Homogeneous Einstein manifolds. An Overview and Recent Results

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A Riemannian manifold (M, g) is called Einstein if $\mathrm{Ric}(g) = cg$ for some $c \in \mathbb{R}$. For a homogeneous space G/H the problem is to prove existence of a G-invariant metric and if possible find all invariant Einstein metrics (up to scale and isometry). I will restrict to the case when $c > 0$ (G/H is compact) and give an overview of recent results for two major classes of homogeneous spaces. For those whose isotropy representation χ decomposes into a direct sum of irreducible and *non equivalent* subrepresentations, and those for which which χ contains *equivalent* subrepresentations. In the last case the description of G-invariant metrics is more complicated, which makes the problem of proving existence of invariant Einstein metrics more complicated. Typical examples in the first class of homogeneous spaces are the generalized flag manifolds, and in the second class the Stiefel manifolds (real, complex, or quaternionic). I will also discuss results about Einstein metrics on homogeneous spaces examples of which belong to both classes, such as generalized Wallach spaces. The case of finding left-invariant Einstein on compact Lie groups requires a special attention and we refer to M. Statha's talk about this.