Abstracts

Topological and variational methods for the Moser-Trudinger equation

Luca Martinazzi (University of Padova)

We discuss the existence of critical points of the Moser-Trudinger functional in dimension 2 with arbitrarily prescribed Dirichlet energy using both degree theory (Leray-Schauder degree) and variational methods (baricenter technique of Djadli-Malchiodi and Struwe monotonicity). This talk is based on joint works with Francesca De Marchis, Olivier Druet, Andrea Malchiodi, Gabriele Mancini and Pierre-Damien Thizy.

On the profile decomposition of a volume functional

Michinori Ishiwata (Osaka University)

The existence of a constant mean curvature surface can be treated by the variational method. To obtain the existence of the surface, it is important to analyse the nature of the breakdown of the compactness property of the corresponding functional which is done by the celebrated papers of Brezis-Coron. In this talk, we shed some light on this issue from the profile-decomposition concerning the Sobolev embedding into BMO together with the compensated-compactness which will give an improved version of Wente' s inequality.

Convex properties of positive solutions for a class of quasi-linear elliptic problems

Tatsuya Watanabe (Kyoto Sangyo University)

We are interested in the convexity of superlevel sets of positive solutions for a class of quasilinear elliptic equations in bounded strictly convex domains. Our first result states that any positive solution has convex superlevel sets for nonlinearities of sublinear type. We further show that any positive semi-stable solution has convex superlevel sets. The existence of positive semi-stable solutions are also investigated for various types of nonlinear terms and quasilinear terms. This talk is based on a joint work with M. Squassina (Universita Cattolica del Sacro Cuore).

The effect of heterogeneity on one-peak stationary solutions to the Schnakenberg model

Yuta Ishii (Tokyo Metropolitan University)

In this talk, we consider the Schnakenberg model on the interval $(-1, 1)$ with heterogeneity $g(x)$ in front of the nonlinear term under the Neumann boundary condition. We first construct one-peak stationary solutions by using the Liapunov-Schmidt reduction method. Moreover, we study the linear stability of the solutions above in details. In previous works, one-peak symmetric solutions have been studied for the symmetric heterogeneity case including $g(x) = 1$. In this work, since $g(x)$ may be not symmetric on the interval $(-1, 1)$, the constructed solution may be not symmetric. In particular, we reveal the effect of the heterogeneity on the location of the concentration point of the solution and the stability, which is a new phenomenon compared with the symmetric heterogeneity case.

Asymptotic property of ground states for a class of quasilinear Schrdinger equations with H^1-critical growth

Masataka Shibata (Tokyo Institute of Technology)

We consider a quasilinear elliptic equation $-\Delta u - \kappa u \Delta(u^2) = u^p - u$ in $\mathbb{R}^N$, where $N \geq 3$, $p > 1$, $\kappa > 0$. The equation appears in the study of standing waves of a modified Schrödinger equation. It has no solution if $p + 1 \geq 2 \cdot 2^*$ and has a unique solution $u_\kappa \in H^1(\mathbb{R}^N)$ if $2 < p + 1 < 2 \cdot 2^*$, where $2^* = 2N/(N-2)$ is the Sobolev critical exponent. We consider the asymptotic behavior of the solution u_κ as $\kappa \to 0$. The asymptotic behavior depends on p deeply. Indeed, in the case $p + 1 < 2^*$ (H^1-subcritical case), u_κ converges to a unique solution of the scalar field equation. In the case $p + 1 > 2^*$ (H^1-supercritical case), u_κ converges to a unique solution of some limiting problem under suitable scaling. In this talk, we discuss the asymptotic behavior in the case $p + 1 = 2^*$ (H^1-critical case).

Variational construction of orbits realizing symbolic sequences in the planar Sitnikov problem

Mitsuru Shibayama (Kyoto University)

Using the variational method, Chenciner and Montgomery (2000 Ann. Math. 152 881-901) proved the existence of an eight-shaped orbit of the planar three-body problem with equal masses. Since then a number of solutions to the N-body problem have been discovered. On the other hand, symbolic dynamics is one of the most useful methods for understanding chaotic dynamics. The Sitnikov problem is a special case of the three-body problem. The system is known to be chaotic and was studied by using symbolic dynamics (J.Moser, Stable and random motions in dynamical systems, Princeton University Press, 1973). In this talk, we study the limiting case of the Sitnikov problem. By using the variational method, we show the existence of various kinds of solutions in the planar Sitnikov problem. For a given symbolic sequence, we show the existence of orbits realizing it. We also prove the existence of periodic orbits.

Strongly perturbed Moser-Trudinger functionals and their critical points in dimension two.

Gabriele Mancini (Sapienza University of Rome)

I will discuss some recent results obtained in collaboration with P-D. Thizy concerning semilinear elliptic problems involving Moser-Trudinger type nonlinearities. After summarizing some known facts regarding the standard Moser-Trudinger critical equation, I will describe how the classical results are affected by the presence of perturbation terms. In particular, I will present the answer to a question formulated by O. Druet, proving the existence of families of nonlinearities with uniformly critical growth for which the corresponding semilinear problem admits a bubbling sequence of solutions having a nontrivial weak limit

Effect of compact term on maximization problem associated with Trudinger-Moser inequality

Masato Hashizume (Ehime University)

It is well known that there exists a extremal for the original Trudinger-Moser inequality. Recently, it was shown that perturbations have an effect on existence and non-existence. In this talk, we consider maximization problem associated with the Trudinger-Moser inequality with lower order term. We obtain a threshold separating existence and non-existence.

The behavior of blow-up solutions for mean field equations with probability measure

Yohei Toyota (National Institute of Technology, Nara College)

In this talk we are concerned with several mean field equations with probability measure, in particular,

$$-\Delta v = \lambda \int_I \frac{\alpha e^{\alpha v}}{\int_\Omega e^{\alpha v} dx} \mathcal{P}(d\alpha) \quad \text{in } \Omega, \quad v = 0 \quad \text{on } \partial\Omega,$$

and

$$-\Delta v = \lambda \frac{\int_I \alpha e^{\alpha v} \mathcal{P}(d\alpha)}{\iint_{\Omega\times I} e^{\alpha v} dx \mathcal{P}(d\alpha)} \quad \text{in } \Omega, \quad v = 0 \quad \text{on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain, $\lambda > 0$ is a constant and $\mathcal{P}(d\alpha)$ is a Borel probability measure on $I = [0, 1]$. The main topic in my talk is the family of unbounded solutions for such mean field equations. In terms of blow-up analysis, we shall show the behavior of blow-up solutions near the blow-up points under the suitable assumptions of $\mathcal{P}$. This is partly joint work with professor T. Suzuki in Osaka University.

Solvability of a semilinear heat equation via a quasi scale invariance

Norisuke Ioku (Tohoku University)

Classification theory on local in time solvability of nonlinear heat equations is investigated. Without assuming a concrete growth rate on a nonlinear term, we reveal the threshold integrability of initial data which classify existence and nonexistence of solutions via a quasi-scaling and its invariant integral. Global in time solvability is also studied. Typical nonlinear terms, for instance polynomial type, exponential type and its sum, product and composition, can be treated as applications. This is a joint work with Yohei Fujishima(Shizuoka University).