

Non-naturally reductive Einstein metrics on compact simple Lie groups

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A Riemannian manifold (M, g) is called Einstein if $\text{Ric}(g) = \lambda g$ for some $\lambda \in \mathbb{R}$. For compact Lie groups, Jensen (1979) proved the existence of left-invariant Einstein metrics. Later (1979) J. E. D'Arti and W. Ziller in work *Naturally reductive metrics and Einstein metrics on compact Lie groups*, they obtained a large number of left-invariant Einstein metrics that are *naturally reductive*. They also asked whether G admits a non-naturally reductive Einstein metric. The problem of finding *non-naturally reductive* left-invariant Einstein metrics on compact simple Lie groups seems to be harder, and in fact this is stressed by the above authors. In the papers of [Mo], [ArMoSa], [ChLi], [ChSa], [YaDe], [CC], [CCD1] and [CCD2] authors found new non-naturally reductive Einstein metrics on several Lie groups.

In the present work we prove existence of new left-invariant Einstein metrics on compact Lie groups $G \in \{\text{SO}(n), \text{Sp}(m)\}$ and $\text{SU}(\ell + 3)$ for $n \geq 7$, $m \geq 3$, and $\ell \geq 2$ which are not naturally reductive. The space of metrics has been studied from the generalized Wallach spaces $G/(G_1 \times G_2 \times G_3)$ where $G_i \in \{\text{SO}(k_i), \text{Sp}(k_i)\}$, $i = 1, 2, 3$ ([ArSaSt1], [ArSaSt2]) $\text{SU}(1 + 2 + \ell)/\text{S}(\text{U}(1) \times \text{U}(2) \times \text{U}(\ell))$ and the homogeneous space $\text{SU}(\ell + 3)/(\text{U}(1) \times \text{SO}(3) \times \text{SU}(\ell))$. In the two last cases the metrics are different from Mori's results.

This presentation is based on joint works with A. Arvanitoyeorgos and Y. Sakane.

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