Grassmann manifold  $G_2(\mathbb{C}^{m+2})$  of all 2-dimensional complex subspaces in  $\mathbb{C}^{m+2}$  has two nice geometric structures - the Kähler structure and the quaternionic Kähler structure. We study totally complex submanifolds of  $G_2(\mathbb{C}^{m+2})$  with respect to the quaternionic Kähler structure. We show that the projective cotangent bundle  $P(T^*\mathbb{C}P^{m+1})$  of a complex projective space  $\mathbb{C}P^{m+1}$  is a twistor space of the quaternionic Kähler manifold  $G_2(\mathbb{C}^{m+2})$ . Applying the twistor theory, we construct maximal totally complex submanifolds of  $G_2(\mathbb{C}^{m+2})$  from complex submanifolds of  $\mathbb{C}P^{m+1}$ . Then we obtain many interesting examples. In particular we classify maximal homogeneous totally complex submanifolds. We show the relationship between the geometry of complex submanifolds of  $\mathbb{C}P^{m+1}$  and that of totally complex submanifolds of  $G_2(\mathbb{C}^{m+2})$ .