International Research Network Project "Symmetry, Topology and Moduli"

International Workshop
"Geometry of Submanifolds and Integrable Systems"

The 15th OCAMI-RIRCM Joint Differential Geometry Workshop &
The 3rd OCAMI-KOBE-WASEDA Joint International Workshop on Differential Geometry and Integrable Systems

Osaka City University (Building E of Faculty of Science, Lecture Room E408)
March 26th–30th, 2018

Organizers

Yoshihiro Ohnita (Osaka City University, OCAMI Director, Japan)
Martin Guest (Waseda University & Visiting Professor of OCAMI, Japan)
Young Jin Suh (Kyungpook National University & RIRCM, Korea)
Masaaki Umehara (Tokyo Institute of Technology, Japan)
Wayne Rossman (Kobe University, Japan)
Takashi Sakai (Tokyo Metropolitan University & Visiting Associate Professor of OCAMI, Japan)
Masashi Yasumoto (chair, OCAMI, Japan)

Contact

Masashi Yasumoto (yasumoto@sci.osaka-cu.ac.jp), Yoshihiro Ohnita (ohnita@sci.osaka-cu.ac.jp)

Supports

– JSPS Grant-in-Aid for Scientific Research (S), No.17H06127
Principal Investigator: Masa-Hiko Saito, 2017-2022
– JSPS Grant-in-Aid for Scientific Research (A), No.26247005
Principal Investigator: Masaaki Umehara, 2014-2019
– JSPS Grant-in-Aid for Scientific Research (C), No.15K04851
Principal Investigator: Yoshihiro Ohnita, 2015-2018
– JSPS Grant-in-Aid for Scientific Research (C), No.17K05223
Principal Investigator: Takashi Sakai, 2017-2020
– Leading Initiative for Excellent Young Researchers
Principal Investigator: Takayuki Koike,
Co-investigator: Yoshihiro Ohnita, 2018-
– Osaka City University Advanced Mathematical Institute
– Research Institute of Real and Complex Manifolds

Schedule

Time & Date	**March 26th**
12:50 – 13:10	**Registration**
13:10 – 13:20	**Opening**
13:20 – 13:50	B. Kim
14:00 – 14:50	Yasumoto
15:00 – 15:30	Tadano
15:30 – 16:10	**Coffee Break**
16:10 – 16:40	M. Morimoto
16:40 – 17:10	G. Kim
17:20 – 17:50	Yamamoto

Time & Date	**March 27th**	**March 28th**	**March 29th**	**March 30th**
9:45 – 10:35	Suh	Dorfmeister	Pedit	Leschke
10:35 – 11:05	**Coffee Break**			
11:05 – 11:55	Sanmartín López	Klein	Lam	Heller
11:55 – 13:30	**Lunch**			
13:30 – 14:20	Sakai	Kajigaya	S. Kobayashi	Moriya
14:30 – 15:00	Lee	Kikuchi	Imada	Ohnita
15:05 – 15:35	Szewieczek	Ogata	Mase	(14:35 – 15:25)
15:35 – 16:05	**Coffee Break**			**Closing**
16:05 – 16:55	Tamaru	Koiso	Umehara	**(15:25 – 15:35)**
17:10 – 18:00	O. Kobayashi	T. Morimoto	Udagawa	
18:30 –		**Party**		

List of Speakers

Plenary Talks

Josef Dorfmeister (Technische Universität München, Germany)
Lynn Heller (Leibniz Universität Hannover, Germany)
Toru Kajigaya (AIST & Tohoku University, Japan)
Sebastian Klein (Universität Mannheim, Germany)
Osamu Kobayashi (Osaka City University Advanced Mathematical Institute, Japan)
Shimpei Kobayashi (Hokkaido University, Japan)
Miyuki Koiso (IMI, Kyushu University, Japan)
Wai Yeung Lam (Brown University, USA)
Katrin Leschke (University of Leicester, United Kingdom)
Tohru Morimoto (Seki Kowa Institute of Mathematics & Oka Mathematical Institute, Japan)
Katsuhiro Moriya (University of Tsukuba, Japan)
Yoshihiro Ohnita (Osaka City University, Japan)
Franz Pedit (University of Massachusetts Amherst, USA)
Takashi Sakai (Tokyo Metropolitan University, Japan)
Victor Sanmartín López (Universidade de Satiago de Compostela, Spain)
Young Jin Suh (Kyungpook National University, Korea)
Hiroshi Tamaru (Hiroshima University, Japan)
Seiichi Udagawa (Nihon University, Japan)
Masaaki Umehara (Tokyo Institute of Technology, Japan)
Masashi Yasumoto (Osaka City University Advanced Mathematical Institute, Japan)

Short Communications

Mitsuhiro Imada (National Institute of Technology, Ibaraki College, Japan)
Keiichi Kikuchi (Tokai University, Japan)
Byung Hak Kim (Kyung Hee University, Korea)
Gyu Jong Kim (RIRCM, Kyungpook National University, Korea)
Hyunjin Lee (RIRCM, Kyungpook National University, Korea)
Makiko Mase (Tokyo Metropolitan University & OCAMI, Japan)
Masahiro Morimoto (Osaka City University, Japan)
Yuta Ogata (Okinawa College, Japan)
Gudrun Szewieczek (Technische Universität Wien & Kobe University, Austria & Japan)
Homare Tadano (Tokyo University of Science, Japan)
Hikaru Yamamoto (Tokyo University of Science, Japan)

International Research Network Project "Symmetry, Topology and Moduli"

International Workshop
"Geometry of Submanifolds and Integrable Systems"

The 15th OCAMI-RIRCM Joint Differential Geometry Workshop &
The 3rd OCAMI-KOBE-WASEDA Joint International Workshop on Differential Geometry and Integrable Systems

Osaka City University (Building E of Faculty of Science, Lecture Room E408)
March 26th–30th, 2018

Program

March 26th, 2018

12:50-13:10	**Registration**
13:10-13:20	**Opening (Masashi Yasumoto & Yoshihiro Ohnita)**
13:20-13:50	Byung Hak Kim (Kyung Hee University) The Ricci soliton and Yamabe soliton in the warped product
14:00-14:50	Masashi Yasumoto (Osaka City University Advanced Mathematical Institute) Discrete timelike minimal surfaces
15:00-15:30	Homare Tadano (Tokyo University of Science) Some Myers-Type Theorems for Ricci Solitons
15:30-16:10	**Coffee Break**
16:10-16:40	Masahiro Morimoto (Osaka City University) The spectral properties of transversally elliptic operators and some applications
16:40-17:10	Imsoon Jeong, Gyu Jong Kim* (RIRCM, Kyungpook National University), Young Jin Suh Real hypersurfaces In complex quadric with Codazzi type-normal Jacobi operator
17:20-17:50	Hikaru Yamamoto (Tokyo University of Science) On correspondence between special Lagrangian submanifolds and deformed Hermitian Yang-Mills connections

March 27th, 2018

9:45-10:35	Young Jin Suh (Kyungpook National University) Isometric Reeb flow and contact hypersurfaces in Hermitian symmeric spaces
10:35-11:05	**Coffee Break**

11:05-11:55	Victor Sanmartín López (Universidade de Satiago de Compostela) Homogeneous and isoparametric hypersurfaces in complex hyperbolic spaces
13:30-14:20	Takashi Sakai (Tokyo Metropolitan University) The intersection of two real flag manifolds in a complex flag manifold
14:30-15:00	Hyunjin Lee* (RIRCM, Kyungpook National Universtiy), Young Jin Suh Hopf real hypersurfaces with cyclic parallel shape operator in complex Grassmannians with rank 2
15:05-15:35	Gudrun Szewieczek (Technische Universität Wien & Kobe University) Combescure transformations of Guichard nets
15:35-16:05	**Coffee Break**
16:05-16:55	Hiroshi Tamaru (Hiroshima University) Left-invariant pseudo-Riemannian metrics on some Lie groups
17:10-18:00	Osamu Kobayashi (Osaka City University Advanced Mathematical Institute) Metrical mass of a Riemannian manifold

March 28th, 2018

9:45-10:35	Josef Dorfmeister (Technische Universität München) Willmore surfaces in spheres and harmonic maps
10:35-11:05	**Coffee Break**
11:05-11:55	Sebastian Klein (Universität Mannheim) Constant mean curvature surfaces and simply-periodic solutions of the sinh-Gordon equation by asymptotic methods
13:30-14:20	Toru Kajigaya (AIST & Tohoku University) On Hamiltonian stable Lagrangian tori in complex space forms
14:30-15:00	Keiichi Kikuchi (Tokai University) Extended harmonic maps and Lagrange equations
15:05-15:35	Yuta Ogata (Okinawa College) Successive Bianchi-Bäcklund transformations with single spectral parameter
15:35-16:05	**Coffee Break**

16:05-16:55	Miyuki Koiso (IMI, Kyushu University) Uniqueness problem for closed non-smooth hypersurfaces with constant anisotropic mean curvature
17:10-18:00	Tohru Morimoto (Seki Kowa Institute of Mathematics & Oka Mathematical Institute) Extrinsic geometries and involutive linear differential equations
18:30-	**Party**

March 29th, 2018

9:45-10:35	Franz Pedit (University of Massachusetts Amherst) Gradient flows of geometric variational problems: the elastic curve flow
10:35-11:05	**Coffee Break**
11:05-11:55	Wai Yeung Lam (Brown University) Discrete holomorphic quadratic differentials in dynamical systems
13:30-14:20	Shimpei Kobayashi (Hokkaido University) Loop group method for constant Gaussian curvature surfaces in the 3-dimensional hyperbolic space
14:30-15:00	Mitsuhiro Imada (National Institute of Technology, Ibaraki College) Complex almost contact metric structures on complex hypersurfaces in hyperkähler manifolds
15:05-15:35	Makiko Mase (Tokyo Metropolitan University & OCAMI) On correspondences among families of K3 surfaces
15:35-16:05	**Coffee Break**
16:05-16:55	Masaaki Umehara (Tokyo Institute of Technology) Surfaces with light-like points in the Lorentz-Minkowski 3-space
17:10-18:00	Seiichi Udagawa (Nihon University) Finite gap solutions for horizontal minimal surfaces of finite type in 5-sphere

March 30th, 2018

9:45-10:35	Katrin Leschke (University of Leicester) TBA
10:35-11:05	**Coffee Break**

11:05-11:55	Lynn Heller (Leibniz Universität Hannover) Recent progress in integrable surface theory
13:30-14:20	Katsuhiro Moriya (University of Tsukuba) Parametrizations of Lagrangian surfaces in the complex Euclidean plane
14:35-15:25	Yoshihiro Ohnita (Osaka City University) TBA
15:25-15:35	**Closing**

International Research Network Project "Symmetry, Topology and Moduli"

International Workshop
"Geometry of Submanifolds and Integrable Systems"

The 15th OCAMI-RIRCM Joint Differential Geometry Workshop &
The 3rd OCAMI-KOBE-WASEDA Joint International Workshop on Differential Geometry and Integrable Systems

Osaka City University (Building E of Faculty of Science, Lecture Room E408)
March 26th–30th, 2018

Titles & Abstracts

March 26th, 2018

Byung Hak Kim (Kyung Hee University)
Title: The Ricci soliton and Yamabe soliton in the warped product

Abstract

In this talk, we introduce and study special solutions, the so called Ricci and Yamabe solitons. It is well known that the Ricci soliton and Yamabe soliton are deeply related with Ricci flow and Yamabe flow, which was introduced by R. Hamilton. It is very interesting to investigate the role of Ricci and scalar curvature in the Ricci and Yamabe solitons respectively. Hence it is meaningful to talk about the recent results for this topics and suggest open problems related to these solitons.

References

[1] H-D Cao, Recent progress on Ricci solitons, Recent advances in geometric analysis, 1-38 Adv. Lect. Math. (ALM), 11 Int Press, Somerville, MA, 2010.

[2] L. Ma, V. Miquel, Remarks on scalar curvature of Yamabe solitons, Ann. Glob. Anal. Geom.42 (2012), 195-205

[3] J-Y Wu, On a class of complete non-compact gradient Yamabe solitons, arxiv:1109.0861v1 math. DG

Masashi Yasumoto (Osaka City University Advanced Mathematical Institute)
Title: Discrete timelike minimal surfaces

Abstract

In the smooth case, the study of timelike minimal surfaces is highly related to linear and nonlinear wave equation. In fact, each coordinate function of a timelike minimal surface solves a 1D wave equation, and the metric function of a timelike minimal surface gives a solution of a nonlinear wave equation. In this

talk we will describe a theory of discrete timelike minimal surfaces in Minkowski 3-space. First we will introduce a Weierstrass-type representation for them. And we will show that each coordinate function of a discrete timelike minimal surface satisfies a discrete version of the 1D wave equation. This result can be regarded as a "reparametrization" of discrete timelike minimal surfaces, providing another Weierstrass-type representation. Finally, we will discuss the convergence of discrete timelike minimal surfaces to the smooth counterparts. Unlike the case of discrete minimal surfaces in the Euclidean 3-space, their convergence can be directly discussed using a discrete version of discrete Dirac operators.

Homare Tadano (Tokyo University of Science, Japan)

Title: Some Myers-Type Theorems for Ricci Solitons

Abstract

An important problem in Riemannian geometry is to investigate the relation between topology and geometric structure on Riemannian manifolds. The celebrated theorem of S. B. Myers guarantees the compactness of a complete Riemannian manifold under some positive lower bound on the Ricci curvature. This theorem may be considered as a topological obstruction for a complete Riemannian manifold to have a positive lower bound on the Ricci curvature. On the other hand, J. Lohkamp proved that in dimension at least three, any manifold admits a complete Riemannian metric of negative Ricci curvature. Hence, in dimension at least three, there are no topological obstructions to the existence of a complete Riemannian metric of negative Ricci curvature. To give an interesting compactness criterion for complete Riemannian manifolds is one of the most important problems in Riemannian geometry, and the Myers theorem has been widely generalized in various directions by many authors.

The aim of this talk is to discuss the compactness of complete Ricci solitons. Ricci solitons were introduced by R. Hamilton in 1982 and are natural generalizations of Einstein manifolds. They correspond to self-similar solutions to the Ricci flow and often arise as singularity models of the flow. The importance of Ricci solitons was demonstrated by G. Perelman, where Ricci solitons played crucial roles in his affirmative resolution of the Poincaré conjecture. In this talk, after reviewing basic facts about Ricci solitons, we shall establish some new compactness theorems for complete Ricci solitons. Our results may be regarded as natural generalizations of the compactness theorem due to J. Cheeger, M. Gromov, and M. Taylor and improve previous compactness theorems obtained by M. Fernández-López and E. García-Río, M. Limoncu, Z. Qian, and G. Wei and W. Wylie.

Masahiro Morimoto (Osaka City University)

Title: The spectral properties of transversally elliptic operators and some applications

Abstract

Let G be a connected compact Lie group. A G-invariant differential operator on a compact G-manifold is said to be transversally elliptic if it is elliptic in the directions transversal to the G-orbits. In this talk, we first review the spectral properties of a transversally elliptic operator. After that we define its heat operator and investigate its character, that is a distribution on G generalizing the trace of the heat operator to the G-equivariant case.

Imsoon Jeong, Gyu Jong Kim* (RIRCM, Kyungpook National University), Young Jin Suh

Title: Real hypersurfaces In complex quadric with Codazzi type-normal Jacobi operator

Abstract

We introduce the notion of normal Jacobi operator of Codazzi type of real hypersurfaces in the complex quadric. This implies the unit normal vector field must be $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic. Taking account of each case, we give a non-existence of real hypersurfaces in complex quadric with normal Jacobi operator of Codazzi type.

Hikaru Yamamoto (Tokyo University of Science)

Title: On correspondence between special Lagrangian submanifolds and deformed Hermitian Yang-Mills connections

Abstract

In 2000, Leung, Yau and Zaslow introduced the notion of deformed Hermitian Yang-Mills connections (dHYM) for connections on a complex line bundle over a Kahler manifold with a Hermitian metric, and they proved that it naturally appears as mirror objects of special Lagrangian submanifolds (sLag) via Fourier-Mukai transform between dual torus fibrations. In their paper, some conditions were imposed for simplicity. In this talk, I will reformulate their results in some more general situations.

March 27th, 2018

Young Jin Suh (Kyungpook National University)

Title: Isometric Reeb flow and contact hypersurfaces in Hermitian symmeric spaces

Abstract

In this talk, first we introduce some notions of isometric Reeb flow and contact for real hypersurfaces in Kaehler manifolds, in particular, complex two-plane Grassamnnians $G_2(\mathbb{C}^{m+2})$ and complex quadrics Q^m, and give some back grounds for the study of such hypersurfaces until now.

Next, by using Lie algebraic method and root systems, we give a new classification of real hypersurfaces with isometric Reeb flow in Hermatian symmetric spaces including $G_2(\mathbb{C}^{m+2})$ and Q^m and give some conjectures for the classification of contact hypersurfaces in Hermitian symmetric spaces.

Victor Sanmartín López (Universidade de Satiago de Compostela & Hiroshima University)

Title: Homogeneous and isoparametric hypersurfaces in complex hyperbolic spaces

Abstract

In this talk, we will focus on isoparametric families of hypersurfaces in the complex hyperbolic space. First, we will introduce and study Hopf and homogeneous examples of hypersurfaces. After that, we will present inhomogeneous examples of isoparametric hypersurfaces. Finally, using their relation with some families of isoparametric hypersurfaces in the anti De Sitter space we deduce a classification in the complex hyperbolic space.

Takashi Sakai (Tokyo Metropolitan University)

Title: The intersection of two real flag manifolds in a complex flag manifold

Abstract

An orbit of the adjoint representation of a compact connected Lie group G admits a G-invariant Kahler structure, and is called a complex flag manifold. In this talk, we define an antipodal set of a complex flag manifold using a torus action, and show that a maximal antipodal set is characterized as an orbit of the Weyl group of G. An orbit of the linear isotropy representation of a Riemannian symmetric pair is embedded in a complex flag manifold as a real form, and is called a real flag manifold. We give a necessary and sufficient condition for two real flag manifolds, which are not necessarily congruent, in a complex flag manifold to intersect transversally in terms of the symmetric triad. Then we show that the intersection of two real flag manifolds is antipodal. As an application, we calculate the Z_2-Lagrangian Floer homology of two real flag manifolds in a complex flag manifold with a Kahler-Einstein metric. The Floer homology is generated by the intersection of two real flag manifolds. This talk is a joint work with O. Ikawa, H. Iriyeh, T. Okuda and H. Tasaki.

Hyunjin Lee* (RIRCM, Kyungpook National Universtiy), Young Jin Suh

Title: Hopf real hypersurfaces with cyclic parallel shape operator in complex Grassmannians with rank 2

Abstract

A main objective in submanifold geometry is the classification of homogeneous hypersurfaces arise as principal orbits of cohmogeneity one actions, and so their classification is equivalent to the classification of cohomogeneity one actions up to orbit equivalence. Actually, the classifications of cohomogeneity one actions in irreducible simply connceted Riemannian symmetric spaces of rank 2 of (non)compact type was obtained by J. Berndt and Y.J. Suh for complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2}) = SU_{2+m}/S(U_2U_m)$ and complex hyperbolic two-plane Grassmannians $G_2^*(\mathbb{C}^{m+2}) = SU_{2,m}/S(U_2U_m)$ (see [1] and [2]).
From these classifications, in [3] Suh classified real hypersurfaces with isometric Reeb flow in $SU_{2,m}/S(U_2U_m)$, $m \geq 3$. Each one can be described as a tube over a totally geodesic $SU_{2,m-1}/S(U_2U_{m-1})$ in $SU_{2,m}/S(U_2U_m)$ or a horosphere whose center at infinity is singular. By using this result, we want to give another characterization for these model spaces with respect to the derivatives of (1,1) type tensor on real hypersurfaces in complex Grassmannians with rank two.

References

[1] J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatsh. Math. **127** (1999), no. 1, 1-14.

[2] J. Berndt and Y.J. Suh, *Hypersurfaces in noncompact complex Grassmannians of rank two*, Internat. J. Math. **23** (2012), no. 10, 1250103(35 pages).

[3] Y.J. Suh, *Hypersurfaces with isometric Reeb flow in complex hyperbolic two-plane Grassmannians*, Adv. Appl. Math. **50** (2013), 645-659.

Gudrun Szewieczek (Technische Universität Wien & Kobe University)

Title: Combescure transformations of Guichard nets

Abstract

The existence of particular Combescure transformations is widely used to characterize different classes of integrable surfaces, e. g. the Christoffel transformation for isothermic surfaces. In this talk we investigate this concept in the realm of triply orthogonal systems and discuss the subclass of Guichard nets - systems with a metric fulfilling a zero trace condition. We define a 1-parameter family of associated systems, whose existence characterizes Guichard nets and examine geometric relations between them.

Hiroshi Tamaru (Hiroshima University)

Title: Left-invariant pseudo-Riemannian metrics on some Lie groups

Abstract

For a given Lie group, the set of all left-invariant pseudo-Riemannian metrics with a fixed signature can be identified with some pseudo-Riemannian symmetric space. We are studying left-invariant pseudo-Riemannian metrics, in terms of the action of the automorphism group of the corresponding Lie algebra on this symmetric space. In this talk, we illustrate this framework, and apply it to some particular solvable Lie groups, e.g., the solvable Lie group of the real hyperbolic space.

Osamu Kobayashi (Osaka City University Advanced Mathematical Institute)

Title: Metrical mass of a Riemannian manifold

Abstract

The Arnowitt-Deser-Misner mass of an asymptotically flat space is an important quantity in the general theory of relativity and is relevant to Riemannian geometry of scalar curvature. In this talk I would like to explain a formula of curvature integral and examine how it looks like mass through examples by Schwarzshild construction. Also I would like to touch upon the Willmore integral of hypersurfaces which is helpful to understand the cases where negative mass appears.

March 28th, 2018

Josef Dorfmeister (Technische Universität München)

Title: Willmore surfaces in spheres and harmonic maps

Abstract

In this talk we will introduce the notion of a Willmore surface in a Euclidean sphere, and we will discuss the relation between special conformally harmonic maps into real Grassmannians and such surfaces. This combines the notion of the "Ruh-Vilms property" of surface classes with the "Hoffmann-Osserman property" of maps into Grassmannians.

Time permitting, we will present a method for the construction of all Willmore surfaces in spheres and point out special subclasses.

Sebastian Klein (Universität Mannheim)

Title: Constant mean curvature surfaces and simply-periodic solutions of the sinh-Gordon equation by asymptotic methods

Abstract

Let M be an immersed surface of constant mean curvature (CMC) H in a 3-dimensional space form $\mathbb{E}^3(\varkappa)$. If $H^2 > -\varkappa$ holds, then there exists near every non-umbilic point of the surface a conformal coordinate z so that the Gauss equation for the induced metric $g = e^u \, \mathrm{d}z \, \mathrm{d}\overline{z}$ of the immersion reduces to the sinh-Gordon equation

$$\triangle u + \sinh(u) = 0$$

for the conformal factor u. Therefore CMC surfaces are at the interface of two classical branches of mathematics: On one hand, they are objects of surface theory and therefore of differential geometry. On the other hand, they give rise to solutions of certain partial differential equations, and these equations played a significant role in the development of the theory of non-linear elliptic partial differential equations.

In this setting CMC tori induce doubly periodic solutions u. Such solutions have been classified by Pinkall/Sterling [4], and independently by Hitchin [1]. With Hitchin's method, a new theme enters, involving complex analysis: He associated to each u so-called *spectral data* (Σ, D). Here the *spectral curve* Σ is a Riemann surface (or a complex curve), and the *spectral divisor* D is a divisor on Σ. It was an important step in Hitchin's classification to show that for doubly periodic u, the spectral curve can be compactified by adding one point each above $\lambda = 0$ and $\lambda = \infty$. Because of this result, Hitchin could apply classical results on compact Riemann surfaces to complete the classification.

In the present talk I will discuss how similar methods can be applied to solutions u which are only simply periodic. Such solutions correspond to a far larger class of CMC surfaces, for example the Lawson surfaces (a family of compact, immersed CMC surfaces of genus $g \geq 2$ with discrete symmetries) are included.

It turns out that Hitchin's construction of the spectral data (Σ, D) transfers to the situation of simply periodic u. However, then the spectral curve Σ generally has infinite genus, and the set of its branch points has two accumulation points: $\lambda = 0$ and $\lambda = \infty$. Therefore it cannot be compactified (unlike in Hitchin's situation). In the talk I will present methods by which to replace the classical results on compact Riemann surfaces that Hitchin had used in his classification. These methods are based on *asymptotic analysis*. In particular it can be shown that the spectral data (Σ, D) for any simply periodic solution u asymptotically approximate (in a precisely quantifiable way) near $\lambda = 0$ and $\lambda = \infty$ the spectral data of the "vacuum solution" $u = 0$.

From the asymptotic results one obtains the important result that a simply periodic solution u is uniquely determined by its spectral data (Σ, D), and I will describe how u can be reconstructed from (Σ, D). Finally I will discuss which pairs (Σ, D) composed of a hyperelliptic spectral curve Σ and a divisor D on Σ occur as spectral data of a simply periodic solution u. In this way, essentially a description of the space of simply periodic solutions of the sinh-Gordon equation, and correspondingly of the immersed CMC surfaces in $\mathbb{E}^3(-\varkappa)$ without umbilic points which are topologically cylinders is obtained.

The results on simply periodic solutions of the sinh-Gordon equation presented here have been obtained by me in my thesis of habilitation [2], which is summarized in [3].

References

[1] N. J. Hitchin, *Harmonic maps from a 2-torus to the 3-sphere*, J. Diff. Geom. **31** (1990), 627–710.

[2] S. Klein, *A spectral theory for simply periodic solutions of the sinh-Gordon equation*, submitted for publication, `arXiv:1607.08792` .

[3] S. Klein, *Spectral data for simply periodic solutions of the sinh-Gordon equation*, Differ. Geom. Appl. **54** (2017), part A, 129–149.

[4] U. Pinkall and I. Sterling, *On the classification of constant mean curvature tori*, Ann. Math. **130** (1989), 407–451.

Toru Kajigaya (AIST & Tohoku University)

Title: On Hamiltonian stable Lagrangian tori in complex space forms

Abstract

The notion of Hamiltonian stability for Lagrangian submanifold in a Kähler manifold was introduced by Y.-G.Oh in 90's, and is of particular interest in the contexts of Hamiltonian volume minimizing problem and Lagrangian mean curvature flow in a Fano manifold. In this talk, I will briefly introduce the notion and the stability criterion, and describe the complete classification of Hamiltonian-stable Lagrangian torus orbits in complex space form M based on the result of Oh (when $M = C^n$), H. Ono (when $M = CP^n$) and my recent work (when $M = CH^n$). Furthermore, I will mention some problems for the Hamiltonian volume minimizing property of the torus orbits.

Keiichi Kikuchi (Tokai University)

Title: Extended harmonic maps and Lagrange equations

Abstract

In my talk, it is described that a conformal and extended harmonic map between connected Riemannian manifolds is homothetic by means of multi-Hamiltonians, via the method of analytical mechanics. Also, it is explained that a conformal and extended harmonic map of a connected, compact Riemannian manifold without boundary to a connected Riemannian manifold is harmonic under a certain condition, by making use of the divergences of stress-energy tensors. Finally, I talk on an example of extended harmonic CMC immersion from the viewpoint of Lagrange formalism.

Yuta Ogata (Okinawa College)

Title: Successive Bianchi-Bäcklund transformations with single spectral parameter

Abstract

Bianchi-Bäcklund transformations of CMC surfaces in the 3-dimensional Euclidean space have been studied by many researchers from various viewpoints including transformation theory, soliton theory and DPW theory. In this talk, we investigate successive Bianchi-Bäcklund transformations with single spectral parameter. We also show some geometric properties of them. This talk is based on the joint work with Joseph Cho in Kobe University.

Miyuki Koiso (IMI, Kyushu University)

Title: Uniqueness problem for closed non-smooth hypersurfaces with constant anisotropic mean curvature

Abstract

An anisotropic surface energy is the integral of an energy density that depends on the surface normal over the considered surface in the 3-dimensional Euclidean space, which was introduced to model the surface tension of a small crystal. The minimizer of such an energy among all closed surfaces enclosing the same volume is unique and it is (up to rescaling) so-called the Wulff shape. The Wulff shape and equilibrium surfaces of this energy for volume-preserving variations are generalizations of the round sphere and constant mean curvature surfaces, respectively. However, they are not smooth in general. In this talk, we show that, if the energy density function is three times continuously differentiable and convex, then any closed stable equilibrium surface is a rescaling of the Wulff shape. Moreover, we show that, for some non- convex energy density functions, there exist closed embedded equilibrium surfaces with genus zero which are not (any homothety of) the Wulff shape. This gives also the non-uniqueness result of closed embedded self -similar shrinking solutions with genus zero of the anisotropic mean curvature flow, which makes a striking contrast to the fact that only spheres are closed embedded self-similar shrinking solutions with genus zero of the mean curvature flow. These concepts and results are naturally generalized to higher dimensions.

Tohru Morimoto (Seki Kowa Institute of Mathematics & Oka Mathematical Institute)

Title: Extrinsic geometries and involutive linear differential equations

Abstract

In this talk I wish to discuss extrinsic geometry of immersions

$$\varphi : (M, \mathfrak{f}) \to L/L^0 \subset Flag(V)$$

and the corresponding L/L^0-category of involutive systems of linear differential equations, where V is a vector space (of finite dimension), L a Lie subgroup of $GL(V)$. L^0 a closed subgroup of L, and $(M, \mathfrak{f})$ a filtered manifold, that is, a differentiable manifold equipped with a tangential filtaration.

Based on our joint work with B. Doubrov and Y. Machida, I will show a general method to find the complete invariants of the immersions φ as well as those of corresponding differential equations.

I wish to mension also classifications of transitive structures, twistor problems and generalizations of Gauss' theorem egregium after our detailed studies in the concrete case of

$$\varphi : (M^3, \mathfrak{f}) \to SO(5,3)/SO(5,3)^0 \subset Flag(V),$$

in relation to the adjoint representation of $\mathfrak{sl}(3, \mathbf{R})$, where the source space $(M^3, \mathfrak{f})$ is a 3-dimensional contact manifold and the target space is an 11-dimensional homogeneous space $SO(5,3)/SO(5,3)^0$.

March 29th, 2018

Franz Pedit (University of Massachusetts Amherst)

Title: Gradient flows of geometric variational problems: the elastic curve flow

Abstract

When considering a variational problem, one is interested in its minimizers, and more generally its critical points given by the Euler-Lagrange equation. The critical points are typically special geometries – Einstein metrics, Yang-Mills fields, harmonic maps, elastic membranes etc. – and one would like to flow a generic initial geometry to a terminal special geometry. This is usually accomplished by considering the gradient flow of the functional at hand. For instance, Riemann used heat flow to minimize the Dirichlet energy, thereby showing the existence of harmonic functions; the Ricci flow limits to Einstein metrics, which was eventually used to solve the Poincare conjecture. The definition of the gradient requires a choice of (Riemannian) metric on the domain of the functional, which to a large extent determines the analytical properties of the gradient flow.

In this lecture, we will discuss these issues in the simplest example (understandable to students at any level) of closed planar curves under the influence of the elastic energy, the average squared curvature of the curve. The critical points are the so-called elastica found by Leonard Euler in a beautiful paper of 1744. The usual gradient flow with respect to the variational L^2 metric is a 4th order non-linear parabolic PDE requiring hard analysis. We show that by using a geometrically defined metric, the gradient flow becomes a proper ODE on a Hilbert manifold, whose longtime existence and convergence to elastica requires nothing more than elementary ODE theory. We will intersperse the talk with numerical experiments carried out via this approach.

Wai Yeung Lam (Brown University)

Title: Discrete holomorphic quadratic differentials in dynamical systems

Abstract

We introduce holomorphic quadratic differentials on graphs. They are the derivative of cross ratios analogous to the Schwarzian derivative. On one hand, they connect discrete harmonic functions, circle packings and Luo's vertex scaling in discrete complex analysis. On the other hand, they unify discrete minimal surfaces via a Weierstass representation formula.

In this talk, we will show how they appear in dynamical systems like the bicycle transformation and the pentagram map. Particularly they play an important role in Miquel dynamics on circle patterns, which is related to Goncharov and Kenyon's cluster integrable systems for dimer models.

Shimpei Kobayashi (Hokkaido University)

Title: Loop group method for constant Gaussian curvature surfaces in the 3-dimensional hyperbolic space

Abstract

This talk is based on a joint work with Jun-ichi Inoguchi. It is known that there is no complete surface in the 3-dimensional hyperbolic space H^3 with constant Gaussian curvature (CGC) $K < -1$, and the only complete CGC $K \geq 0$ surfaces in H^3 are totally umbilic spheres or equidistant surfaces from geodesics. On the other hand, there are many complete CGC $-1 \leq K < 0$ surfaces in H^3, (e.g. J. Spruck and H. Rosenberg). In this talk we characterize a surface with CGC $-1 < K < 0$ (resp. $K > 0$) by a harmonic map into the hyperbolic plane (resp. 2-sphere), which is given by a spectral-parametric deformation of a natural harmonic Gauss map of the surface. As an application, we show the existence of equivariant (weakly) complete CGC $-1 < K < 0$ surfaces in H^3. Our works completes loop group methods for all the classes of constant curvature surfaces in 3-dimensional space forms.

Mitsuhiro Imada (National Institute of Technology, Ibaraki College)

Title: Complex almost contact metric structures on complex hypersurfaces in hyperkähler manifolds

Abstract

The notion of complex contact geometry is an analogy of that of (real) contact geometry. Tashiro showed that every smooth orientable real hypersurface of an almost complex manifold has an almost contact structure. As a similar result, we find that we can construct a complex almost contact metric structure on every complex hypersurface in a hyperkähler manifold.

Makiko Mase (Tokyo Metropolitan University & OCAMI)

Title: On correspondences among families of K3 surfaces

Abstract

I will review the study in algebraic geometry I have done so far. The first part will be dedicated to explaining a birational correspondence among families of K3 surfaces that are general anticanonical sections of Fano 3-folds. In the second and last part, after introducing K3 surfaces associated to singularities, will be devoted to explaining dualities among families of K3 surfaces.

Masaaki Umehara (Tokyo Institute of Technology)

Title: Surfaces with light-like points in the Lorentz-Minkowski 3-space

Abstract

This is a joint work with Kotaro Yamada. With several concrete examples of zero mean curvature surfaces in the Lorentz-Minkowski 3-space R^3_1 containing a light-like line recently having been found, here we construct all real analytic germs of zero mean curvature surfaces by applying the Cauchy-Kovalevski theorem for partial differential equations.

We next consider an immersion $F : U(\subset R^2) \to R^3_1$, where U is a domain of R^2. A point $o \in U$ is called a *light-like point* if the first fundamental form ds^2 of F is degenerate at o. We denote by B_F the determinant function of the symmetric matrix associated to ds^2 with respect to the canonical coordinate system of U. A light-like point o is said to be *degenerate* if the gradient vector of B_F vanishes at o. We show that if o is a degenerate light-like point, then the image of F contains a light-like line segment of R^3_1 passing through $f(o)$, under the assumption that the mean curvature form is smoothly extended at o. This explains why several known examples of zero mean curvature surface in R^3_1 often contain light-like lines on their set of light-like points. Several related results are also given.

Seiichi Udagawa (Nihon University)

Title: Finite gap solutions for horizontal minimal surfaces of finite type in 5-sphere

Abstract

We consider a horizontal minimal surface in 5-sphere $s_0 : M \longrightarrow S^5 \subset \mathbf{C}^3$. The Gauss, Codazzi and Ricci equations for s_0 becomes a unified equation called "Tzitéica equation"of elliptic type as follows.

$$\partial_{\overline{z}}\partial_z \mathfrak{u} = e^{-2\mathfrak{u}} - e^{\mathfrak{u}}, \tag{1}$$

where $z = x + \sqrt{-1}y$ a local coordinate system for a Riemann surface M and $\mathfrak{u} = \mathfrak{u}(z, \overline{z})$ is a real valued function on M. The induced metric g on M is given by $g = 2e^{\mathfrak{u}} dz d\overline{z}$. Let $< , >$ be a Hermitian fibre metric on $M \times \mathbf{C}^3$ compatible with g. Since s_0 is a minimal isometric immersion, we have

$$< s_0, s_0 >= 1, \quad \partial_{\overline{z}}\partial_z s_0 = -e^u s_0. \tag{2}$$

The horizontality of s_0 with respect to the Hopf fibration $S^5 \longrightarrow \mathbf{C}P^2$ means that

$$< \partial_z s_0, s_0 >= 0, \quad < \partial_{\overline{z}} s_0, s_0 >= 0. \tag{3}$$

We also have $< s_0, \partial_{\overline{z}} s_0 >= 0, < s_0, \partial_z s_0 >= 0$. The conformality of s_0 means that

$$< \partial_z s_0, \partial_{\overline{z}} s_0 >= 0, \quad < \partial_{\overline{z}} s_0, \partial_z s_0 >= 0. \tag{4}$$

We set

$$s_1 = e^{-\frac{\mathfrak{u}}{2}} \partial_z s_0, \quad s_2 = e^{-\frac{\mathfrak{u}}{2}} \partial_{\overline{z}} s_0, \quad \phi = e^{\frac{\mathfrak{u}}{2}} < \partial_z \partial_z s_0, s_2 > . \tag{5}$$

It then follows from (2), (3), (4) and (5) that $F = (s_0\ s_1\ s_2)$ is a unitary frame on $M \times \mathbf{C}^3$. We have

$$F^{-1}\partial_z F = \begin{pmatrix} 0 & 0 & -e^{\frac{\mathfrak{u}}{2}} \\ e^{\frac{\mathfrak{u}}{2}} & \frac{\mathfrak{u}_z}{2} & 0 \\ 0 & \phi e^{-\mathfrak{u}} & -\frac{\mathfrak{u}_z}{2} \end{pmatrix}, \quad F^{-1}\partial_{\overline{z}} F = \begin{pmatrix} 0 & -e^{\frac{\mathfrak{u}}{2}} & 0 \\ 0 & -\frac{\mathfrak{u}_{\overline{z}}}{2} & -\overline{\phi} e^{-\mathfrak{u}} \\ e^{\frac{\mathfrak{u}}{2}} & 0 & \frac{\mathfrak{u}_{\overline{z}}}{2} \end{pmatrix}, \tag{6}$$

where we have set $\mathfrak{u}_z = \partial_z \mathfrak{u}, \mathfrak{u}_{\overline{z}} = \partial_{\overline{z}} \mathfrak{u}$. If we set $U = F^{-1}\partial_z F, V = F^{-1}\partial_{\overline{z}} F$, then the compatibility condition for (6) is given by $\partial_{\overline{z}} U - \partial_z V = [U, V]$, which is equivalent to the following equations.

$$\begin{cases} \partial_{\overline{z}}\partial_z \mathfrak{u} & = |\phi|^2 e^{-2\mathfrak{u}} - e^{\mathfrak{u}}, \\ \partial_{\overline{z}} \phi & = 0. \end{cases}$$

Changing the local complex coordinate apropriately, we may assume that $\phi = -1$. We then obtain the Tzitzéica equation stated in (1). This is a special case of the famous Toda equation in the theory of the integrable systems. We first give an explicit solution of the Tzitzéica equation in terms of the Jacobi elliptic function. Secondly, we express the solution in terms of the Riemann theta function, which is so-called a finite gap solution. Moreover, some examples of horizontal minimal surfaces in 5-sphere can be described in terms of the Jacobi elliptic functions, which are also described explicitly in terms of the Baker-Akhiezer function. In this work, we give a spectal curve explicitly, which is a hyperelliptic curve of genus 2 and given in the affine coordinate by $\hat{\mathcal{C}} : \widetilde{\nu}^2 = \prod_{j=1}^{3} (\mu - \mu_j)(\mu + \mu_j)$. We also give Abelian differentials of second kind explicitly. Our Baker-Akhiezer function is given by

$$\hat{\Psi}(z, \overline{z}, \hat{P}, \mathbf{e}) = \frac{\theta(\mathcal{B}(\hat{P}) - (z + \overline{z})\mathbf{U}^0 - \mathbf{e})\, \theta(\mathbf{e})}{\theta(\mathcal{B}(\hat{P}) - \mathbf{e})\, \theta((z + \overline{z})\mathbf{U}^0 + \mathbf{e})}\, \Phi_e(z, \overline{z}, \hat{P}),$$

where $\mathbf{e} = \pi\sqrt{-1}, \mathbf{U}^0 = \dfrac{\pi\sqrt{-1}}{2\omega^0}$, $\hat{P} \in \hat{\mathcal{C}}$ and $\mathcal{B} : \hat{\mathcal{C}} \longrightarrow \mathrm{Prym}(\hat{\mathcal{C}})$ is the Prym-Abel map. $\Phi_e(z, \overline{z}, \hat{P})$ is given by

$$\Phi_e(z, \overline{z}, \hat{P}) = \exp\left(z \left(\int_{\hat{P}_1}^{\hat{P}} \hat{\Omega}_\infty - \frac{\sqrt{-1}}{2}\mu_1 \right) - \overline{z} \left(\int_{\hat{P}_1}^{\hat{P}} \hat{\Omega}_0 - \frac{\sqrt{-1}}{2}\mu_1 \right) \right).$$

Finally, remark that a generalization of the reconstruction from some spectral curve of higher genus is also possible.

References

[1] F.E. Burstall, *Harmonic tori in spheres and complex projective spaces*, J. Reine Angew. Math. **469** (1995), 149–177.

[2] I. Castro and F. Urbano, *New examples of minimal Lagrangian tori in the complex projective plane*, Manuscripta Math. **85** (1994), 265–281.

[3] I. Yu. Cherdantsev and R. A. Sharipov, *Finite-gap solutions of the Bullough-Dodd-Zhiber-Shabat equation*, Teor i Mathema. Fizika **82**(1990), 155-160. English translation: Theoretȧnd MathṖhys. **82** (1990), no. 1, 108–111.

[4] N. Ejiri, *A generalization of minimal cones*, Trans. Amer. Math. Soc. **276** (1983), 347–360.

[5] H. Hashimoto, T. Taniguchi and S. Udagawa, *Constructions of almost complex 2-tori of type (III) in the nearly Kähler 6-sphere*, Differential Geom. and its Appl. **21** (2004), 127-145.

[6] H. Ma and Y. Ma, *Totally real minimal tori in* $\mathbf{C}P^2$, Math. Z. **249** (2005), 241-267.

[7] Y. Ohnita and S. Udagawa, *Harmonic maps of finite type into generalized flag manifolds, and twistor fibrations*, Integrable Systems in Differential Geometry (Proceedings of the 9th MSJ-IRI, Tokyo, 2000), Contemporary Math. **308** (2002), 245–270, American Math. Soc. Providence.

[8] R.A. Sharipov, *Minimal tori in the five dimensional sphere in* $\mathbf{C}^3$, Theor. Math. Phys. **87** (1991), 363-369.

[9] K. Ueno and K. Takasaki, *Recent development in gauge theory and integrable systems (in Japanese)*, RIMS Koukyuuroku **469** (1982), 72-93.

March 30th, 2018

Katrin Leschke (University of Leicester)

Title: TBA

Abstract

TBA

Lynn Heller (Leibniz Universität Hannover)

Title: Recent progress in integrable surface theory

Abstract

I consider surfaces in 3-space which are critical with respect to certain geometric variational problems, such as CMC and minimal surfaces and (constrained) Willmore surfaces. In this talk I want to give an overview on recent results on the construction of new examples of higher genus CMC surfaces and on the identification of constrained Willmore minimizers in the class of conformal tori. Moreover, by viewing minimal surfaces in different space forms within the constrained Willmore integrable system, counterexamples to a question of Simpson are constructed. This suggests a deeper connection between Willmore surfaces, i.e., rank 4 harmonic maps theory, and the rank 2 theory of Hitchin's self-duality equations.

This talk is based on joint work with Cheikh Birahim Ndiaye, Sebastian Heller and Nicholas Schmitt.

Katsuhiro Moriya (University of Tsukuba)

Title: Parametrizations of Lagrangian surfaces in the complex Euclidean plane

Abstract

Helein and Romon showed that the set of Hamiltonian stationary Lagrangian surfaces in the complex Euclidean plane forms a completely integrable system. Using the Weierstrass representation, they classified tori.

We give some examples of parametrizations of local Lagrangian surfaces those are not Hamiltonian stationary. Parametrizations are regarded as parallel sections of flat connections.

Yoshihiro Ohnita (Osaka City University)

Title: TBA

Abstract

TBA